Testing Gravity (General Relativity) in Cosmology

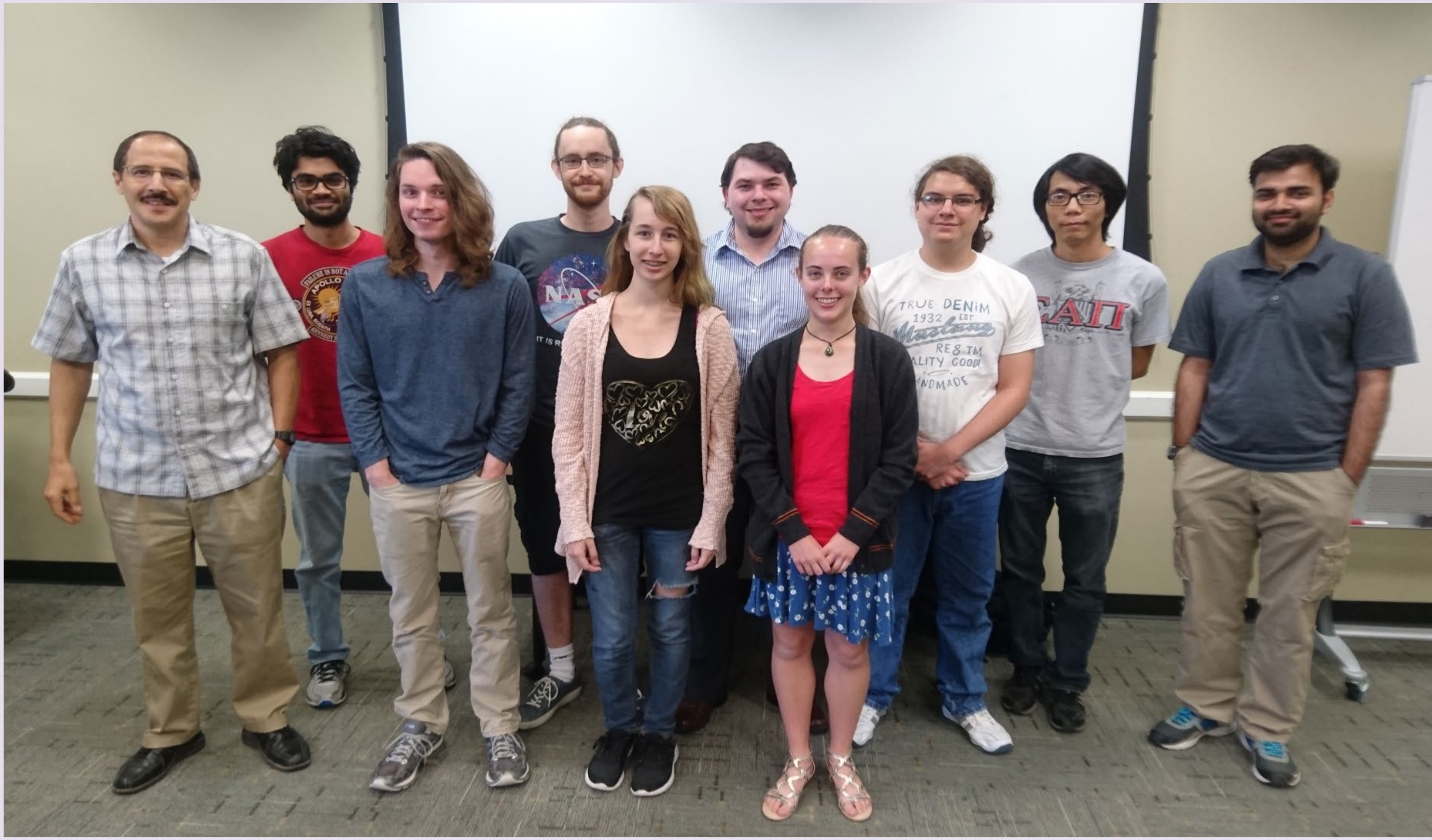
Prof. Mustapha Ishak-Boushaki

Cosmology, Astrophysics and Relativity Group

Department of Physics

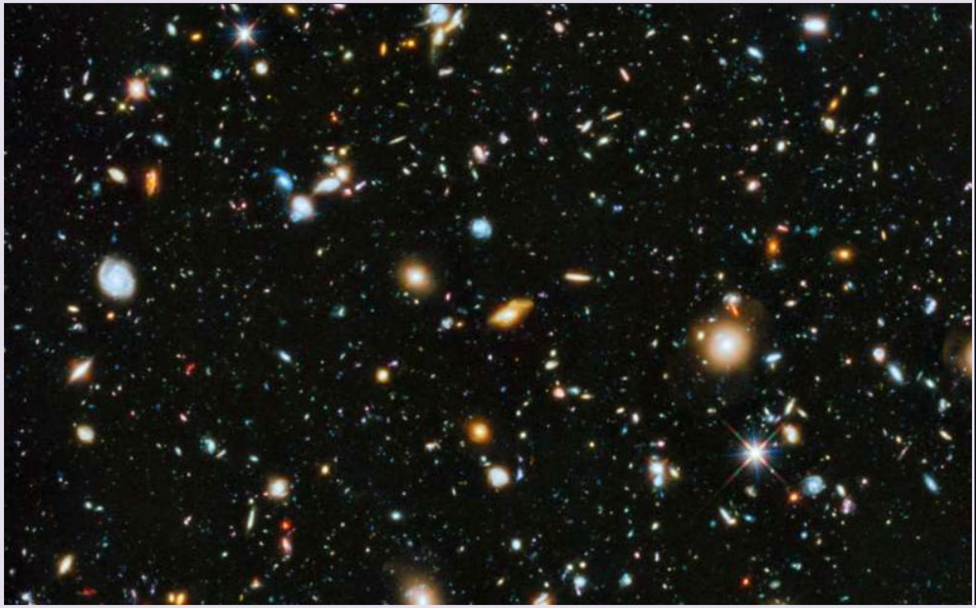
The University of Texas at Dallas

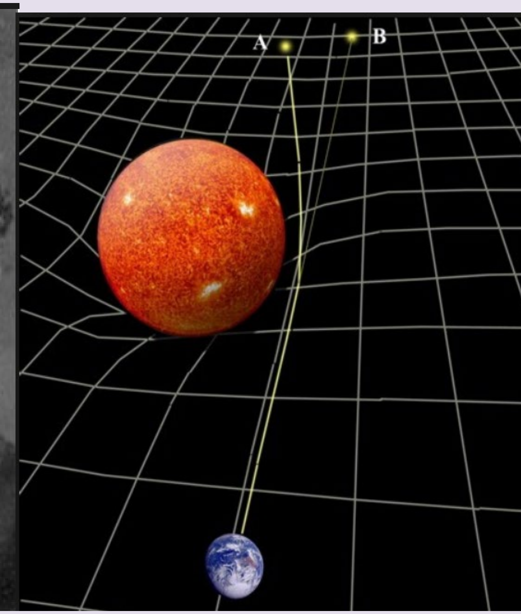
My research group in summer 2018: 5 graduate students, 4 undergraduate students/interns



projects described sponsored by NASA, NSF, and DOE

In 1915, Einstein proposed his General Relativity (GR) theory of gravity





- revolutionized our understanding of space and time
- Gave gravity theory that is more accurate than Newton's
- Gave Big Bang Cosmology of an expanding Universe
- Predicted existence of Black Holes and gravitational waves
- Makes our GPS work today … otherwise 10 km off per day!

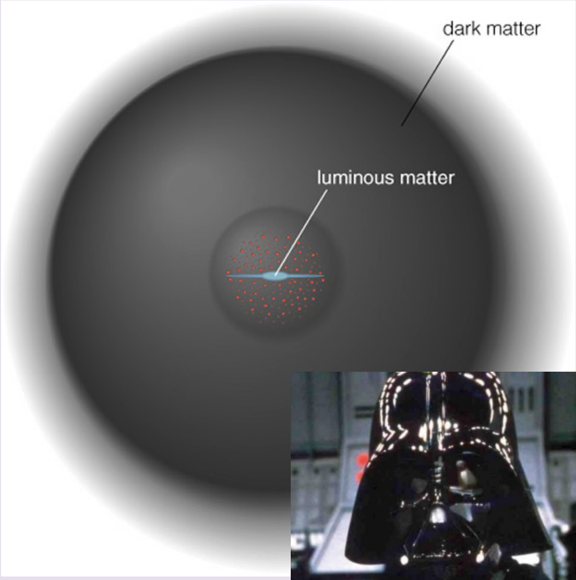
Einstein's Equations intact for over a century!
with a successful cosmological model

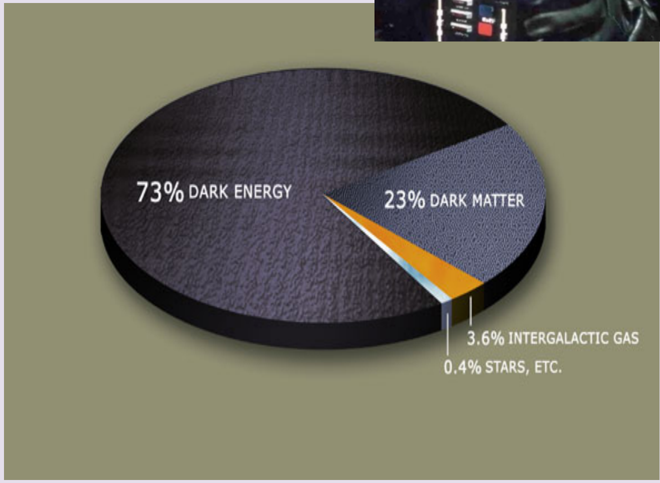
$$G^a_b + \Lambda \delta^a_b = \kappa T^a_b$$

But not without puzzles and surprises

Dark Matter

- 80-90% or more of the gravitationally attractive matter
- Beside gravity, no other interactions with photons or baryons (ordinary matter) only maybe weakly

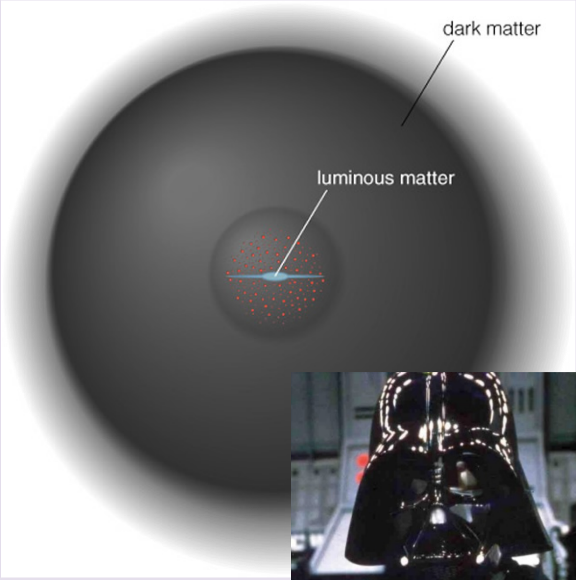




But not without puzzles and surprises

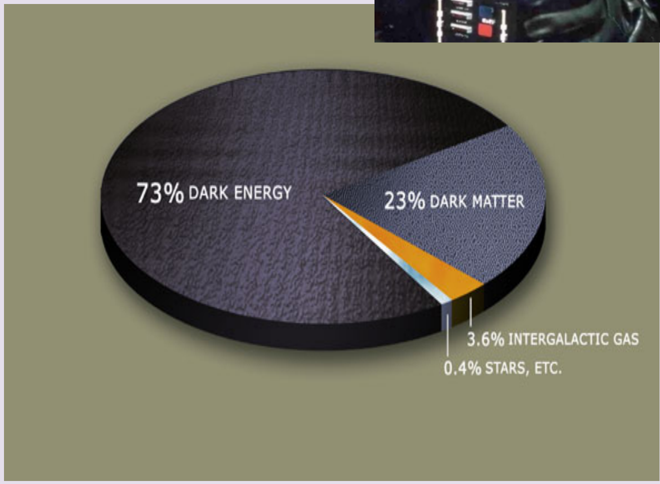
Dark Matter

- 80-90% or more of the gravitationally attractive matter
- Beside gravity, no other interactions with photons or baryons (ordinary matter) only maybe weakly



Cosmic acceleration and Dark Energy

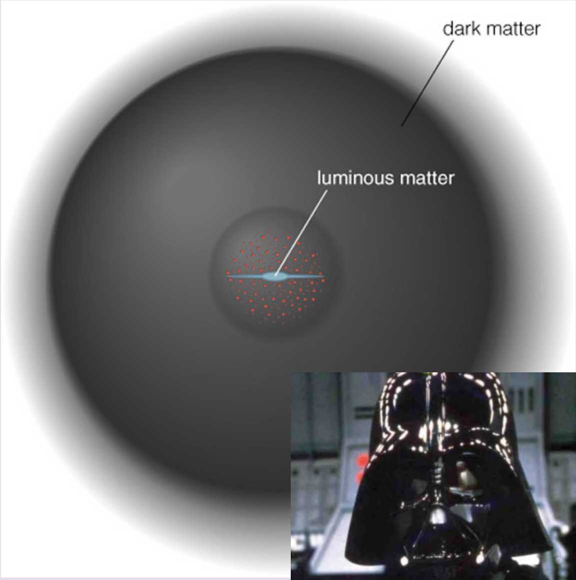
- expansion of the universe is speeding up while expected to be slowing down
- Complementary observations have been indicating this for 22 years! (1998-2020)



But not without puzzles and surprises

Dark Matter

- 80-90% or more of the gravitationally attractive matter
- Beside gravity, no other interactions with photons or baryons (ordinary matter) only maybe weakly

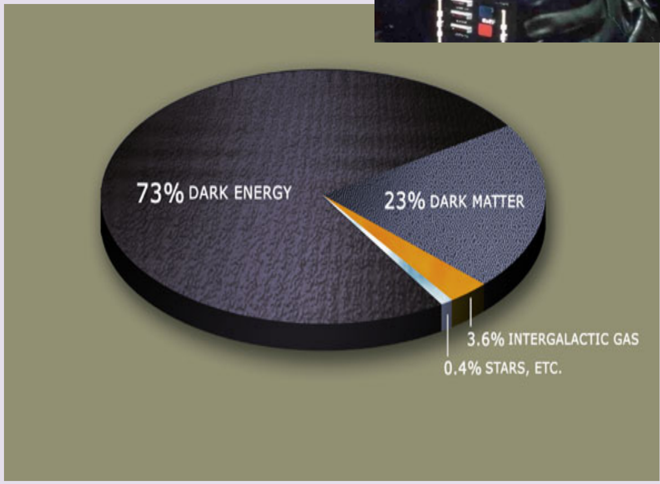


Cosmic acceleration and Dark Energy

- expansion of the universe is speeding up while expected to be slowing down
- Complementary observations have been indicating this for 22 years! (1998-2020)

Cosmological discordances

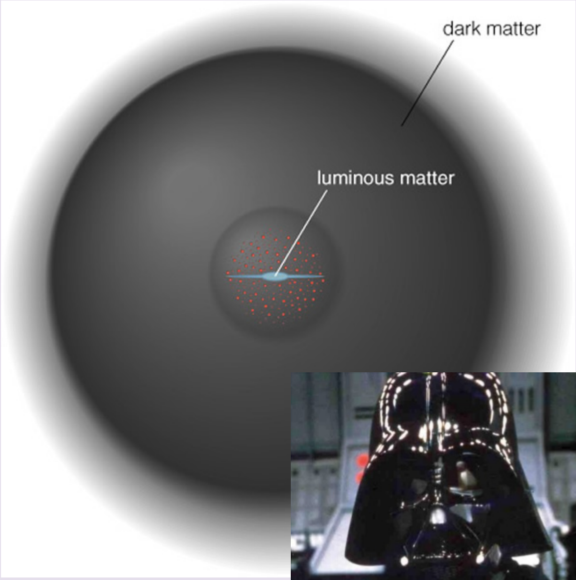
- Discrepant cosmological parameters from different datasets, H0 , σ_8, now MG parameters



But not without puzzles and surprises

Dark Matter

- 80-90% or more of the gravitationally attractive matter
- Beside gravity, no other interactions with photons or baryons (ordinary matter) only maybe weakly

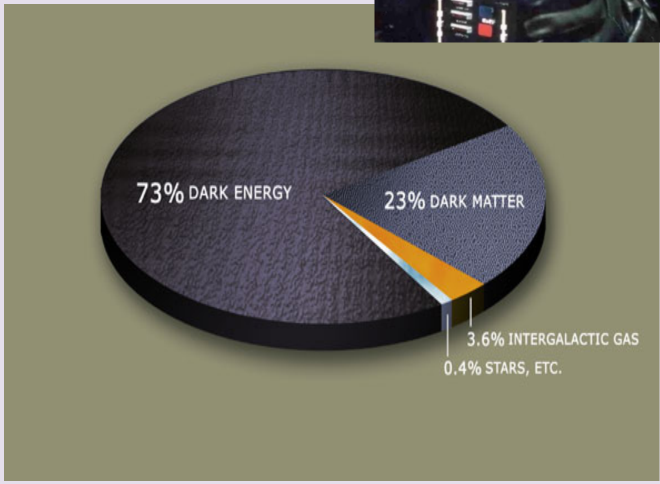


Cosmic acceleration and Dark Energy

- expansion of the universe is speeding up while expected to be slowing down
- Complementary observations have been indicating this for 22 years! (1998-2020)

Cosmological discordances

- Discrepant cosmological parameters from different datasets, H0 , sigma8, now MG parameters



Are these related?

Why testing Gravity (General Relativity) at Cosmological Scale?

Motivated by:

- Dark sector and the problem of cosmic acceleration
- Cosmological discordances
- Intrinsic open questions in GR (e.g. singularities)
- Quest for unified theories
- Simple fact that General Relativity can now be tested at cosmological scales

Approaches to Testing Gravity at Cosmological Scales

I. Phenomenological parameterizations of deviations from General Relativity

II. Testing specific modified gravity models

III. Consistency tests using datasets

See for example: Ishak, M., "Testing general relativity in cosmology", *Living Review in Relativity* (2019) 22: 1, 204 pages https://doi.org/10.1007/s41114-018-0017-4, arXiv:1806.10122v2

Approach I: Phenomenological parameterizations of deviations from General Relativity

- Modified Gravity (MG) models can produce cosmic acceleration and be indistinguishable from wCDM at the level of the expansion history probes

Approach I: Phenomenological parameterizations of deviations from General Relativity

- Modified Gravity (MG) models can produce cosmic acceleration and be indistinguishable from wCDM at the level of the expansion history probes
- Large scale structure constraints can break this degeneracy

Approach I: Phenomenological parameterizations of deviations from General Relativity

- Modified Gravity (MG) models can produce cosmic acceleration and be indistinguishable from wCDM at the level of the expansion history probes
- Large scale structure constraints can break this degeneracy
- Framework for this is the Perturbed Einstein's Equations

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)\delta_{ij}dx^i dx^j].$$

Approach I: Phenomenological parameterizations of deviations from General Relativity

- Modified Gravity (MG) models can produce cosmic acceleration and be indistinguishable from wCDM at the level of the expansion history probes
- Large scale structure constraints can break this degeneracy
- Framework for this is the Perturbed Einstein's Equations

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)\delta_{ij}dx^i dx^j].$$

$$k^2\Psi = -4\pi G a^2 \mu(a,k) \sum_i [\rho_i \Delta_i + 3\rho_i(1+w_i)\sigma_i],$$ for motion of matter

$$k^2[\Psi - \eta(a,k)\Phi] = -12\pi G a^2 \mu(a,k) \sum_i \rho_i(1+w_i)\sigma_i.$$ slip parameter

$$k^2(\Phi+\Psi) = -4\pi G a^2 \Sigma(a,k) \sum_i [2\rho_i \Delta_i + 3\rho_i(1+w_i)\sigma_i].$$ for motion of light

$$\Sigma(k,a) = \frac{\mu(k,a)[1+\gamma(k,a)]}{2}.$$ only two are needed

Approach I: Phenomenological parameterizations of deviations from General Relativity

- Modified Gravity (MG) models can produce cosmic acceleration and be indistinguishable from wCDM at the level of the expansion history probes
- Large scale structure constraints can break this degeneracy
- Framework for this is the Perturbed Einstein's Equations

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)\delta_{ij}dx^i dx^j].$$

$$k^2\Psi = -4\pi G a^2 \mu(a,k) \sum_i \left[\rho_i \Delta_i + 3\rho_i(1+w_i)\sigma_i\right],$$ for motion of matter

$$k^2[\Psi - \eta(a,k)\Phi] = -12\pi G a^2 \mu(a,k) \sum_i \rho_i(1+w_i)\sigma_i.$$ slip parameter

$$k^2(\Phi+\Psi) = -4\pi G a^2 \Sigma(a,k) \sum_i \left[2\rho_i \Delta_i + 3\rho_i(1+w_i)\sigma_i\right].$$ for motion of light

$$\Sigma(k,a) = \frac{\mu(k,a)[1+\gamma(k,a)]}{2}.$$ only two are needed

- MG parameters are inserted in the perturbed Einstein's Equations
- For GR, MG parameters take values of 1 so no effect

Approach I: Phenomenological parameterizations of deviations from General Relativity

- Modified Gravity (MG) models can produce cosmic acceleration and be indistinguishable from wCDM at the level of the expansion history probes
- Large scale structure constraints can break this degeneracy
- Framework for this is the Perturbed Einstein's Equations

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)\delta_{ij}dx^i dx^j].$$

$$k^2\Psi = -4\pi G a^2 \mu(a,k) \sum_i [\rho_i \Delta_i + 3\rho_i(1+w_i)\sigma_i]\,,$$ for motion of matter

$$k^2[\Psi - \eta(a,k)\Phi] = -12\pi G a^2 \mu(a,k) \sum_i \rho_i (1+w_i)\sigma_i.$$ slip parameter

$$k^2(\Phi+\Psi) = -4\pi G a^2 \Sigma(a,k) \sum_i [2\rho_i \Delta_i + 3\rho_i(1+w_i)\sigma_i]\,.$$ for motion of light

$$\Sigma(k,a) = \frac{\mu(k,a)[1+\gamma(k,a)]}{2}.$$ only two are needed

- MG parameters are inserted in the perturbed Einstein's Equations
- For GR, MG parameters take values of 1 so no effect
- A significant measurement of MG parameter deviation from 1 will be indicative of troubles

Approach I: MG parameterizations in redshift and scale

scale-dependence is important to distinguish MG from DE

Functional form parametrizations

- Redshift dependence only

$$\mu(a) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda}$$

$$\Sigma(a) = 1 + \Sigma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda},$$

- Redshift and scale

$$\mu(a,k) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda} \left[\frac{1 + c_1 \left(\lambda H(a)/k\right)^2}{1 + \left(\lambda H(a)/k\right)^2} \right]$$

$$\Sigma(a,k) = 1 + \Sigma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda} \left[\frac{1 + c_2 \left(\lambda H(a)/k\right)^2}{1 + \left(\lambda H(a)/k\right)^2} \right]$$

- Do these functional forms bring limitations?

Approach I: MG parameterizations in redshift and scale

scale-dependence is important to distinguish MG from DE

Functional form parametrizations

- Redshift dependence only

$$\mu(a) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda}$$

$$\Sigma(a) = 1 + \Sigma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda},$$

- Redshift and scale

$$\mu(a,k) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda} \left[\frac{1 + c_1 (\lambda H(a)/k)^2}{1 + (\lambda H(a)/k)^2} \right]$$

$$\Sigma(a,k) = 1 + \Sigma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda} \left[\frac{1 + c_2 (\lambda H(a)/k)^2}{1 + (\lambda H(a)/k)^2} \right]$$

- Do these functional forms bring limitations?

Binned forms

- Free from assumed anzats

	Redshift bins		
Scale bins	$0 \le z < 1$	$1 \le z < 2$	$z \ge 2$
$0 \le k < 0.01$	μ_1, Σ_1	μ_3, Σ_3	GR is assumed
$0.01 \le k < \infty$	μ_2, Σ_2	μ_4, Σ_4	GR is assumed

- Less used because not implemented in most available MG codes

Approach I: MG parameterizations in redshift and scale

scale-dependence is important to distinguish MG from DE

Functional form parametrizations

- Redshift dependence only

$$\mu(a) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda}$$

$$\Sigma(a) = 1 + \Sigma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda},$$

- Redshift and scale

$$\mu(a,k) = 1 + \mu_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda} \left[\frac{1 + c_1 \left(\lambda H(a)/k\right)^2}{1 + \left(\lambda H(a)/k\right)^2} \right]$$

$$\Sigma(a,k) = 1 + \Sigma_0 \frac{\Omega_{\rm DE}(a)}{\Omega_\Lambda} \left[\frac{1 + c_2 \left(\lambda H(a)/k\right)^2}{1 + \left(\lambda H(a)/k\right)^2} \right]$$

- Do these functional forms bring limitations?

Binned forms

- Free from assumed anzats

	Redshift bins		
Scale bins	$0 \le z < 1$	$1 \le z < 2$	$z \ge 2$
$0 \le k < 0.01$	μ_1, Σ_1	μ_3, Σ_3	GR is assumed
$0.01 \le k < \infty$	μ_2, Σ_2	μ_4, Σ_4	GR is assumed

- Less used because not implemented in most available MG codes

These parameters enter various observable functions and power spectra

Constraints on MG parameters using latest cosmological datasets

(Garcia-Quinterro, Ishak, Fox and Dossett, PRD, 2019 **arXiv:1908.00290;**
Garcia-Quinterro & Ishak, 2020, **arXiv:2009.01189**)

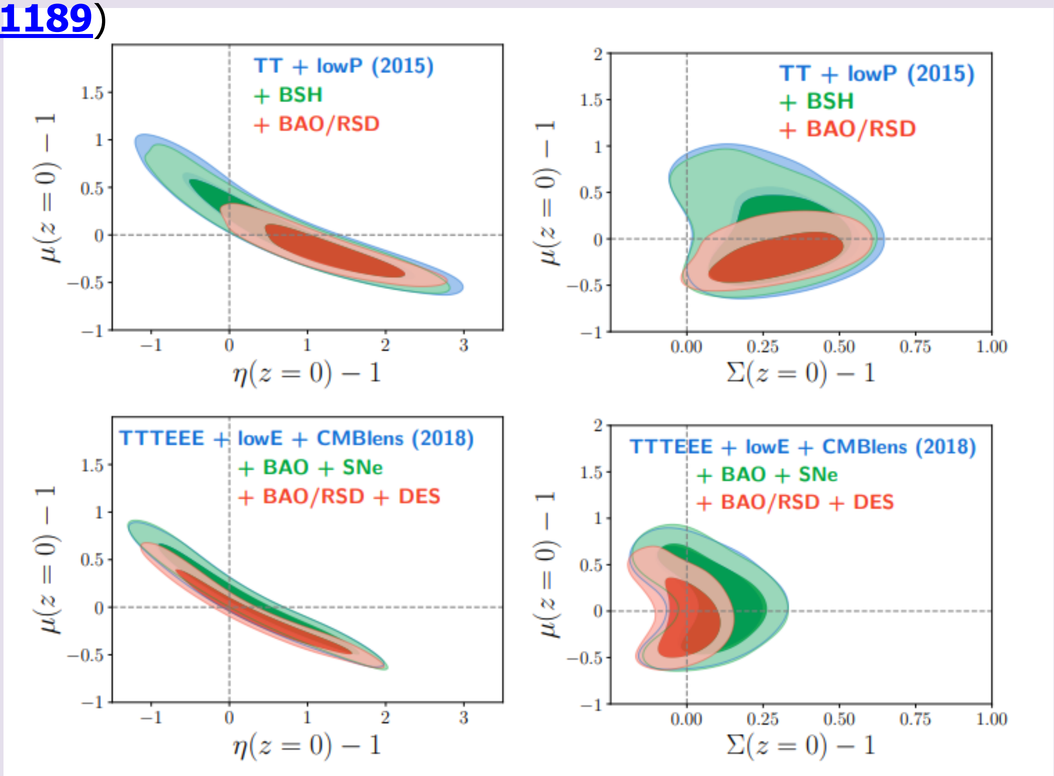
Datasets used:

Datasets	Description
1. Individual datasets	
TTTEEE	Planck high-ℓ temperature and polarization spectra, and low-ℓ temperature Commander likelihood [31]
lowE	Planck low-ℓ SimAll E-mode polarization spectra likelihood [31]
CMBL	Light deflection measurements from the CMB [32]
SNe	Pantheon supernovae type Ia compilation [33]
6dFGS	BAO measurements from the 6dFGS [34]
MGS	BAO measurements from the SDSS MGS [35]
BOSS	BAO consensus results from BOSS DR12 [36]
Ly-α	BAO measurements from the correlation of Lyman-α forest absorption and quasars [37]
RSD	SDSS III galaxy clustering data from BAO spectroscopic survey [36]
BBN	One percent determination of the primordial deuterium abundance [38]
HST	Hubble Space Telescope local measurements of H_0 [39]
DES	Dark Energy Survey Year 1 clustering and lensing analysis [40]
2. Combined datasets	
P18	TTTEEE+lowE
BAO	BOSS+6DFGS+MGS+Ly-α
SBB	SNe+BAO+BBN

Constraints on MG parameters using latest cosmological datasets

(Garcia-Quinterro, Ishak, Fox and Dossett, PRD, 2019 **arXiv:1908.00290**;
Garcia-Quinterro & Ishak, 2020, **arXiv:2009.01189**)

Using Plank 2015 and other datasets from SN, BAO, and RSD,



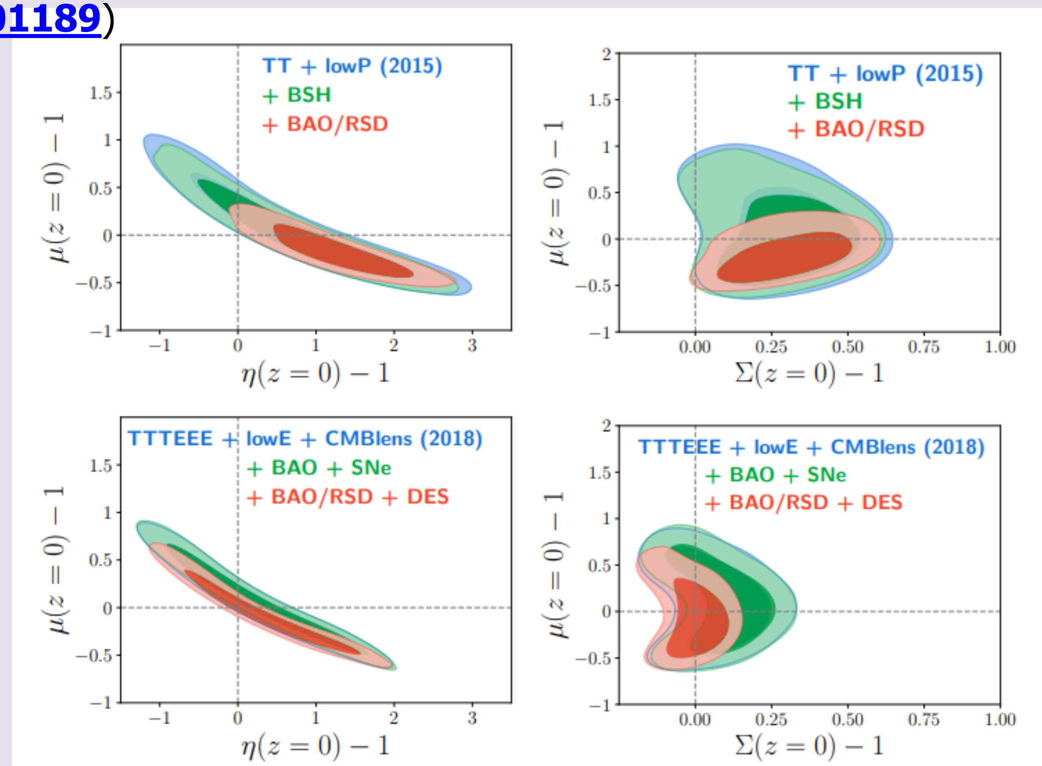
Constraints on MG parameters using latest cosmological datasets

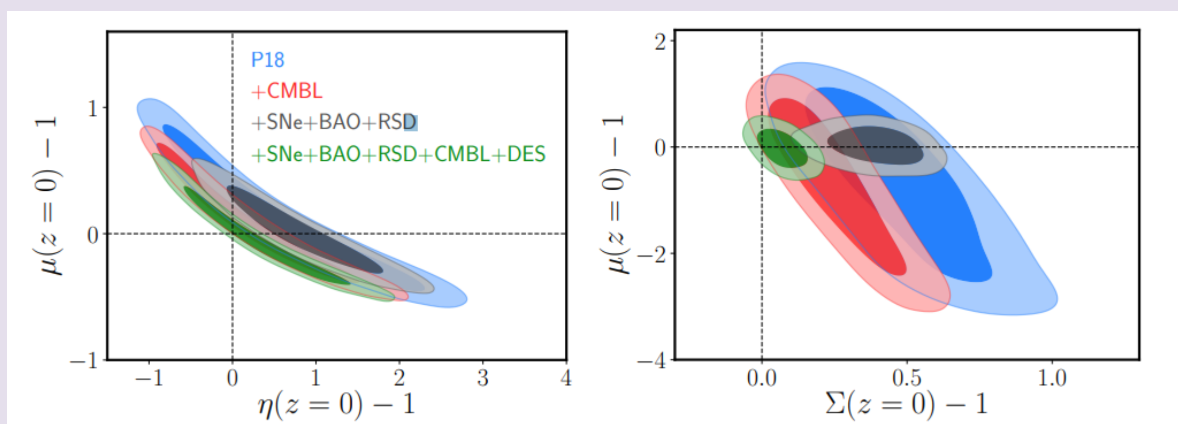
(Garcia-Quinterro, Ishak, Fox and Dossett, PRD, 2019 **arXiv:1908.00290;**
Garcia-Quinterro & Ishak, 2020, **arXiv:2009.01189**)

Using Plank 2015 and other datasets from SN, BAO, and RSD,

Or later below

Using Plank 2018 and other datasets from SN, BAO and RSD.





Constraints on MG parameters using latest cosmological datasets

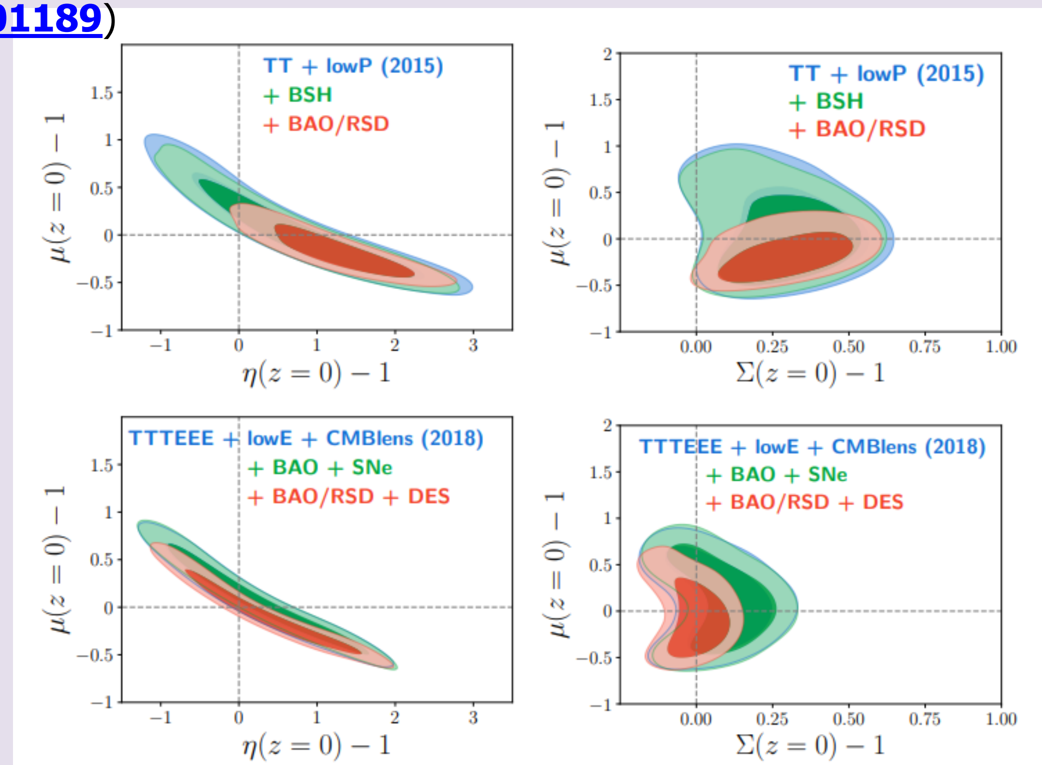
(Garcia-Quinterro, Ishak, Fox and Dossett, PRD, 2019 **arXiv:1908.00290**;
Garcia-Quinterro & Ishak, 2020, **arXiv:2009.01189**)

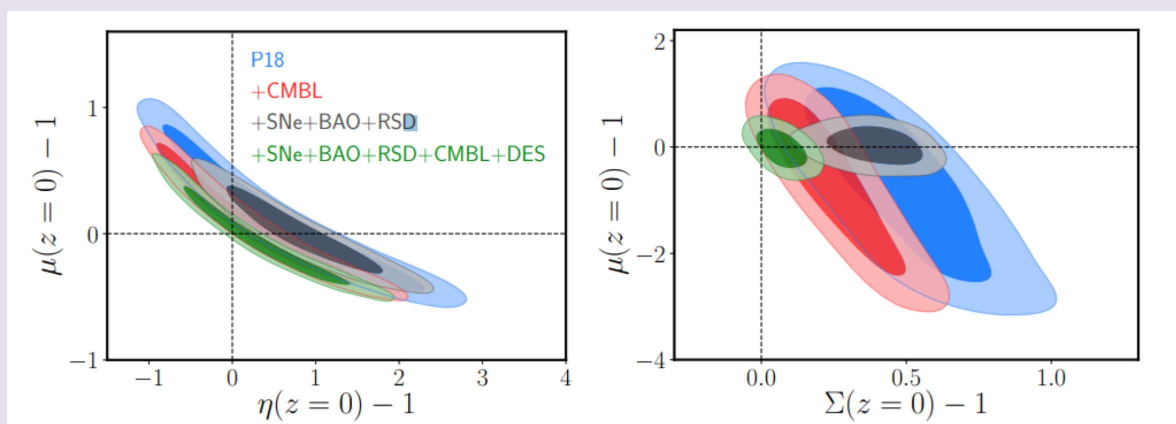
Using Plank 2015 and other datasets from SN, BAO, and RSD,

Or later below

Using Plank 2018 and other datasets from SN, BAO and RSD.

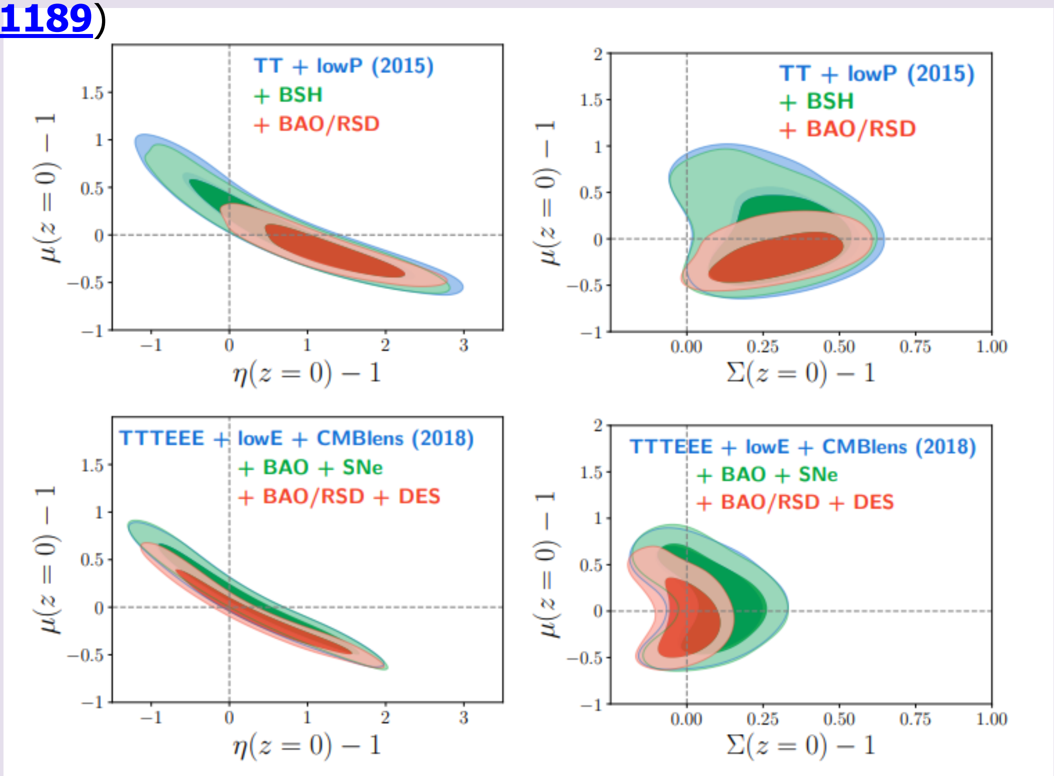
Gives consistently over a **3-σ** tension with GR





Constraints on MG parameters using latest cosmological datasets

(Garcia-Quinterro, Ishak, Fox and Dossett, PRD, 2019 arXiv:1908.00290;
Garcia-Quinterro & Ishak, 2020, arXiv:2009.01189)



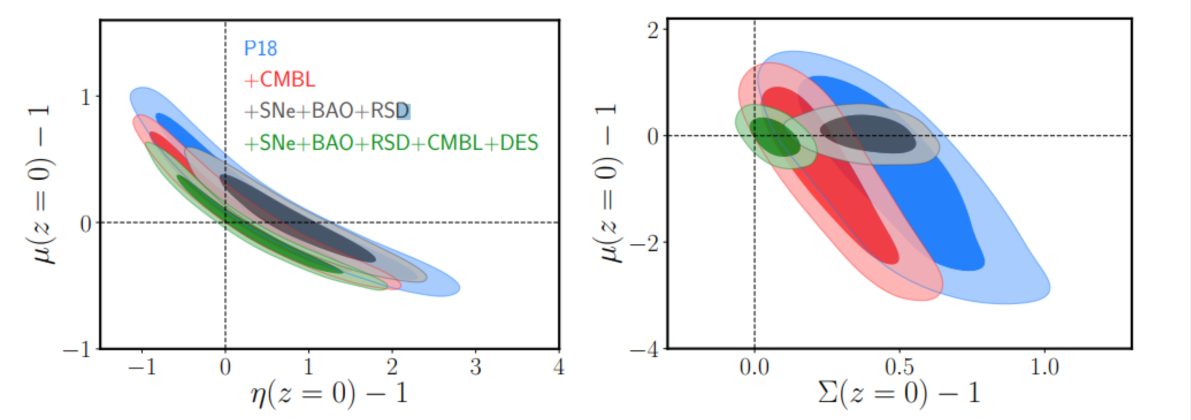
Using Plank 2015 and other datasets from SN, BAO, and RSD,

Or later below

Using Plank 2018 and other datasets from SN, BAO and RSD.

Gives consistently over a **3-σ** tension with GR

adding CMB Lensing or DES restores consistency with GR

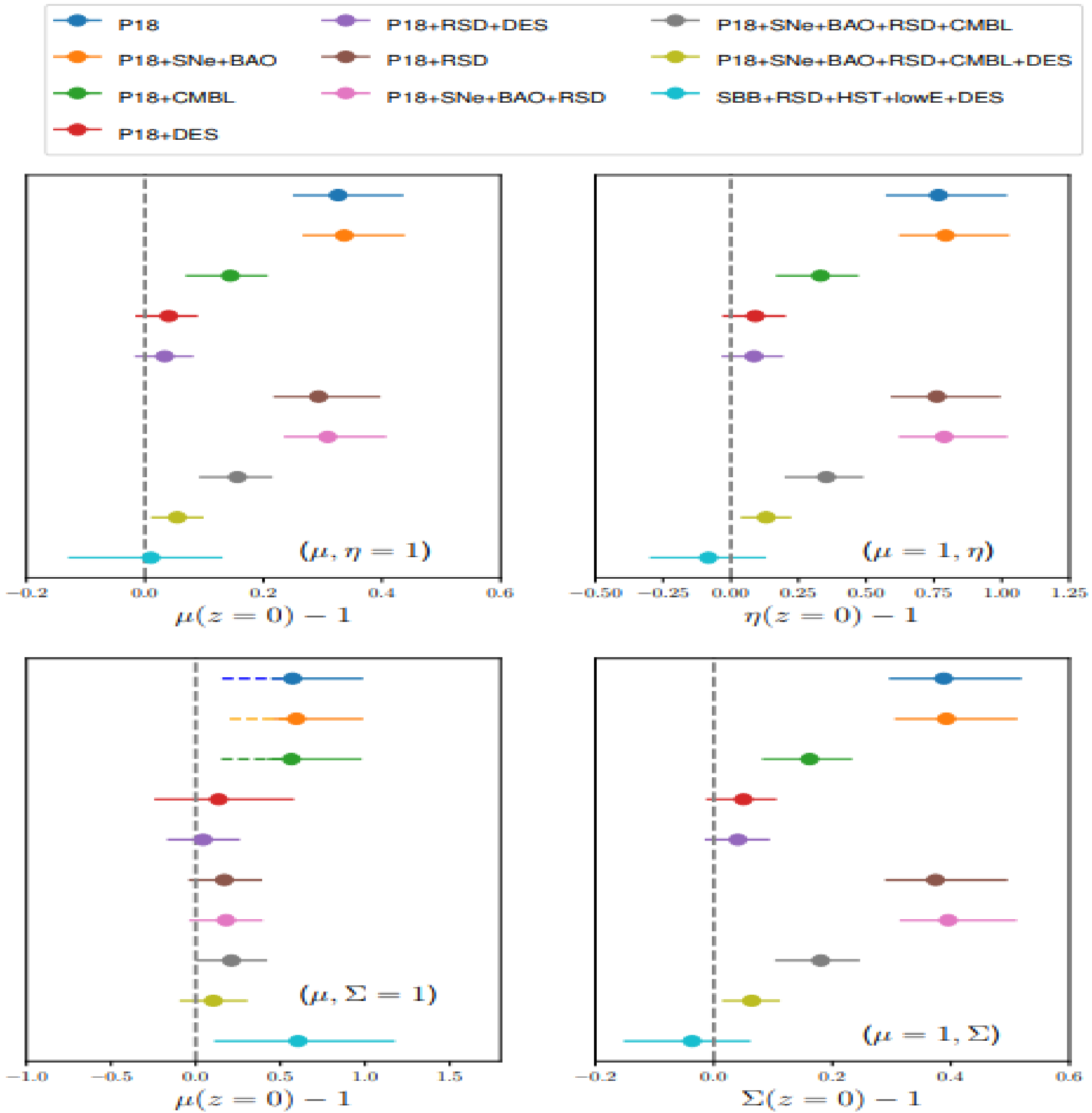


Singling out modified gravity parameters and datasets

(Garcia-Quinterro & Ishak, 2020, arXiv:2009.01189)

Getting more power from the same datasets

Dissecting the tensions in MG parameters



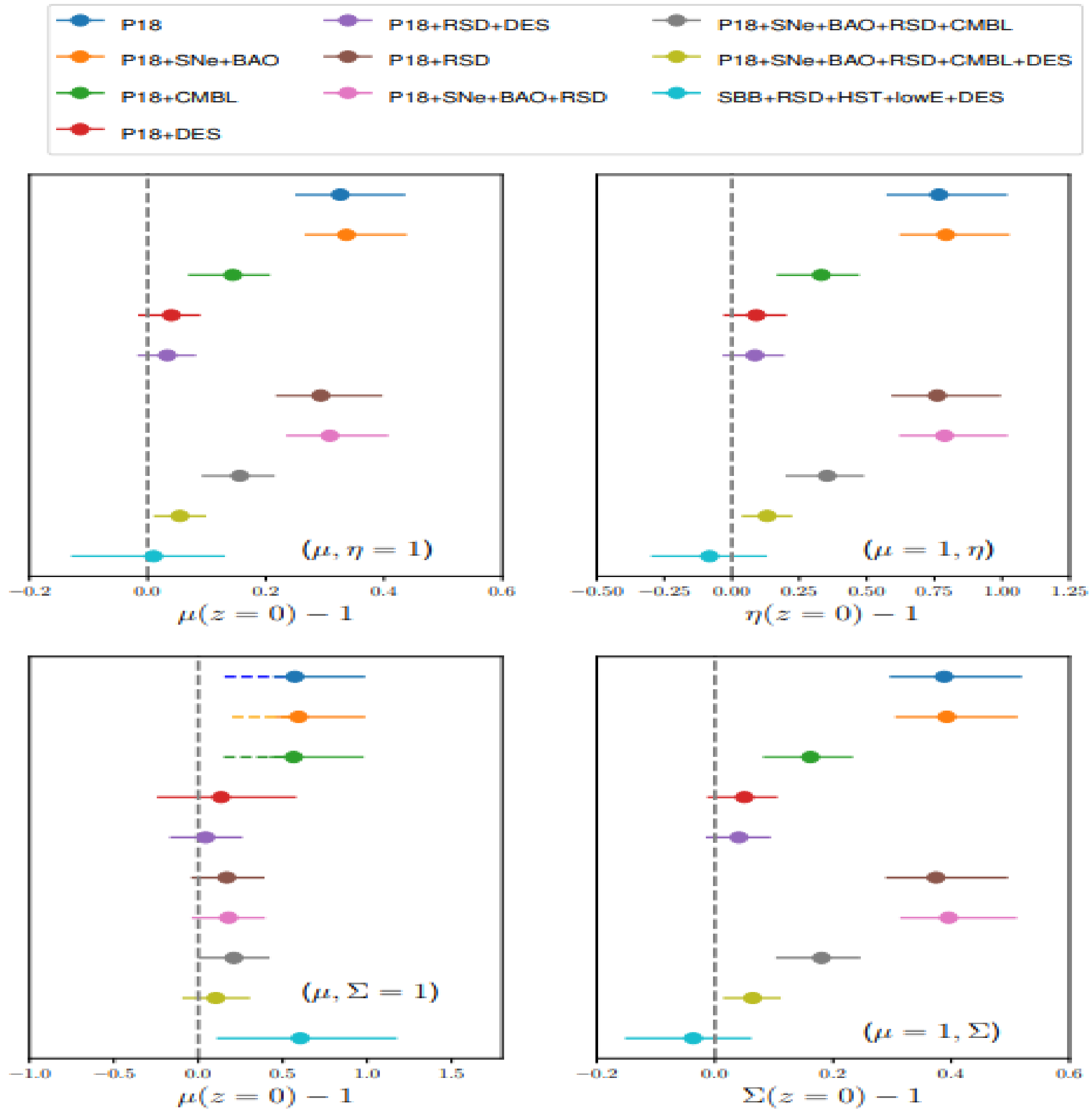
Singling out modified gravity parameters and datasets

(Garcia-Quinterro & Ishak, 2020, arXiv:2009.01189)

Getting more power from the same datasets

Dissecting the tensions in MG parameters

tension with GR increased to 3.91-σ for Σ and 3.87-σ for η



Singling out modified gravity parameters and datasets

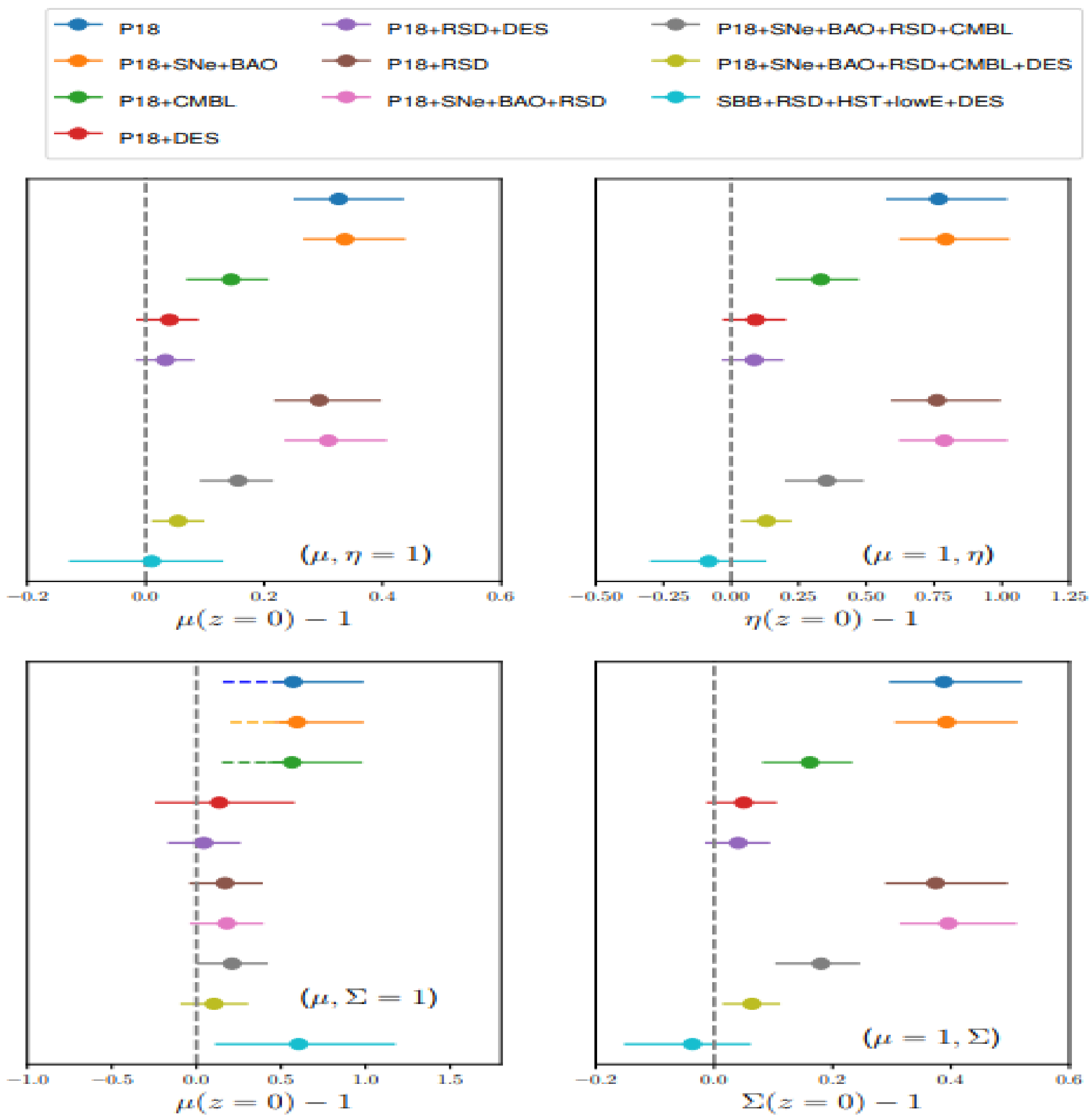
(Garcia-Quinterro & Ishak, 2020, arXiv:2009.01189)

Getting more power from the same datasets

Dissecting the tensions in MG parameters

tension with GR increased to 3.91-σ for Σ and 3.87-σ for η

Found a dichotomy between Planck and lensing for deviation from GR!



Singling out modified gravity parameters and datasets

(Garcia-Quinterro & Ishak, 2020, arXiv:2009.01189)

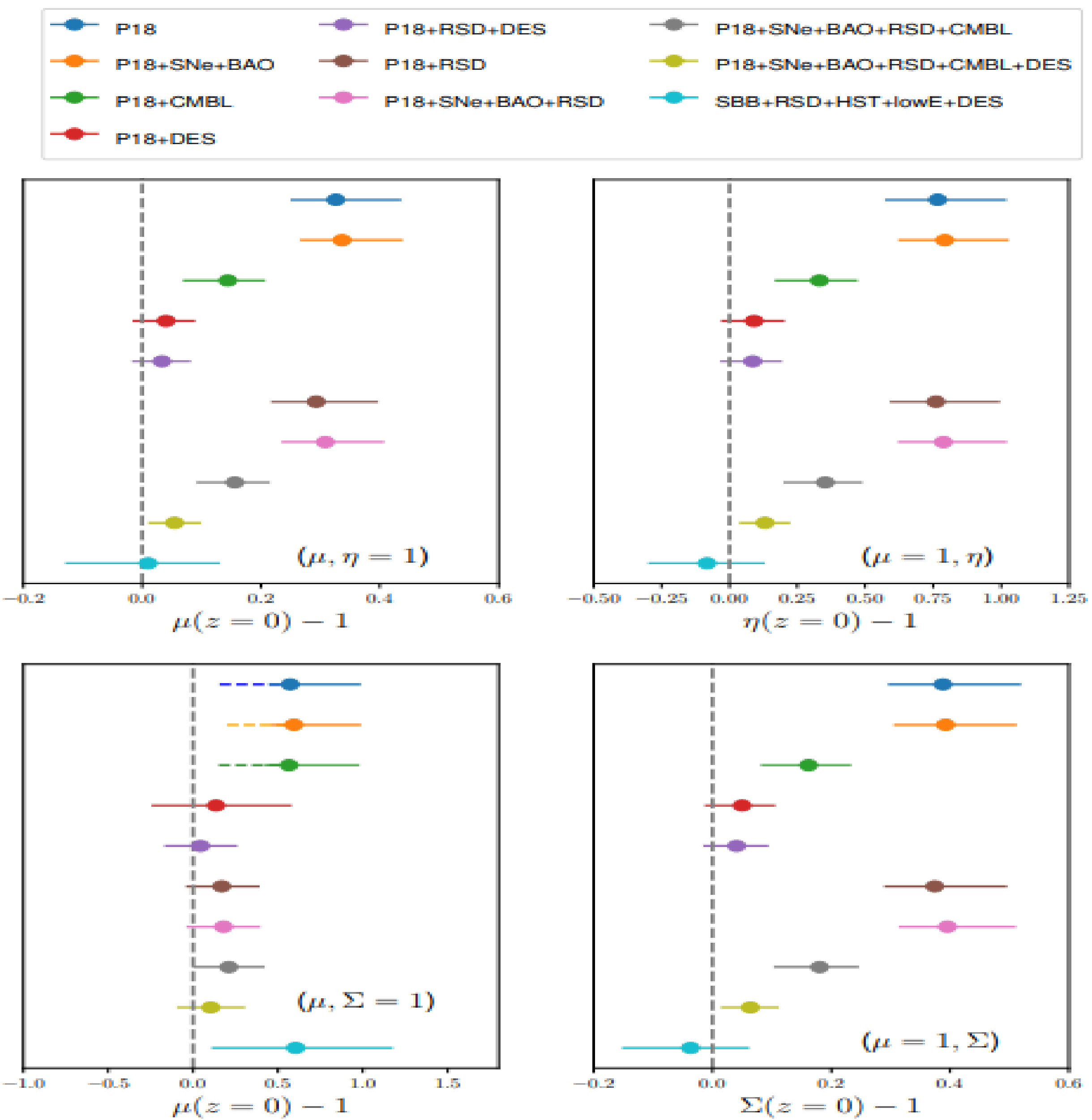
Getting more power from the same datasets

Dissecting the tensions in MG parameters

tension with GR increased to 3.91-σ for Σ and 3.87-σ for η

Found a dichotomy between Planck and lensing for deviation from GR!

Lensing more consistent with GR than Planck: why?



Singling out modified gravity parameters and datasets

(Garcia-Quinterro & Ishak, 2020, **arXiv:2009.01189**)

$\ln B_{i0}$	**P18**	**P18+SNe+BAO**	P18+CMBL	P18+DES	P18+CMBL+DES	**P18+RSD**	P18+RSD+CMBL	P18+RSD+DES	P18+RSD+CMBL+DES	**P18+SNe+BAO+RSD**	P18+SNe+BAO+RSD+CMBL	P18+SNe+BAO+RSD+DES	P18+SNe+BAO+RSD+CMBL+DES	SBB+RSD+HST+lowE	SBB+RSD+HST+lowE+DES
(μ, η)	-0.29	0.38	-2.83	-5.21	-4.35	-0.77	-3.19	-6.31	-5.03	-0.14	-2.59	-5.83	-5.39	-1.02	-4.41
(μ, Σ)	0.29	0.78	-2.22	-5.02	-3.89	-1.13	-3.56	-7.08	-5.77	-0.4	-3.05	-6.82	-5.47	-1.25	-3.95
$(\mu, \eta = 1)$	0.78	**1.56**	-1.5	-4.49	-3.46	0.35	-1.2	-3.98	-3.92	0.97	-0.82	-4.84	-3.09	-0.21	-3.46
$(\mu = 1, \eta)$	**1.81**	**2.56**	-0.75	-4.05	-2.42	**2.04**	-0.45	-4.09	-2.88	**2.64**	0.11	-3.79	-2.32	-0.64	-3.02
$(\mu = 1, \Sigma)$	**1.23**	**1.92**	-1.36	-4.48	-3.47	**1.66**	-1.07	-3.74	-3.02	**2.09**	-0.36	-4.42	-3.28	-0.83	-3.56
$(\mu, \Sigma = 1)$	-1.49	-1.23	-0.74	-2.48	-1.83	-3.15	-2.2	-3.5	-1.38	-3.15	-2.37	-3.83	-2.79	-1.07	-0.63

- Some MG models moderately favored over LCDM!
- Fixing Σ to 1 restores GR and its model preference regardless of μ : why?

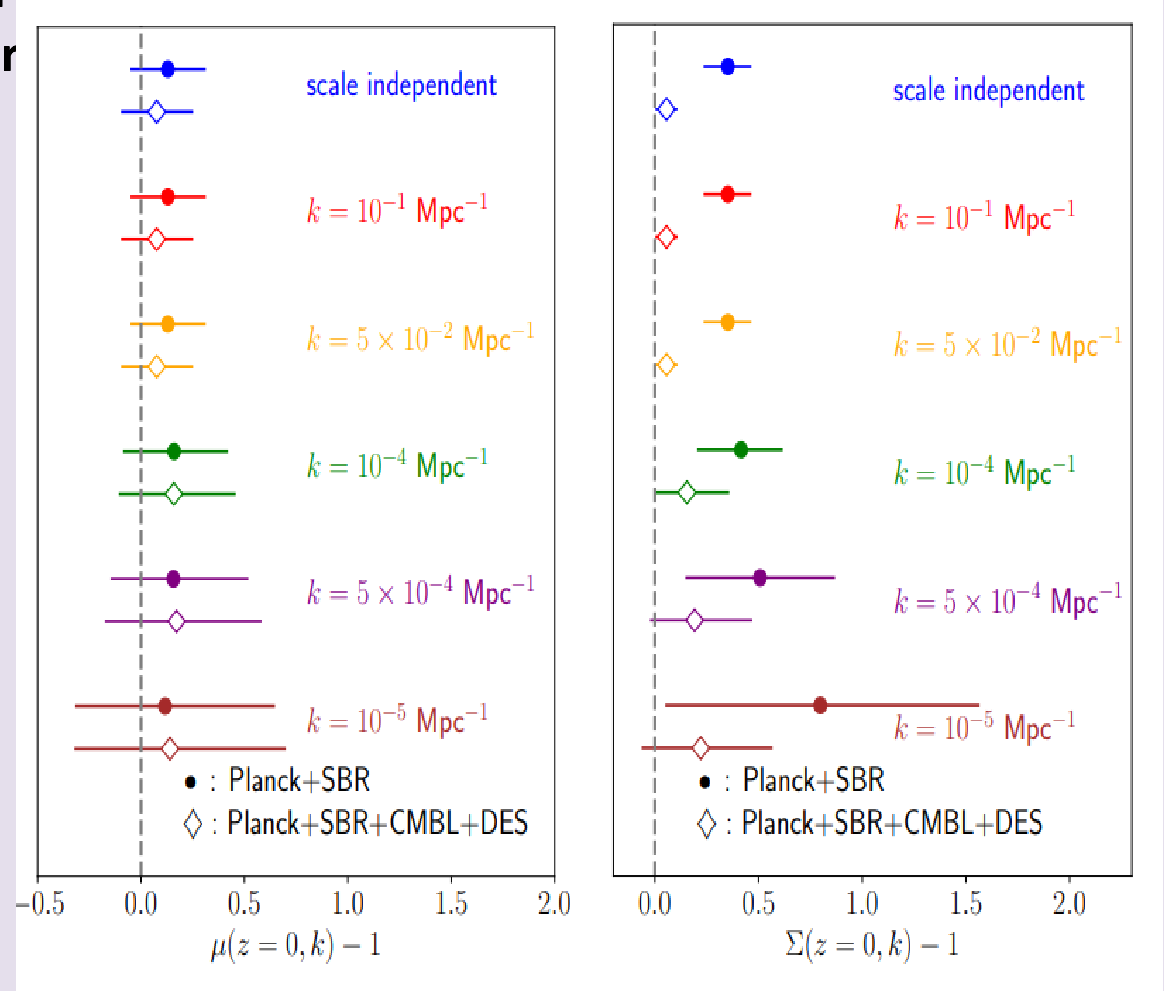
Ranges	Interpretation
$0 < \|\ln B_{i0}\| < 1$	Not worth more than a bare mention
$1 \leq \|\ln B_{i0}\| < 3$	Moderate
$3 \leq \|\ln B_{i0}\| < 5$	Strong
$5 \leq \|\ln B_{i0}\|$	Very strong

Constraining deviations from GR using functional and binning forms in redshift and scale (Garcia-Quinterro, Ishak and Ning, JCAP 2020, arXiv:2010.12519)

Functional form for redshift and scale, but fixing 1 MG parameter

$\Sigma(z = 0, k)$ shows a 3.5-σ tension with GR when using Planck+SBR and going toward smaller scales with $k \geq 10^{-2}$ Mpc^{-1}

tension goes away when we add lensing data.



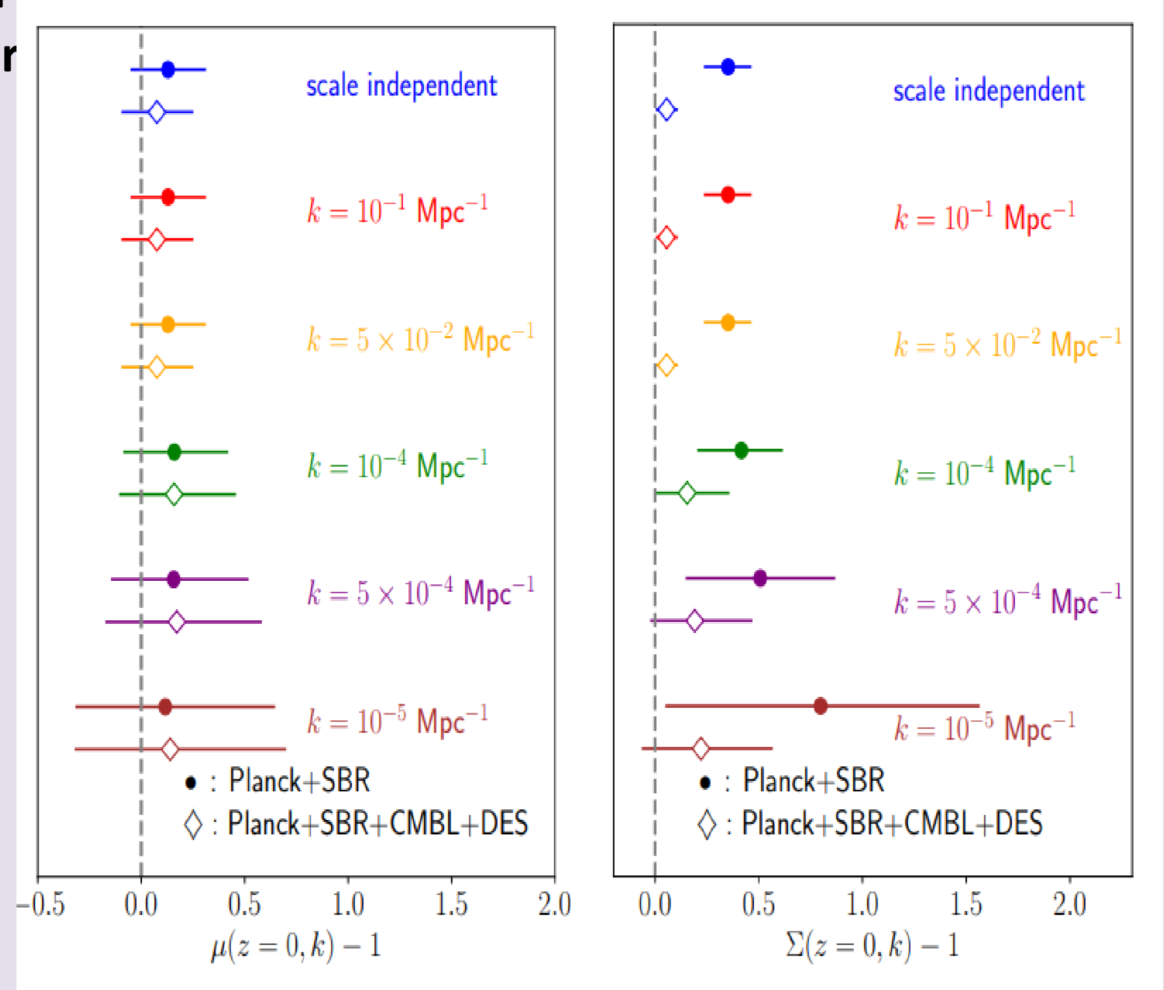
Constraining deviations from GR using functional and binning forms in redshift and scale (Garcia-Quinterro, Ishak and Ning, JCAP 2020, arXiv:2010.12519)

Functional form for redshift and scale, but fixing 1 MG parameter

$\Sigma(z = 0, k)$ shows a 3.5-σ tension with GR when using Planck+SBR and going toward smaller scales with $k \geq 10^{-2}$ Mpc^{-1}

tension goes away when we add lensing data.

Same dichotomy between Planck and Lensing data



Constraining deviations from GR using functional and binning forms in redshift and scale (Garcia-Quinterro, Ishak and Ning, JCAP 2020, **arXiv:2010.12519**)

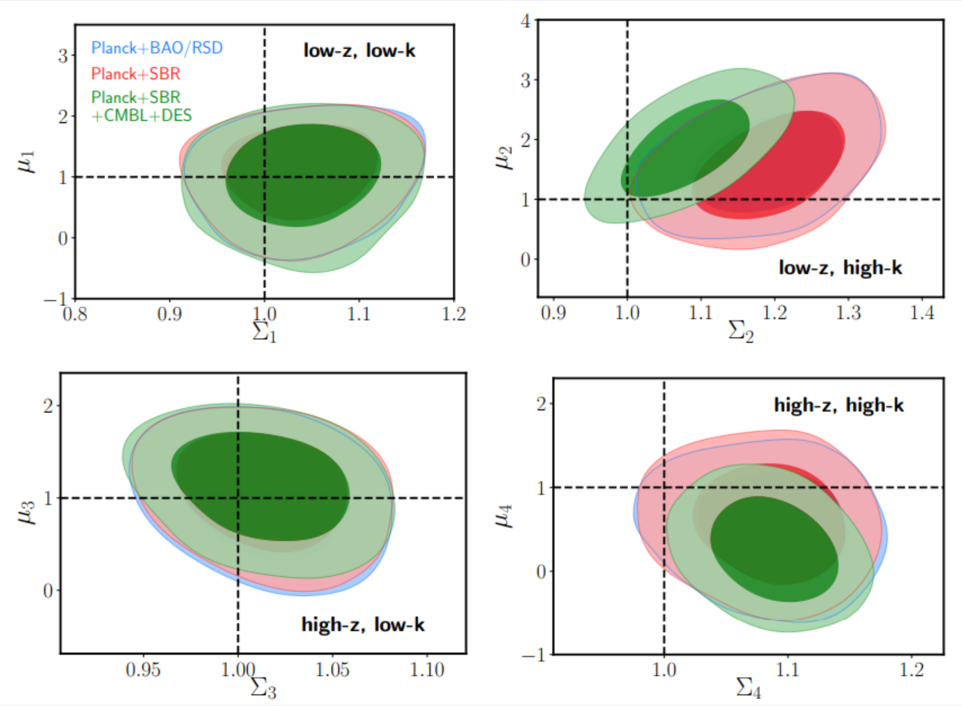
	Redshift bins		
Scale bins	$0 \leq z < 1$	$1 \leq z < 2$	$z \geq 2$
$0 \leq k < 0.01$	μ_1, Σ_1	μ_3, Σ_3	GR is assumed
$0.01 \leq k < \infty$	μ_2, Σ_2	μ_4, Σ_4	GR is assumed

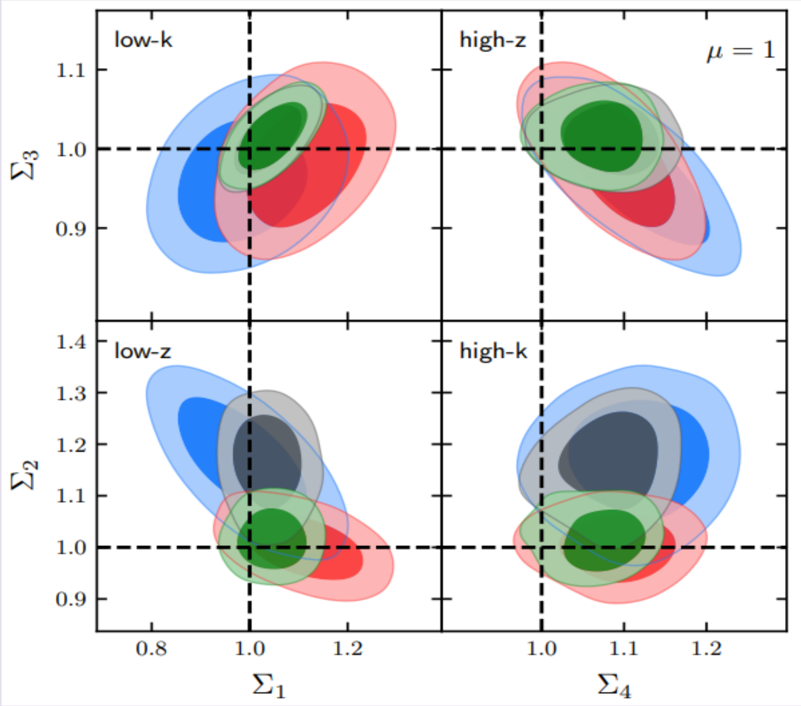
-

Constraining deviations from GR using functional and binning forms in redshift and scale (Garcia-Quinterro, Ishak and Ning, JCAP 2020, arXiv:2010.12519)

	Redshift bins		
Scale bins	$0 \leq z < 1$	$1 \leq z < 2$	$z \geq 2$
$0 \leq k < 0.01$	μ_1, Σ_1	μ_3, Σ_3	GR is assumed
$0.01 \leq k < \infty$	μ_2, Σ_2	μ_4, Σ_4	GR is assumed

- At smaller scales (high-k), Σ_2 and Σ_4 in 2.2-σ and 2.4-σ tension with GR

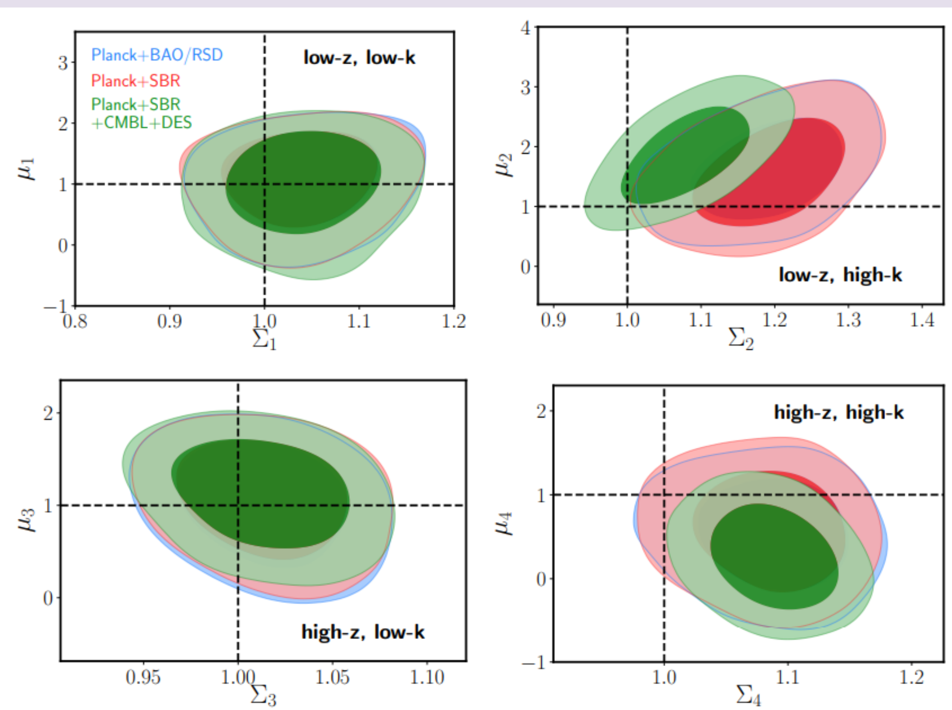


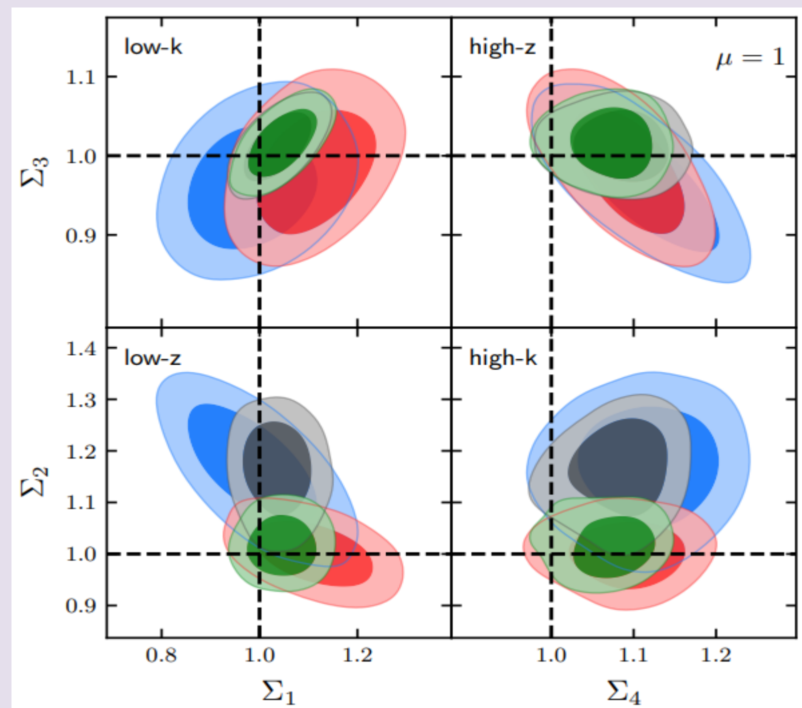


Constraining deviations from GR using functional and binning forms in redshift and scale (Garcia-Quinterro, Ishak and Ning, JCAP 2020, [arXiv:2010.12519](https://arxiv.org/abs/2010.12519))

	Redshift bins		
Scale bins	$0 \leq z < 1$	$1 \leq z < 2$	$z \geq 2$
$0 \leq k < 0.01$	μ_1, Σ_1	μ_3, Σ_3	GR is assumed
$0.01 \leq k < \infty$	μ_2, Σ_2	μ_4, Σ_4	GR is assumed

- At smaller scales (high-k), Σ_2 and Σ_4 in 2.2-σ and 2.4-σ tension with GR
- Model with Σ is marginally preferred over LCDM

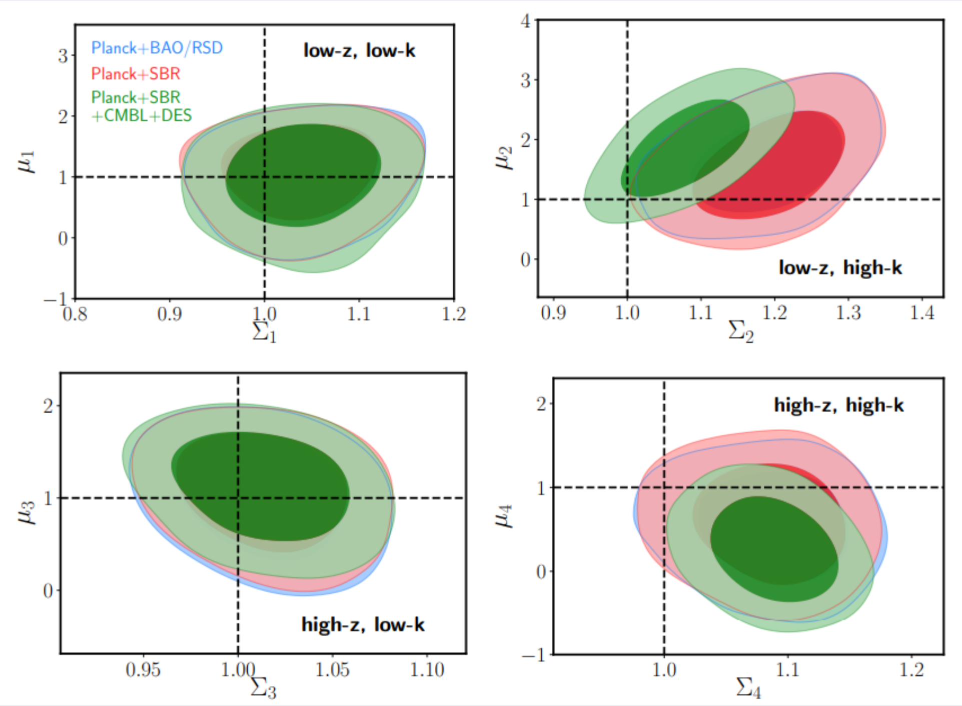


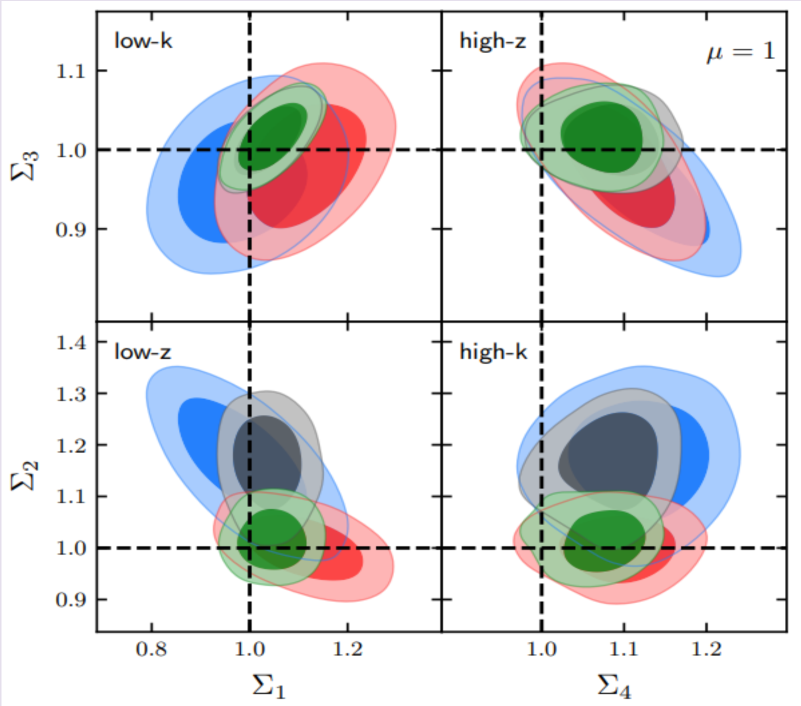


Constraining deviations from GR using functional and binning forms in redshift and scale (Garcia-Quinterro, Ishak and Ning, JCAP 2020, arXiv:2010.12519)

	Redshift bins		
Scale bins	$0 \leq z < 1$	$1 \leq z < 2$	$z \geq 2$
$0 \leq k < 0.01$	μ_1, Σ_1	μ_3, Σ_3	GR is assumed
$0.01 \leq k < \infty$	μ_2, Σ_2	μ_4, Σ_4	GR is assumed

- At smaller scales (high-k), Σ_2 and Σ_4 in 2.2-σ and 2.4-σ tension with GR
- Model with Σ is marginally preferred over LCDM
- All these tensions need to be watched with incoming more constraining data





ISiTGR: Integrated Software in Testing General Relativity

- Developed ISiTGR based on modifications to software CAMB and CosmoMC

- Available at https://github.com/mishakb/ISiTGR (**arXiv:1109.4583v3** ; **arXiv:1908.00290**)

- Has a number of useful features:
  - Dynamical dark energy parametrizations with a constant or time-dependent equation of state
  - A consistent implementation of anisotropic shear to model massive neutrinos throughout the full formalism
  - Multiple commonly used parameterizations of modified growth (MG) parameters
  - **Functional and binned** redshift and scale dependencies for MG parameters
  - Spatially **flat or curved** backgrounds (present in previous version as well).

Approach I: but using the growth index parameter

(Linder, 2005; Linder and Cahn, 2007; Gong, Ishak, Wang, 2009; Ishak, Dossett, 2009)

- The growth rate function $f = d\ln D / d\ln a$ *can be* approximated using the form

$$f = \Omega_m^\gamma$$

where γ is the growth index parameter

- γ is a discriminator between theories of gravity and can be measured from, for example, Redshift Space Distortion observations

- γ =6/11=0.545 for the standard LCDM model

- γ =11/16=0.687 for the flat DGP modified gravity model

Approach II: Testing specific modified gravity models

- **A number of gravity models beyond GR have been proposed like the**
  - **f(R) and f(G)**
  - **DGP**
  - **Bigravity theory**
  - **Non-Local gravity**
  - **Horava-Lifshitz theory; etc**

Approach II: Testing specific modified gravity models

- **A number of gravity models beyond GR have been proposed like the**
  - **f(R) and f(G)**
  - **DGP**
  - **Bigravity theory**
  - **Non-Local gravity**
  - **Horava-Lifshitz theory; etc**

- difficult to constrain and costly because one needs to derive the observables and also produce simulations for the non-linear regime

Approach II: Testing specific modified gravity models

- **A number of gravity models beyond GR have been proposed like the**
  - **f(R) and f(G)**
  - **DGP**
  - **Bigravity theory**
  - **Non-Local gravity**
  - **Horava-Lifshitz theory; etc**

- Costly and difficult to constrain because one needs to derive the observables and also produce simulations for the non-linear regime

- Progress is being made: fast COLA (COmoving Lagrangian Acceleration) simulations (e.g. Valogiannis & Bean, 2017; Winther et al., 2017; for f(R) models)

Approach II: Testing specific modified gravity models

- **A number of gravity models beyond GR have been proposed like the**
  - **f(R) and f(G)**
  - **DGP**
  - **Bigravity theory**
  - **Non-Local gravity**
  - **Horava-Lifshitz theory; etc**

- Costly and difficult to constrain because one needs to derive the observables and also produce simulations for the non-linear regime

- Progress is being made: fast COLA (COmoving Lagrangian Acceleration) simulations (e.g. Valogiannis & Bean, 2017; Winther et al., 2017; for f(R) models)

- A number of cosmological analyses for f(R), Non-Local gravity, Horndeski and Beyond Horndeski models (e.g. see reviews Clifton et al. 2012; Ishak, 2019)

Approach II: Testing specific modified gravity models

- Matter Power Spectrum Emulator for f(R) Modified Gravity Cosmologies. (Ramachandra, Valogiannis, Ishak, and Heitmann, 2020 and the LSST Dark Energy Science Collaboration, arXiv:2010.00596)
- Emulator based on COLA simulations
- Evaluation time for the emulator prediction of f(R) matter power spectrum is less than a milliseconds per computation on an Intel Core i5 Processor (10^6 faster than COLA approach).
- Ongoing work to incorporate this in the LSST-DESC Core-Cosmology-Library

Approach II: Testing specific modified gravity models

- Some recent work: Matter Power Spectrum Emulator for f(R) Modified Gravity Cosmologies. (Ramachandra, Valogiannis, Ishak, and Heitmann, 2020 and the LSST Dark Energy Science Collaboration, arXiv:2010.00596)

- Emulator based on COLA simulations

- Evaluation time for an emulator prediction of f(R) matter power spectrum is less than a milliseconds per computation on an Intel Core i5 Processor (10^6 faster than COLA approach).

- Ongoing work to incorporate this in the LSST-DESC Core-Cosmology-Library

- Other older work: testing f(G) using available observations (G=Gauss-Bonnet curvature invariant).

- Comparison to observations (e.g. Ishak and Moldenhauer, JCAP 2009a; Moldenhauer and Ishak, JCAP 2009b, 2010). Instabilities in structure formation, but some renewal on other stable variants of the theory (Clifton et al. 2020).

Approach III. Inconsistency tests between cosmological parameters from different datasets

- Inconsistencies/tensions between the parameter values as determined from different data sets can signal
  - systematic errors in the data or
  - problems with the underlying gravity theory

Approach III. Inconsistency tests between cosmological parameters from different datasets

- Inconsistencies/tensions between the parameter values as determined from different data sets can signal
  - systematic errors in the data or
  - problems with the underlying gravity theory

- How to characterize and quantify the level of inconsistency (Lin & Ishak, 2017a; 2017b; Garcia-Quinterro, Ishak and Lin, 2019)

Approach III. Inconsistency tests between cosmological parameters from different datasets

- Inconsistencies/tensions between the parameter values as determined from different data sets can signal
  - systematic errors in the data or
  - problems with the underlying gravity theory

- How to characterize and quantify the level of inconsistency (Lin & Ishak, 2017a; 2017b; Garcia-Quinterro, Ishak and Lin, 2019)

- Some pioneering work: Consistency relation between the expansion history and the growth rate of large scale structure as a test of gravity (Ishak, Upadhye, Spergel, PRD 2006; arXiv:astro-ph/0507184v2)
  - Showed that using the wrong gravity model in a cosmological analysis can lead to discordances between the dark energy parameters!!

Approach III: How to quantify the degree of inconsistency?

Lin & Ishak, PRD 96, 023532 (2017) **arXiv:1705.05303**

Need to introduce a mathematical measure that takes into account at least 3 aspects of inconsistencies:

a) deviation between likelihood maxima

b) volume of covariance matrices (ellipsoid sizes)

c) degeneracy directions (ellipsoid orientations)

Index of Inconsistency (IOI)

Lin & MI, PRD 96, 023532 (2017) arXiv:1705.05303

- We consider two experiments and define

$$\tfrac{1}{2}\Delta\chi^2(\boldsymbol{\mu}) \equiv \tfrac{1}{2}\Delta\chi^2_{(1)}(\boldsymbol{\mu}) + \tfrac{1}{2}\Delta\chi^2_{(2)}(\boldsymbol{\mu}) \ ,$$

where $\Delta\chi^2_{(i)}(\boldsymbol{\mu}) = \chi^2_{(i)}(\boldsymbol{\mu}) - \chi^2_{(i)}(\boldsymbol{\mu}^{(i)})$.

$\frac{1}{2}\Delta\chi^2_{(1)}(\boldsymbol{\mu})$: The 'difficulty' for the 1st experiment to support the mean of joint analysis.

$\frac{1}{2}\Delta\chi^2_{(2)}(\boldsymbol{\mu})$: The 'difficulty' for the 2nd experiment to support the mean of joint analysis.

Ranges	IOI<1	$1<$IOI<2.5	$2.5<$IOI<5	IOI>5
Interpretation	No significant inconsistency	Weak inconsistency	Moderate inconsistency	Strong inconsistency
Confidence level (one dimension only)	<1.4-σ	1.4-σ $-$ 2.2-σ	2.2-σ $-$ 3.2-σ	>3.2-σ

TABLE I: The values of IOI can be interpreted using the Jeffreys' scale. We use an overall more conservative terminology than the original one

Multi-experiment Index of Inconsistency (*IOI*)

Lin & MI, PRD 96, 023532 (2017) [arXiv:1705.05303]

- In the Gaussian limit $\Delta\chi^2_{(i)} = (\boldsymbol{\lambda} - \boldsymbol{\mu}^{(i)})\boldsymbol{L}^{(i)}(\boldsymbol{\lambda} - \boldsymbol{\mu}^{(i)})$

- We define the IOI as: $\frac{1}{2}\Delta\chi^2 \xrightarrow{Gaussian} \frac{1}{2}\boldsymbol{\Delta G \Delta} \equiv \mathrm{I\,OI}$

where $\boldsymbol{\Delta} = \boldsymbol{\mu}^{(2)} - \boldsymbol{\mu}^{(1)}$, and $\boldsymbol{G} = \left((\boldsymbol{L}^{(1)})^{-1} + (\boldsymbol{L}^{(2)})^{-1}\right)^{-1} = (\boldsymbol{C}^{(1)} + \boldsymbol{C}^{(2)})^{-1}$.

- Generalized to multiple experiments:

$$\frac{1}{2}\sum_i \Delta\chi^2_{(i)}(\mu) \xrightarrow{Gaussian} \frac{1}{2}\left(\sum_i \boldsymbol{\mu}^{(i)}\boldsymbol{L}^{(i)}\boldsymbol{\mu}^{(i)} - \boldsymbol{\mu L \mu}\right) \equiv \mathrm{IOI}$$

- Where $\boldsymbol{\mu} = \boldsymbol{L}^{-1}\left(\sum_i \boldsymbol{L}^{(i)}\boldsymbol{\mu}^{(i)}\right)$ and $\boldsymbol{L} = \sum \boldsymbol{L}^{(i)}$.

Application to current data sets: cosmological discordances

Data sets	Description	Added probes
1. Background		
SH0ES	Locally measured Hubble constant [1]	N/A
H0LiCOW	Strong gravitational lensing [101]	N/A
CCHP-TGRB[a]	Tip of the Red Giant Branch (TRGB) applied to SNe Ia [102]	N/A
TRGB-2	TRGB+SNe Ia distance ladder as re-analyzed with different methods by [5]	N/A
SNe	SNe Ia joint analysis data from Pantheon compilation [89]	BBN[b]
BAO	Six Degree Field Galactic Survey (6dF) ($z_{eff} = 0.106$) [87], SDSS main galaxy sample (MGS) ($z_{eff} = 0.15$) [88] and BAO consensus constraints [86]	BBN
2. Planck-2018 temperature and polarization		
TT+lowE	Planck high-ℓ[c] temperature auto correlation [83]	lowE[d]
TE+lowE	Planck high-ℓ temperature-E polarization cross correlation [83]	lowE
EE+lowE	Planck high-ℓ E-mode polarization auto correlation [83]	lowE
TTTEEE+lowE	Planck high-ℓ temperature and E-mode polarization joint data set [83]	lowE
3. Large-Scale-Structure data sets		
DES	Dark Energy Survey Year 1 clustering and lensing analysis [85]	lowE+SNe+BBN
CMB lens	Planck-2018 CMB lensing measurements [84]	lowE+SNe+BBN
SDSS RSD	SDSS III galaxy clustering data from BAO spectroscopic survey [86]	lowE+SNe+BBN
Joint LSS	Joint analysis combining DES+CMB lens+SDSS RSD	lowE(or with priors)+BBN

[a] CCHP stands for Carnegie-Chicago Hubble Program

[b] Primordial deuterium abundance $D/H = 2.527 \pm 0.030 \times 10^{-5}$ [90].

[c] High-ℓ represents the range $30 \leq \ell \leq 2058$.

[d] Planck-2018 low-ℓ ($2 \leq \ell \leq 29$) polarization [83]. Opposite to our previous work, here we only use the EE likelihood since a poor statistical consistency of the TE spectrum was reported in [83].

Application 1: Hubble constant tension: physics or systematics?

Lin & MI, Phys. Rev. D 96, 083532 (2017) **arXiv:1708.09813**

Garcia-Quintero, Ishak, Fox, Lin, PRD 2019, arXiv:1910.01608

Methods	Planck	SH0ES	Joint LSS (with priors)	H0LiCOW	SNe+BAO+BBN	CCHP	TRGB-2
$H_0\left(\frac{\mathrm{km/sec}}{\mathrm{Mpc}}\right)$	67.4 ± 0.50	74.03 ± 1.42	67.72 ± 0.97	73.30 ± 1.75	68.90 ± 1.70	69.80 ± 1.90	72.4 ± 1.90

Application 1: Hubble constant tension: physics or systematics?

Lin & MI, Phys. Rev. D 96, 083532 (2017) **arXiv:1708.09813**

Garcia-Quintero, Ishak, Fox, Lin, PRD 2019, arXiv:1910.01608

Methods	Planck	SH0ES	Joint LSS (with priors)	H0LiCOW	SNe+BAO+BBN	CCHP	TRGB-2
$H_0\left(\frac{\text{km/sec}}{\text{Mpc}}\right)$	67.4 ± 0.50	74.03 ± 1.42	67.72 ± 0.97	73.30 ± 1.75	68.90 ± 1.70	69.80 ± 1.90	72.4 ± 1.90

	Planck	SH0ES	Joint LSS (with priors)	H0LiCOW	SNe+BAO +BBN	CCHP-TRGB	TRGB-2
Planck	–	9.70	0.04	5.25	0.36	0.75	3.24
SH0ES[a]	9.70	–	6.73	0.05	2.68	1.59	0.24
Joint LSS	0.04	6.73	–	3.89	0.18	0.48	2.41
H0LiCOW	5.25	0.05	3.89	–	1.63	0.92	0.06
SNe+BAO+BBN	0.36	2.68	0.18	1.63	–	0.06	0.94
CCHP-TRGB[a]	0.75	1.59	0.48	0.92	0.06	–	0.47
TRGB-2[a]	3.24	0.24	2.41	0.06	0.94	0.47	–

TABLE VIII: Two-experiment IOI values for the seven data sets used to measure H_0.

Ranges	IOI<1	$1<$IOI<2.5	$2.5<$IOI<5	IOI>5
Interpretation	No significant inconsistency	Weak inconsistency	Moderate inconsistency	Strong inconsistency
Confidence level (one dimension only)	<1.4-σ	1.4-σ – 2.2-σ	2.2-σ – 3.2-σ	>3.2-σ

TABLE I: The values of IOI can be interpreted using the Jeffreys' scale. We use an overall more conservative terminology than the original one

Application 2: Planck CMB versus Large Scale Structure data sets

A persistent tension from moderate to strong range

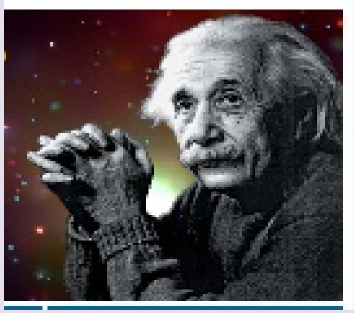
	CMB lens	DES	SDSS RSD	Joint LSS
TT+lowE	1.83	3.28	0.90	5.01
TE+lowE	1.54	1.54	0.40	3.90
EE+lowE	2.07	3.13	1.39	3.97
TTTEEE+lowE	1.60	3.14	0.70	5.27

This is consistent with other reported tensions between Planck and LSS (e.g, Hildebrandt et al. 2016; Joudaki et al. 2017; Leauthaud, et al. 2016 and others studies using the amplitude of matter fluctuation as parameter)

Is this due to systematic effects or this is the symptom of deviation from General Relativity?

Need to be explored using more precise data, e.g. LSST, DESI, WFIRST, JWST, Euclid

Concluding remarks



1. Testing gravity (General Relativity) at cosmological scales has several motivations with cosmic acceleration at the front

2. At least three approaches to testing gravity in cosmology
   - MG parameters of deviations from GR
   - Testing specific proposed models
   - Pursuing cosmological inconsistencies

3. Binning methods for MG parameters offer an alternative to using functional methods

4. Current datasets show persistent tension well above 3-σ when using Planck or Planck+SN+BAO+RSD

5. Tensions with GR go away when adding Lensing data from DES and CMB Lensing

6. Why such dichotomy between Planck and Lensing for MG parameters?

7. Emulators will be helpful in allowing the testing of specific MG models like Horndeski and beyond

8. Are the persistent tensions early symptoms of model/gravity problems or due to data systematics?

9. Promising surveys and experiment will give data soon to test decisively some of these questions!