The PDF of cosmic density fluctuations - a way beyond 2-point statistics in analyses of the large scale structure

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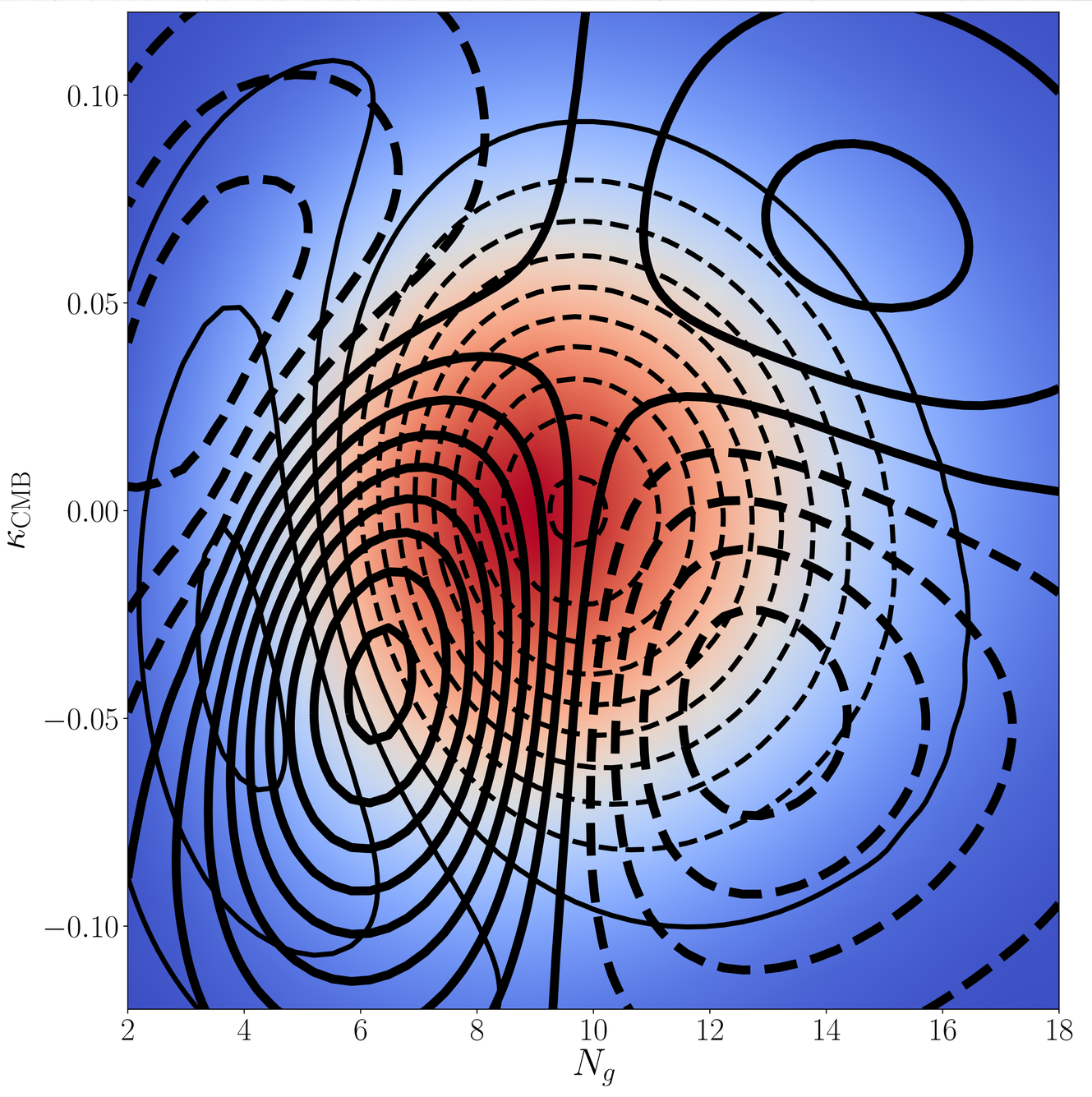
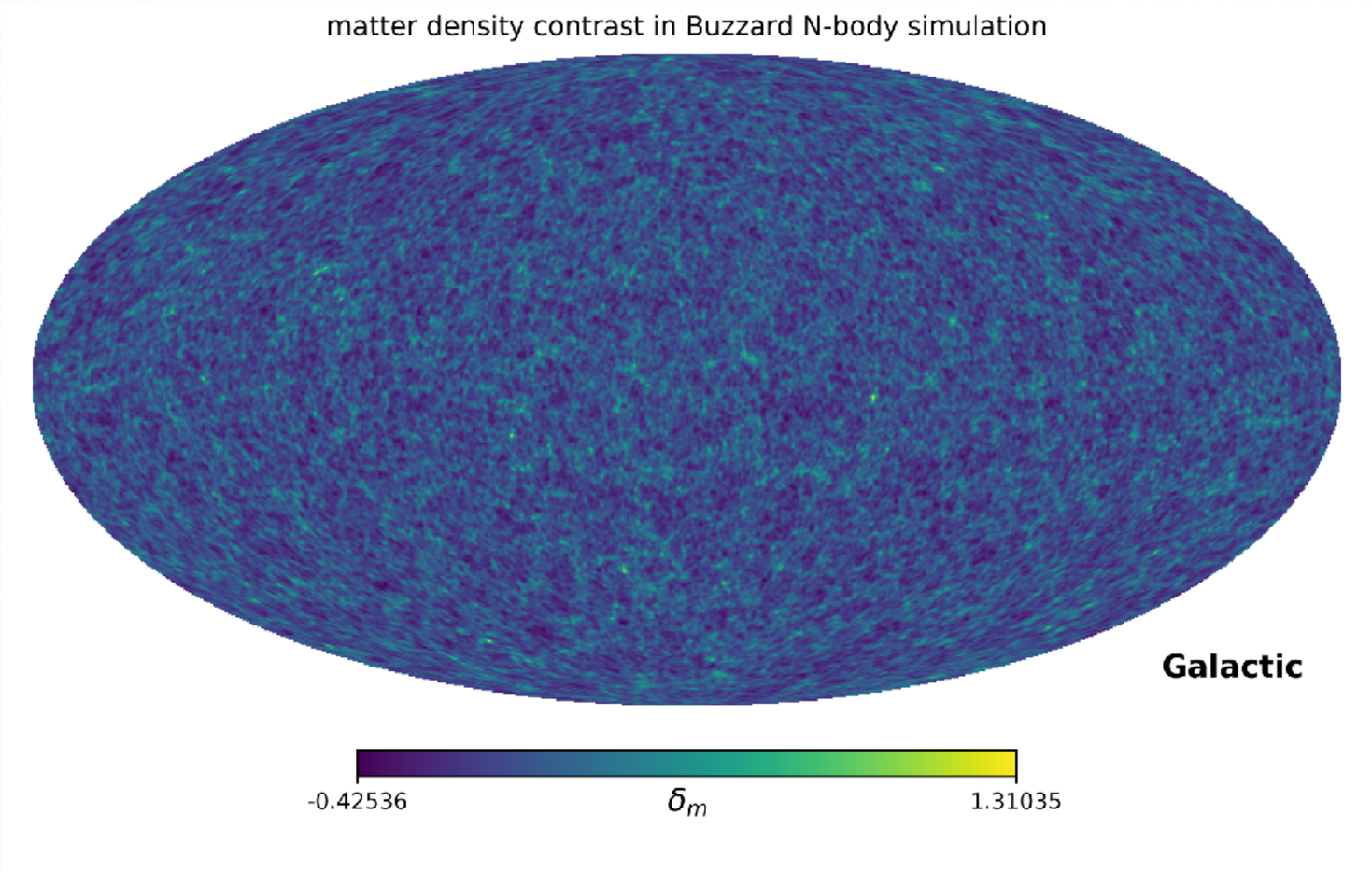
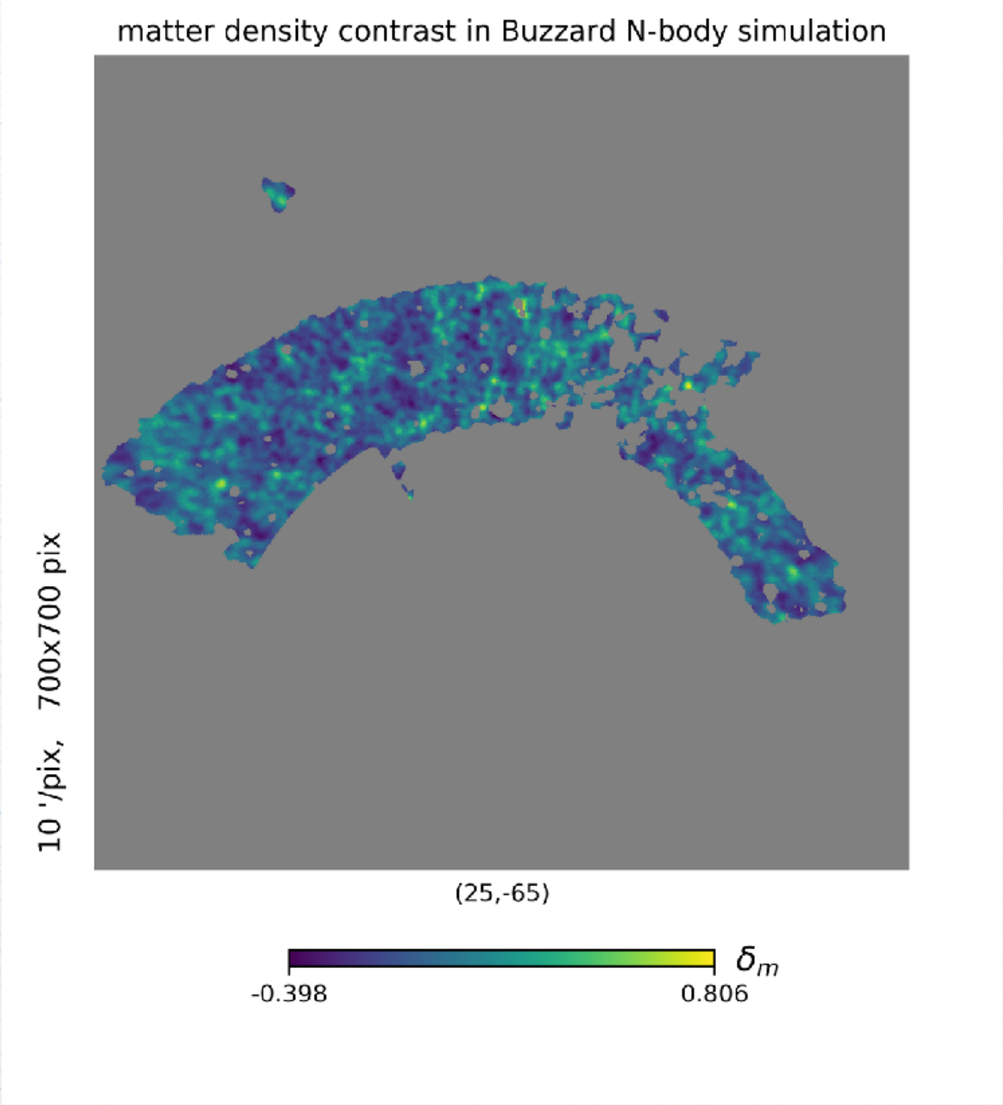
KIC Cambridge

work with Anik Halder, Daniel Gruen, Cora Uhlemann, Joe DeRose, Elisabeth Krause, Tobias Baldauf, Francisco Villaescusa-Navarro, Marc Manera, Takahiro Nishimichi and more!



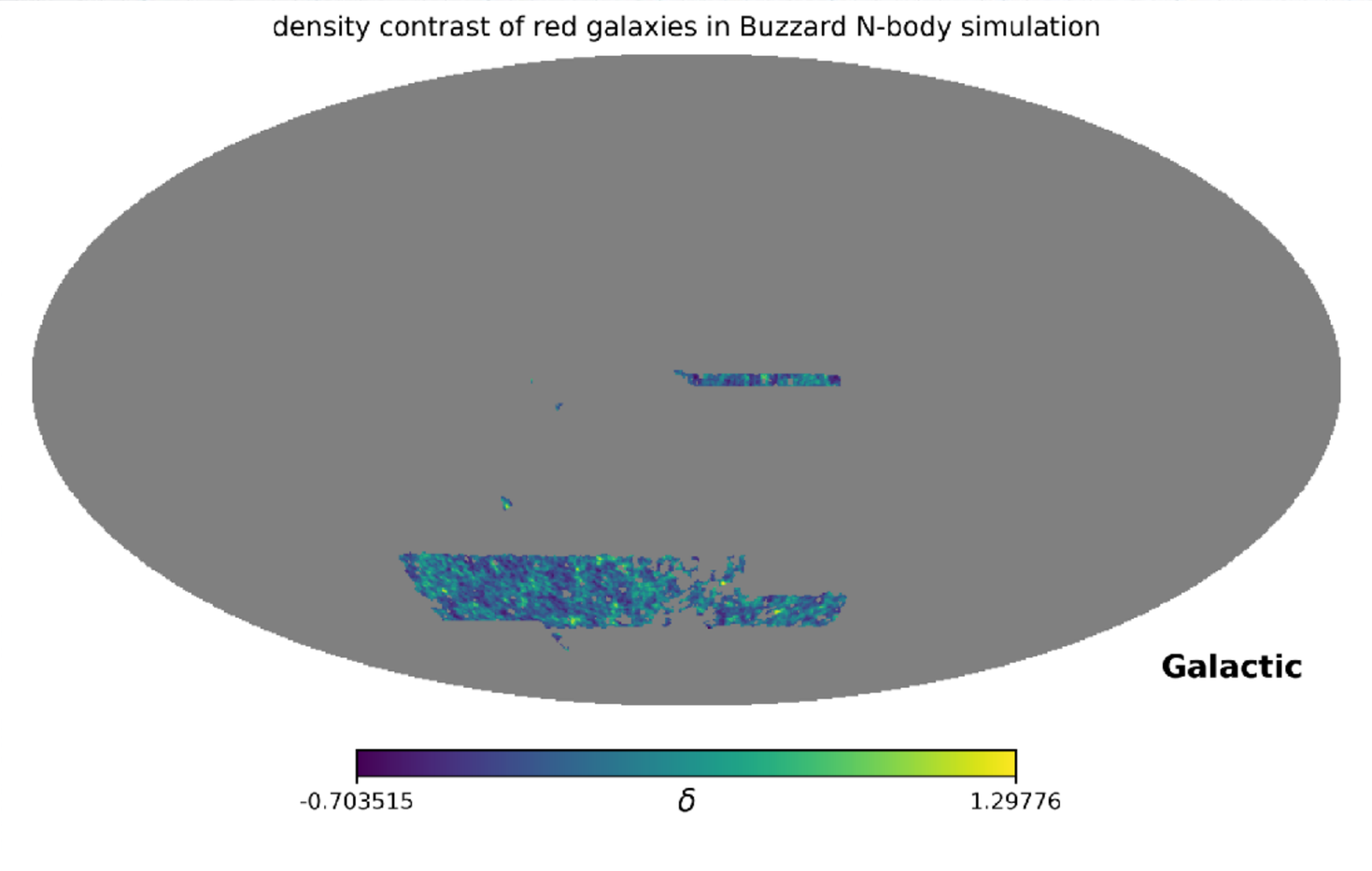
Matter density contrast in Buzzard N-body simulation:

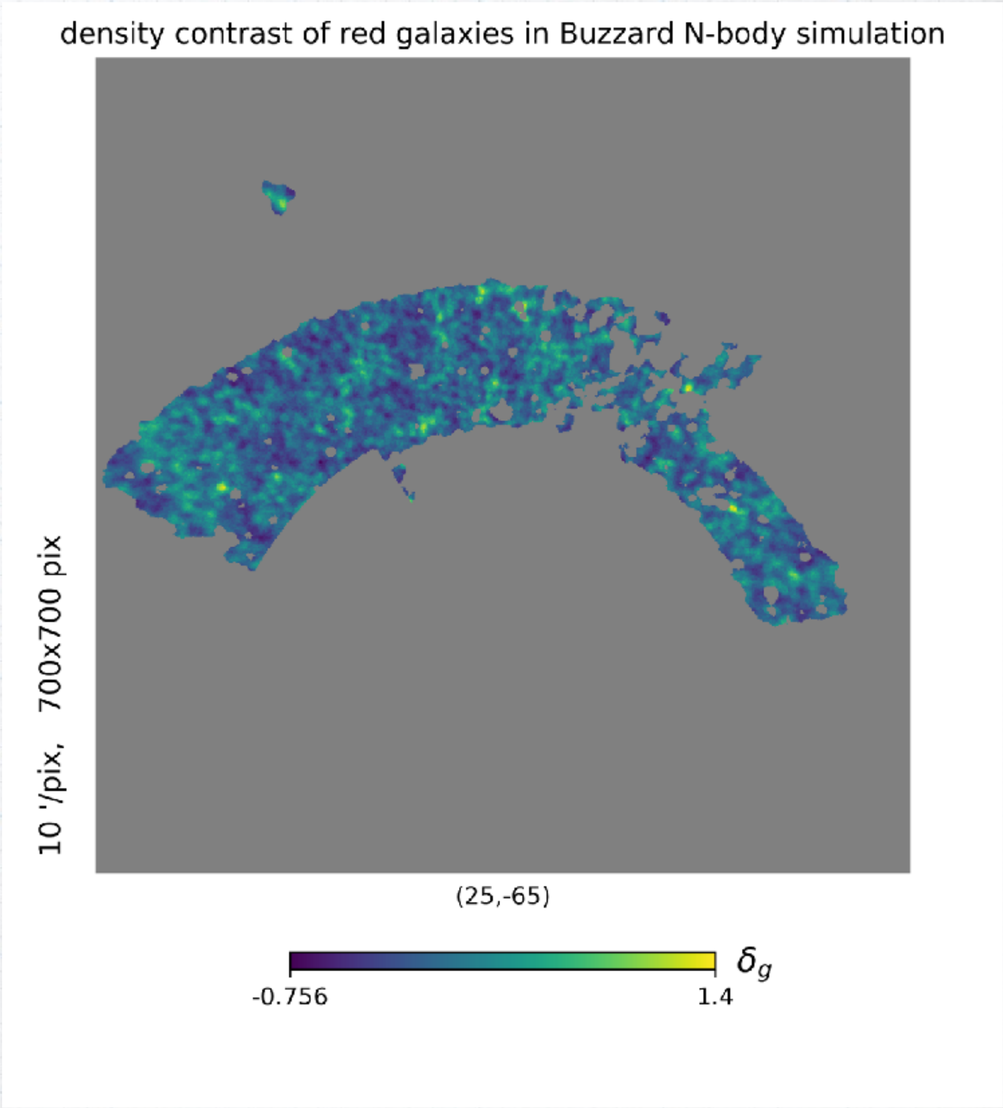




Y1 Buzzard Flock, DeRose++ (2019)

Galaxy density contrast in Buzzard N-body simulation:





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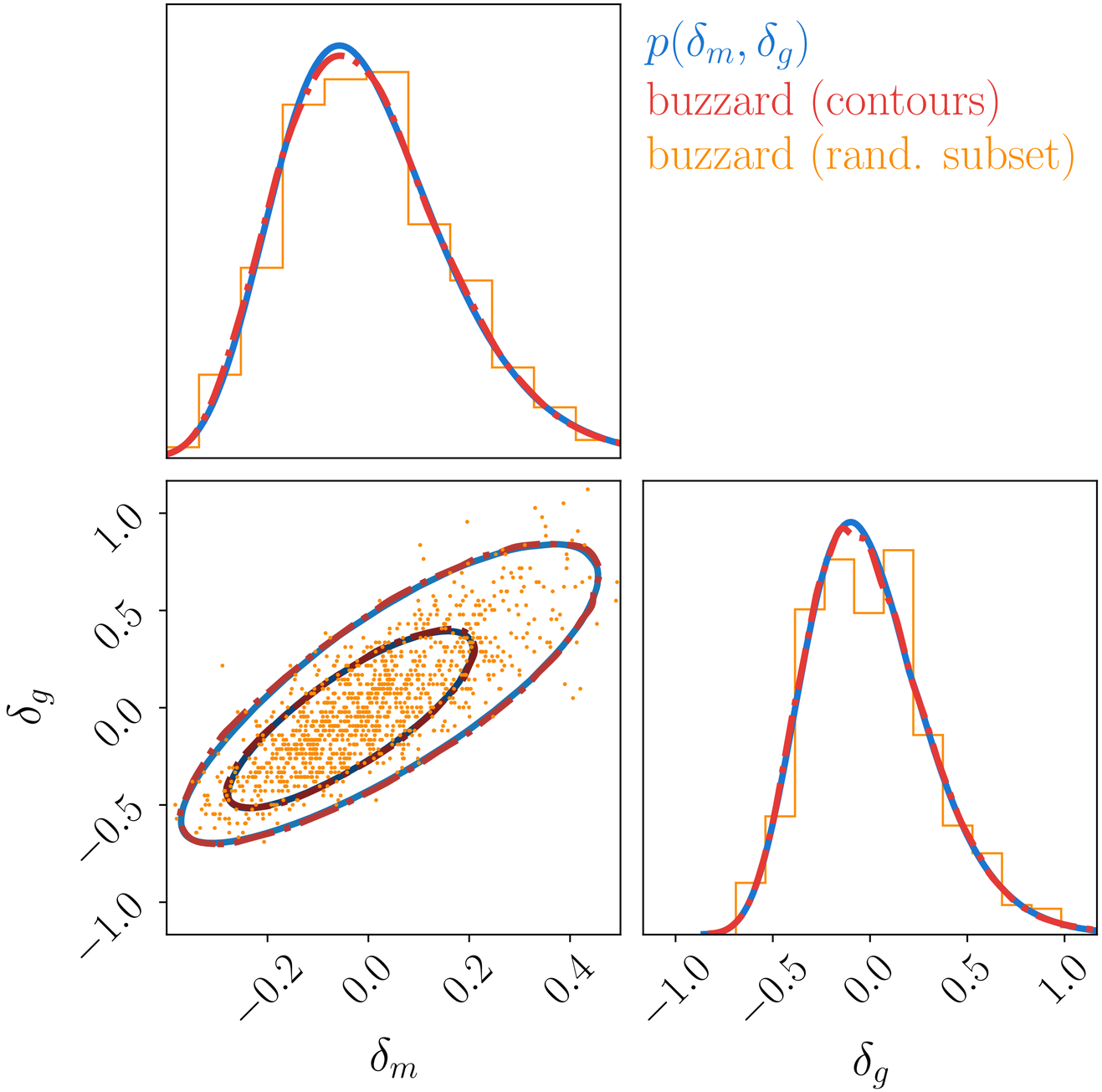


galaxies are biased & stochastic tracers of matter density

2-point statistics only extracts three numbers from this!

→ way to do better: analyse full shape of the PDF

Blue: model predictions calculated in **Friedrich++(2018)**
Red: data from Buzzard N-body sims (DeRose, Wechsler++2019)

$$\delta_m = (\rho_m - \bar{\rho}_m)/\bar{\rho}_m$$
$$\delta_g = (\rho_g - \bar{\rho}_g)/\bar{\rho}_g$$

PDF / cumulants

- Cumulant generating function (CGF):

$$\varphi_R(\lambda) = \sum_n \langle \delta_R^n \rangle_c \frac{\lambda^n}{n!}$$

$\delta_R =$ density contrast smoothed by radius R

$\langle \delta_R^n \rangle_c =$ Cumulant (part of the nth moment that vanishes for Gaussian field)

2-point statistics

- Power spectrum:

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{q}} \rangle \sim P(k)\ \delta_D(\mathbf{k} + \mathbf{q})$$

PDF / cumulants

- Actual observable:

  PDF $p(\delta_R)$

  related to CGF via Laplace transform

$$e^{\varphi_R(\lambda)} = \int \mathrm{d}\delta_R \; p(\delta_R) \; e^{\lambda \delta_R}$$

2-point statistics

- Alternative observable:

  2-point function $\xi(r) \equiv \langle \delta(\mathbf{x}) \delta(\mathbf{x}+\mathbf{r}) \rangle$

  related to power spectrum via Fourier transform:

$$P(|\mathbf{k}|) \sim \int \mathrm{d}^3 r \; \xi(|\mathbf{r}|) \; e^{-i\mathbf{r}\mathbf{k}}$$

PDF / cumulants

- Leading order perturbation theory
  Friedrich++ 2020, arxiv.org/abs/1912.06621 :

$$\varphi_R(\lambda) \approx \text{ minimum of function}$$

$$s_\lambda(\delta_i, j) = -\lambda F(\delta_i) + j\delta_i + \varphi_{R(1+F(\delta_i))^{1/3},init}(j)$$

Spherical or cylindrical collapse of an initial fluctuation

Initial CGF at initial radius

"step 1: Legendre transform initial CGF"
"step 2: Legendre trans. back, but wrt transformed variable

See also:
Valageas (2002), Bernardeau et al. (2015)
Uhlemann et al. (2017), Friedrich et al. (2018)
Ivanov et al. (2019) … and more!

2-point statistics

- Leading order perturbation theory

$$P(k,t) = P(k,t_i)\,\frac{D^2(t)}{D^2(t_i)}$$

PDF / cumulants

- line-of-sight projections:

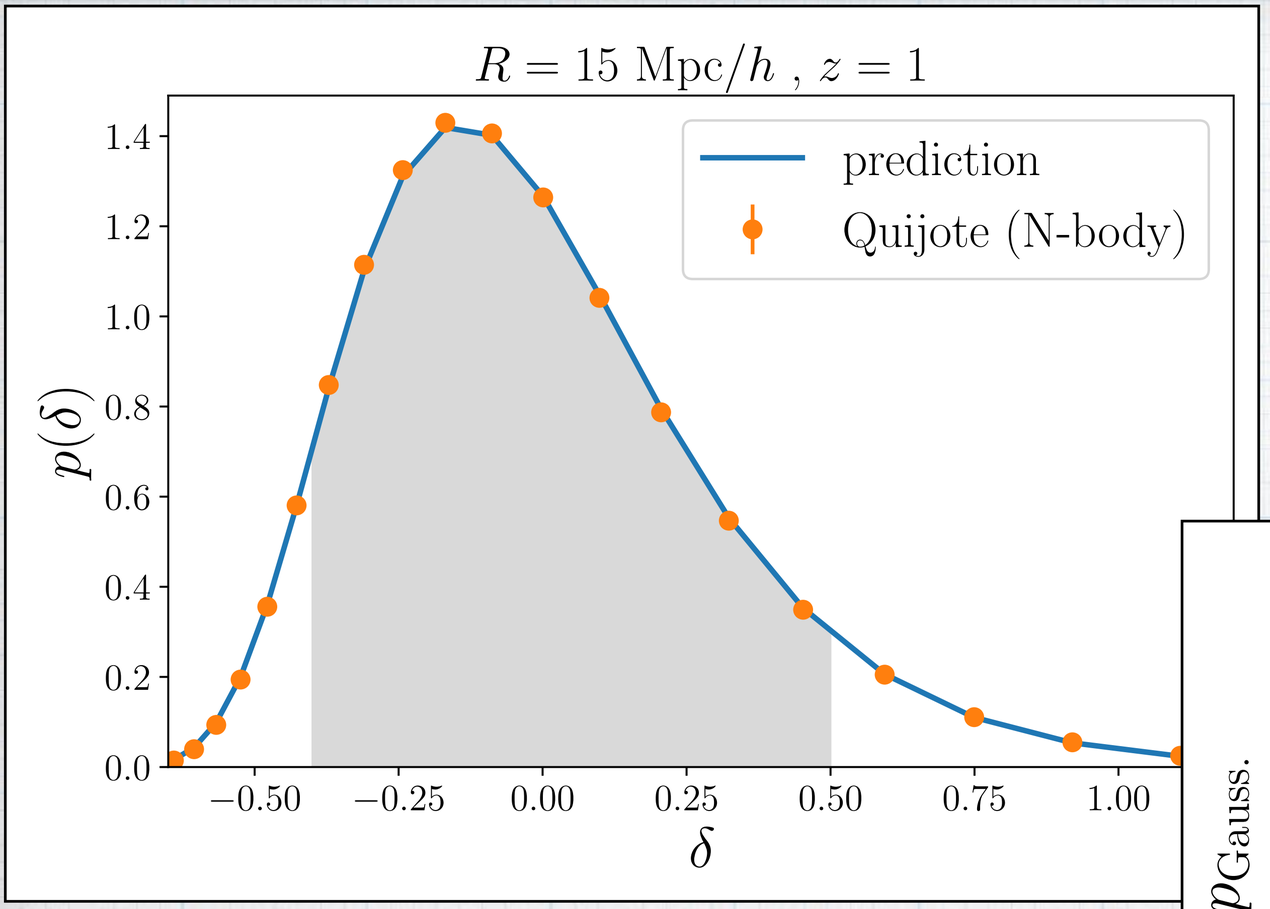
$$\varphi_{q,\theta}(\lambda) \approx \int \mathrm{d}w \lim_{L\to\infty} \frac{\varphi_{\mathrm{cy},w\theta,L}\left(q(w)L\lambda, w\right)}{L}$$

CGF of cylindrical filter

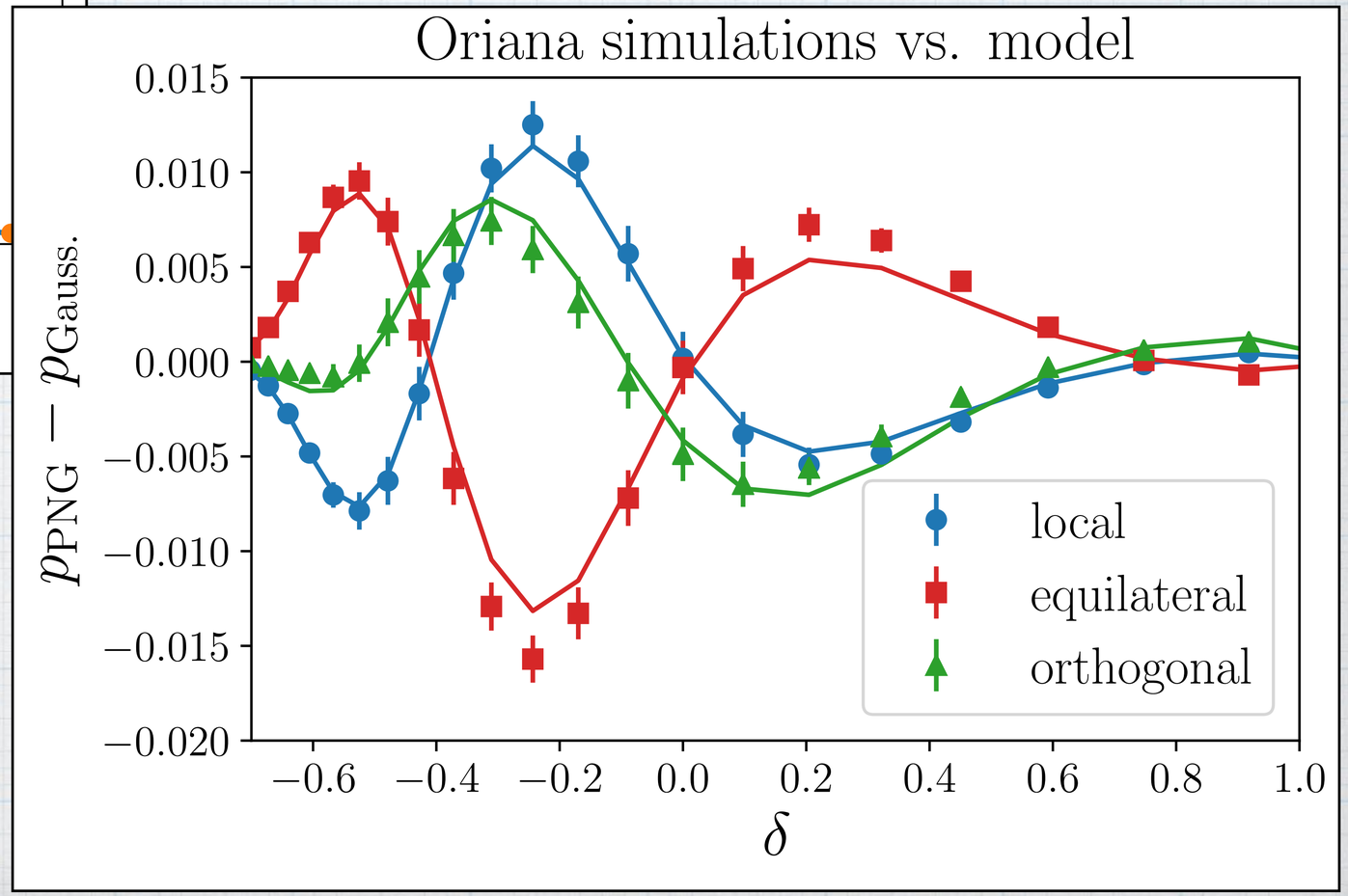
2-point statistics

- line-of-sight projections:

$$P_q(\ell) \approx \int \mathrm{d}w \left(\frac{q(w)}{w}\right)^2 P_{\mathrm{3D}}\left(\frac{\ell+\frac{1}{2}}{w}, \eta_0 - w\right)$$



Theory prediction
(Friedrich et al. 2020)
compared to the PDF measured in the Quijote N-body simulations
(Villaescusa-Navarro et al. 2020)



Prediction for impact of primordial non-Gaussianity in the PDF
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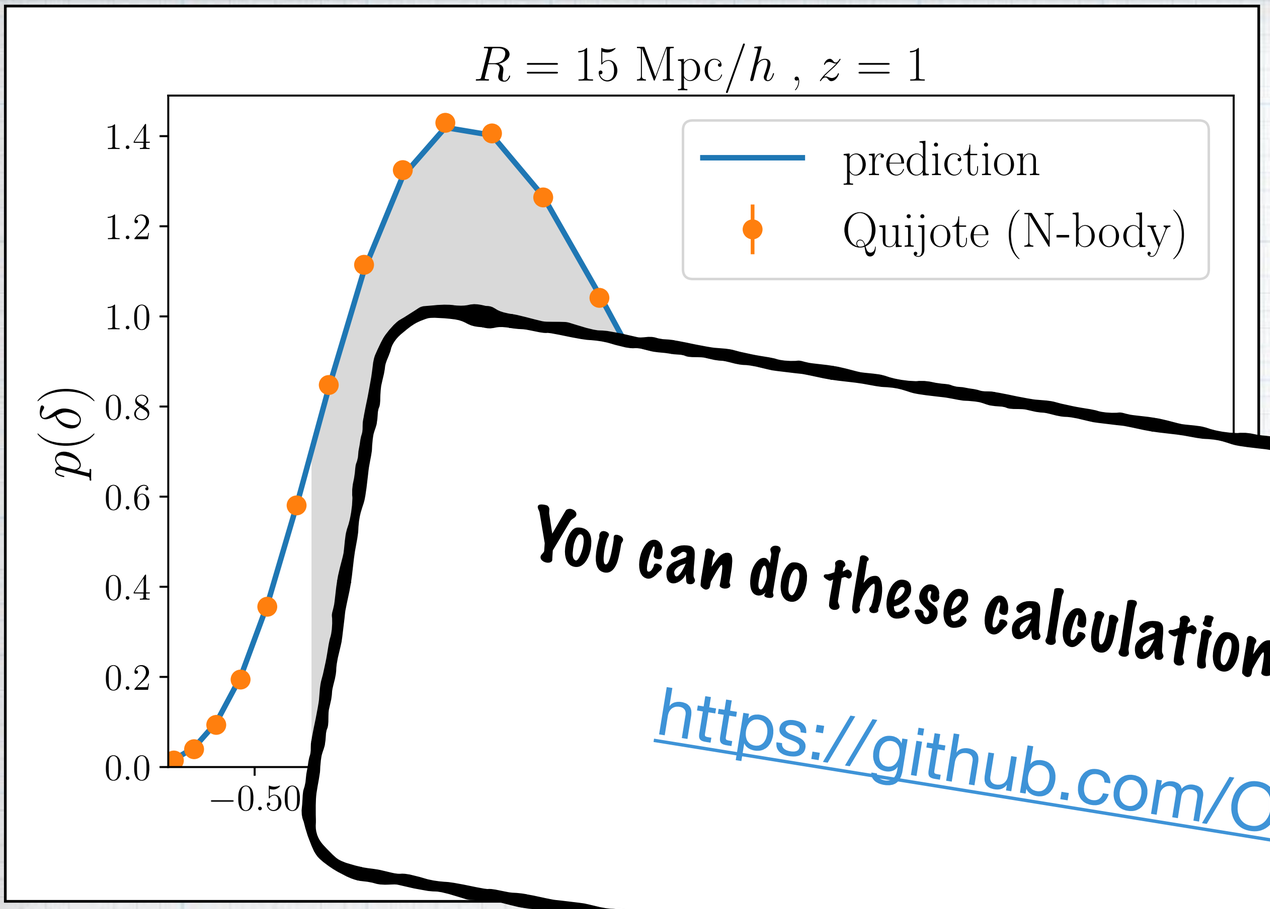
compared to Oriana N-body simulations **(Mao et al. 2014)**



Theory prediction
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You can do these calculations yourself with CosMomentum:

https://github.com/OliverFHD/CosMomentum

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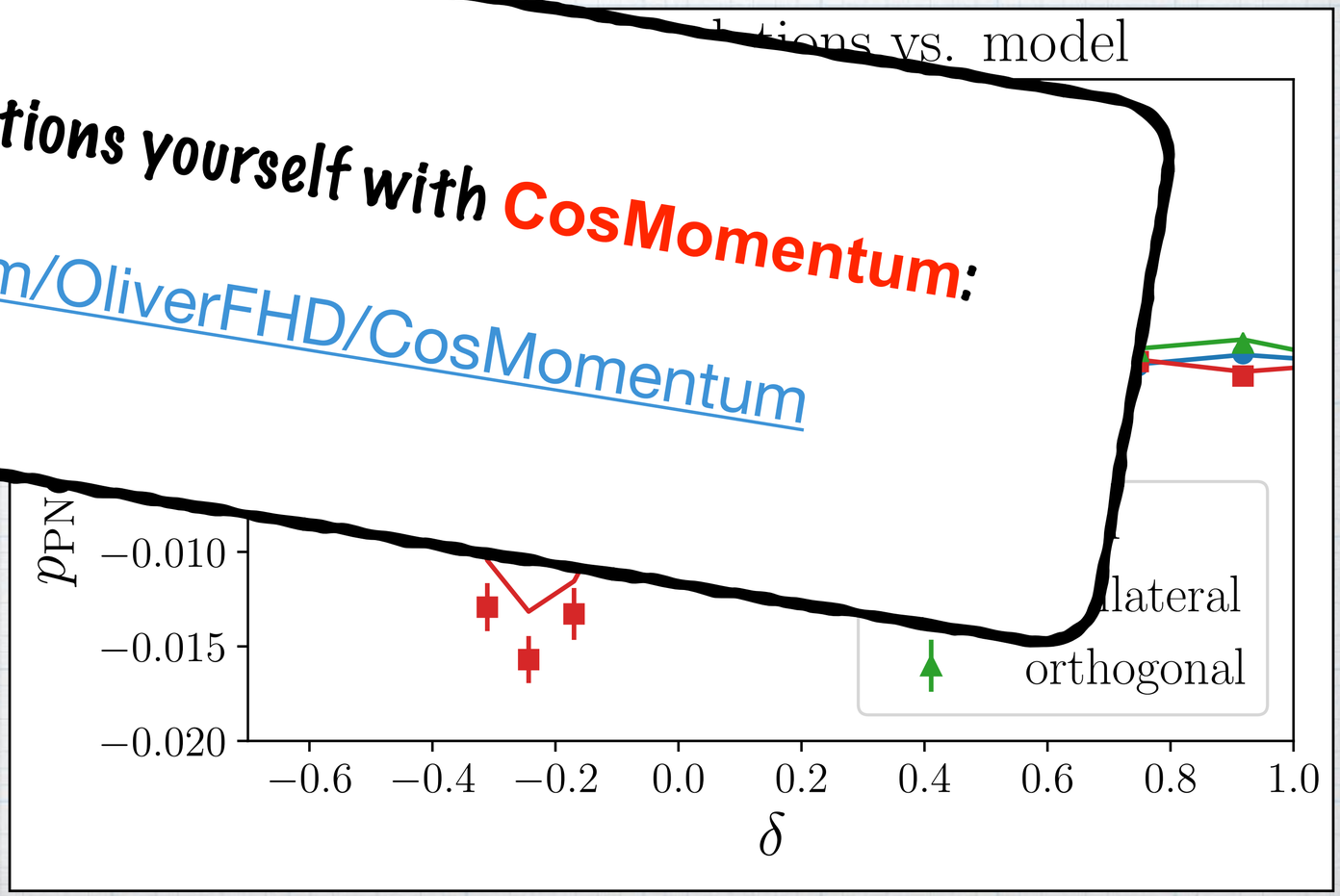

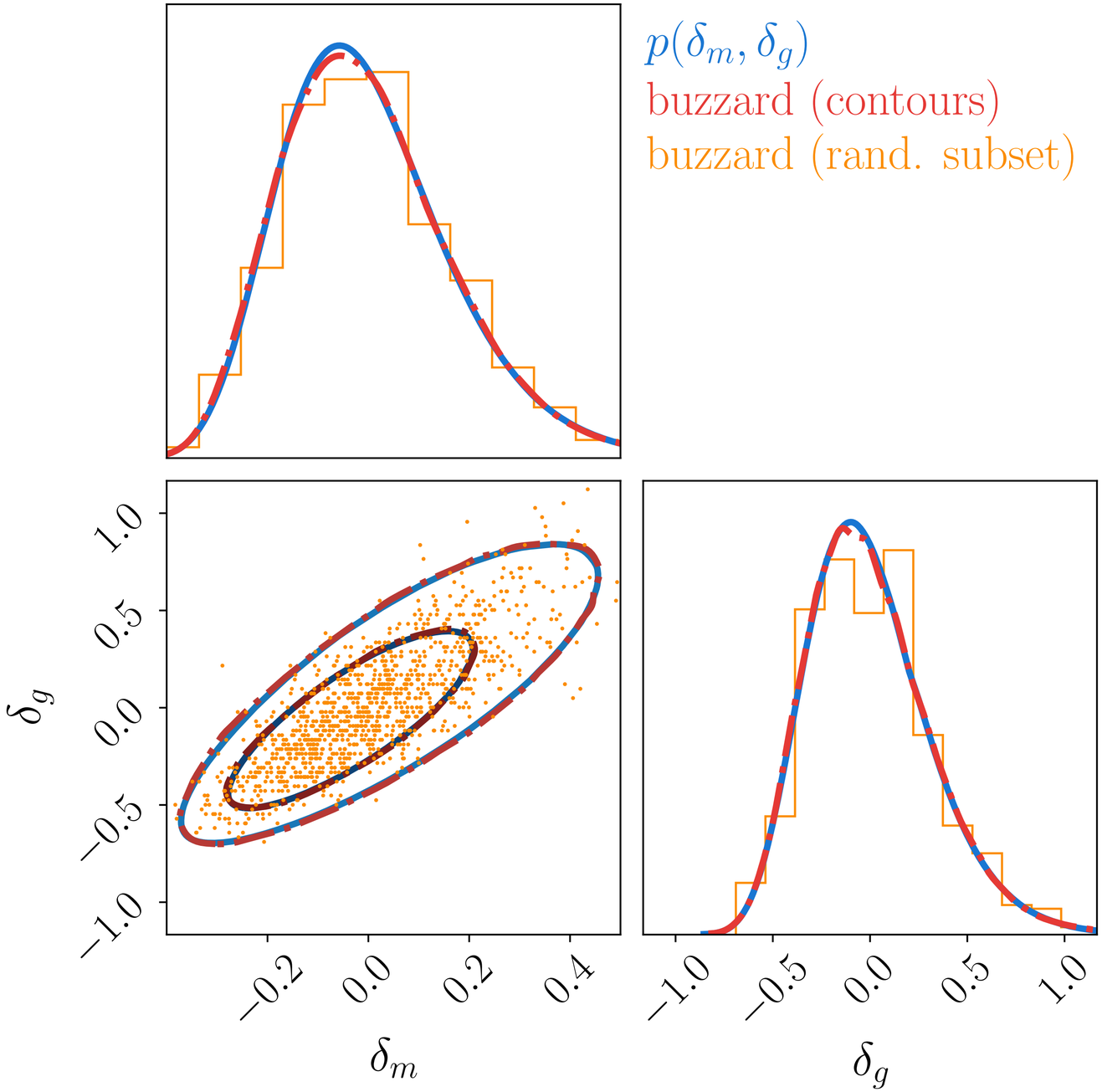

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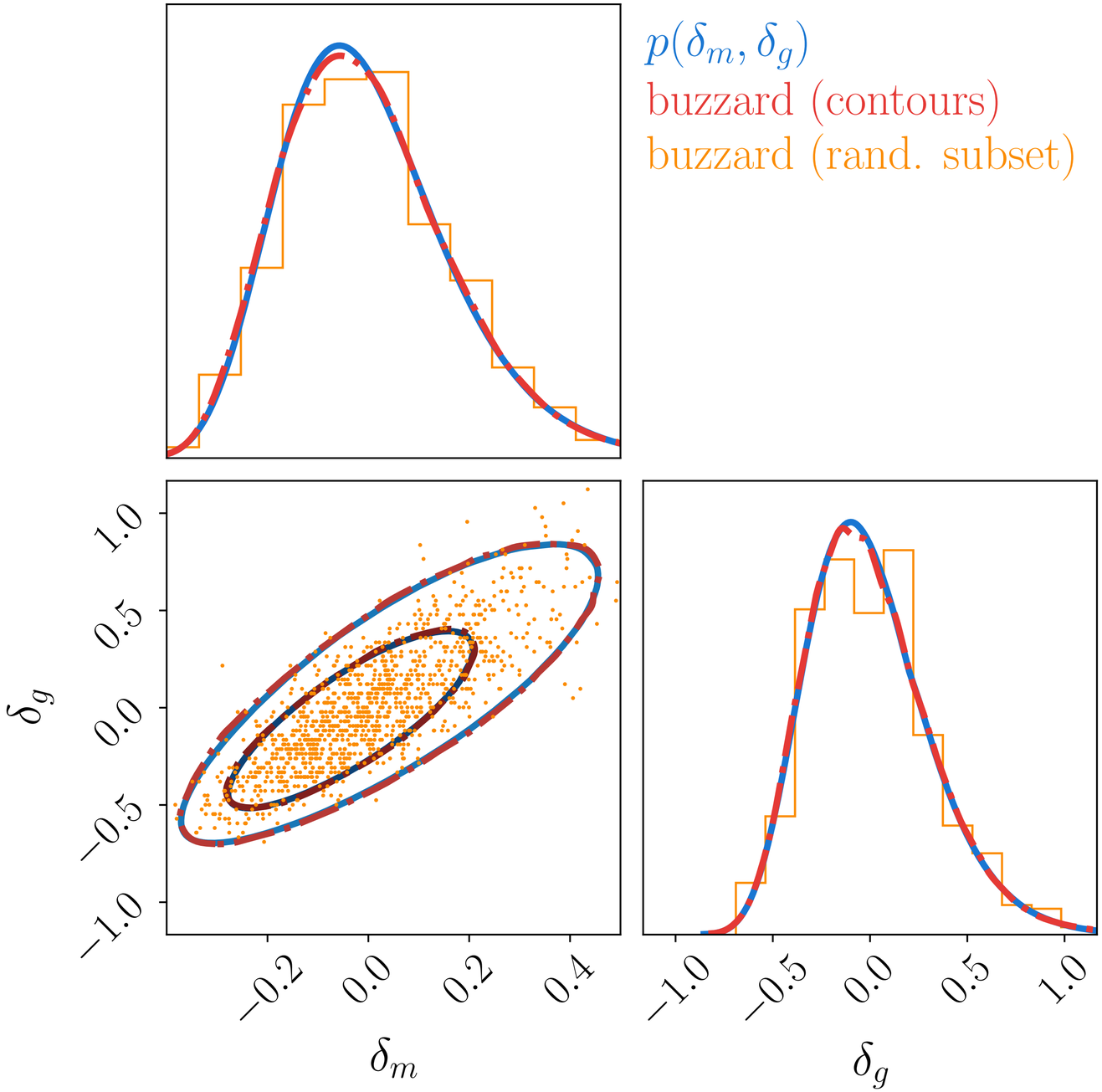

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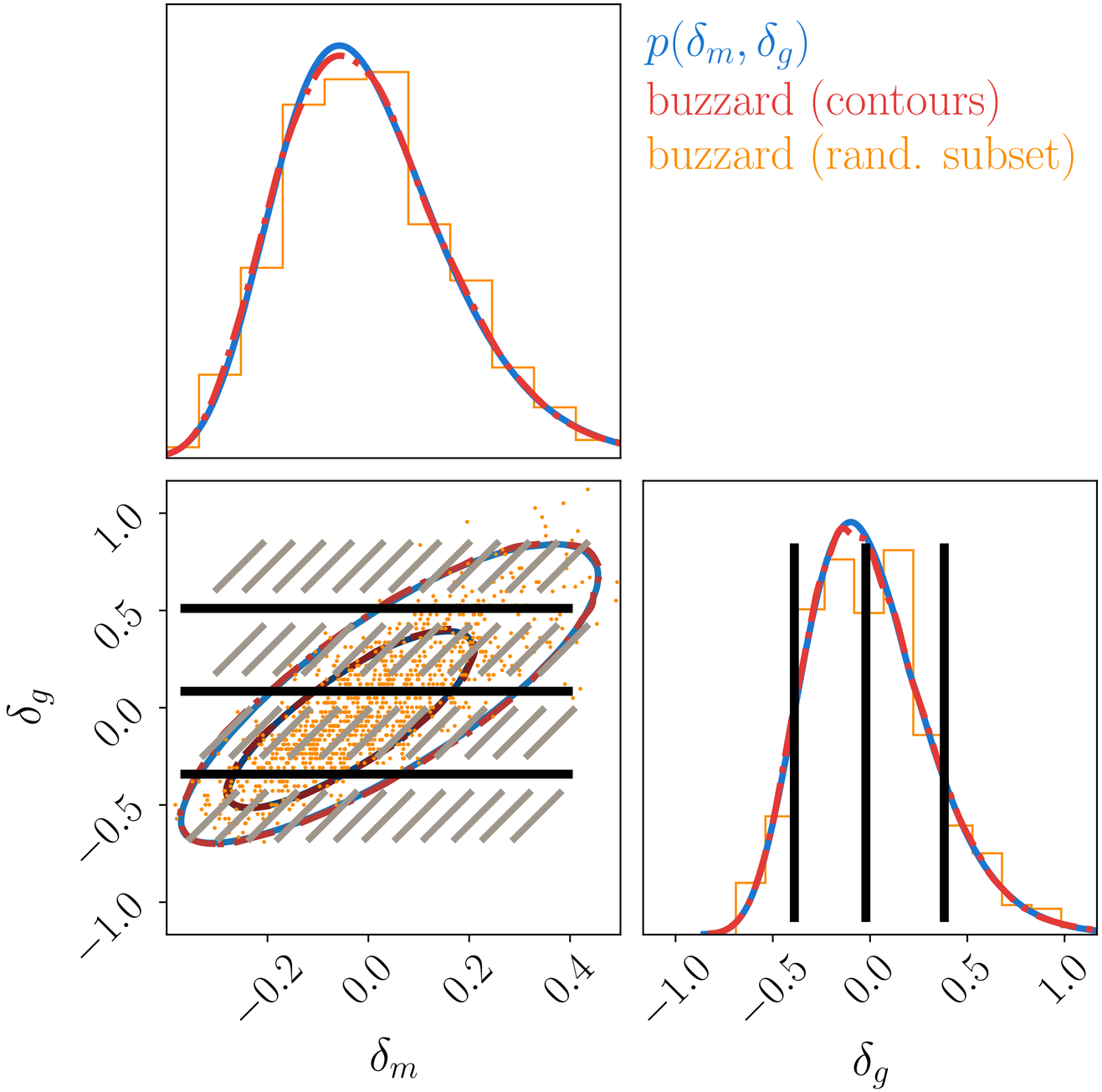
But:
does this work in real data?



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But:
does this work in real data?

We already (partly) did this with DES year-1 data!

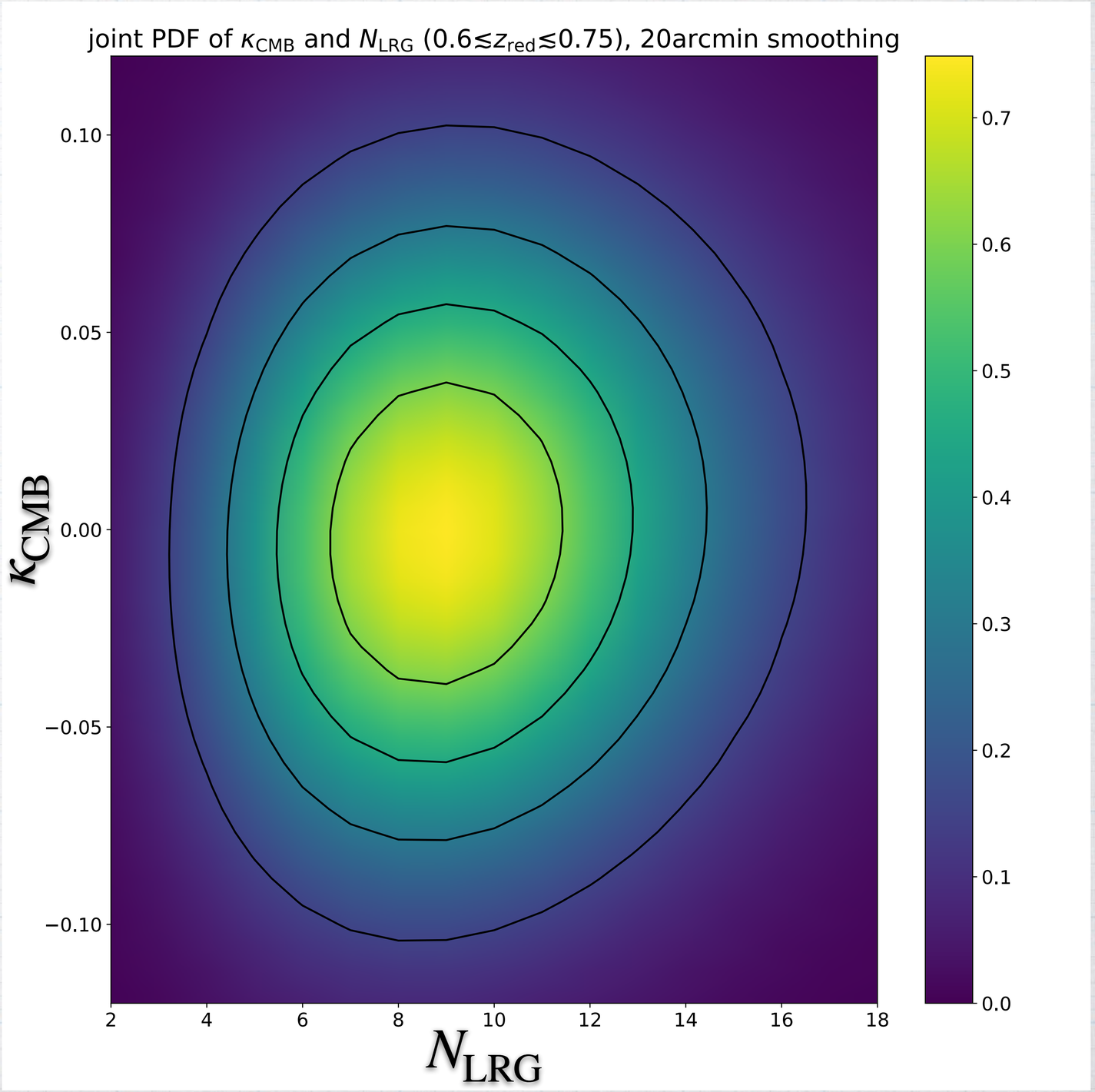
Density split statistics uses gravitational lensing to scan through galaxy density PDF “quantile-by-quantile” to connect it to underlying matter density PDF

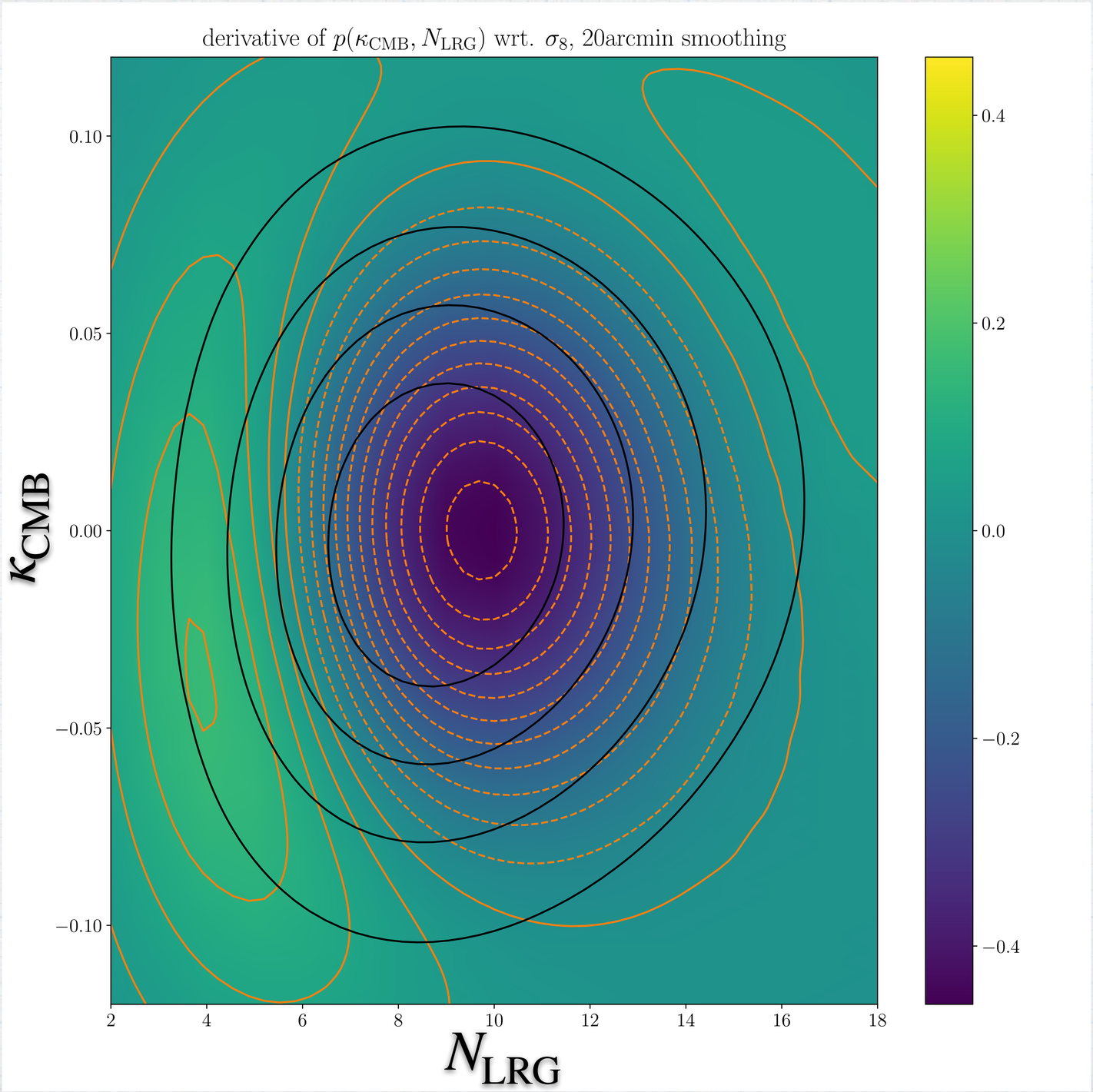
Friedrich et al. (2018)
Gruen et al. (2018)

- Where from here?
  Analyse full shape of the joint PDF!
  (**Friedrich ++ in prep**.)

- in the following assume:
  - LRGs within 0.6 < z < 0.9
  - CMB lensing with SPT-like noise
  - 20 arcmin smoothing scale
  - 5000 square degrees on the sky

- Data vector:
  - pixelizing PDF, 20x20
  - use pixels that include 90%
    of probability



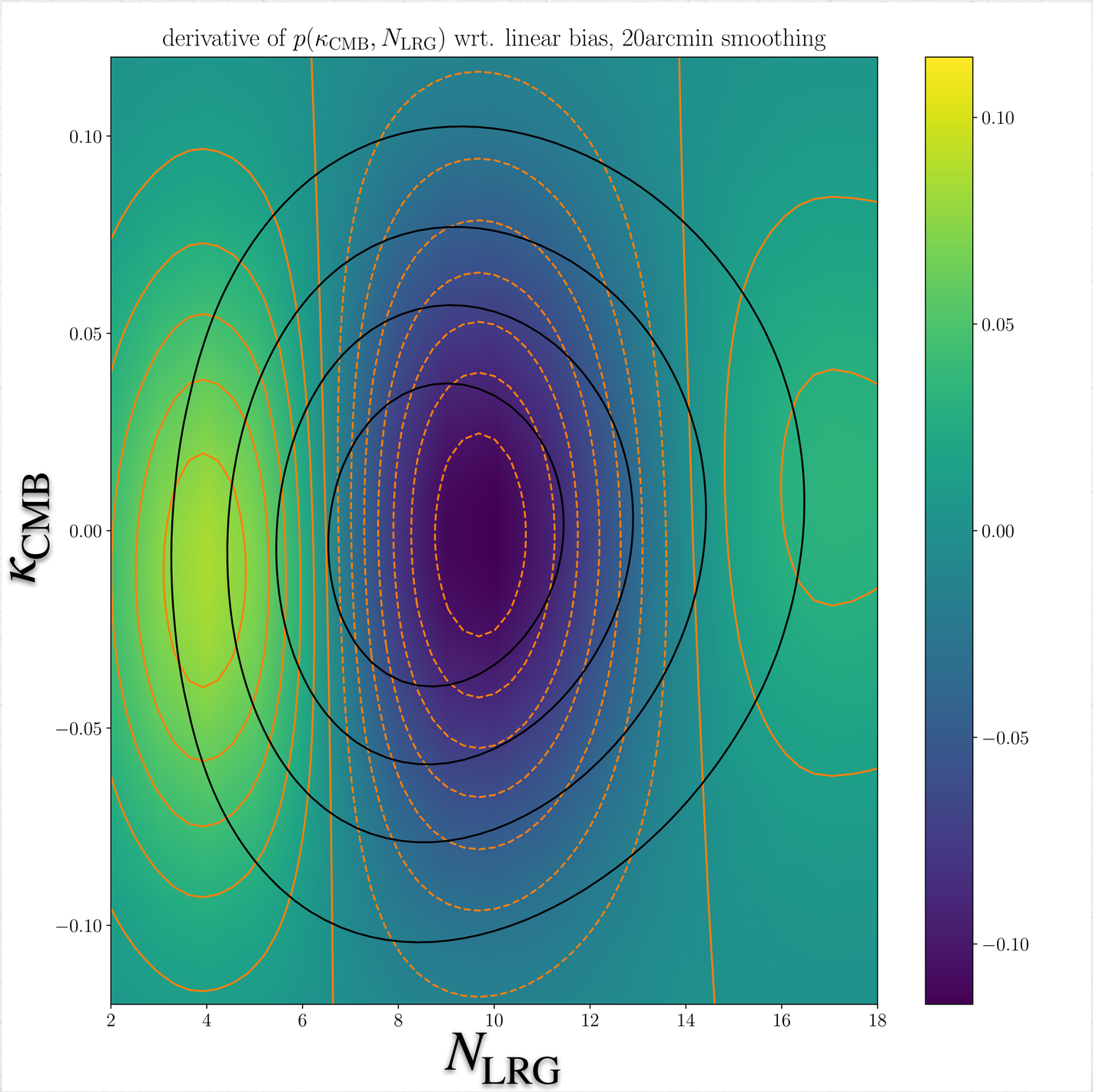


Black contours: fiducial PDF

Color map and orange contours: derivative wrt.

σ_8

(dashed = negative)

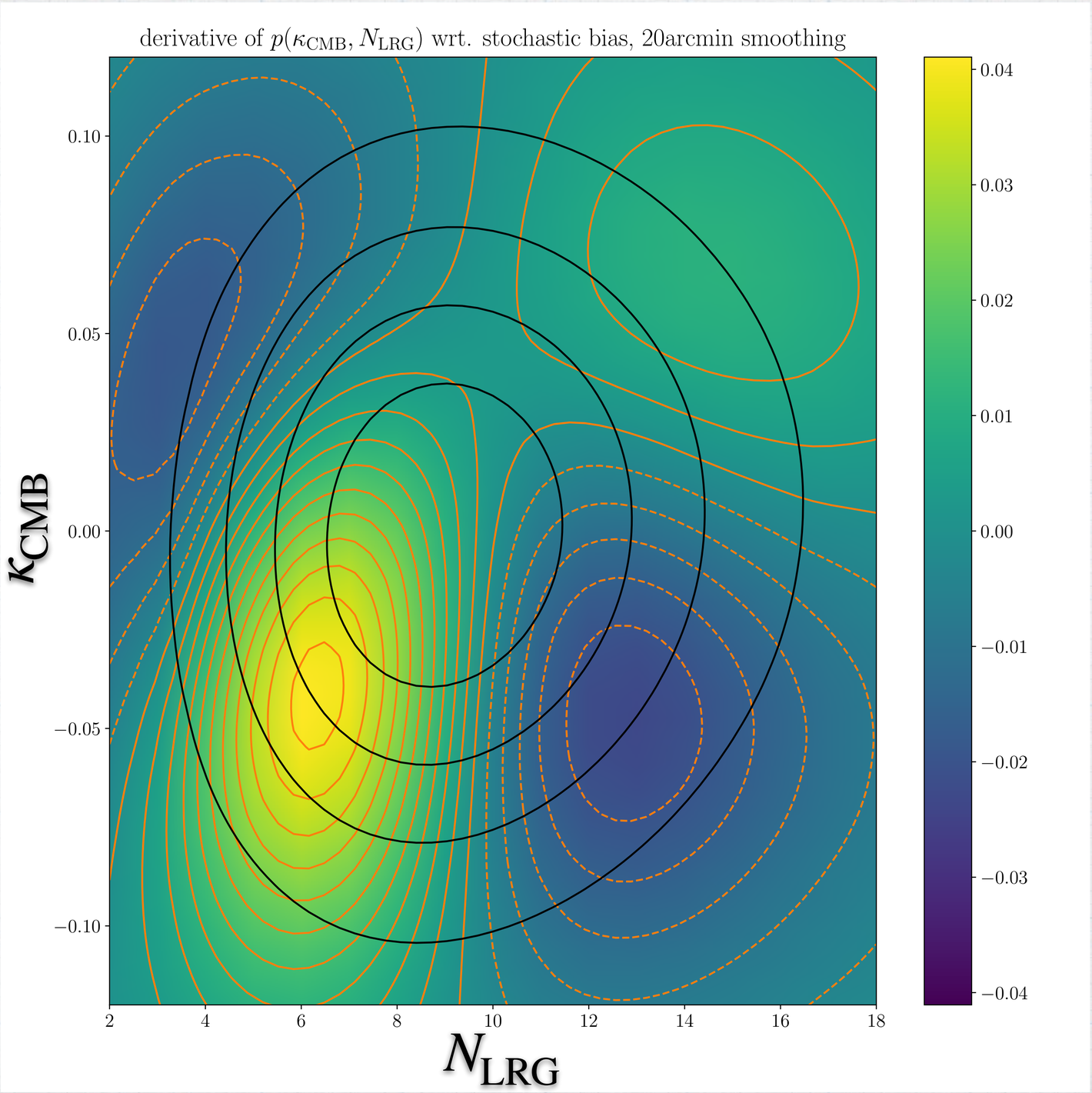


Black contours: fiducial PDF

Color map and orange contours: derivative wrt.

$$b_1$$

(dashed = negative)



Black contours: fiducial PDF

Color map and orange contours: derivative wrt.

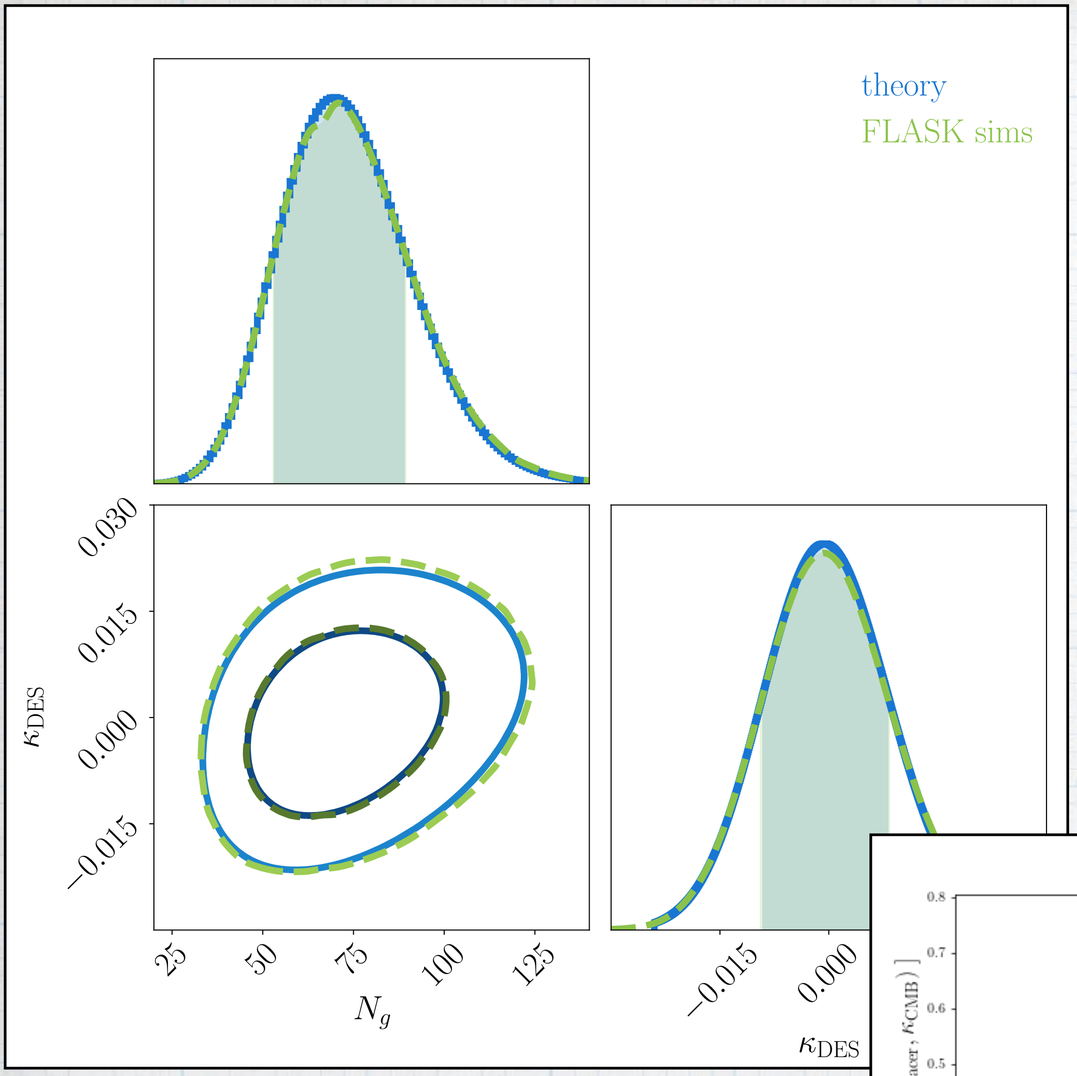
$r_{\rm stoch}$

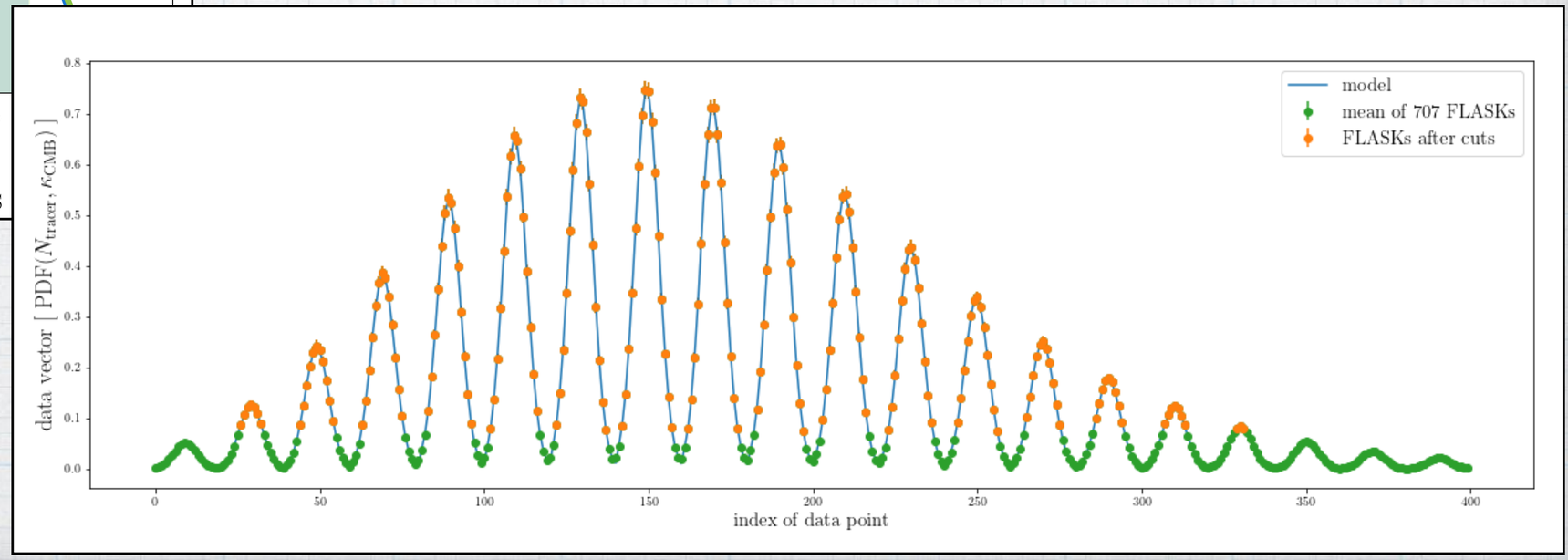
(dashed = negative)

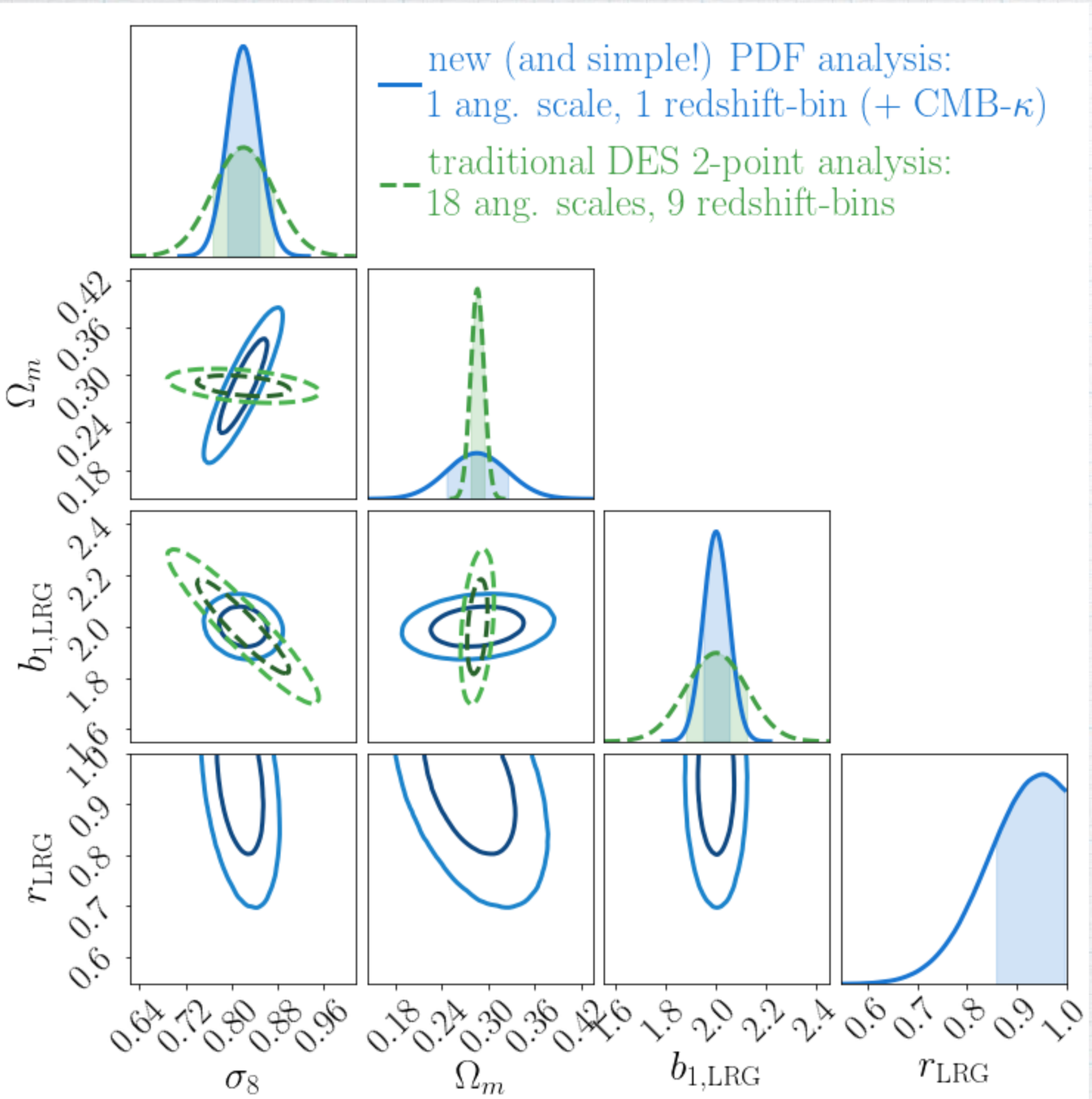
Covariance from log-normal Simulations

- have 700 FLASK simulations (Xavier et al. 2016)
- Combining several log-normal fields to capture non-Gaussian correlation structure

Work by Anik Halder, LMU Munich





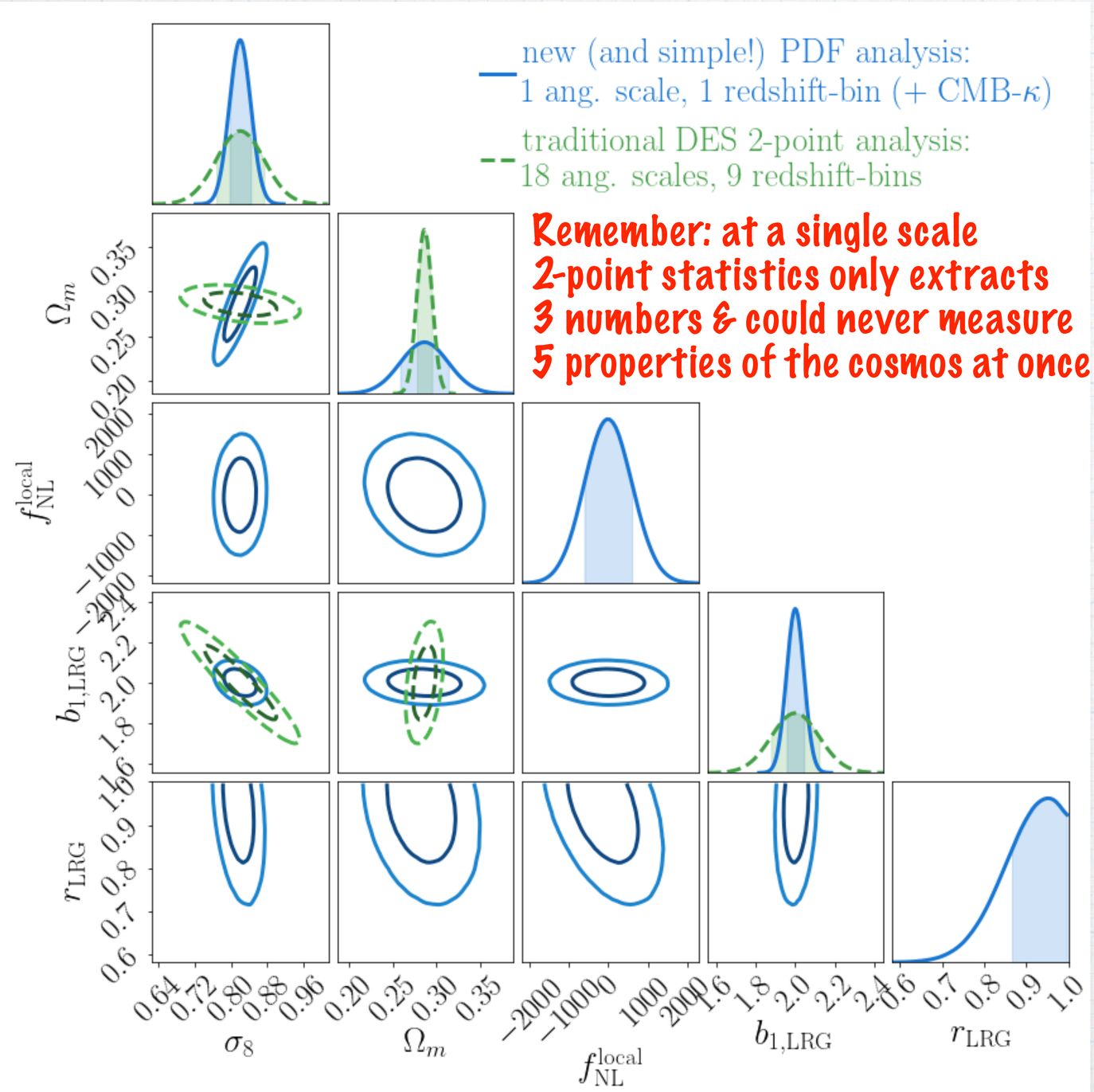


PDF can measure

- Omega_m
- sigma_8
- linear bias
- stochastic bias

using only one angular scale & 1 redshift bin!

(recall: **2point statistics only extract three numbers** from PDF)

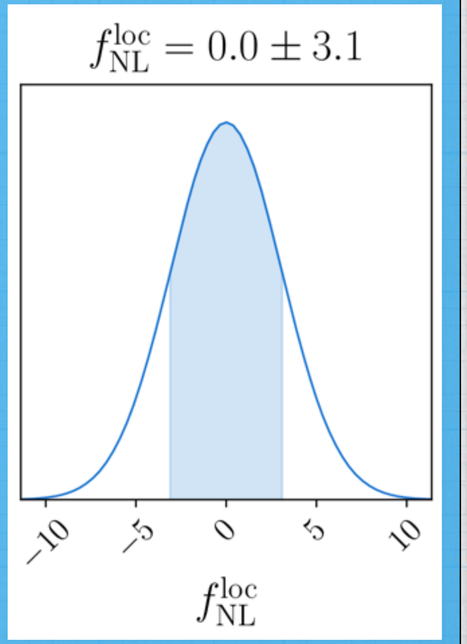


Can also attempt to measure primordial non-Gaussianity

But: 0.6 < z < 0.9 seems to be to broad to yield competitive results…

(Friedrich++2020: need scales of ~ 30 Mpc/h)

With spectroscopic data will be able to do much better, see **Friedrich++ 2020**:



Conclusions & Outlook

- Can open up complete “plane of perturbations” for LSS studies
- complete LCMD analysis possible even with complicated bias model and simple analysis setup
- Potential to break degeneracies of 3x2pt
- Much more!
  (see e.g. neutrino physics from PDF in **Uhlemann, Friedrich et al. 2020**)
- Most of these tools are already publicly available in the **CosMomentum** package:
  https://github.com/OliverFHD/CosMomentum

  - 3D PDFs and cumulant generating functions
  - PDF of gravitational lensing convergence
  - 2D and 3D halo counts-in-cells PDFs