Learning about inflation from the three-point functions

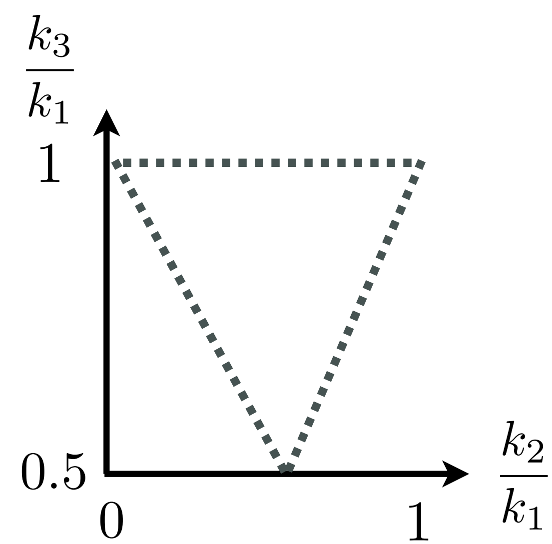
Jorge Noreña
Pontificia Universidad Católica de Valparaíso

Outline

- Non-Gaussianity
- Current and future constraints
- Relativistic galaxy power spectrum and bispectrum

3-Point Functions

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)B(k_1,k_2,k_3)$$



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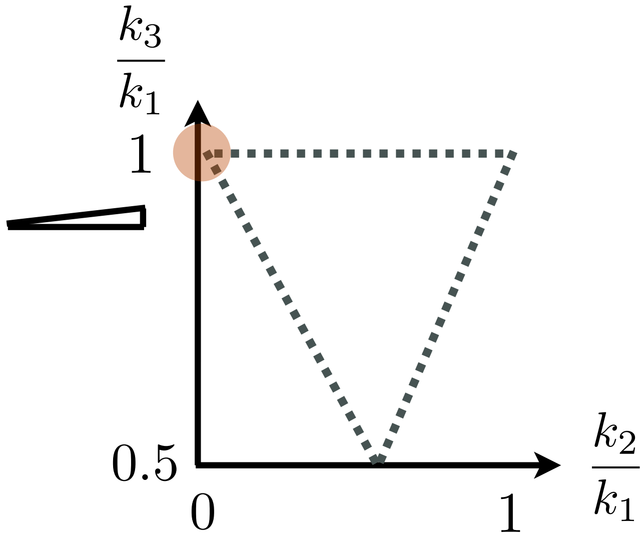
Squeezed limit: $k_1 \ll k_2, k_3$



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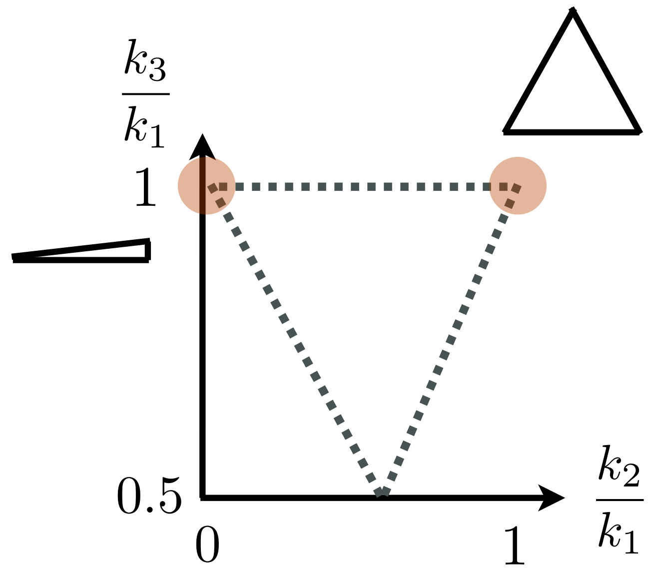

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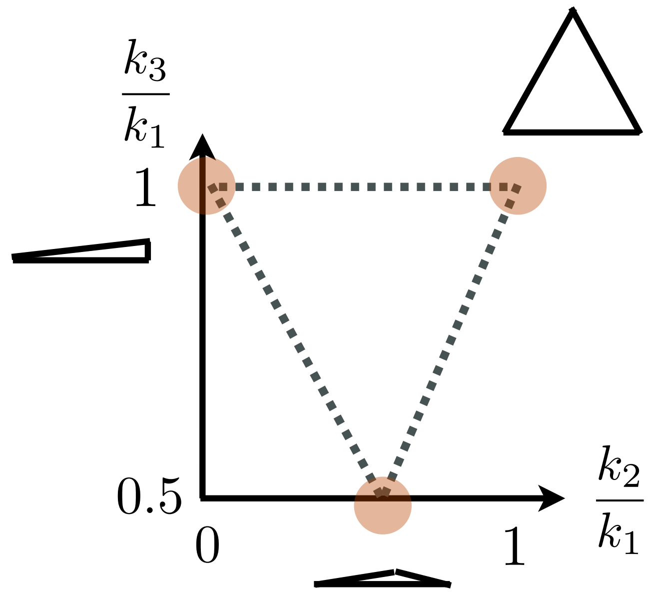
Equilateral configurations: $k_1 = k_2 = k_3$

Enfolded configurations: $k_1 = 2k_2 = 2k_3$

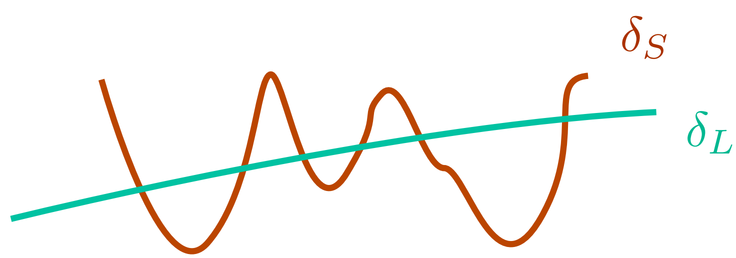


Squeezed limit

The information about the non-linearity of the evolution of the perturbations from inflation all the way to the LSS is contained in higher-order correlation functions

$$\langle \delta(\mathbf{q})\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{q}+\mathbf{k}_1+\mathbf{k}_2)B(q,k_1,k_2)$$

We will be interested in the limit $q \ll k_1, k_2$



$$\langle \delta(\mathbf{q})\delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_n)\rangle \stackrel{q\to 0}{=} \langle \delta(\mathbf{q})\langle\delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_n)\rangle_{\delta_L}\rangle$$

Khouri, Hinterbickler, Hui, Joyce, 2012

Khouri, Hinterbickler, Hui, Joyce, 2013

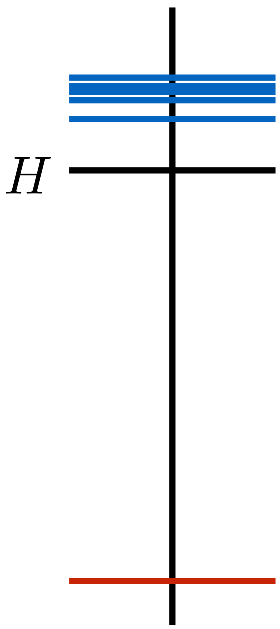
Ghosh, Kundu, Raju, Trivedi, 2014

Goldberger, Hui, Nicolis, 2013

Squeezed limit information

The squeezed limit contains model independent information about the physics during inflation

Single field



$$B(q, k_1, k_2) \overset{q\to 0}{\sim} \frac{1}{q}$$

J. Maldacena, 2003
P. Creminelli, M. Zaldarriaga, 2004
P. Creminelli, G. D'Amico, M. Musso, JN, 2011
P. Creminelli, JN, M. Simonovic, 2012

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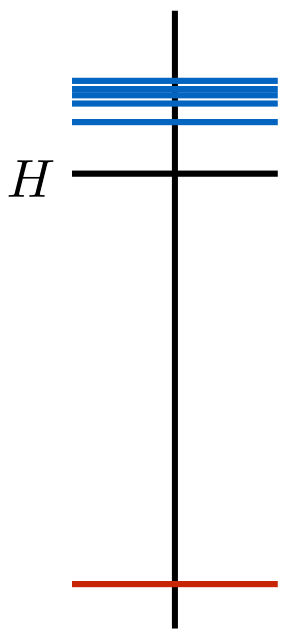
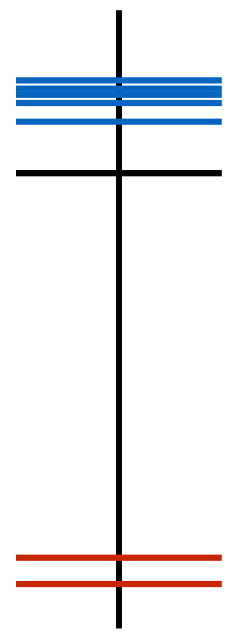


Multi field



$$B(q,k_1,k_2) \overset{q\to 0}{\sim} \quad \frac{1}{q} \qquad\qquad \frac{1}{q^3}$$

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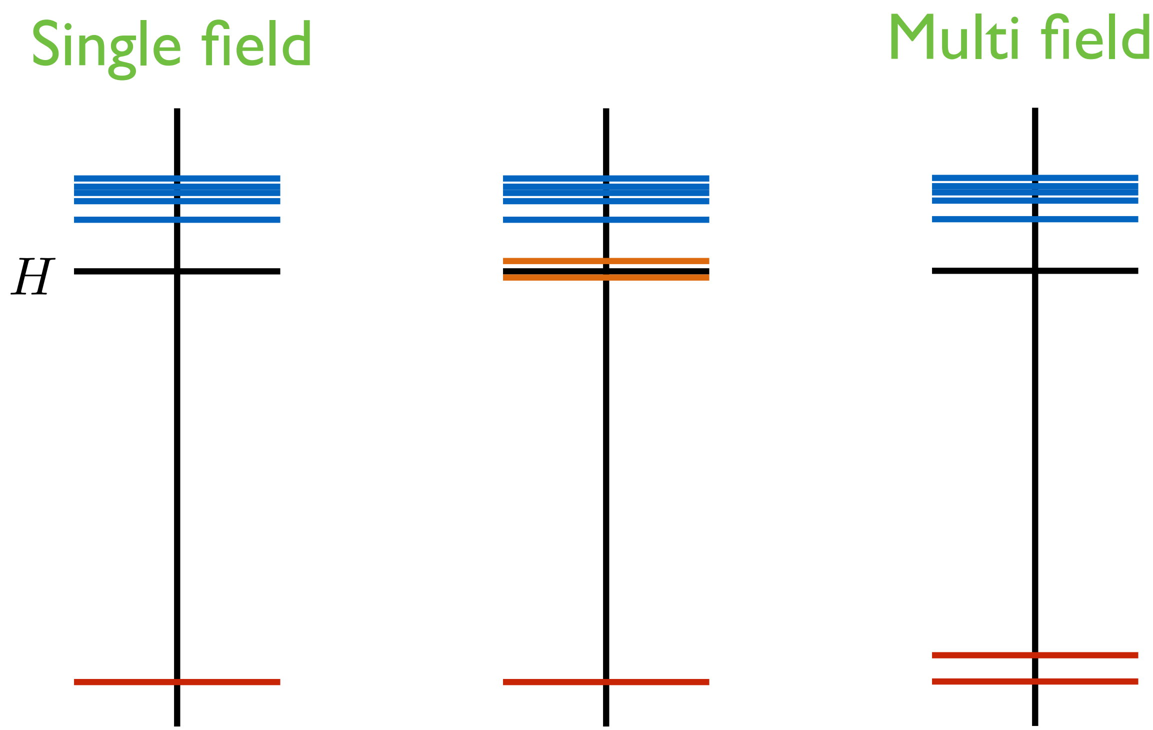

$B(q, k_1, k_2) \overset{q \to 0}{\sim}$ $\quad \frac{1}{q} \quad\quad \frac{1}{q^\alpha} \quad 1 < \alpha < 3 \quad\quad \frac{1}{q^3}$

X. Chen, J. Wang, 2009

Squeezed limit information

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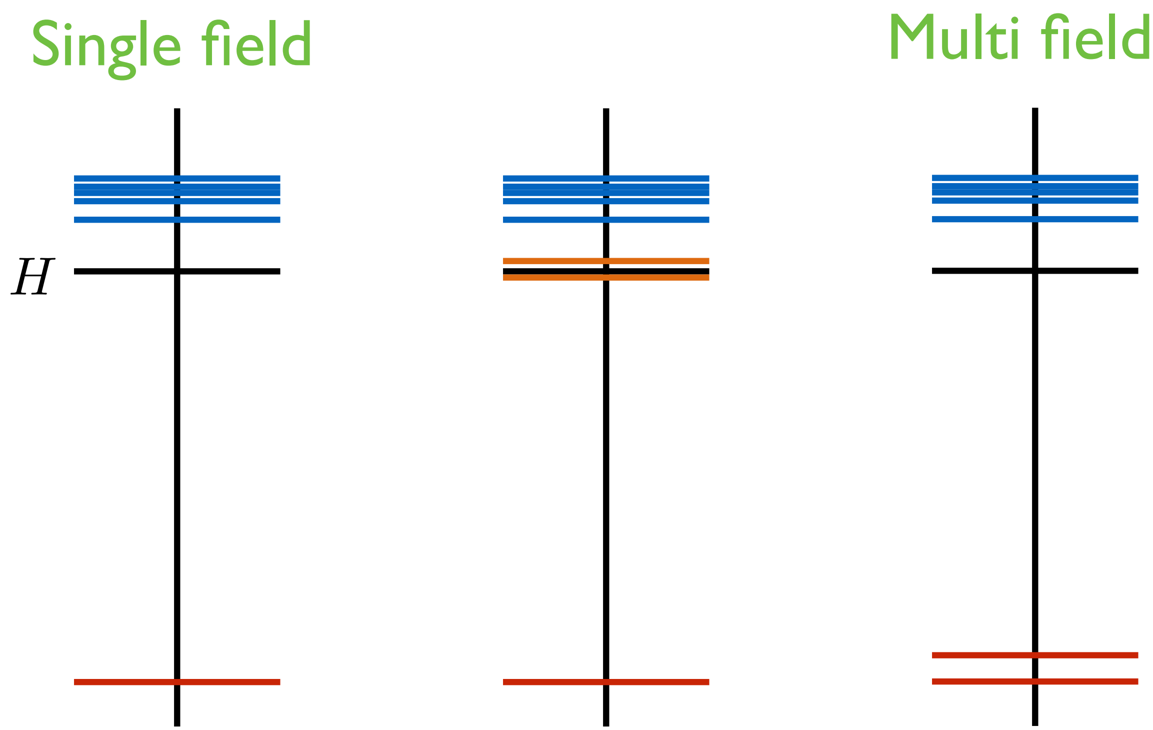

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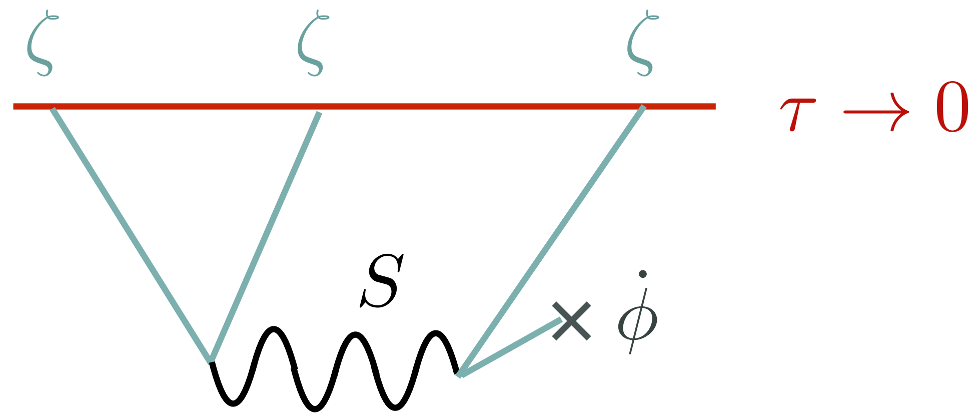
Figure: Assassi, Baumann, Green, 2012

Other fields



$$\langle \zeta(q)\zeta(k)\zeta(k) \rangle \sim e^{-\pi\mu} \left[e^{i\delta(\mu)} \left(\frac{q}{k}\right)^{\frac{3}{2}+i\mu} + e^{-i\delta(\mu)} \left(\frac{q}{k}\right)^{\frac{3}{2}-i\mu} \right] P_s(\cos\theta)$$

Characteristic angle dependence

$$\mu = \sqrt{\frac{m^2}{H^2} - \left(s - \frac{1}{2}\right)}$$

J. Maldacena, N. Arkani-Hamed, 2015

H. Lee, D. Baumann, G. Pimentel, 2016

A. Riotto, A. Kehagias, 2017

A. Moradinezhad, H. Lee, J. Muñoz, C. Dvorkin, 2018

L. Bordin, P. Creminelli, A. Khlemintsky, L. Senatore 2018

EFT of Inflation

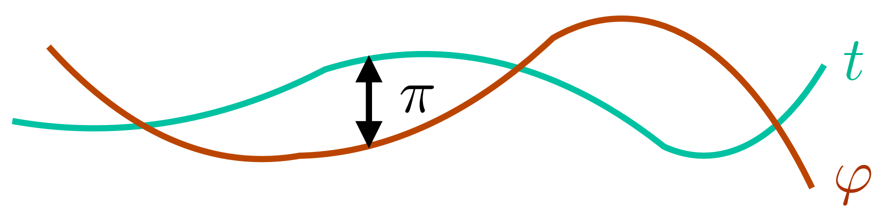
A way of writing the EFT for inflation

$$\mathcal{L} = -\frac{M_{\rm pl}^2 \dot{H}}{c_s^2}\left(\dot{\pi}^2 - \frac{c_s^2}{a^2}(\partial_i \pi)^2\right)$$

$$+\frac{M_{\rm pl}^2 \dot{H}}{c_s^2}(1-c_s^2)\frac{1}{a^2}\dot{\pi}(\partial_i\pi)^2 + \frac{M_{\rm pl}^2 \dot{H}}{c_s^2}A\dot{\pi}^3$$

I define the field perturbations in the following way

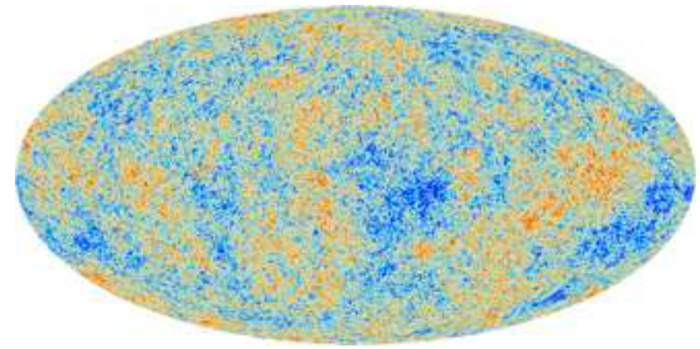
$$\varphi(t,\vec{x}) = \varphi_o(t+\pi(t,\vec{x}))$$



Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore, 2008

How can it be observed?

$$\left\langle \frac{\delta T}{\bar{T}} \frac{\delta T}{\bar{T}} \frac{\delta T}{\bar{T}} \right\rangle \longleftarrow$$



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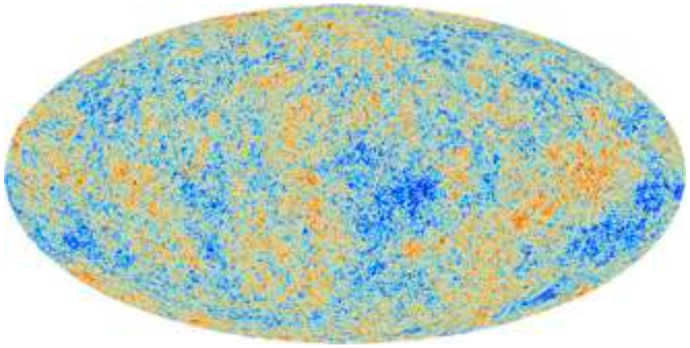
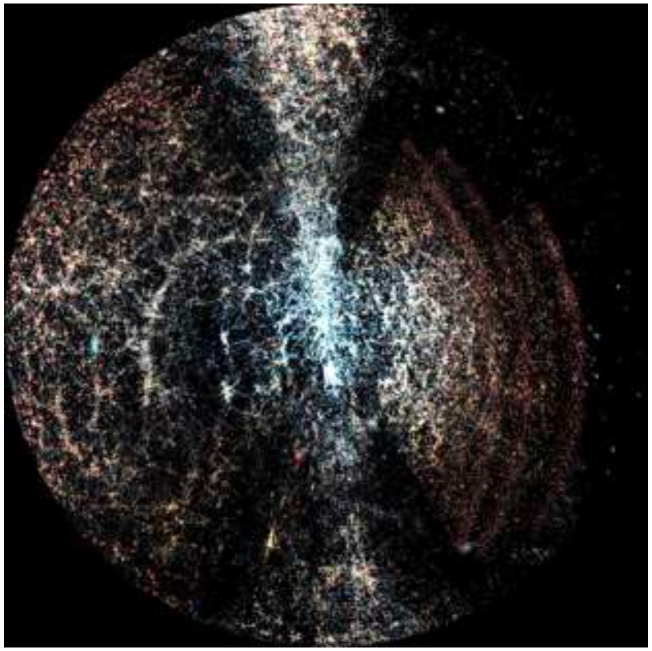


$$\Delta_g = \frac{n_g(z,\hat{n}) - \bar{n}_g(z)}{\bar{n}_g(z)}$$

$$\Delta_g \sim \delta_m$$

$\longleftarrow$



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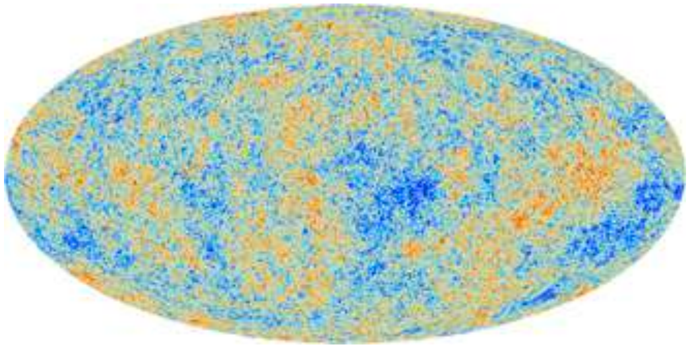

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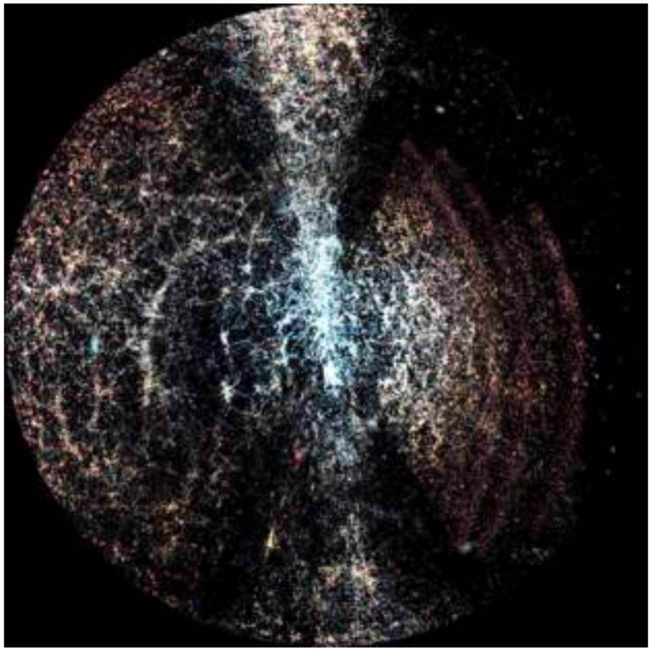

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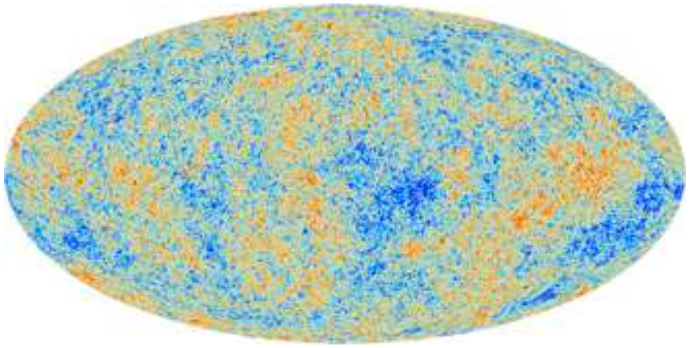

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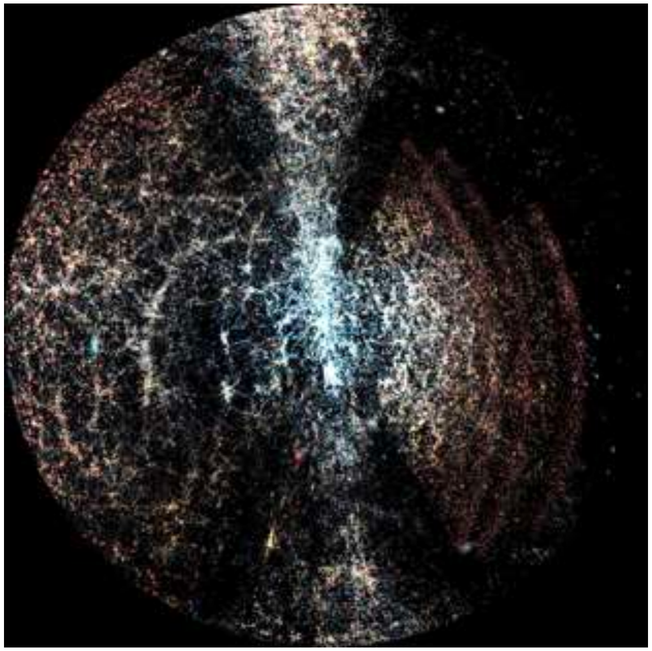

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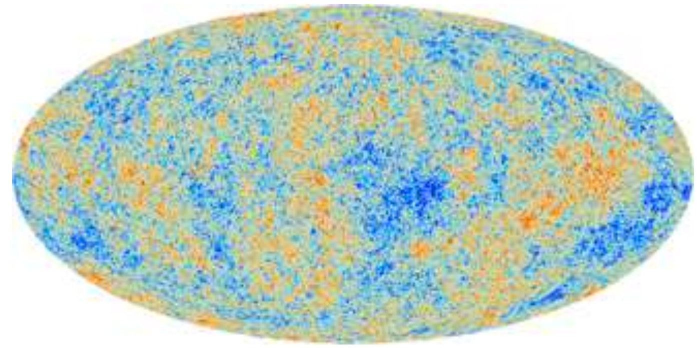

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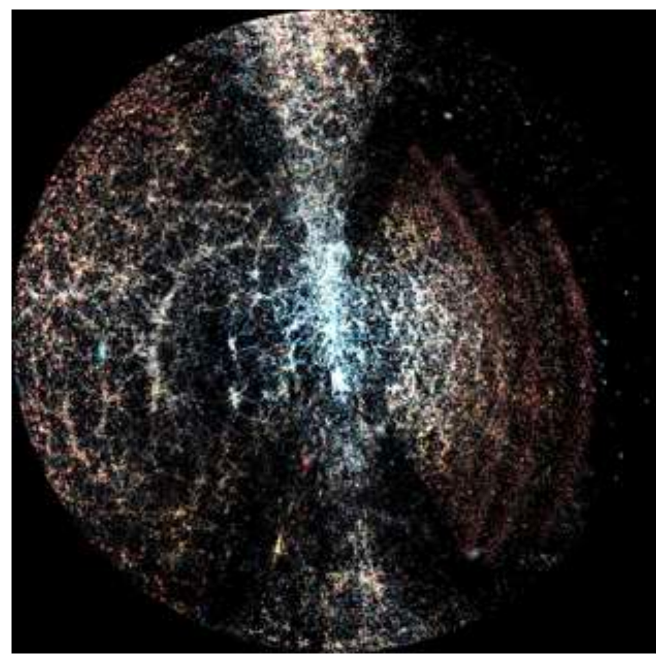
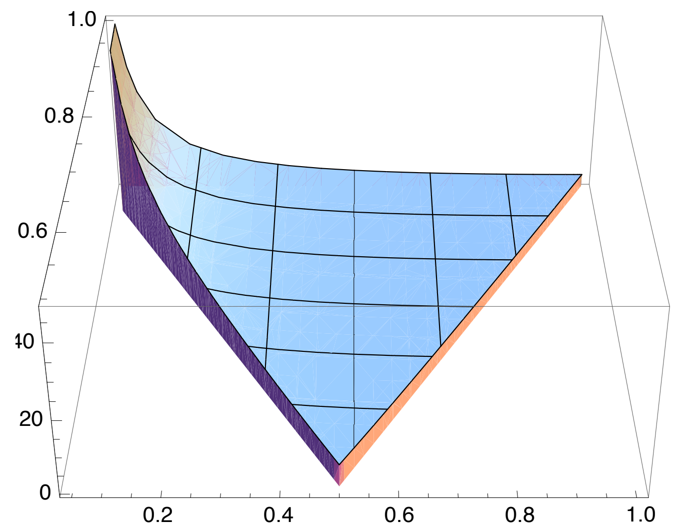


CMB Constraints

Consider a phenomenological model:

$$\zeta = \zeta_g + \frac{3}{5} f_{NL}^{local} \zeta_g^2$$

$$F^{local}(k_1, k_2, k_3) = -2\frac{3}{5} f_{NL}^{local} A^2 \frac{1}{k_1^3 k_2^3} + 3\ perms.$$



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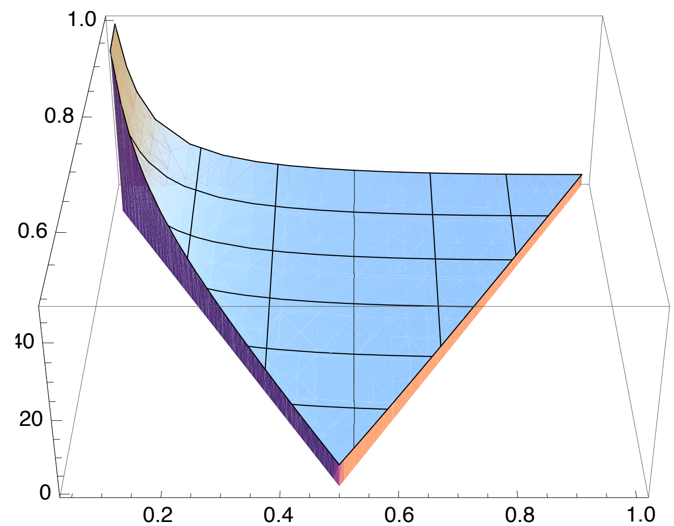

shape "Overlap"

$$F_1 \cdot F_2 \equiv \sum_{k_1, k_2, k_3} \frac{F_1(k_1, k_2, k_3) F_2(k_1, k_2, k_3)}{\sigma^2(k_1)\sigma^2(k_2)\sigma^2(k_3)}$$

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Data is analyzed for simple shapes.

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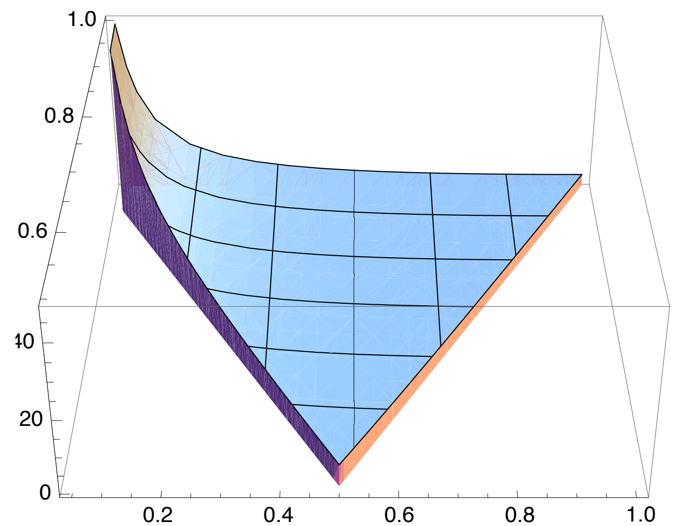

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Data is analyzed for simple shapes.

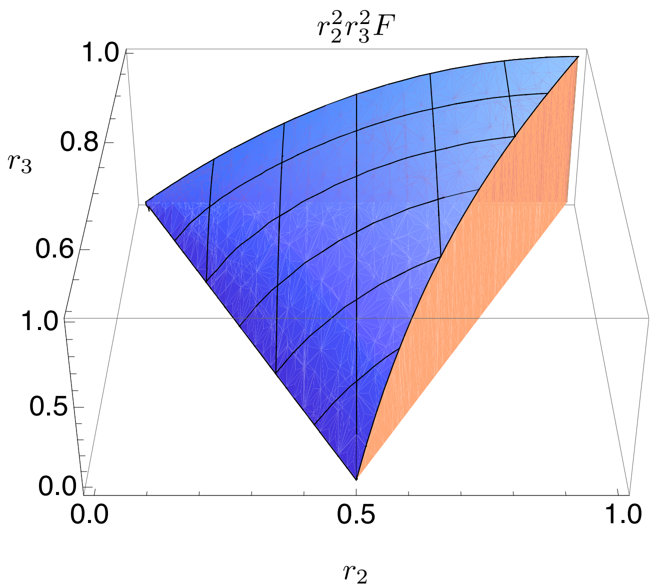
This shape is the one produced by multi-field models.

CMB Constraints

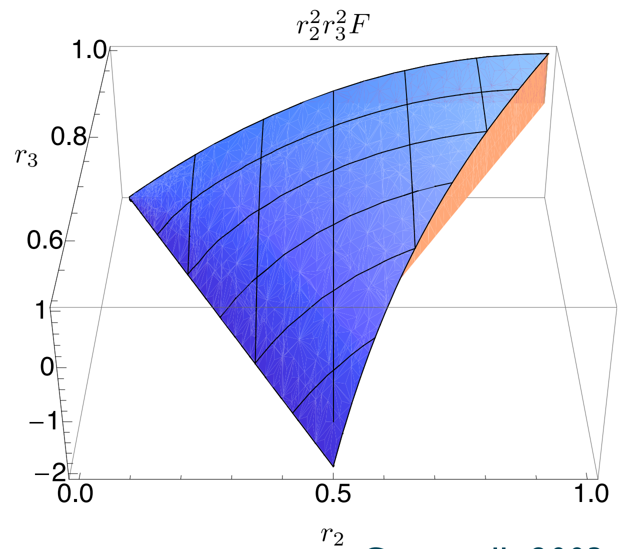
General single field models

Find templates that are like the NG produced by the 2 EFT operators

Equilateral



Orthogonal



Creminelli, 2003 [arXiv: astro-ph/0306122]
Cheung et. al., 2008 [arXiv: 0709.0293]
Senatore et. al., 2010 [arXiv: 09053746]

CMB Constraints

(68% CL)

Single-field EFT	Equilateral	$f_{\rm NL}^{equi} = -26 \pm 47$	Planck collaboration 2019
	Orthogonal	$f_{\rm NL}^{orth} = -38 \pm 24$	
Multi-field	Local	$f_{\rm NL}^{loc} = -0.9 \pm 5.1$	

Implications for theory

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$f_{\rm NL}^{loc} = \mathcal{O}(0.1)$ if coupled to inflaton.

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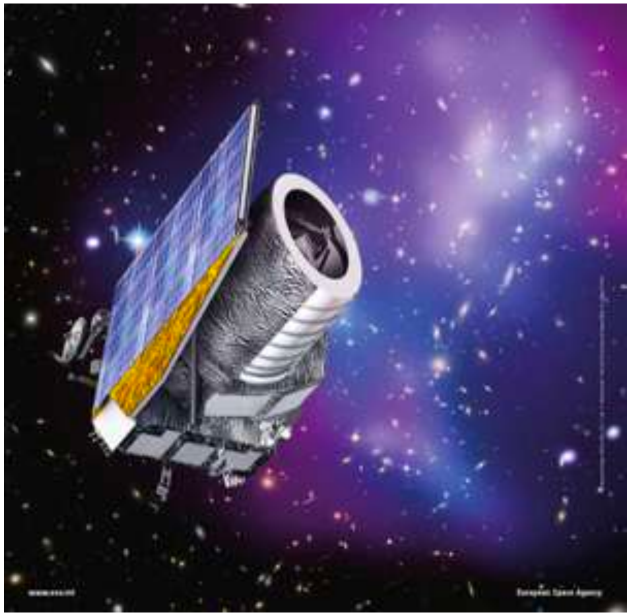
$f_{\rm NL}^{loc} = \mathcal{O}(0.1)$ if coupled to inflaton.

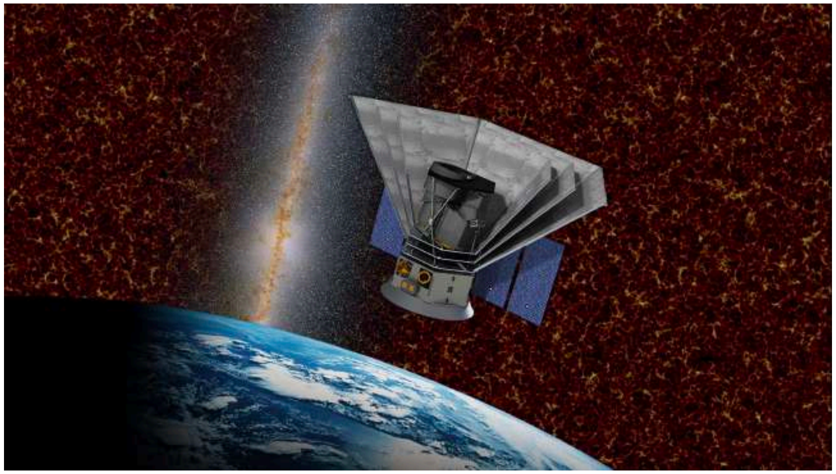
Consistent with weakly coupled inflation

The EFT of inflation teaches us that $f_{\rm NL}^{equil} \propto (H/\Lambda)^2$

Current constraints imply $\Lambda \gtrsim \mathcal{O}(10) H$

Future: LSS





The Scale-dependent bias

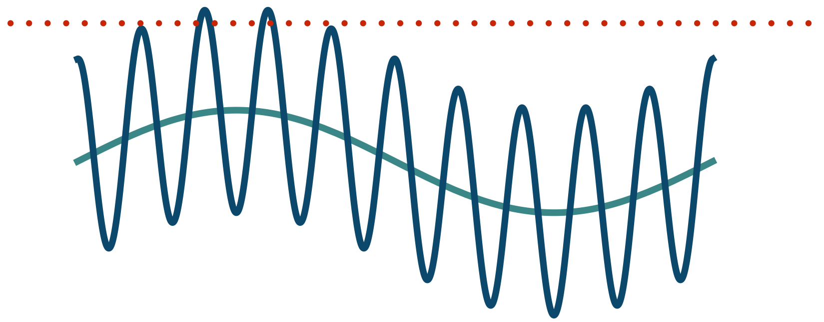
Bias is the connection of galaxies and matter $\delta_g = b\delta$

Dalal, et. al., 2008

Matarrese, Verde, et. al., 2008

Slosar, et. al., 2008

For the local model: $\Phi = \Phi_g + f_{\rm NL}\Phi_g^2$



$f_{\rm NL} = 0$

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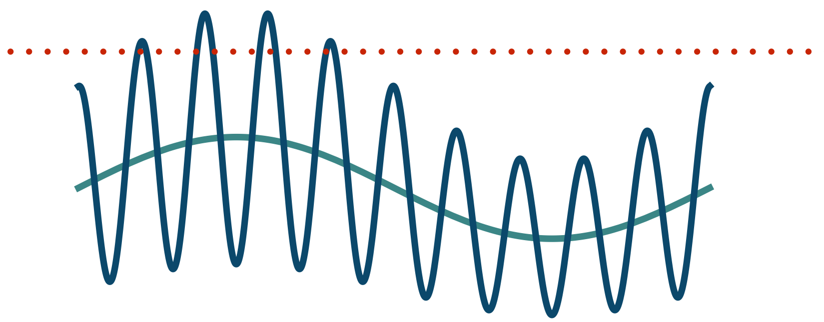

$f_{\rm NL} > 0$

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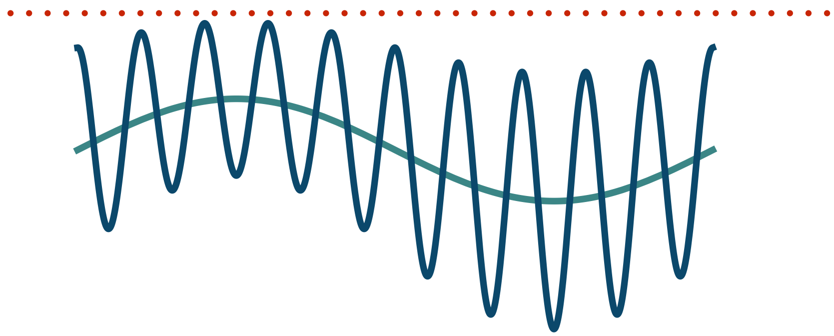


$f_{\rm NL} < 0$

There is a correlation between Φ and the number of galaxies Δ_g.

$$\langle \Delta_g \Delta_g \rangle \subset \langle \Phi\delta \rangle \sim \frac{1}{k^2}\langle \delta\delta \rangle$$

Sensitive to the squeezed limit!

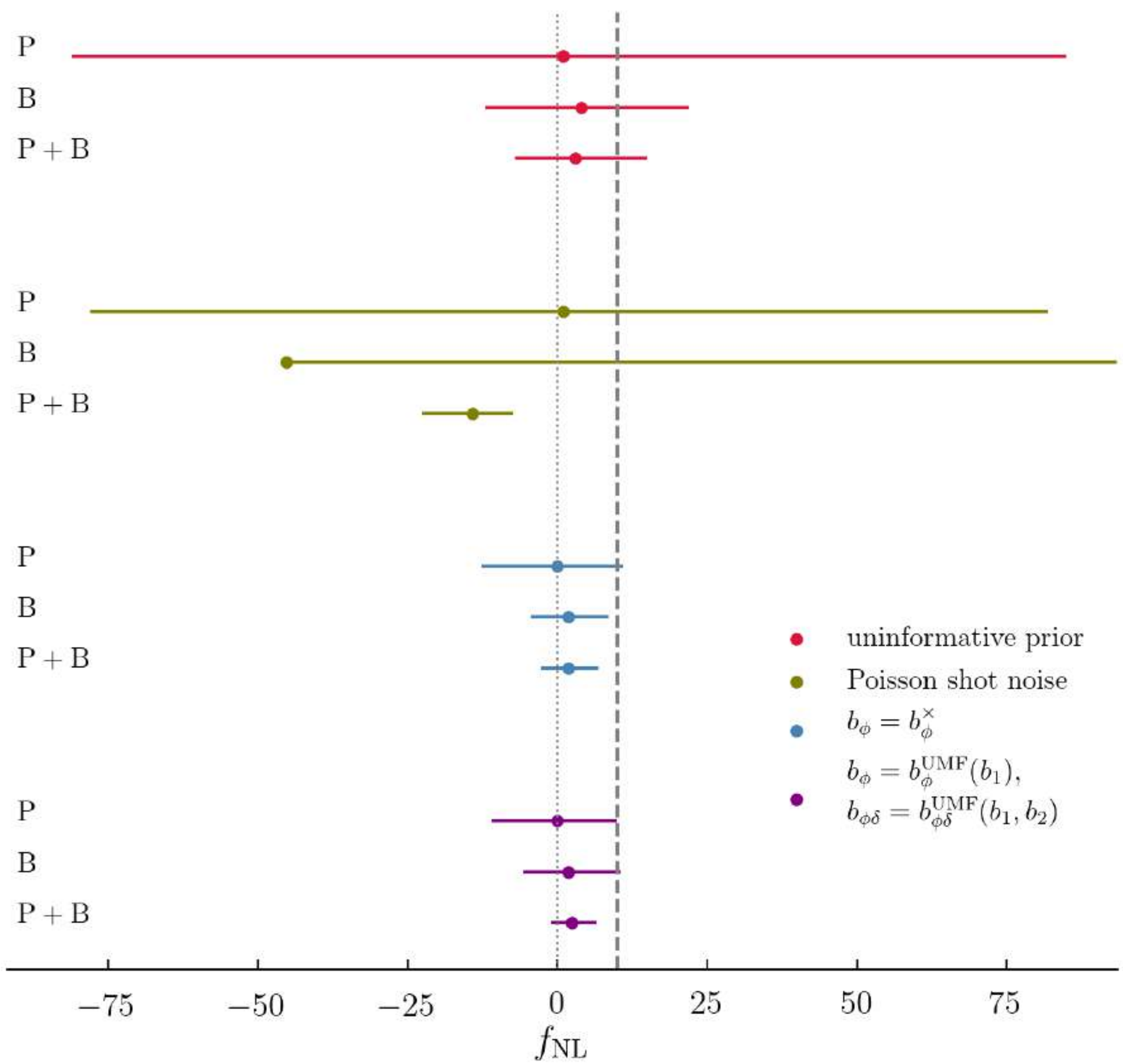
Forecasts

Karagiannis et. al., 2018

Doré et. al., 2014

	"Euclid-like"	"LSST inspired"	SPHEREx
P	~ 6	~ 1	~ 1
B	~ 6	~ 0.5	~ 0.2
$P+B$	~ 5	~ 0.5	
	Single tracer	Single tracer	Multi tracer

Can we improve on CMB?



Moradinezhad Dizgah, Biagetti, et. al., 2020

No coupling with potential

A homogeneus gravitational potential has no physical meaning

Peloso, Pietroni, 2013 Kehagias, Riotto, 2013 Creminelli, JN, Simonović, Vernizzi, 2014

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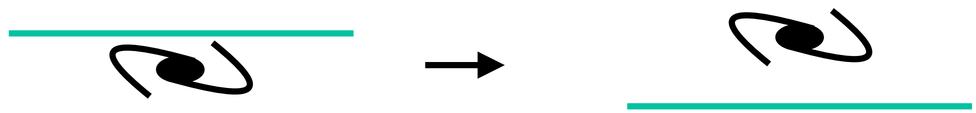
$$\Phi \to 0$$



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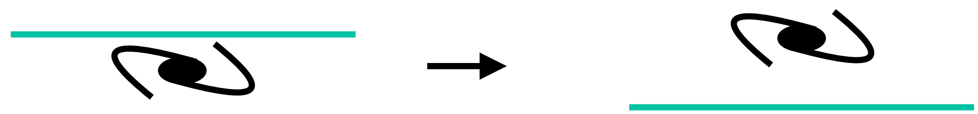
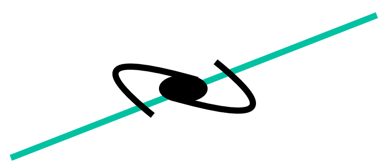


A homogeneus gravitational force can be set to zero by going to a freely falling frame



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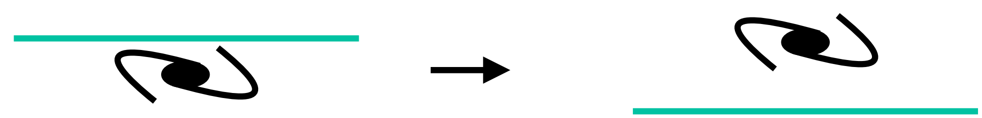

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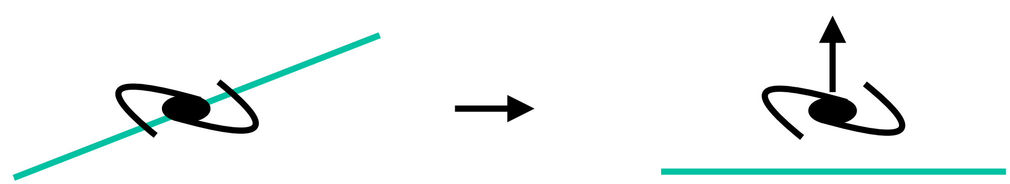
$$\nabla\Phi \to 0$$

$$\vec{V} \to \vec{V} - t\nabla\Phi$$



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Exploiting the consistency relation

Express the bispectrum as a series in the soft mode

$$B(q, k, \theta) = \sum_n a_n(k, \theta) q^n P(q)$$

Esposito, Hui, Scocimarro, 2019

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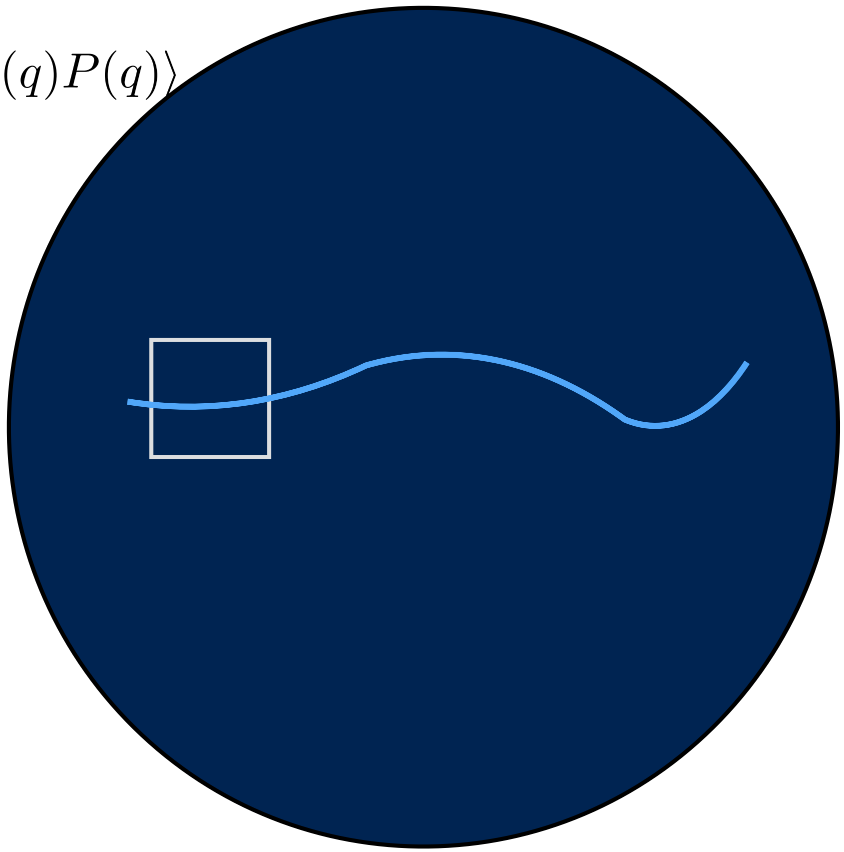
Non-perturbative scales.

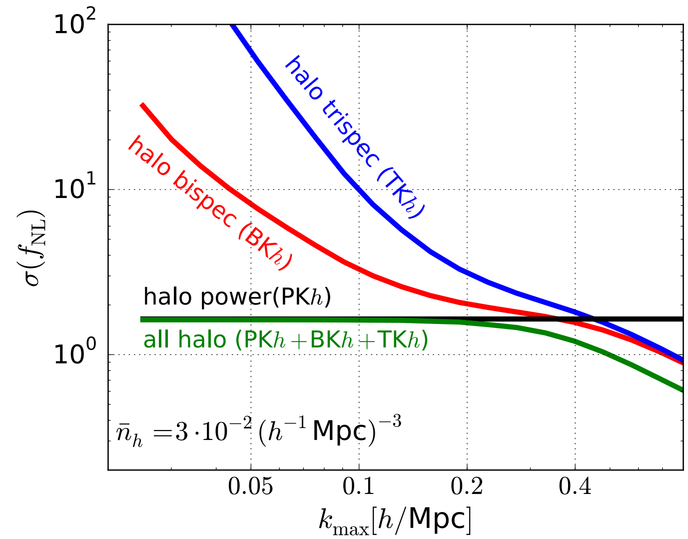
Few points → Simpler covariance

Esposito, Hui, Scocimarro, 2019

Position dependent PS

$\langle \delta(q) P(q) \rangle$





Chiang, Wagner, Schmidt, Komatsu, 2015

De Puttter, 2018

Other techniques

Skew-spectra

Moradinezhad, et. al., 2019

Dai, Verde, Xia, 2020

Topological…

Biagetti, Cole, Shiu, 2020

Reconstruction

Shirasaki, et. al., 2020

However: Projection effects

We observe the number density of galaxies in a direction $\hat{n}$ and a redshift z

$$n_g(z,\hat{n}) = \bar{n}_g(z)(1+\Delta_g(z,\hat{n})) \qquad \bar{n}_g(z) = \frac{\bar{N}_g(z)}{V}$$

For example, the frequency of the photon is sensitive to Φ

$$\delta z \supset (1+z)(\Phi_e - \Phi_o)$$

The separation between photon propagation and "dynamics" is gauge-dependent.

Yoo, Fitzpatrick, Zaldarriaga, 2009, Durrer, Bonvin, 2011, Jeong, Schmidt, Hirata, 2011

Relativistic power spectrum

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→ The distortion in redshift affects our measurement of the average number of galaxies at a given z. Parametrised by

$$e = \frac{d \log \bar{N}_g}{d \log z}$$

Evolution bias.

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Evolution bias.

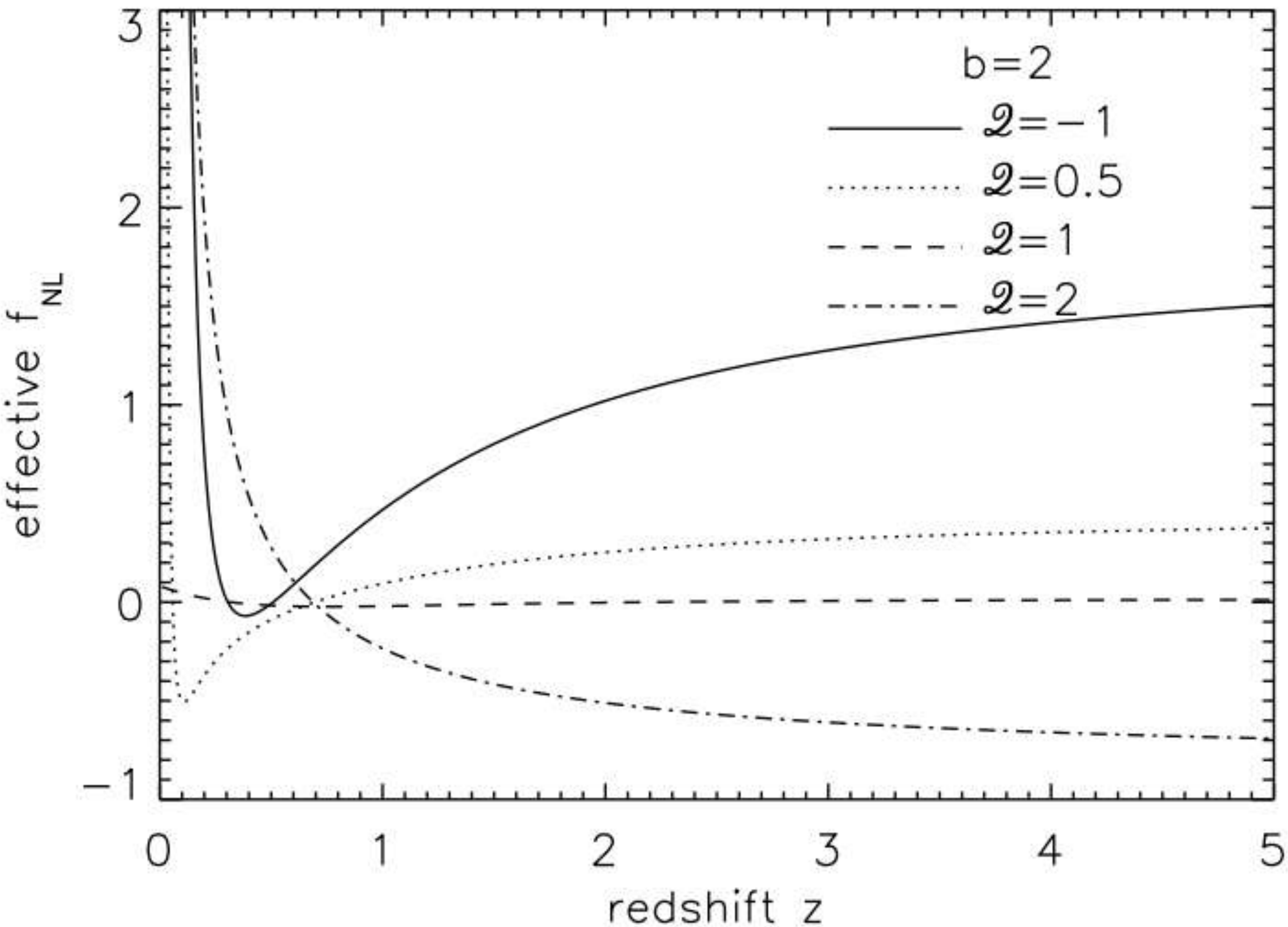
→ The coordinate volume is different from the physical volume.

→ Lensing magnification makes galaxies appear fainter or brighter. At the threshold of observation, some may appear or disappear.

Magnification bias t

Yoo, Fitzpatrick, Zaldarriaga, 2009, Durrer, Bonvin, 2011, Jeong, Schmidt, Hirata, 2011

Relativistic power spectrum



Jeong, Schmidt, Hirata, 2011

Relativistic bispectrum

Standard steps to compute the bispectrum

- Expand the metric and stress tensor in perturbations

$$ds^2 = -(1+2\Phi)dt^2 + 2\omega_i dx^i dt + a^2((1+2\Psi)\delta_{ij} + \gamma_{ij})dx^i dx^j \qquad \rho = \bar{\rho}(1+\delta)$$

Bartolo, Matarrese, Verde, …, … 2020
Jolicoeur, Umeh, Maartens, Clarkson, 2014 … 2020,
Di Dio, Durrer, Marozzi, Montanari, 2014, 2015
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- Fix a gauge:

Poisson:	Synchronous-comoving:
$\partial_i \omega_i = 0$	$u^0 = \frac{dx^0}{d\tau} = 1$
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- Take all fields to be small, expand Einstein's and fluid equations to second order, and solve.

$$\delta \sim \Phi \sim v \ll 1$$

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- Solve for the photon geodesic at second order to get lensing and redshift space distortions.

Bartolo, Matarrese, Verde, …, … 2020
Jolicoeur, Umeh, Maartens, Clarkson, 2014 … 2020,
Di Dio, Durrer, Marozzi, Montanari, 2014, 2015
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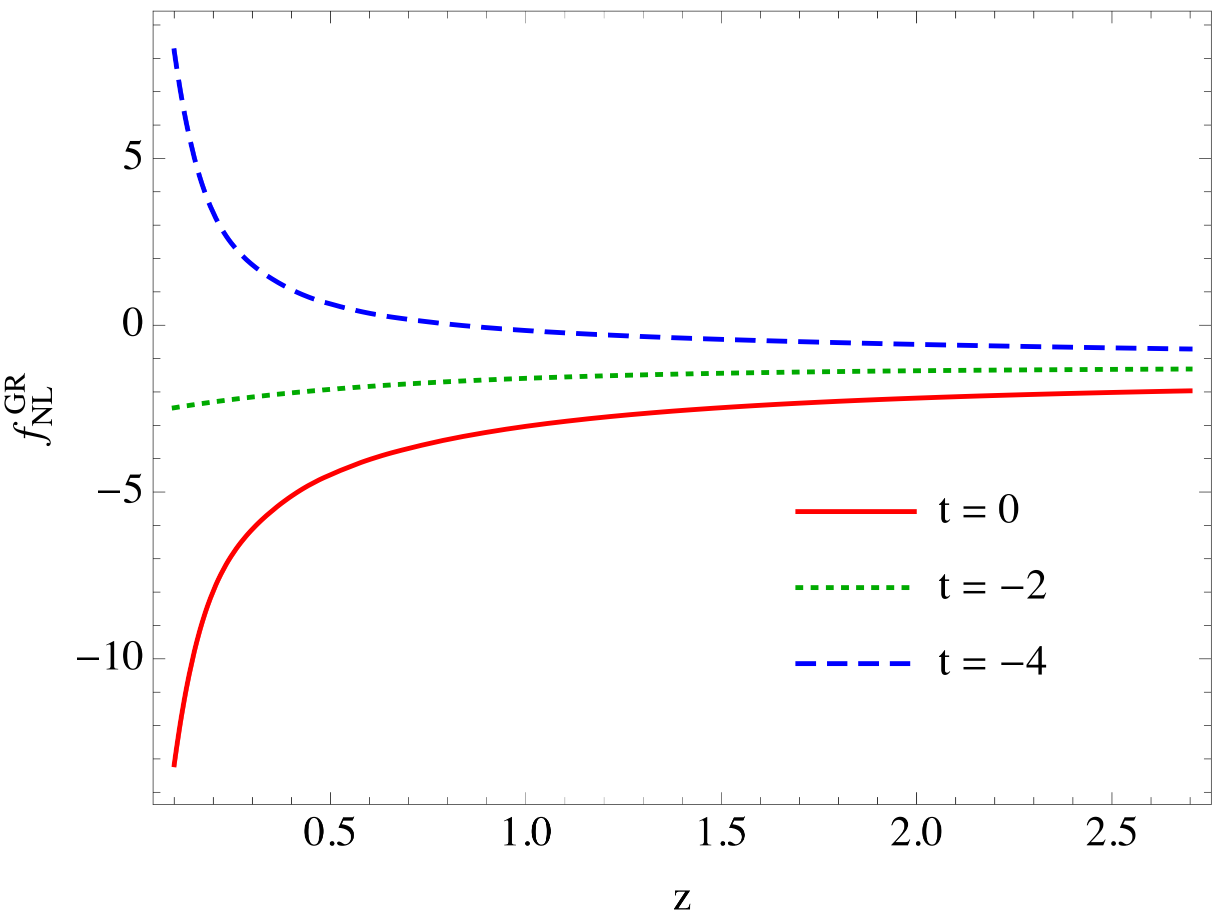
The very squeezed limit

Sum of all terms going like $\langle \Phi \delta \delta \rangle$



A. Kehagias, A. Moradinezhad, JN, H. Perrier, A. Riotto, 2015

Castiblanco, Gannouji, JN, Stahl, 2018

Weak field approximation

The 1-loop bispectrum requires 4th order perturbation theory…

Seems impossible in GR, but… even on small scales:

$$\Phi \sim \mathcal{O}(10^{-5}) \qquad \vec{v} \sim \mathcal{O}(10^{-3})$$

Density, however, becomes non-linear $\delta = \frac{2k^2}{3a^2H^2}\Phi$

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For simplicity take the universe to be Einstein de Sitter.

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Castiblanco, Gannouji, JN, Stahl, 2018
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Differences with Newtonian case

Newtonian case

Relativistic corrections

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It is correct to use linear initial conditions (non-linearities grow).

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The background is fixed

$$\langle \delta(\vec{k}) \rangle = (2\pi)^3 \delta_D(\vec{k}) \int_{\vec{q}} F_2(-\vec{q}, \vec{q}) P(q)$$

$$\lim_{\vec{q}_1 \to -\vec{q}_2} F_2(\vec{q}_1, \vec{q}_2) \propto (\vec{q}_1 + \vec{q}_2)^2$$

$$\implies \langle \delta \rangle = 0$$

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Now $F_2^R \sim a$, so initial conditions must be fixed at 2nd order.

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Reabsorb $\langle \delta \rangle$ in $\bar{\rho}$.

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Reabsorb $\langle\delta\rangle$ in $\bar{\rho}$.

Reabsorb $\langle\Phi\rangle$ with $\bar{p}$.

Conclusions

- Non-Gaussianity is a way to detect the fields active during the very early universe, especially in the soft limit.
- The CMB has already taught us a lot. But we are still far from the natural values expected from theory.
- The LSS has the potential to improve on CMB observations by an order of magnitude.
- GR effects are important if you hope to achieve

$$\Delta f_{\rm NL} \sim \mathcal{O}(1)$$

THE
END