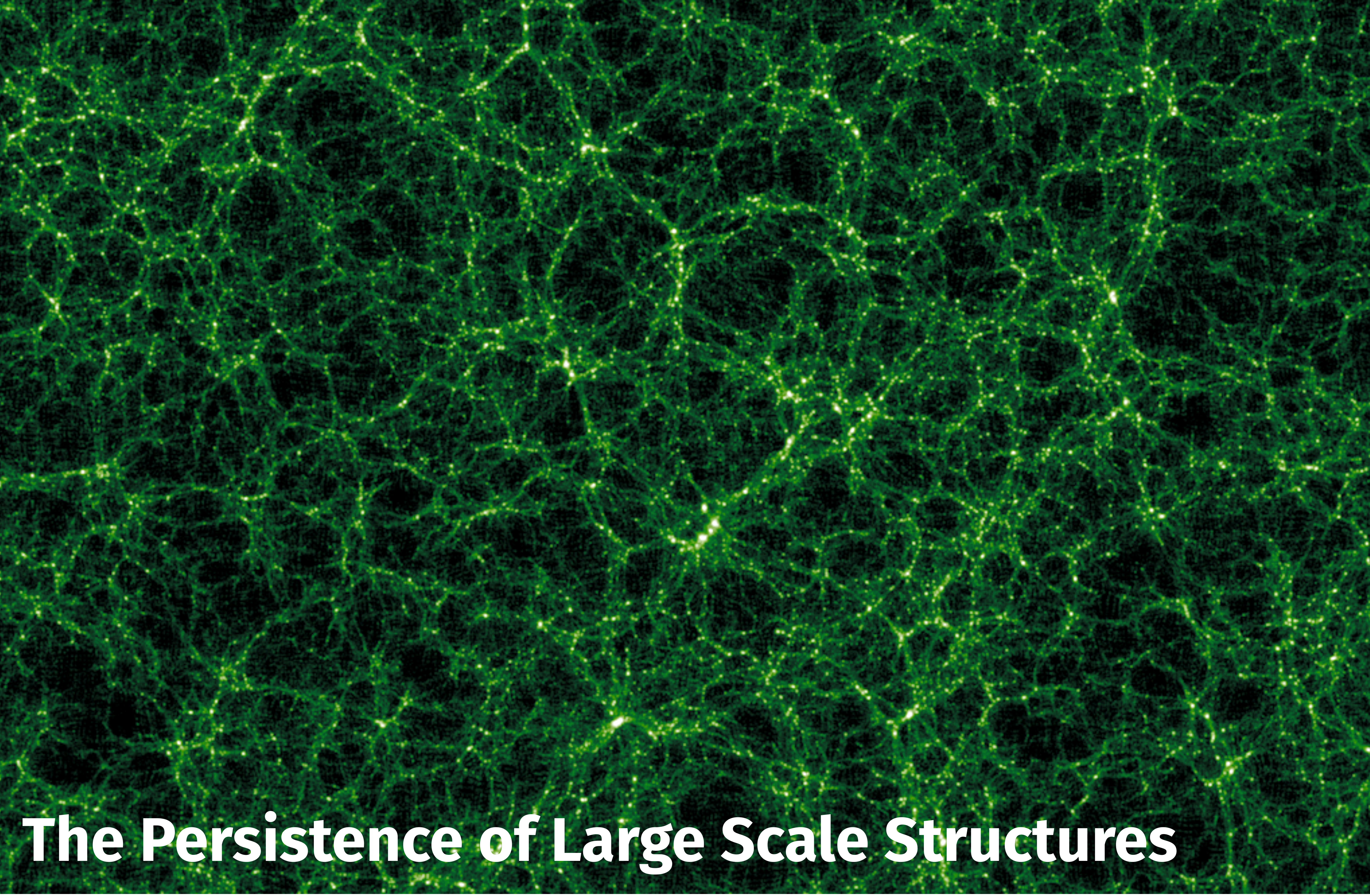


The Persistence of Large Scale Structures

Matteo Biagetti

with Alex Cole and Gary Shiu

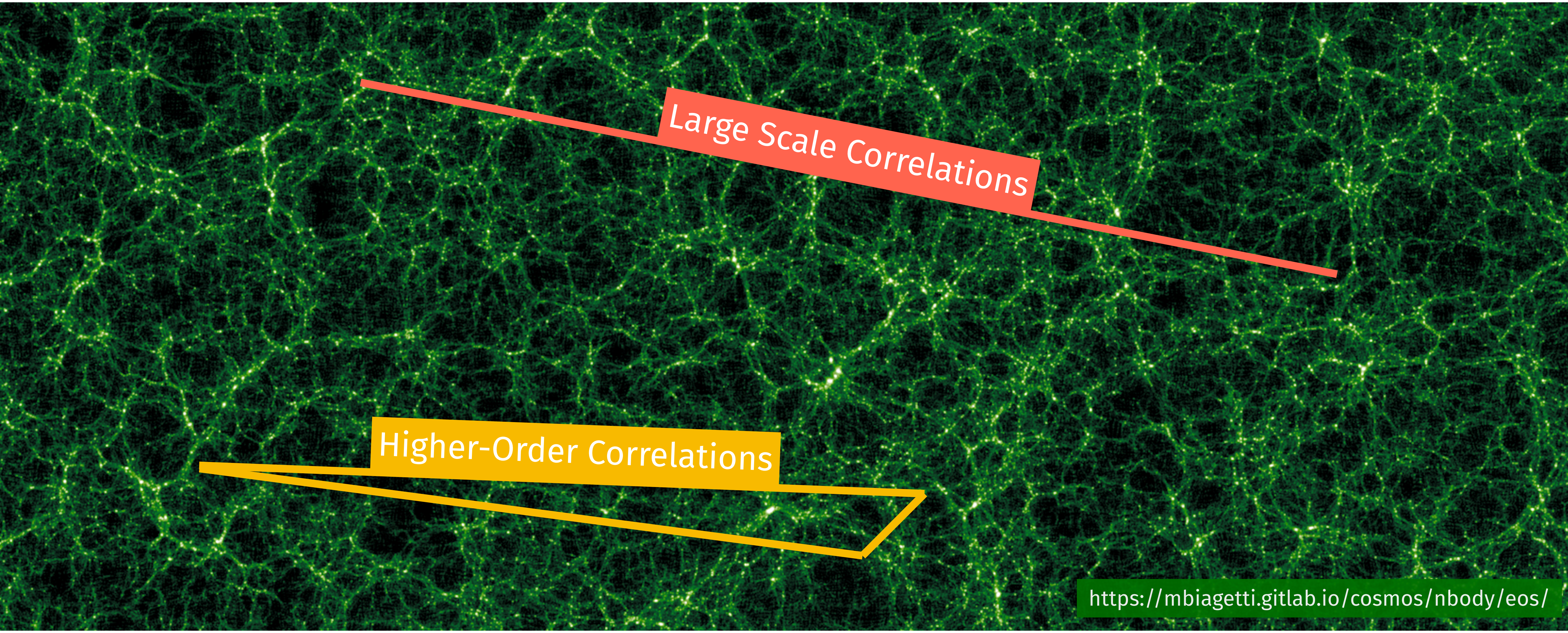
arXiv: 2009.04819

ifpU

NWO VENI

Motivation

Cosmology from biased tracers



Challenges

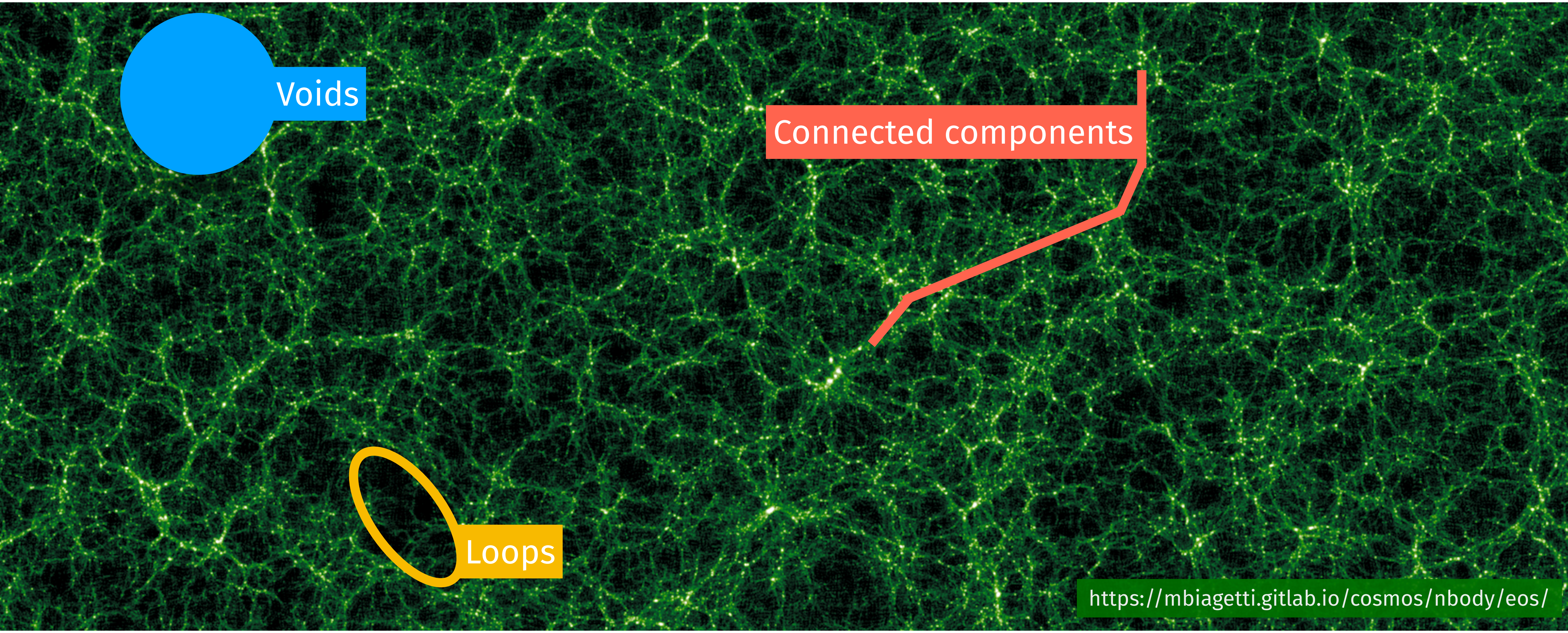
- Dark matter nonlinearities
- Galaxy bias
- Redshift-space distortions

Advantages

- Theory under control
- Handling of uncertainties
- Complementarity with Nbody

Motivation

Cosmology from biased tracers



Challenges

- Theoretical predictions
- Computational expense

Advantages

- Connection to LSS observables
- High-order correlations

Topological Data Analysis

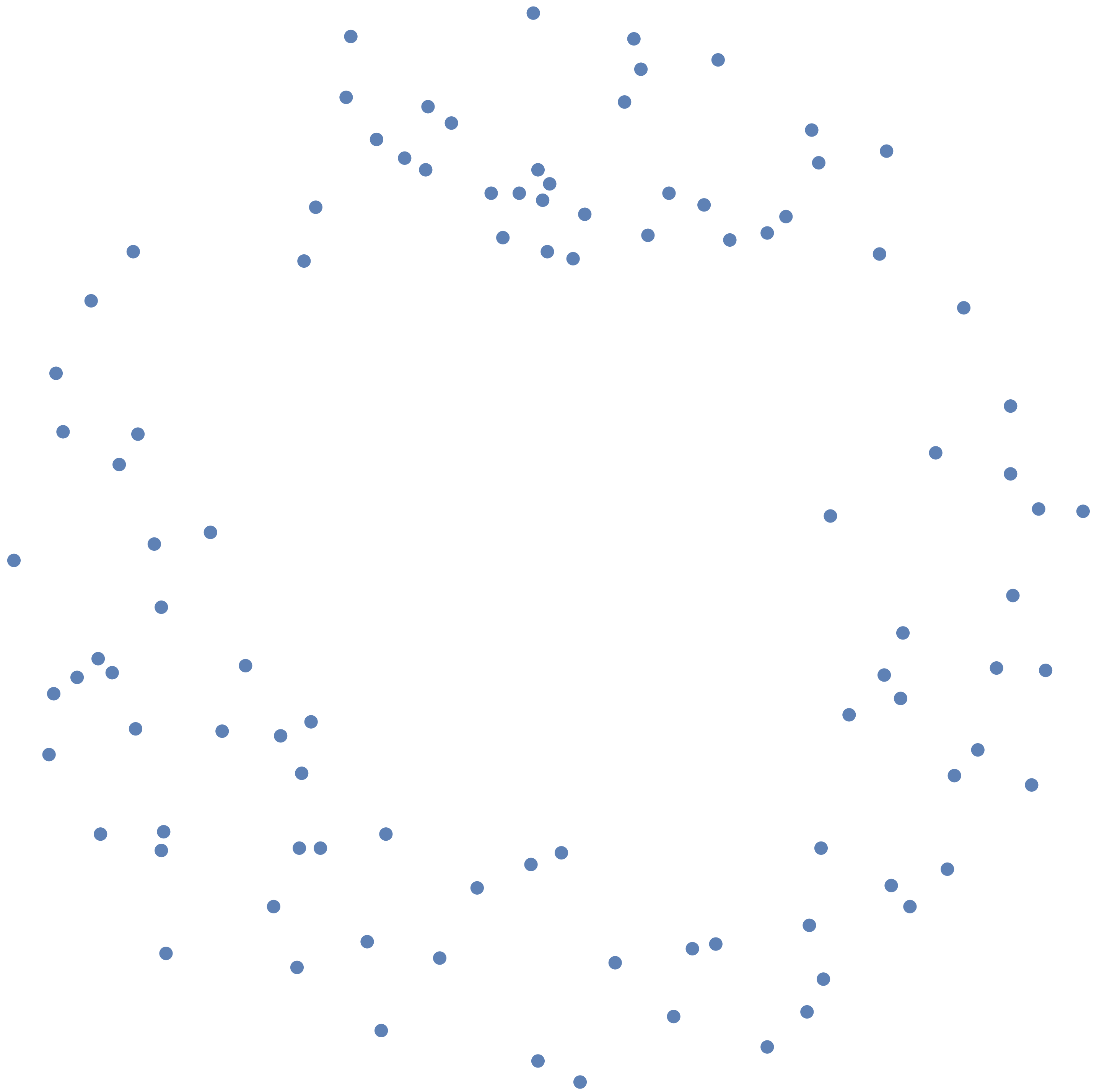
Compute the shape of discrete data via its multiscale topology

Track individual topological features as they are created and destroyed by successive coarse-graining transformations

Applications:

- Sensor networks
- Image processing
- Genomics
- Protein structure
- Neuroscience
- Physics

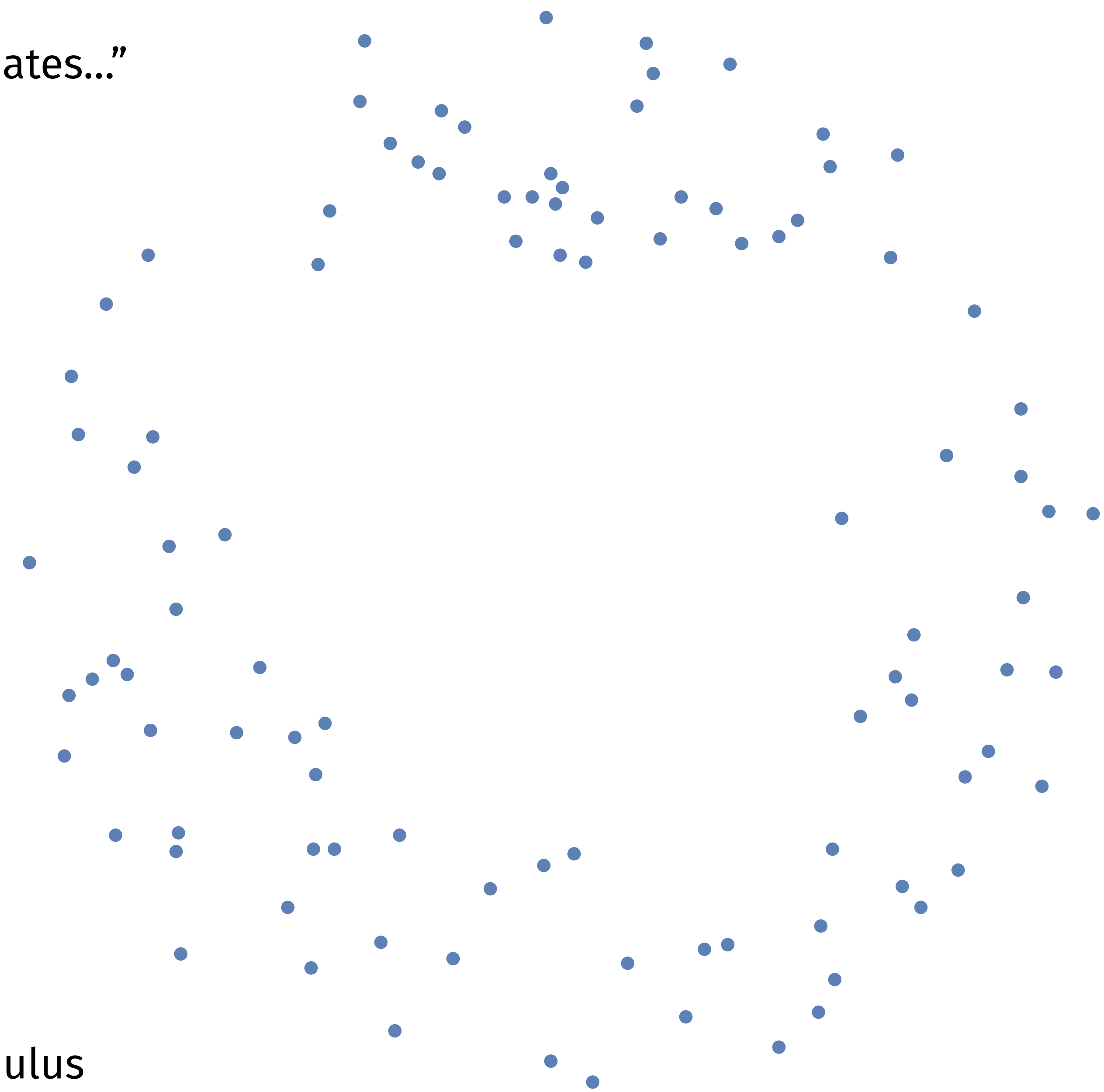
Persistent Homology



Courtesy of Alex Cole

Persistent Homology

Computer answer:
“100 points with
the following coordinates...”



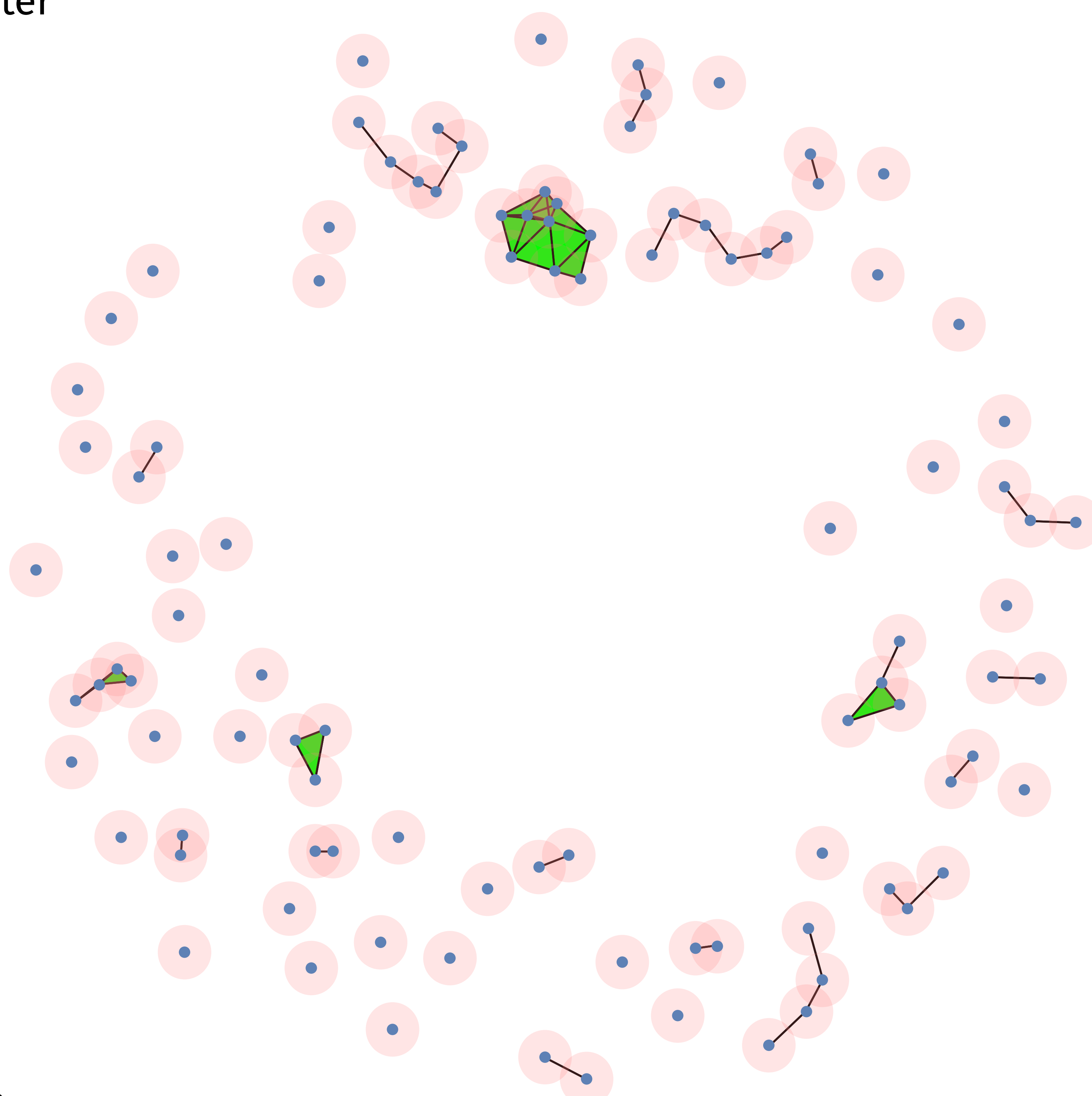
Human answer:
a noisy circle/ points
sampled from an annulus

Courtesy of Alex Cole

Persistent Homology

Filtration parameter

$\nu = 1$



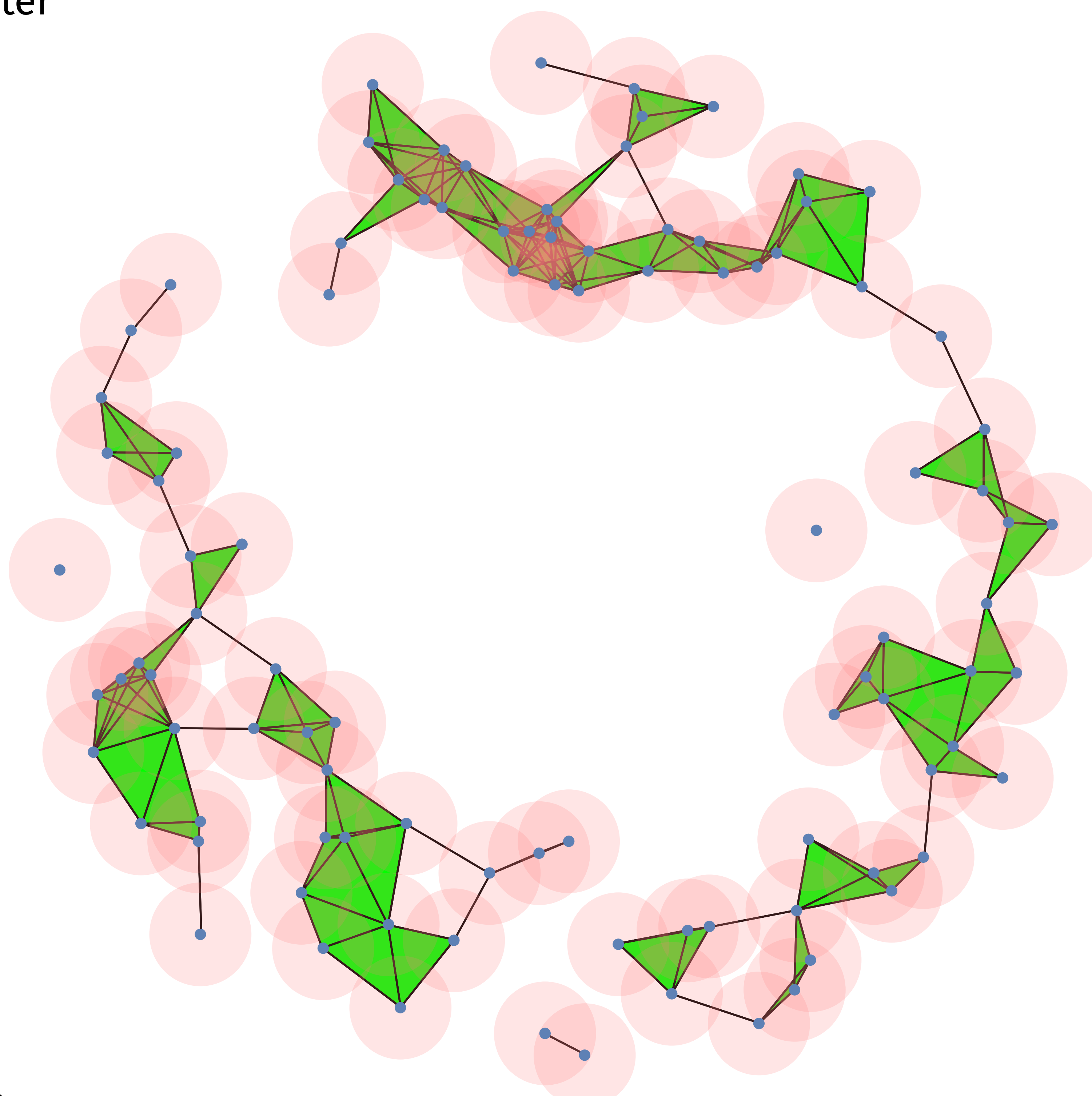
Vietoris-Rips filtration

Courtesy of Alex Cole

Persistent Homology

Filtration parameter

$\nu = 2$



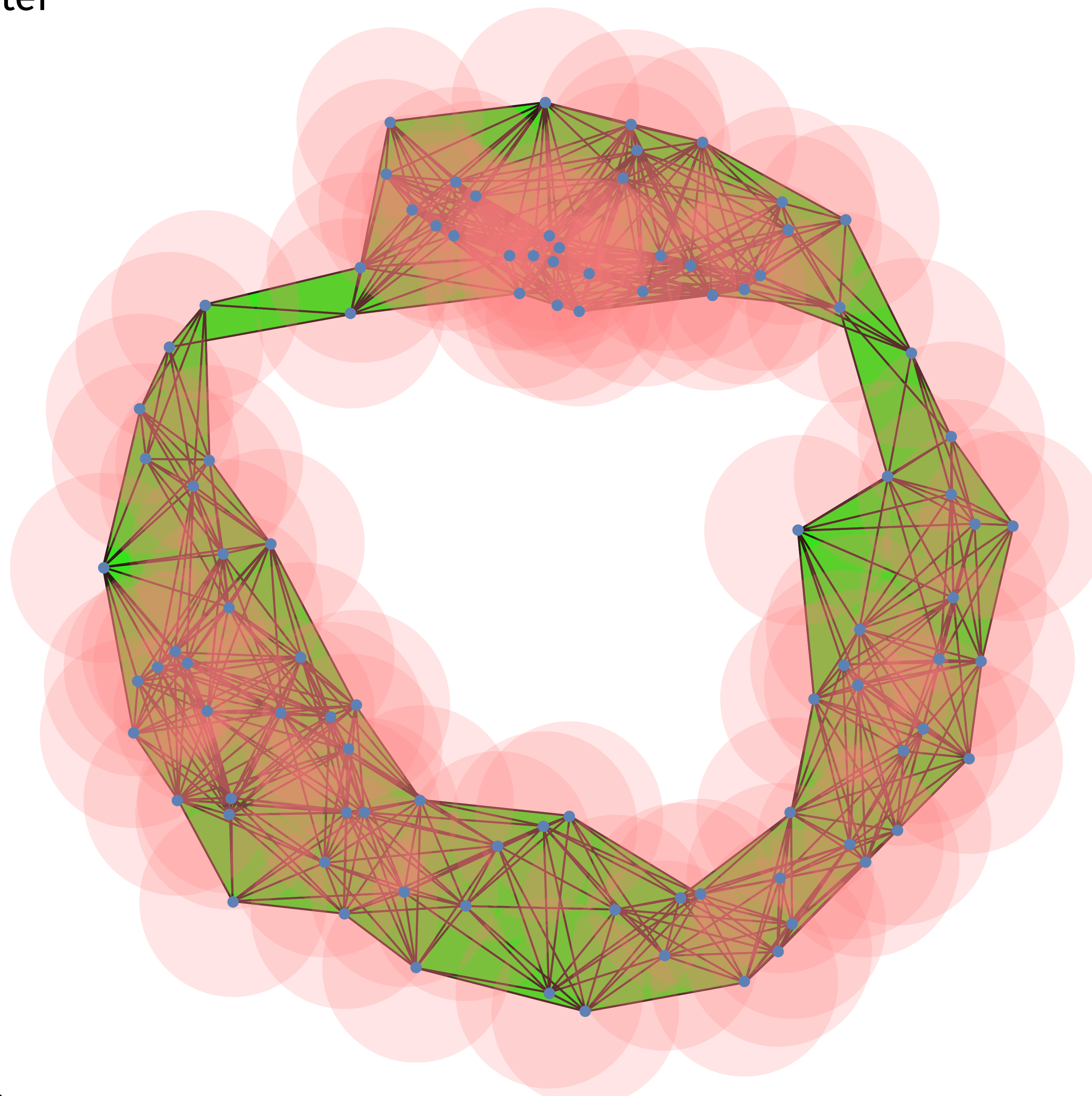
Vietoris-Rips filtration

Courtesy of Alex Cole

Persistent Homology

Filtration parameter

$\nu = 3$



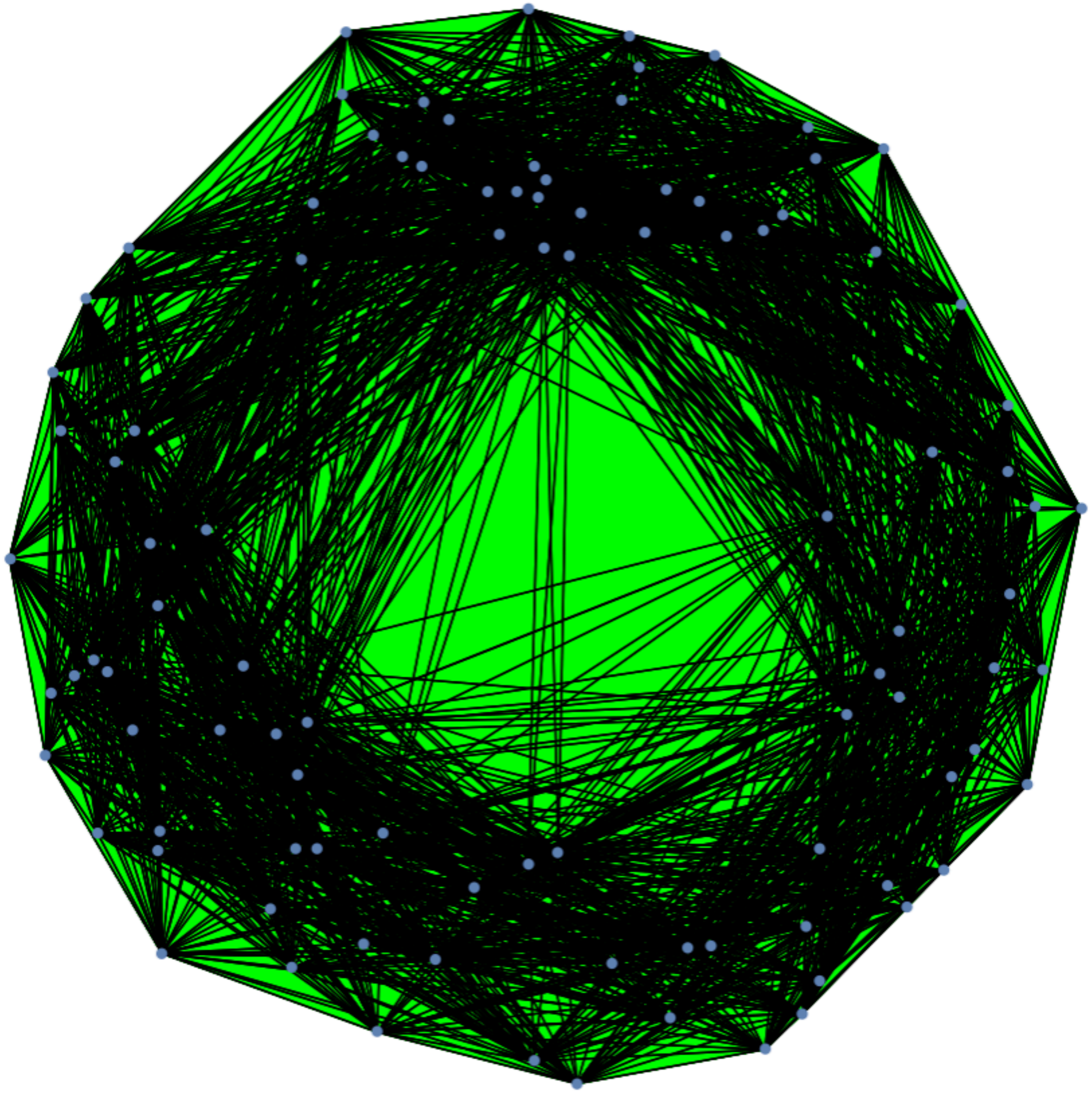
Vietoris-Rips filtration

Courtesy of Alex Cole

Persistent Homology

Filtration parameter

$\nu = 5$



Vietoris-Rips filtration

Courtesy of Alex Cole

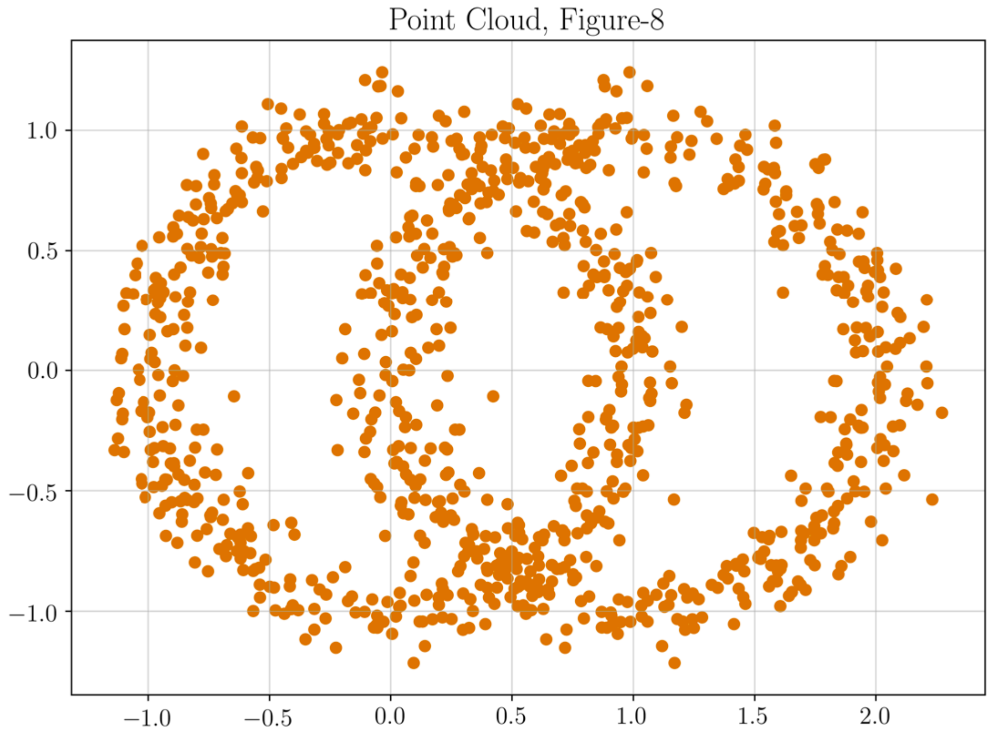
Persistent Homology

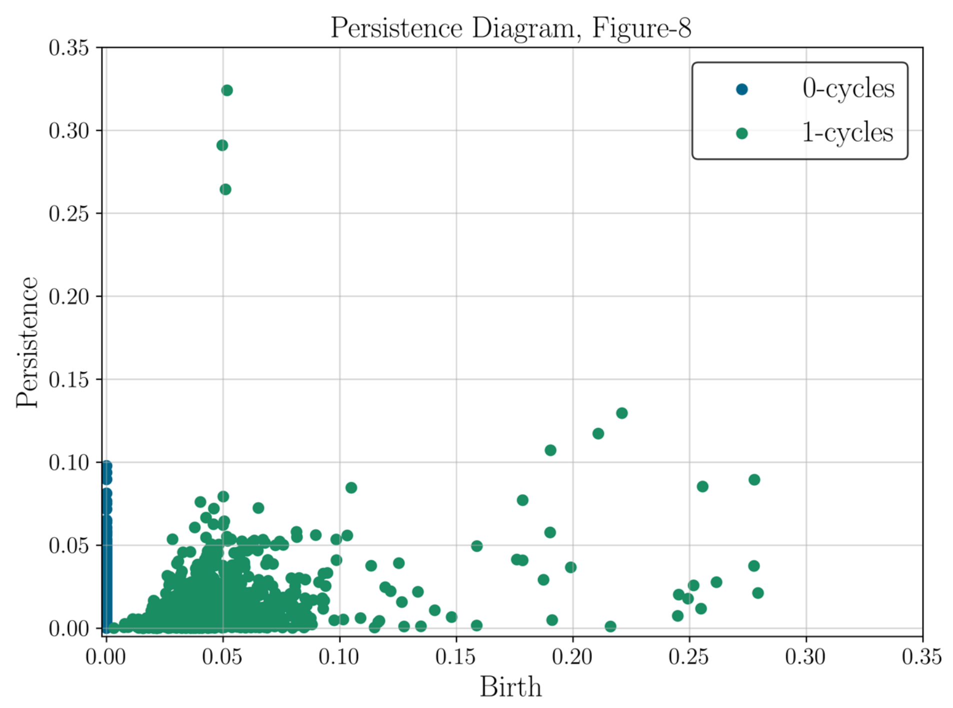
Persistence diagrams

- Persistent homology: vary simplicial representation Σ_ν of data with parameter ν such that $\Sigma_{\nu_1} \subseteq \Sigma_{\nu_2}$ for $\nu_1 < \nu_2$. Σ_ν is called a filtration.

- Scatter plots of birth and death times for individual homology generators

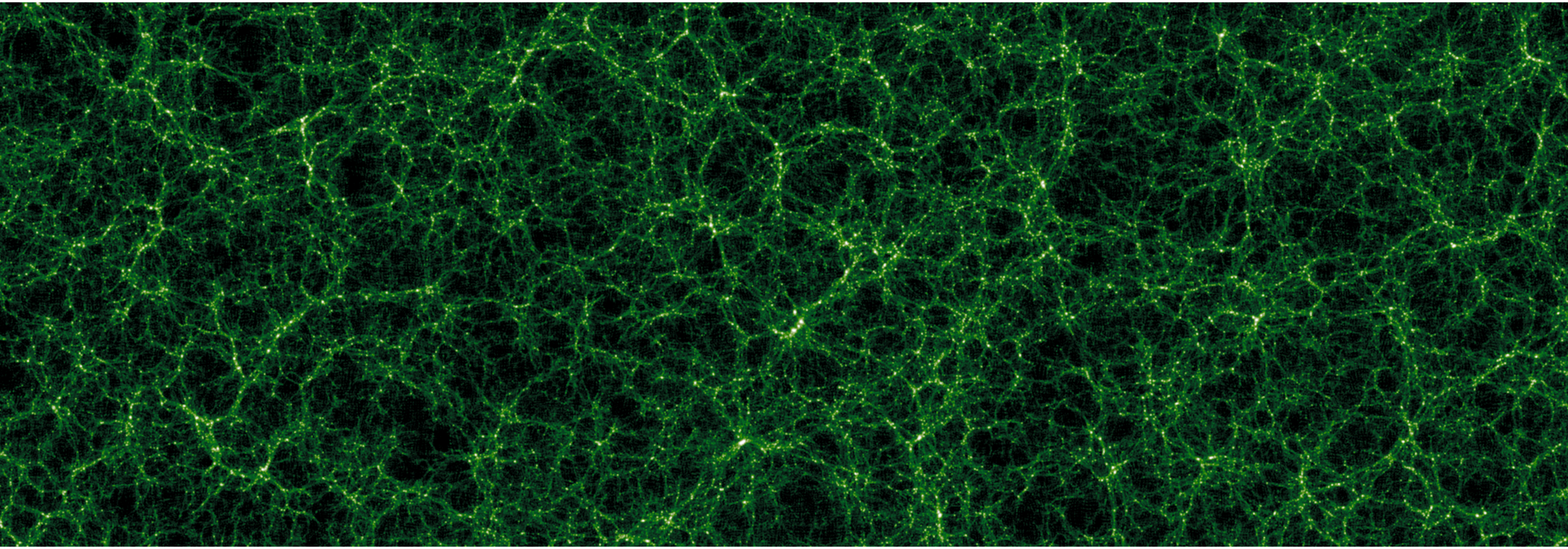
- Intuition*: long-lived features are "real," short-lived features are "noise."





Courtesy of Alex Cole

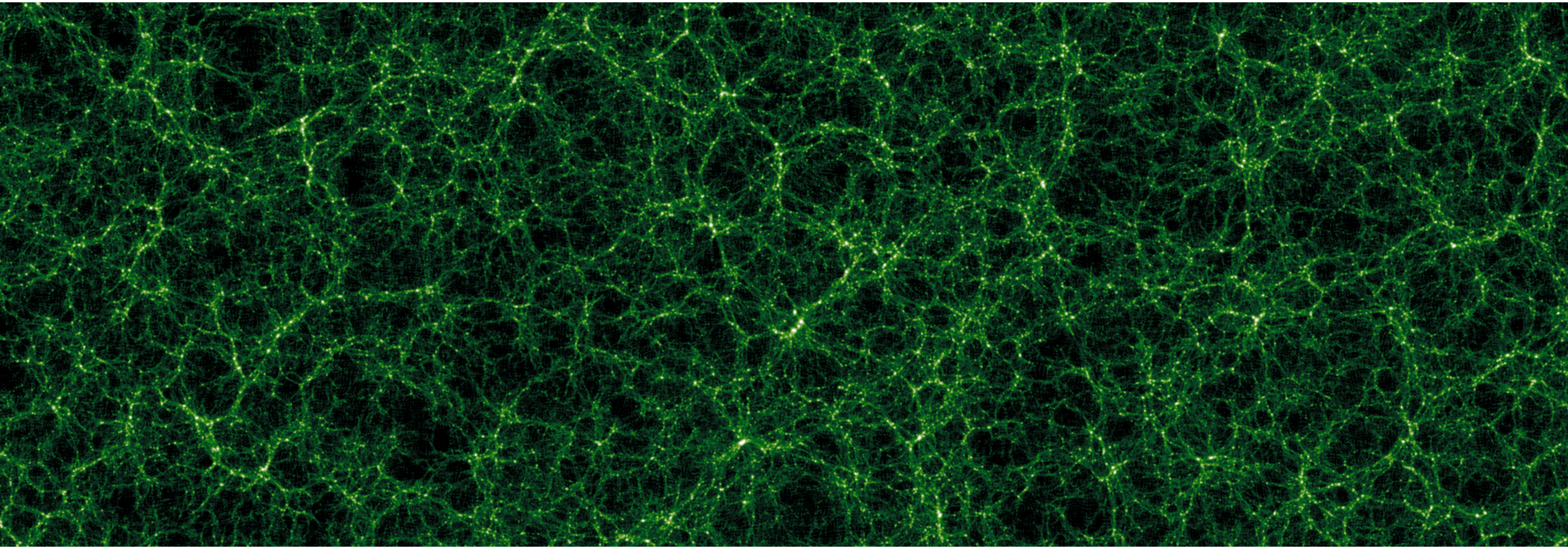
Persistence of Large Scale Structure



Apply persistent homology to a halo catalog:

- Pipeline on Nbody to constrain a cosmological parameter ($f_{\mathrm{NL}}^{\mathrm{loc}} = 10$)
- Preliminary study of multiple cosmological parameters and degeneracies
- Study physics of topological signatures

Persistence of Large Scale Structure



Apply persistent homology to a halo catalog:

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Primordial non-Gaussianity

A one-slide summary on primordial non-Gaussianity

- Primordial non-Gaussianity refers to the non-Gaussianity statistical distribution of primordial perturbations

- Non-zero N-point function is related to interactions among fields taking place during inflation

- We have contraints on the free theory (power spectrum), but none on interactions (we have not detected primordial non-Gaussianity yet)

- Constraints on primordial non-Gaussianity allow to tell something about the mechanism of production of primordial perturbations and the particle content during inflation

[See overview on Jorge Noreña's talk]

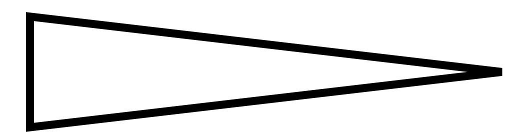
Primordial non-Gaussianity

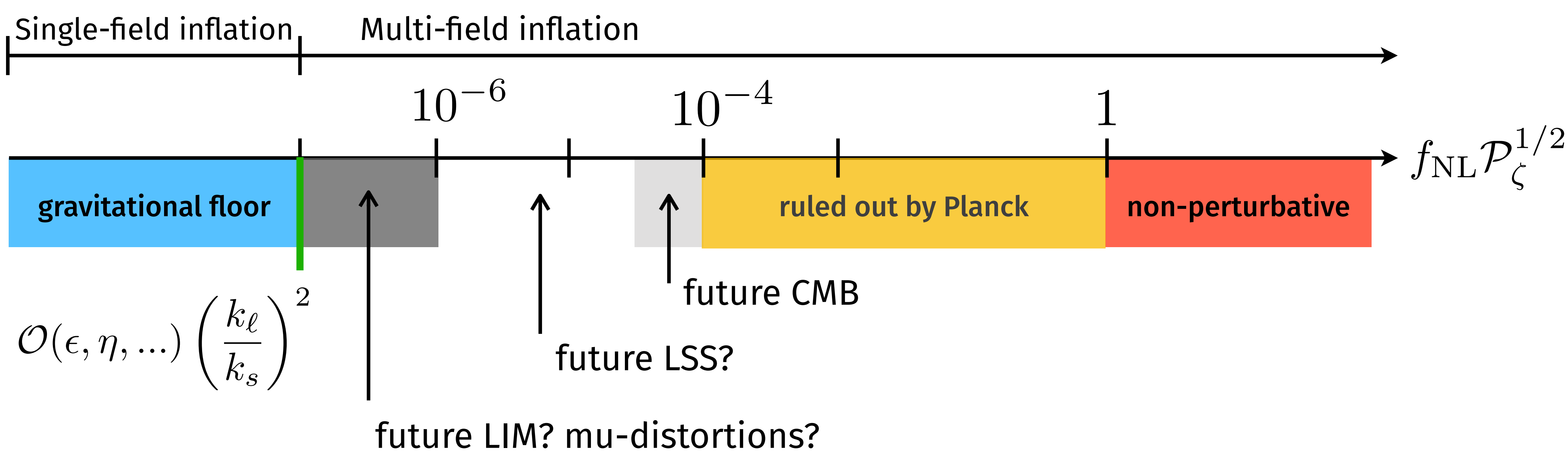
Local type

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL}^{loc} \left(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2 \rangle\right) + \mathcal{O}\left(\zeta_G^3\right).$$

Bispectrum $\quad \dfrac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^2} \propto f_{NL}$

Squeezed configuration



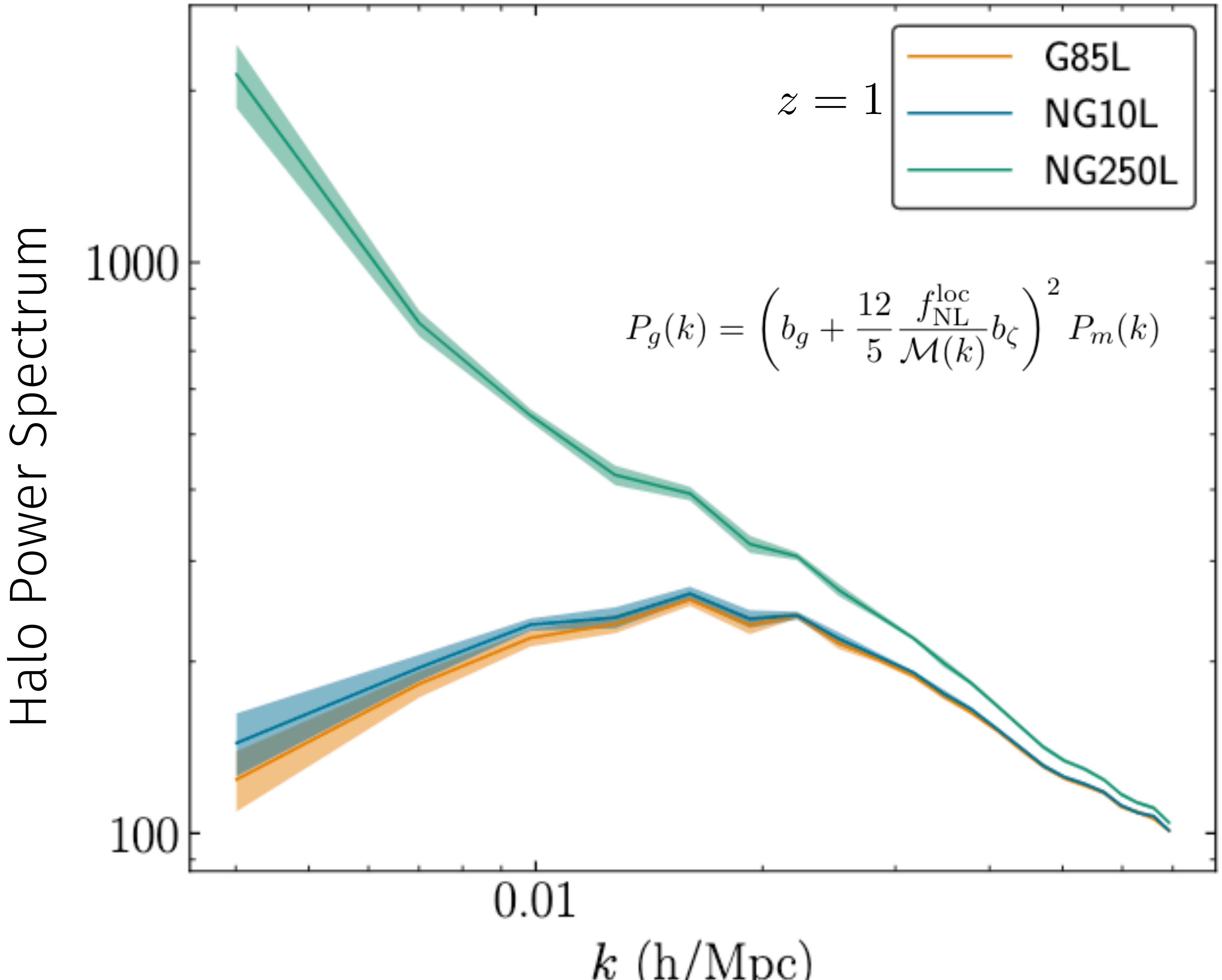


current constraints

current constraints	$f_{NL}^{loc} = -0.9 \pm 5.1,$	@68% CL	Planck (2018)
	$-51 < f_{NL}^{loc} < 21$	@95% CL	eBOSS (2019)

Primordial non-Gaussianity

Effect of local primordial non-Gaussianity on biased tracers



Dalal et al. (2007)
Slosar et al. (2007)
Matarrese and Verde (2007)

Persistence of Large Scale Structure

Dataset:

- **G85L**: 15 realizations, $120\,(\mathrm{Gpc/h})^3$ total volume, **LCDM** → **Constraints**
- **NG10L**: 15 realizations, $120\,(\mathrm{Gpc/h})^3$ total volume, $f_{\mathrm{NL}} = 10$ → **Constraints**
- **NG250L**: 10 realizations, $80\,(\mathrm{Gpc/h})^3$ total volume, $f_{\mathrm{NL}} = 250$ → **Physical Intuition**
- **NG250S**: 5 realizations, $5\,(\mathrm{Gpc/h})^3$ total volume, $f_{\mathrm{NL}} = 250$ → **Templates**

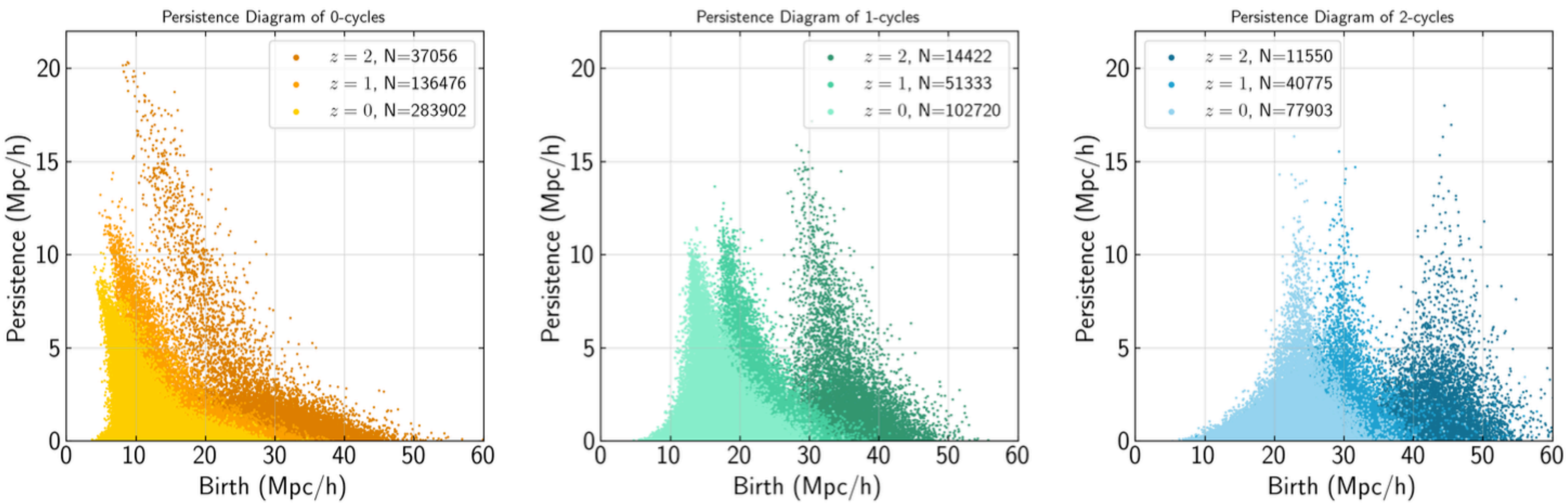
LCDM cosmology: $n_s = 0.967$ $\Omega_m = 0.3$ $h = 0.7$

~ 10 millions halos at $z = 0,\ 1,\ 2$ **with masses** $10^{13} M_\odot \lesssim M \lesssim 10^{15} M_\odot$

https://mbiagetti.gitlab.io/cosmos/nbody/eos/

Persistence of Large Scale Structure

Persistence diagrams



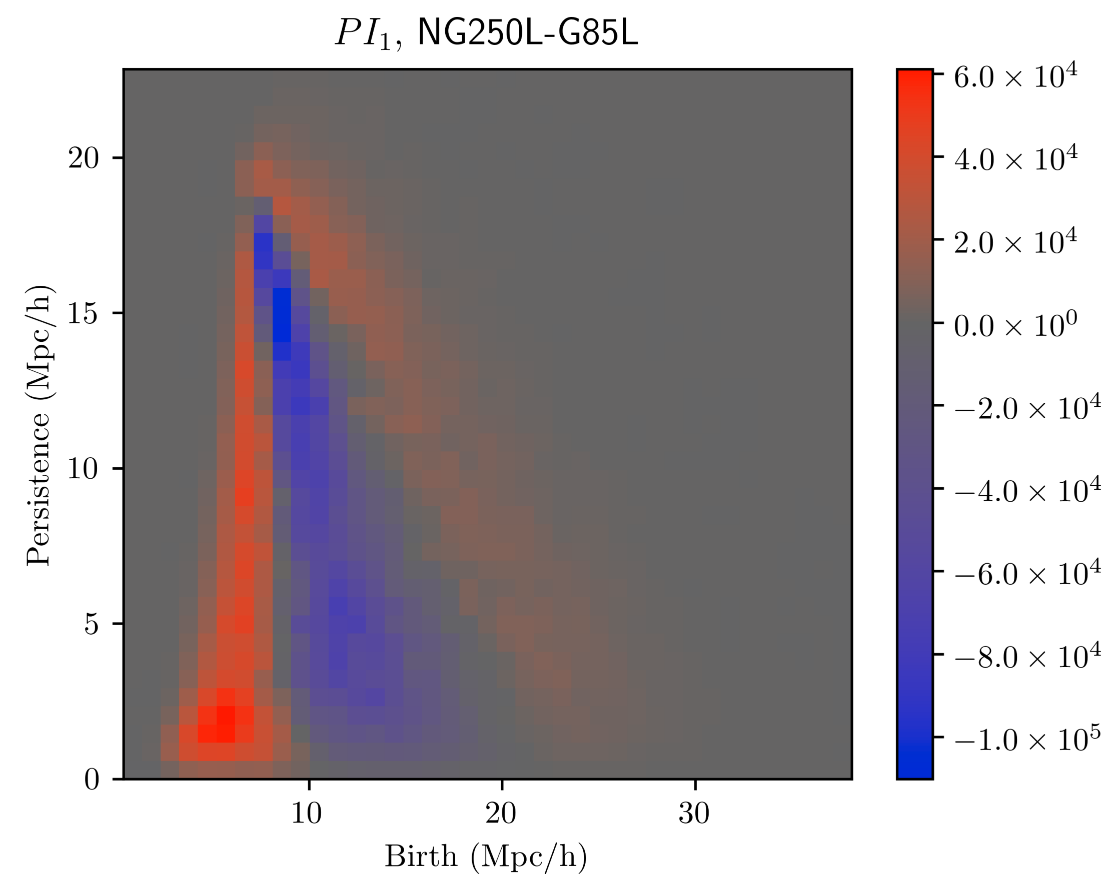
Each dot corresponds to a topological feature in the LCDM simulation

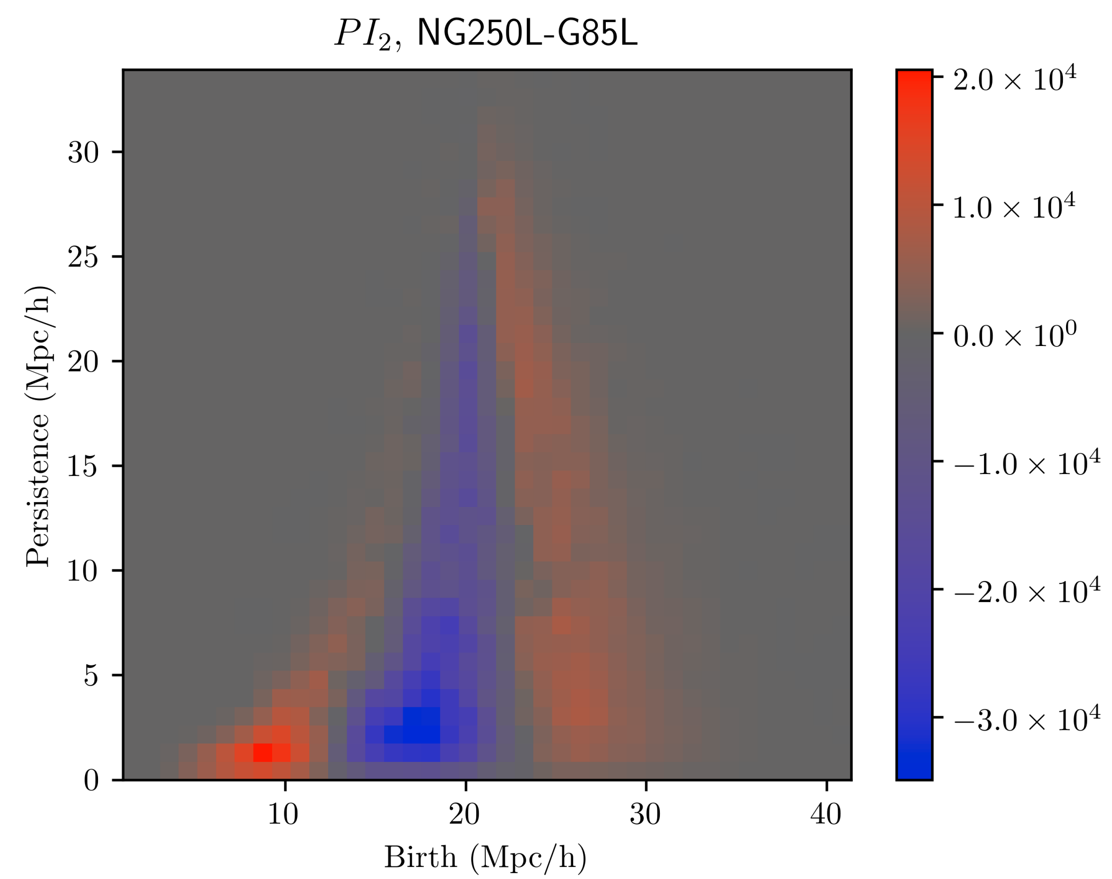
0-cycles are clusters of halos, 1-cycles are loops from connecting halos and 2-cycles are voids

Persistence of Large Scale Structure

Persistence Images

- For physical intuition, look at strong non-Gaussinianity $f_{\rm NL}^{\rm loc} = 250$
- Obvious sensitivity of births to $f_{\rm NL}^{\rm loc}$. For positive $f_{\rm NL}^{\rm loc}$, more features born early and late.
- Hints toward which slicings of PD will be statistically useful.
- Notice the scale of deviations: much smaller than scale dependent bias.

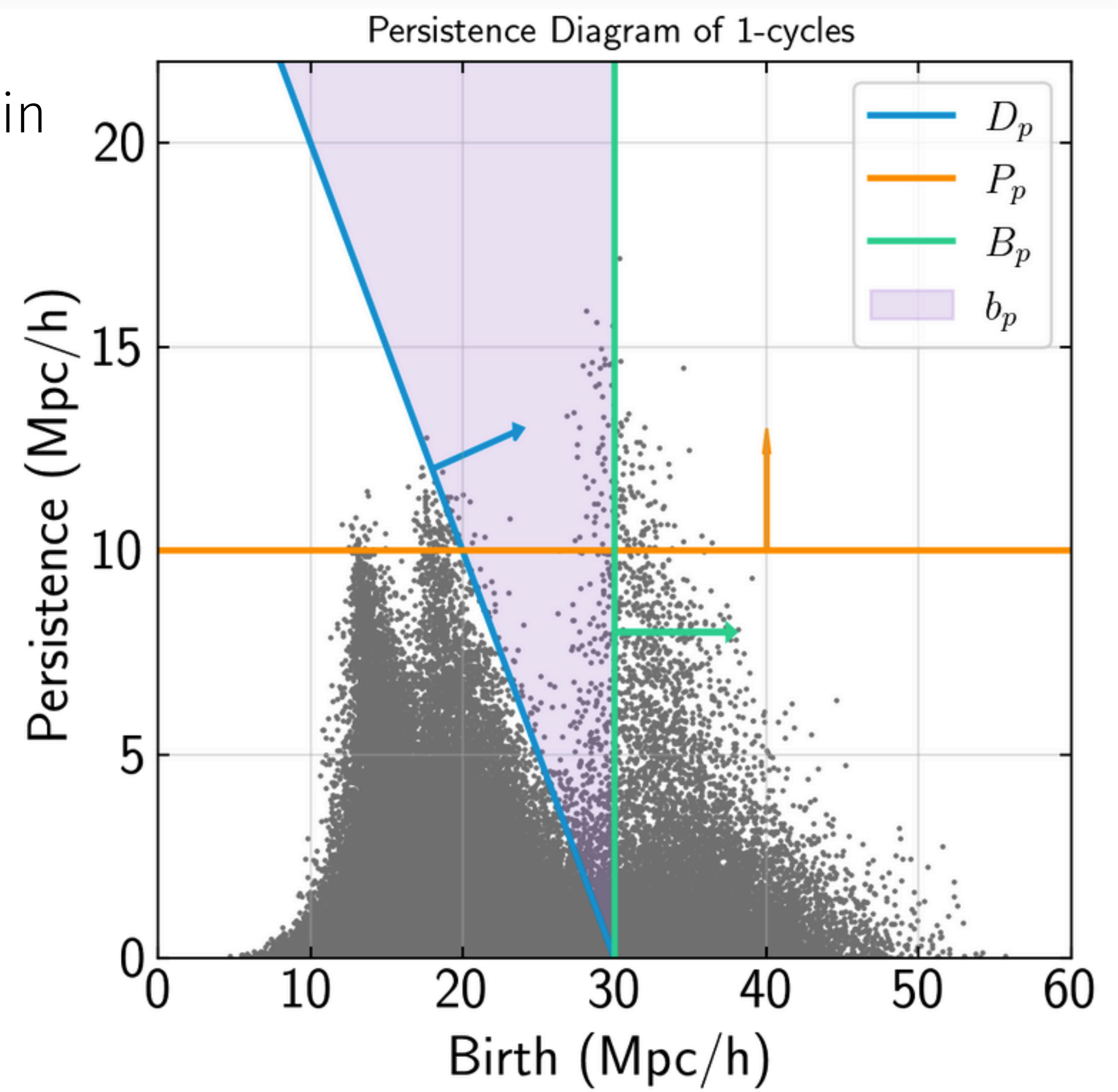




Persistence of Large Scale Structure

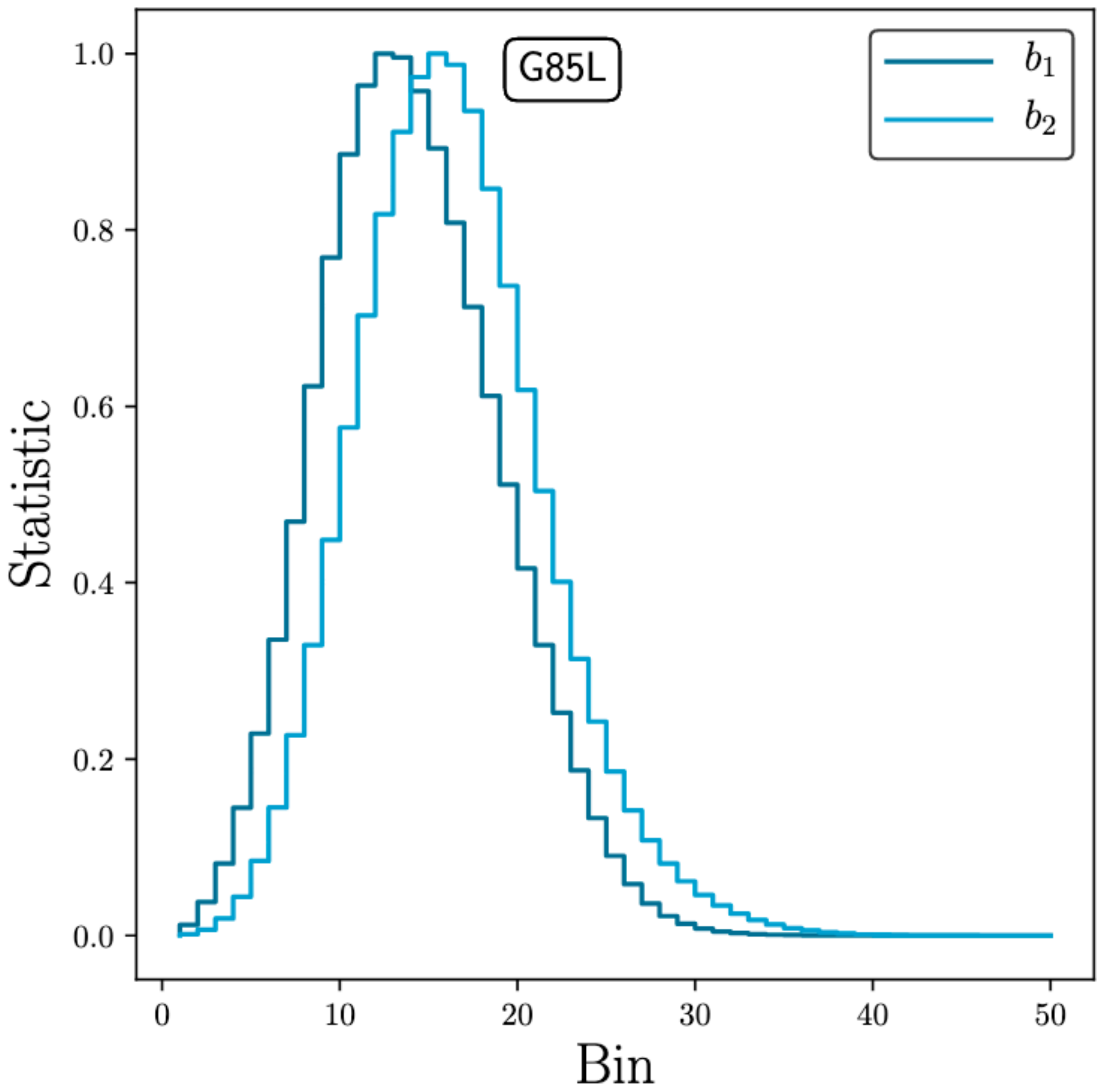
Topological Curves

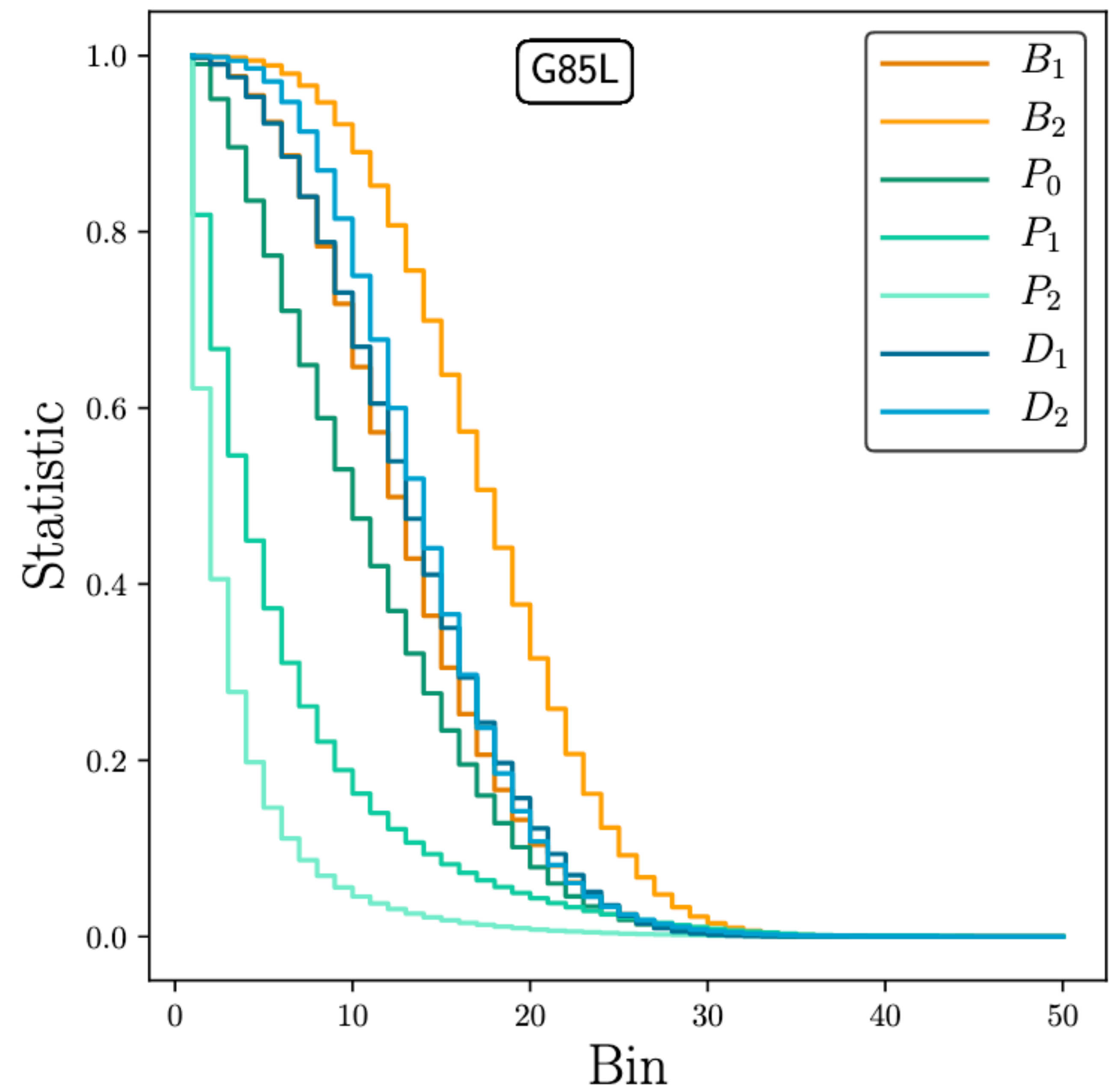
- Topological curves: count cycles in particular regions of diagram.
- $D_p(\nu)$: deaths
- $P_p(\nu)$: persistence
- $B_p(\nu)$: births
- $b_p(\nu) = B_p(\nu) - D_p(\nu)$: Betti numbers



Persistence of Large Scale Structure

Topological Curves





D_p, P_p, B_p have interpretation of empirical distribution functions for deaths, etc. of cycles

Persistence of Large Scale Structure

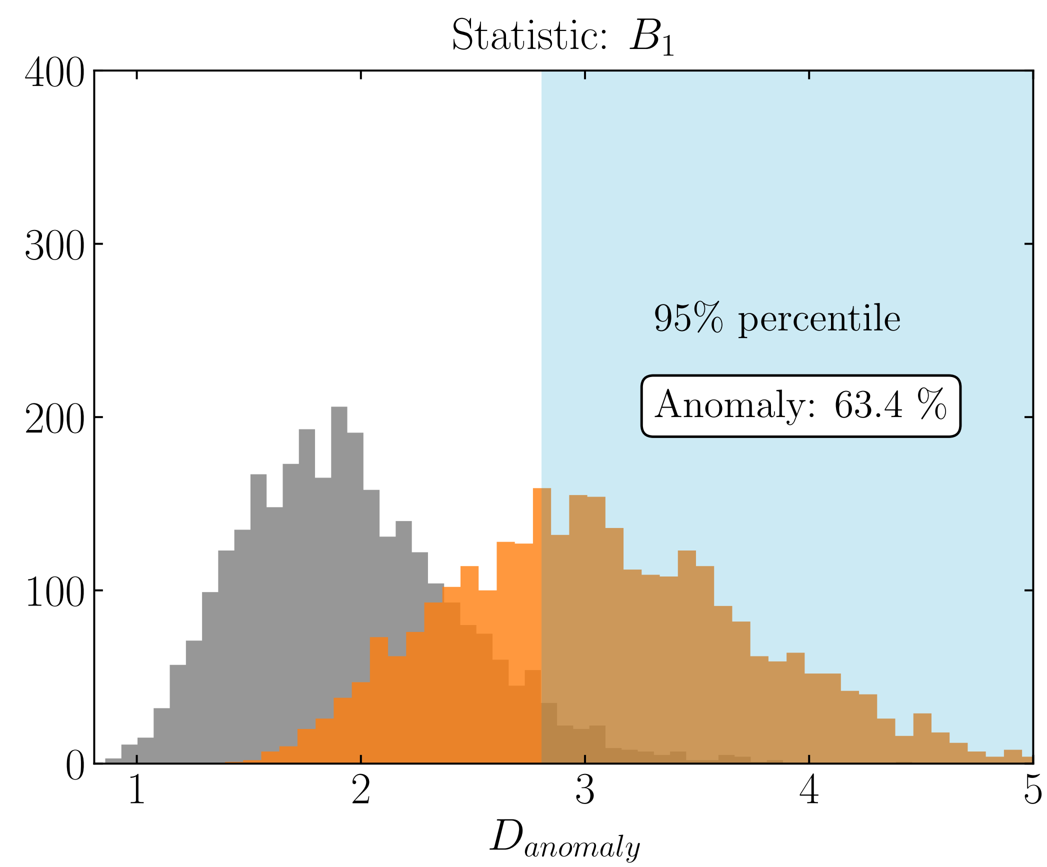
Anomaly detection

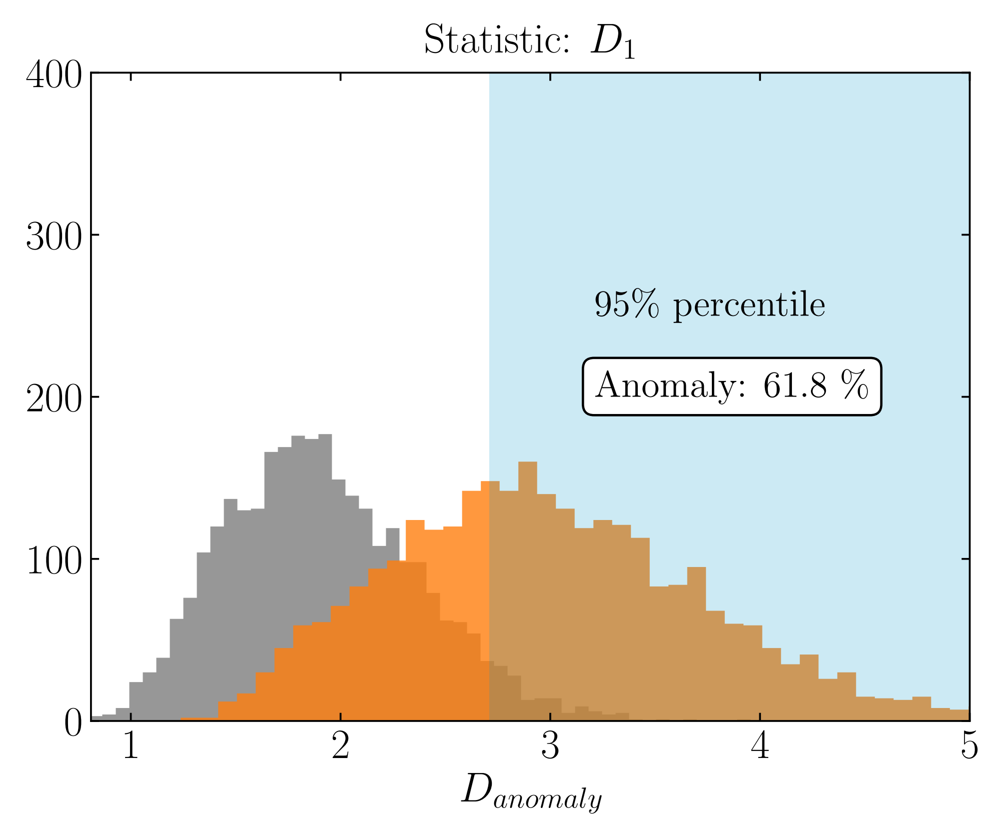
- Anomaly detection: given a statistic X from a simulation with $f_{\rm NL}^{\rm loc} = 10$, compute probability that it arises when $f_{\rm NL}^{\rm loc} = 0$.

$$D^X_{\rm anomaly} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(d_i - \mu_i)^2}{\sigma_i^2}}$$

- Compare to results from fiducial cosmology to account for cosmic variance. Use to set threshold for anomaly, e.g. at 95 percentile.

- Features scales ~10 Mpc/h, so complementary to usual scale dependent bias





Persistence of Large Scale Structure

Constraint using template

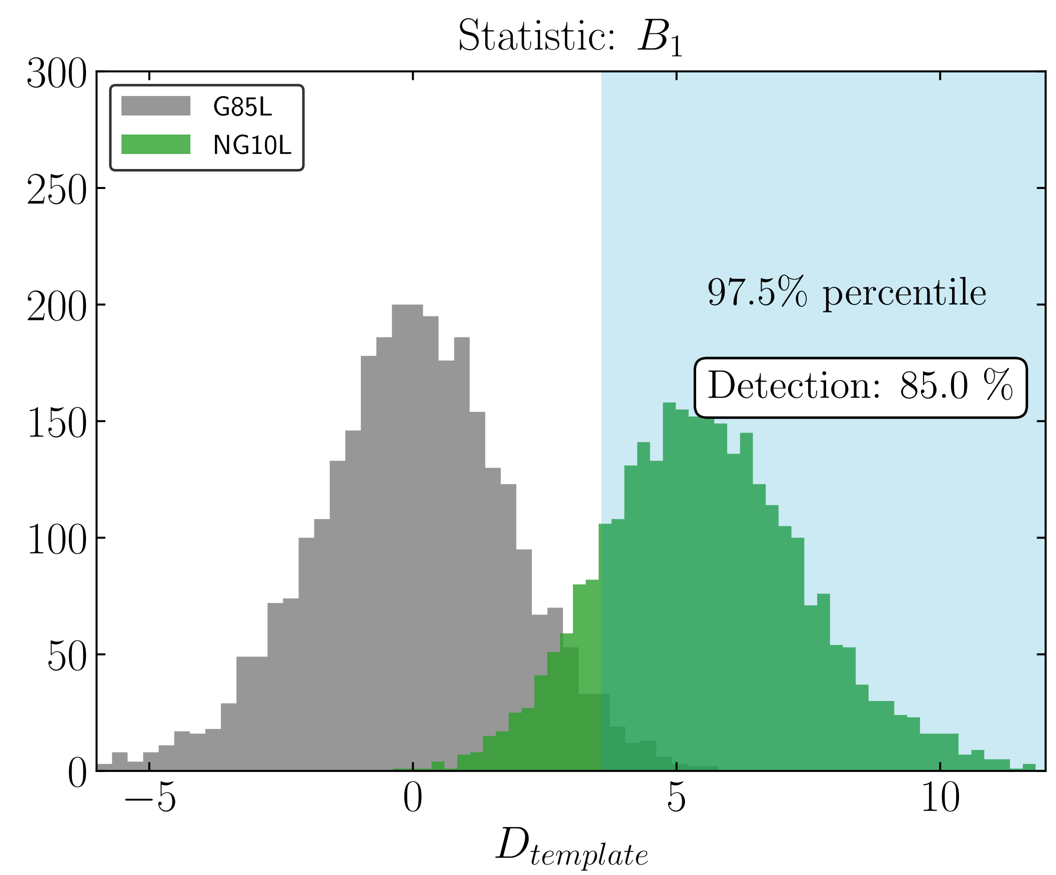
- Template method: compute templates corresponding to deviations from the fiducial cosmology.

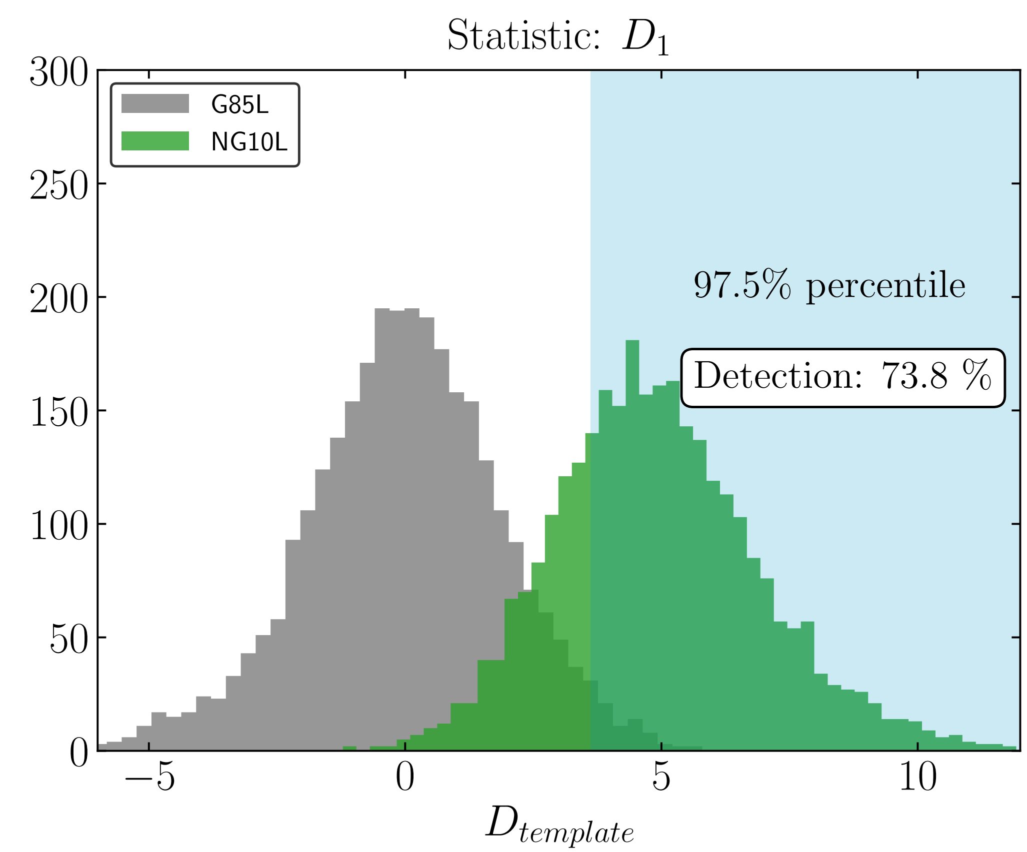
$$\overrightarrow{T^X} \equiv \frac{1}{N_r} \sum_{i=1}^{N_r} \overrightarrow{S}^X_{NG_i} - \overrightarrow{S}^X_{G_i}$$

- Compare D_{template} to results from fiducial cosmology to account for cosmic variance. Set threshold for detection at e.g. 97.5%.

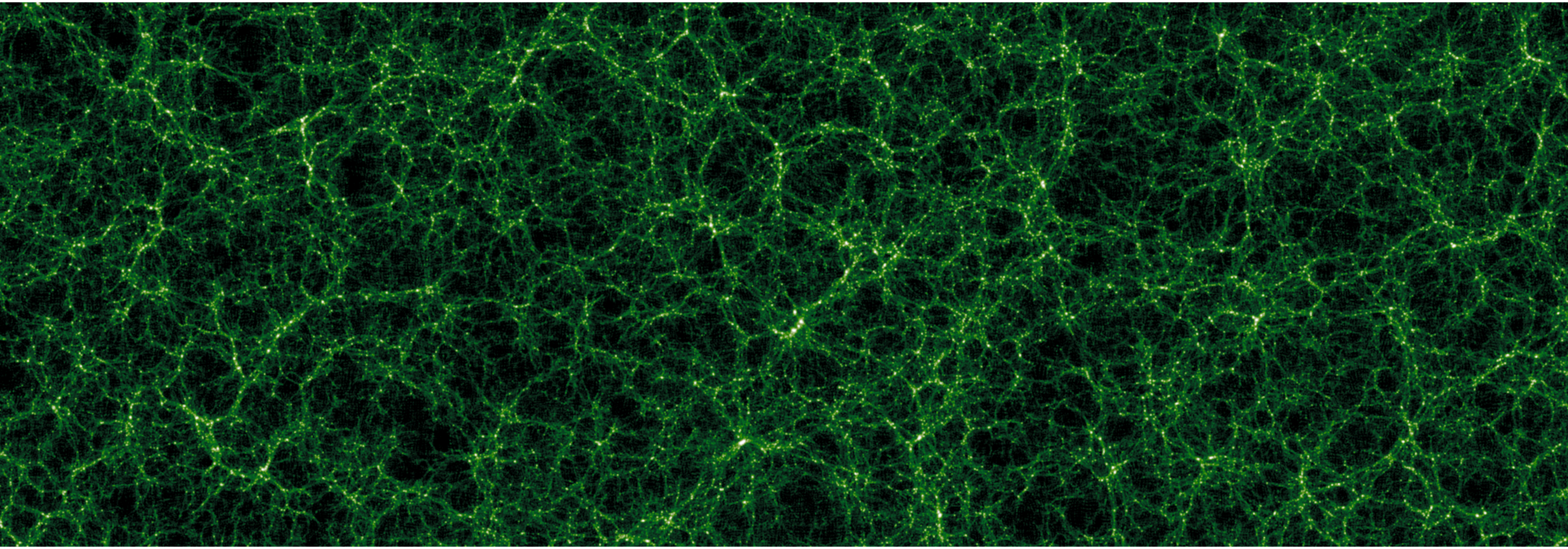
$$D_{\text{template}} = \frac{\vec{S}_{\text{survey}} \cdot \vec{T} - \vec{S}_{\text{fid,avg}} \cdot \vec{T}}{\sigma}$$

- Results improve because our anomaly detection model was simplistic.





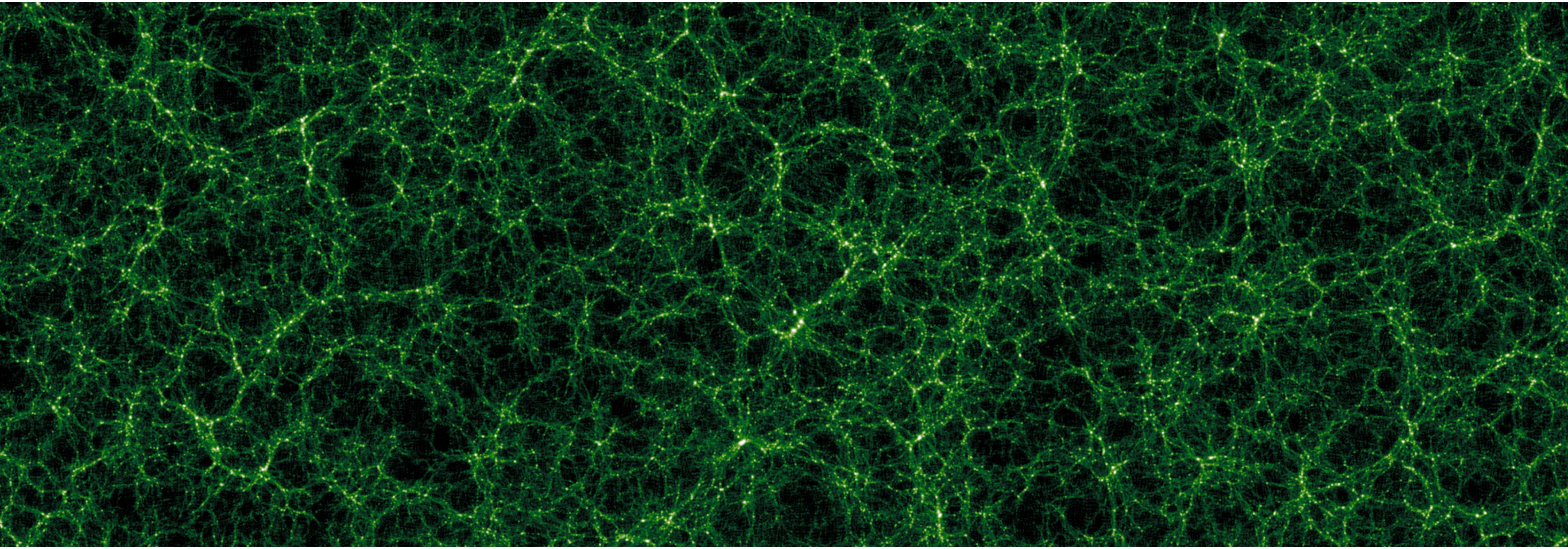
Persistence of Large Scale Structure



Main results:

- See $f_{\mathrm{NL}}^{\mathrm{loc}} = 10$ with a degree of significance
- Physics at complementary scales wrt conventional methods (large volumes, but not so large scales)
- Pipeline applicable to any cosmological parameters

Persistence of Large Scale Structure



Future plans

- Constraining cosmological parameters
- Real data
- Other types of primordial non-Gaussianity