A number of gravitational microlensing surveys have been conducted to search for primordial black holes (PBHs) in nearby dark matter halos (Niklura et al, Nat. Astron. 2018). In many DM models, spatially extended structures are predicted in the form of exotic compact objects and subhalos. Examples include axion mini-halos, ultracompact mini-halos, boson stars, mirror stars, etc. Establishing microlensing limits on these extended DM structures requires us to recast the existing microlensing limits on PBHs. One essential step is to investigate the geometry of the setup depicted in the figure below, with $\mu$ being the lens-source distance, $\bar{u}(\varphi) = \sqrt{u^2 + r_s^2 + 2ru_0 \cos \varphi}$ being the distance from a point on the edge of the source to lens.

For every infinitesimal point on the edge of the source, the lensing equation which relates the true position of the point and the image can be written as $\bar{u}(\varphi) = \bar{u}(\varphi) = m(\bar{u}(\varphi))/t(\varphi)$, where $m(t)$ is the projected lens mass enclosed within $t$. The magnification of each images is given by

$$\mu = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\sqrt{1 + \frac{2}{\pi} \eta}} d\varphi,$$

where $\eta = -\text{sign}(d\bar{u}/d\varphi)|_{\varphi=0}$ is the “parity” of the image. The total magnification $\mu_{\text{tot}}$ is the sum of individual $\mu_i$.

In typical microlensing surveys, an event is called if the magnification exceeds 1.34. By numerically solving these equations, we find the impact parameter $a_{1.34}$ corresponding to $\mu_{1.34} = 1.34$. For NFW subhalos and boson stars, we obtain $a_{1.34}$ as a function of $r_0$ and $v_\text{c}$ in the figure below.

If the mass fraction of lenses in the DM halo is $f_{\text{DM}}$ and all the lenses have an identical mass $M$ and size $R_0$, the lensing rate of a particular background star with size $R_\star$ is given by (Griest, APJ 1990)

$$\frac{d\Gamma}{dM} = f_{\text{DM}}(R_\star/R_0)^2 \int d\bar{u} x^2 \frac{d\varphi}{d\bar{u}} \frac{d\mu}{d\bar{u}} \frac{d\bar{u}}{d\bar{u}},$$

where $\rho(x)$ is the local DM density projected along the line of sight, $x$ is the detector efficiency, $\gamma_\text{L}(x) = 2a_{1.34}(x)/t_E$ and $r_E$ is the DM circular velocity. The expected number of events is

$$N_{\text{events}} = N_s T_{\text{obs}} \int d\bar{u} x^2 \frac{d\varphi}{d\bar{u}} \frac{d\mu}{d\bar{u}} \frac{d\bar{u}}{d\bar{u}},$$

where $N_s$ is the number of stars, $T_{\text{obs}}$ is the net observation time, and $dn/dR_\star$ is the (normalized) stellar radius distribution of the source stars.

The Subaru-HSC survey is towards M31, and we take $dn/dR_\star$ in the figure below. Deviations from PBHs limits become visible for lens size larger than $O(0.1 R_\odot)$, and lenses up to $O(10^2 R_\odot)$ can be probed by Subaru-HSC. Constraints from EROS-2 and OGLE-IV surveys are also shown.