On the role of rotation in galactic dynamics

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Abstract

Although the existence of dark matter would allow good explanations for various cosmic phenomena, the nature of such matter is unknown and it still lacks of direct detection. This limitation has stimulated the investigation of alternative gravity models to explain phenomena such as the galactic dynamics that yields rotation curves of galaxies. These models depart from general relativity in different ways, like in f(R) theories or in brane-world approaches, or as in the MOND framework, in which an acceleration is a fundamental constant. Other approaches propose that the dark matter phenomena can be attributed to an elastic response of a dark energy medium permeating the universe, or that spacetime would consist of an anistropic fluid. Several alternative models investigate scenarios where galaxies are described as static, spherically symmetric sistems. However, galaxies are rotating systems and studies have signaled the possible relevance of this rotation on the explanation of the flat rotation curves of galaxies. In this work we apply general relativity to analyse the behavior of a mass probe moving in the gravitational field of a massive, rotating body. We focus on the motion far from the source, where the gravitational field is weak, looking for clues that might help understand the role of rotation in galactic dynamics.

Introduction

The existence of dark matter (DM) is a powerful assumption aimed at explaining the motion of stars in galaxies, among other cosmic phenomena [1, 13]. In this work we focus on the unexplained rotation curves of galaxies, where stars far from the center of spiral galaxies display nearly constant and relatively high rotational speeds in contrast to the theoretical ones expected in Newtonian gravity (NG), as sketched in Fig. 1.

Normally, the DM content in galaxies is estimated using NG under the assumption that star velocities are non-relativistic and the gravitational field is weak in the galactic interior. Calculations are then normally performed assuming a steady-state configuration and a spherically symmetric DM mass distribution.

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In this work we argue that general relativity may be a good candidate to explain galactic RC.

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This argument is based on the success of the theory in explaining other phenomena. Some works have been developed with this goal and we will briefly comment on them.

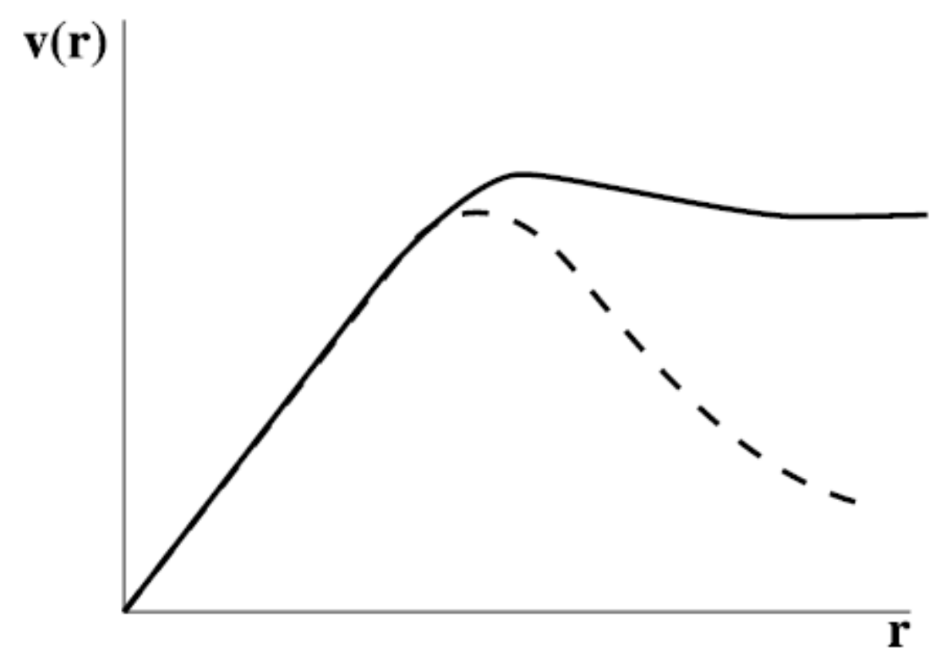

Figure 1: Sketch of a generic rotation curve for a spiral galaxy. The solid line represents the actual rotation curve while the dashed line represents a theoretical rotation curve based on Newtonian gravity considering only visible matter. Image reproduced from [13].

Galactic dynamics and Newtonian gravity

When NG is applied to galactic dynamics, the determination of DM mass distribution is guided by the virial theorem applied to a star (i=1) with mass m positioned at a galactocentric radius r with a speed, v, that belongs in the flat part of the RC [13] :

$$< \sum_i m_i v_i^2 >= - < \sum_i \mathbf{F_i} \cdot \mathbf{r_i} > \Rightarrow \mathbf{mv^2} = \frac{\mathbf{GmM(r)}}{\mathbf{r}},$$

where $<>$ denotes a time average and G is Newton's gravitational constant.

In this context, the gravitational interaction between the star and the rest of the galaxy is mediated solely by the Newtonian potential. The DM mass distribution is obtained from an adequate fit that yields the function $M(r)$ that best reproduces the respective rotation curve. This procedure obviously leads to variations of the DM content from galaxy to galaxy, hindering the development of a general theory that would explain galactic RC.

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The application of Newtonian gravity to the rotation curve problem has been justified in view of the weak gravity throughout galaxies and the non-relativistic velocities of their stars. However, these conditions are also present in various important phenomena where GR has a role to play.

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Stars in galaxies move exclusively under gravity (they are gravitationally bound), and it has been known for quite some time that this kind of system yields nonlinear equations of motion in GR, in contrast to the linear equations of NG. Two phenomena where GR is required are the precession of Mercury's perihelion and the Lense-Thirring effect, about which we comment below.

Galactic dynamics and general relativity

The explanation of the precession of Mercury's perihelion demanded the use of general relativity despite the fact that the motions of the planet and of the Sun are non-relativistic and that the gravitational fields involved are weak. The relativistic correction is very small yet essential, and NG is not able to supply it because this classical theory involves a flat space, while GR involves a *curved* space-time. It is the curved space in GR that contributes decisively to the correct precession [12].

Another phenomenon that results from GR is the Lense-Thirring effect, in which *extra* gravitational effect is created by the motion of the source, and felt only by moving test particles. In this case, a rotating object causes the surrounding space-time also to rotate, thus influencing the motion of the moving test particle. Orbiting satellite projects were designed to detect this GR prediction on the gravitational field of the Earth, as it should be manifested even under weak fields and non-relativistic speeds [7, 11].

Recently this effect was reported under a new scenario, induced by a fast-rotating white dwarf in a binary pulsar system [8].

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Such examples suggest that the rotation curve problem in galaxies could be solved with appropriate applications of general relativity.

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Preliminary works in this direction have been reported, as we present next.

First relativistic models for galaxies

A galaxy is an extremely complex system and the application of general relativity to it is expected to be a difficult task.

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The challenge to describe galaxies using general relativity started to be faced more than a decade ago, when a galaxy was described by dust rotating under axial symmetry.

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That pioneering work, authored by Cooperstock and Tieu (CT) [4], was able to fit RC with amazing precision using the metric

$$ds^2 = e^{\nu - w}(u - dz^2) + dr^2 + r^2 e - w d\phi^2 - e^w (dt - N d\phi)^2,$$

where $c = 1$ and the symbols are explained in [5].

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A striking aspect of this metric is that star velocities are proportional to N, the metric coefficient that describes rotation. Therefore, as in the Lense-Thirring effect, rotation may be key to the description of rotation curves, an ingredient that Newtonian gravity lacks.

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The CT model was criticized by several theoreticians, especially because of a discontinuity in one derivative. This would, according to some critics, render the solution unphysical. This and other criticisms were answered by those authors in later papers [5]. Cooperstock was aware that dust is a crude model for a galaxy, but he envisioned it as a first approximation to the RC problem using GR [3]. The excellent RC fits given by the model allowed estimates of galactic masses, which suggested that no DM would be necessary to explain RC using GR.

Amidst that debate another relativistic dust model for spiral galaxies was proposed by Balasin and Grumiller (BG) [2], which had a singularity at the galactic center, hence being as unphysical as its precedent. Nevertheless, the authors were able to gain by the model the insight that the Newtonian approximation breaks down in an extended rotating region, as well as overestimates the amount of matter needed to explain RC.

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Both relativistic dust models for spiral galaxies suggest that the use of Newtonian gravity to solve the rotation curve problem may overestimate the amount of matter necessary for the solution.

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To date, CT and BG dust models seem to be the only ones that try to address the RC problem using GR. Their mathematical issues were investigated in detail by Neill [10], who also proposed ways to overcome them, concluding that one should attempt to solve the Einstein field equations for dust more generally than CT and BG did, setting physically appropriate boundary conditions and perhaps involving numerical integration. The resulting solution could yield not only a better picture of the DM distribution, but also boost our understanding of the current cosmological model.

Astrophysics from relativistic models of galaxies

Despite the limitations of the simplified CT and BG models, astrophysical investigations were pursued in search for information about any differences in behavior between GR and NG. Moreover, the predictive success of the CT model was yet to be tested, as pointed out by Neill. For example, one should use CT's expression for the mass density with the coefficients produced in the velocity fit and compare the calculated density profile with the observed density.

Such project was developed by this author and Cooperstock [9], where the CT model still fitted RC very well but was not able to fully describe observed mass densities, demanding extra mass to correct the general-relativistic distribution. However, the amount of extra mass was significantly smaller than the amount required using NG.

A recent work investigated similar issues using Milky Way data under the BG model[6], suggesting that GR could explain rotation curves without DM.

As pointed out before, in GR *curved space* is responsible for important physical contributions to gravitating systems. Therefore, as more suited metrics are used to describe galaxies - with as many spatial details as possible - more contributions to rotation curves in the flat regime are expected to appear from spatial curvature, perhaps totally fulfilling the role that dark matter plays in Newtonian gravity.

It is possible that a relativistic galactic model, more accurate than CT and BG simple models, might explain galactic rotation curves and yield observed mass densities using only barionic matter. Such investigation could be carried on with, for example, the methodology proposed by Neill.

Concluding remarks

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