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Why it is interesting to consider PBH as Dark Matter

Where it is interesting to look for PBH as Dark Matter

...some “NO-SEE-UMS”, “SPACE COWS”, “PYRAMIDS”
Can there be enough PBH around to be the DM?

What is the maximal fraction of dark matter in PBH?

THIS PLOT IS WRONG

Carr et al, 2017
The fraction of PBH that could be the dark matter depends on the mass function!

...what is the mathematical function that maximizes the mass fraction of primordial black holes compatibly with constraints?

Carr et al, 2017
The Maximal-Density Mass Function for Primordial Black Hole Dark Matter

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Abstract. The advent of gravitational wave astronomy has rekindled interest in primordial black holes (PBH) as a dark matter candidate. As there are many different observational probes of the PBH density across different masses, constraints on PBH models are dependent on the functional form of the PBH mass function. This complicates general statements about

Answer: with \( N \) independent constraints, the optimal function is a linear combination of \( N \) delta functions with calculable relative weights

\[
\min \left\{ \|x\| \mid x \in \text{conv} \left\{ g(M) \mid M \in U \right\} \right\}
\]

* Lehmann, Profumo and Yant, JCAP 2018
Answer: with $N$ independent constraints, the optimal function is a linear combination of $N$ delta functions with calculable relative weights.

* Lehmann, Profumo and Yant, JCAP 2018
Numerical validation

* Lehmann, Profumo and Yant, JCAP 2018
* Lehmann, Profumo and Yant, JCAP 2018
So **YES**, depending on the constraints choice, PBH can be **100%** of the dark matter!

* Lehmann, Profumo and Yant, JCAP 2018
Is there a goldilocks signature of PBH?

Yes! BH merger with a sub-Chandrasekhar mass (1.4 $M_{\text{sun}}$)

LIGO search results are out*

Is there a goldilocks signature of PBH?

Yes! BH merger with a sub-Chandrasekhar mass (1.4 $M_{\text{sun}}$)

LIGO search results are out*

Given a mass function, one can calculate:

1. Rate of “goldilocks events”

   $$R_{DP}(\psi) = \int_{DP^2} dm_1 \, dm_2 \, \mathcal{R}(m_1, m_2) V_{\text{eff}}(m_1, m_2).$$

2. Mass fraction of light+detectable BHs

   $$r_{DP} = \frac{1}{f_{\text{PBH}}} \int_{m_{DP}^{\text{min}}}^{m_{DP}^{\text{max}}} \, dm \, \psi(m).$$

We can numerically compute the maximal and minimal possible “goldilocks event rate”

* Lehmann, Profumo and Yant, MNRAS
We can numerically compute the maximal and minimal possible “goldilocks event rate”

* Lehmann, Profumo and Yant, MNRAS
We can numerically compute the maximal and minimal possible “goldilocks event rate”

\[ \int dm \psi(m) = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} \equiv f_{\text{PBH}}. \]

\[ r_{\text{DP}} = \frac{1}{f_{\text{PBH}}} \int_{m_{\text{DP}}^{\text{min}}}^{m_{\text{DP}}^{\text{max}}} dm \psi(m). \]

* Lehmann, Profumo and Yant, MNRAS
Besides the mass, LIGO informs us about the spin of BHs...
Besides the mass, LIGO informs us about the spin of BHs...

<table>
<thead>
<tr>
<th>Event</th>
<th>$m_1/M_\odot$</th>
<th>$m_2/M_\odot$</th>
<th>$M/M_\odot$</th>
<th>$\chi_{\text{eff}}$</th>
<th>$M_\text{f}/M_\odot$</th>
<th>$a_\text{f}$</th>
<th>$E_{\text{rad}}/\left(\text{M}_\odot\text{c}^2\right)$</th>
<th>$\ell_{\text{peak}}/\left(\text{erg s}^{-1}\right)$</th>
<th>$d_L/\text{Mpc}$</th>
<th>$z$</th>
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<td>63.1$^{+3.3}_{-3.0}$</td>
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<td>2.2$^{+0.5}_{-0.5}$</td>
<td>$3.3^{+0.6} \times 10^{56}$</td>
<td>960$^{+430}_{-410}$</td>
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<td>56.4$^{+5.2}_{-3.7}$</td>
<td>0.70$^{+0.08}_{-0.07}$</td>
<td>2.7$^{+0.6}_{-0.6}$</td>
<td>$3.5^{+0.6} \times 10^{56}$</td>
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<td>53.4$^{+3.2}_{-2.4}$</td>
<td>0.72$^{+0.07}_{-0.05}$</td>
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<td>$3.7^{+0.4} \times 10^{56}$</td>
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<td>$\leq 2.8$</td>
<td>$\leq 0.89$</td>
<td>$\geq 0.04$</td>
<td>$\geq 0.1 \times 10^{56}$</td>
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<td>26.7$^{+2.1}_{-1.7}$</td>
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<td>59.8$^{+4.8}_{-3.8}$</td>
<td>0.67$^{+0.07}_{-0.08}$</td>
<td>2.7$^{+0.5}_{-0.5}$</td>
<td>$3.4^{+0.5} \times 10^{56}$</td>
<td>1020$^{+430}_{-360}$</td>
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<td>29.3$^{+4.2}_{-3.2}$</td>
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<td>0.71$^{+0.08}_{-0.10}$</td>
<td>3.3$^{+0.9}_{-0.8}$</td>
<td>$3.6^{+0.6} \times 10^{56}$</td>
<td>1850$^{+440}_{-840}$</td>
<td>0.34$^{+0.13}_{-0.14}$</td>
<td>1651</td>
</tr>
</tbody>
</table>

Slide credit: Nico Fernandez (UCSC → UIUC)
Effective Spin

\[ \chi = \frac{|\vec{S}|}{Gm^2} \]

Dimensionless spin parameter

\[ \chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_1 + m_2 \chi_2 \cos \theta_2}{m_1 + m_2} \]

Information about:

- Direction. +++
- Spin magnitude. ++
- Masses. +

Slide credit: Nico Fernandez (UCSC → UIUC)
Effective Spin = 1

\[ \chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_1 + m_2 \chi_2 \cos \theta_2}{m_1 + m_2} \]

\[ \cos \theta = \hat{\chi} \cdot \hat{L} \]

Most black holes from stellar binaries probably start off with their spins aligned

Slide credit: Nico Fernandez (UCSC \(\rightarrow\) UIUC)
Effective Spin = 0

\[ \chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_1 + m_2 \chi_2 \cos \theta_2}{m_1 + m_2} \]

\[ \cos \theta = \hat{\chi} \cdot \hat{L} \]

Spins are essentially isotropic in the dynamical formation scenario. Binary was probably formed in a cluster

\[ \chi_{\text{eff}} \approx 0 \]
Effective Spin = 0

\[ \chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_1 + m_2 \chi_2 \cos \theta_2}{m_1 + m_2} \]

\[ \cos \theta = \hat{\chi} \cdot \hat{L} \]

Spin magnitudes are close to zero (expected from PBHs).
Effective Spin = -1

\[ \chi_{\text{eff}} = \frac{m_1 \chi_1 \cos \theta_1 + m_2 \chi_2 \cos \theta_2}{m_1 + m_2} \]

\[ \cos \theta = \hat{\chi} \cdot \hat{L} \]

Both spins are anti-aligned with its orbit (rare)
Magnitude Spin Priors

Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
Model Selection

- Spin magnitude: Low (L), Flat (F), High (H) and PBH

- Spin orientations: Isotropic (I) and Aligned (A)

Example:

FI = Flat spin magnitude and isotropic spins (LIGO)
FA = Flat spin magnitude and align spins
Effective Spin Priors

\[ p(\chi_{\text{eff}}) \]

Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
Evolution of the Odds ratios

Truth: PBH

Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
Evolution of the Odds ratios

Truth: Low-isotropic

Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
What about mixed models?

Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
What about mixed models?

projection
truth=0.5

Fernandez and Profumo, 1905.13109 (JCAP); Slide credit: Nico Fernandez (UCSC → UIUC)
What else could **fake** a low-spin PBH? **Super-radiance!**

Assuming an initial **spin** and **alignment** distribution, one can compute the “**best-fit**” axion mass.

Similarly, spin measurements can put **constraints** on axion-like particles.
What else could fake a low-spin PBH? Super-radiance!

Regge plot (effective spin vs mass) assuming Flat priors for both mass and spin*

*Fernandez, Ghalsasy, Profumo, 1911.07862
What else could fake a low-spin PBH? Super-radiance!

...but these are massive, so high-l is non-super-radiant!

*Fernandez, Ghalsasy, Profumo, 1911.07862*
What else could fake a low-spin PBH? Super-radiance!

Posterior Probability for ALP mass

*Fernandez, Ghalsasy, Profumo, 1911.07862*
THIS PLOT IS WRONG
SUBARU HSC microlensing, 1701.02151 VERSION 1
wacky constraints (WD, NS) have disappeared
This constraint also non-existent* 

* Katz et al, 1807.11495

SUBARU HSC microlensing, VERSION 3: finite source AND wave effects

...but assuming all stars have $R = R_{sun}$!
...but are these bounds robust?

A few (worrisome) assumptions:

- All stars are at the same distance
- All stars have the same size \( (1 \, R_{\text{sun}}) \)
- DM is completely smooth

* Smyth, Profumo et al, 1910.01285, PRD
Sun-like stars are however too dim for HSC!

Stars that contribute to the microlensing constraints are ~ 100x larger in the sky than the Sun!

* Profumo, Smyth+ PRD 2020
The bigger the star, the more important finite-source-size effects!

* Profumo, Smyth+ PRD 2020
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* Profumo, Smyth+ PRD 2020
The bigger the star, the more important finite-source-size effects!

* Profumo, Smyth+ PRD 2020
The bigger the star, the more important finite-source-size effects!

* Profumo, Smyth+ PRD 2020
How do we go after them? Capture and perturbation around PSR?

* Profumo, Smyth+ PRD 2020
Lightest PBH that can be dark matter...

\[ \tau(M) \approx 200 \tau_U \left( \frac{M}{10^{15} \text{ g}} \right)^3 \approx 200 \tau_U \left( \frac{10 \text{ MeV}}{T_H} \right)^3 \]

- are \sim asteroid/comet/PYRAMID mass
- can’t be much hotter than 10 MeV

Coogan, Morrison & Profumo, 2010.04797
$T_H = 20 \text{ MeV}, M = 5.3 \times 10^{14}\text{g}$

$T_H = 3 \text{ MeV}, M = 3.5 \times 10^{15}\text{g}$

$T_H = 0.3 \text{ MeV}, M = 3.5 \times 10^{16}\text{g}$

$T_H = 0.06 \text{ MeV}, M = 1.8 \times 10^{17}\text{g}$

$\frac{dN_\gamma}{dE_\gamma dt}$ (MeVs$^{-1}$)

$\frac{dN_\gamma}{dE_\gamma dt}$ (MeVs$^{-1}$)

$E_\gamma$ (MeV)

$E_\gamma$ (MeV)

Coogan, Morrison & Profumo, 2010.04797
Our new COMPTEL constraints are among strongest/robust

Coogan, Morrison & Profumo, 2010.04797
New MeV Telescopes could discover Hawking evaporation!

Coogan, Morrison & Profumo, 2010.04797
New MeV Telescopes could discover Hawking evaporation!

Snowmass2021 - Letter of Interest

**Searching for Dark Matter and New Physics with GECCO**

**Thematic Areas:**
- (CF1) Dark Matter: Particle Like
- (CF2) Dark Matter: Wavelike
- (CF3) Dark Matter: Cosmic Probes
- (CF4) Dark Energy and Cosmic Acceleration: The Modern Universe
- (CF5) Dark Energy and Cosmic Acceleration: Cosmic Dawn and Before
- (CF6) Dark Energy and Cosmic Acceleration: Complementarity of Probes and New Facilities
- (CF7) Cosmic Probes of Fundamental Physics

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**Authors:** Alexander Moiseev (CRESST, Greenbelt and NASA, Goddard and Maryland University), Stefano Profumo (UC Santa Cruz and Santa Cruz Institute for Particle Physics), Adam Coogan (Gravitation Astroparticle Physics Amsterdam (GRAPPA), Institute for Theoretical Physics Amsterdam and Delta Institute for Theoretical Physics, University of Amsterdam), Logan Morrison (UC Santa Cruz and Santa Cruz Institute for Particle Physics)

**Abstract:** We outline the potential science opportunities offered by a future MeV gamma-ray telescope. We point out that such an instrument would play a critical role in opening up a discovery window for particle dark matter with mass in the MeV or sub-MeV range, in disentangling the origin of the mysterious 511 keV line emission in the Galactic center region, and in potentially discovering Hawking evaporation from light primordial black holes. We refer to a new, proposed MeV gamma-ray telescope, the Galactic Explorer with a Coded Aperture Mask Compton Telescope (GECCO) that could deliver on all of those science objectives in the search for new physics and specifically for the nature of dark matter.

Coogan, Morrison & Profumo, 2010.04797
...even if PBH are **NOT** the dark matter, they can **PRODUCE** the dark matter via **Hawking evaporation**!
...even if PBH are **NOT** the dark matter, they can **PRODUCE** the dark matter via **Hawking evaporation**!

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>$T_H$ (GeV)</th>
<th>$\tau$ (s)</th>
<th>$T_{evap} = T(\tau)$ (GeV)</th>
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<tr>
<td>$5M_P \approx 10^{-4}$</td>
<td>$1.7 \times 10^{17}$</td>
<td>$10^{-41}$</td>
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<td>1</td>
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<td>$10^3$</td>
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<td>$10^9$</td>
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<td>$10^{12}$</td>
<td>17</td>
<td>$4 \times 10^7 \sim 1$ yr</td>
<td>$\sim 1$ keV</td>
</tr>
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</table>

* Morrison, Profumo and Yu (JCAP, 2019)
Relative initial abundance of PBH to everything else

PBH (eventually) dominate universe energy density

Mass of decaying RH neutrino producing baryon asymmetry

Ruled out by CMB limits on $H_1$

RH neutrinos thermalize

Ruled out by CMB limits on $H_1$

Dark Matter too fast

* Morrison, Profumo and Yu (JCAP, 2019)
Dark Matter can be a mix of Planck-scale relics from PBH evaporation, and stuff the PBH evaporated into!

* Morrison, Profumo and Yu (JCAP, 2019)*
As BH approach the Planck scale, they can acquire a significant relic electric charge

(under simple assumptions) the relic charge is approximately Gaussian*  

\[ P(Q) \sim \exp \left( -4\pi \alpha \frac{Q}{e} \right) \]

\[ (8\pi \alpha)^{-1/2} \approx 2.34 \]

If evaporation stops around the Planck scale (because of extremality, or because of quantum gravity) we are left with a population of charged, Planck-scale relics!

* Page, 1977
** Lehmann, Johnson, Profumo and Schwemberger, 1906.06348
* Lehmann, Johnson, Profumo and Schwemberger, 1906.06348
“Stellar-Mass” (10^{35} g) Black Holes

- Spins look a lot like PBH!
- ...or maybe they are low because of superradiance?
- Do they disrupt CMB*?

* Gaspari, Lehmann, Profumo, in preparation
“Asteroid-Mass” (10^{22} g) Black Holes

Microlensing a lot trickier than previously thought!

Detection strategies? PTA?
“Pyramid-Mass” (10^{16} g)  
“Evanescent” Black Holes

Best constraints: COMPTEL
✓ Future MeV telescopes
Decays can produce DM, BAU, Planck relics

Ton-size "Space-cow" Black Holes
Grain-of-Salt
“No-see-ums”
Black Holes

- Likely (partly) charged
- Detectable!
In the era of gravitational wave astronomy, the physics of macroscopic DM candidates offers many opportunities for the ingenuity of theorists and the craft of observers.
Merger rate calculation (Cheng+Huang, 2018; Raidal +, 2017)

\[
\bar{\tau}(m_1, m_2, m_3) = \frac{348 \alpha^4 \beta^7 a_{eq} m_3^7 \bar{x}(m_1, m_2)^4}{85 G^3 m_1 m_2 (m_1 + m_2)^8}.
\]

\[
\mathcal{G}(\psi; m_1, m_2, m_3) = \Gamma \left( \frac{58}{37}, \frac{\bar{N}(\psi; m_1, m_2) t^{3/16}}{\bar{\tau}(m_1, m_2, m_3)^{3/16}} \right)
- \Gamma \left( \frac{58}{37}, \frac{\bar{N}(\psi; m_1, m_2) t^{-1/7}}{\bar{\tau}(m_1, m_2, m_3)^{-1/7}} \right),
\]

\[
\mathcal{R}(m_1, m_2) = \frac{9 \bar{m}(\psi)^3 \bar{N}(\psi; m_1, m_2)^{\frac{33}{37}}}{296 \pi \delta_{dc} \bar{x}(m_1, m_2)^3 t^{34/37}}
\times \frac{\psi(m_1) \psi(m_2)}{m_1 m_2} \int dm_3 \frac{\mathcal{G}(\psi; m_1, m_2, m_3) \psi(m_3)}{\bar{\tau}(m_1, m_2, m_3)^{3/37} m_3}.
\]
Mass range where PBHs can form the entire dark matter density

$f_{\text{PBH}} = \text{fraction of the dark matter in the form of PBHs}$

Global constraints on primordial black hole dark matter

Dasgupta, Laha, and Ray 1912.01014