Inflation & the origin of perturbations

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classical : rapid expansion i.th. early Universe → homogeneity on large scales

quantum : quantum fluctuations become classical → seeds for inhomogeneities

Overview

1) • recap : some GR
   • Big Bang Puzzles & cosmic inflation

2) • classical background dynamics : slow-roll inflation
   • cosmological perturbation theory

3) • quantum fluctuations (during inflation)

4) • post-inflationary evolution

5) • tests of cosmic inflation

Literature

+ "Advanced Cosmology" ↓

- Daniel Baumann, arxiv: 0907.5424, "Inflation"
- Scott Dodelson, "Modern Cosmology"
- Julien Lesgourgues, arxiv: 1302.4640, "Cosmological Perturbations"
- Michele Maggiore, arxiv: gr-qc/9909001, "Gravitational waves"

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Lecture I : Big Bang Puzzles

1) Some GR

- homogeneity + isotropy $\longrightarrow$ FRW universe

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$$= a^2(\tau)\left[-d\tau^2 + \left(\qquad\right)\right], \quad d\tau = \frac{dt}{a(t)}$$

- Einstein's equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{FRW metric}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{perfect fluid}}$$

$T^{\mu}{}_{\nu} = \mathrm{diag}(\rho, -p, -p, -p)$

Friedmann equations:

H = Hubble parameter

(F1) $$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{1}{3}\rho - \frac{k}{a^2}$$

(F2) $$\frac{\ddot a}{a} = \dot H + H^2 = -\frac{1}{6}(\rho + 3\underbrace{p}_{w\rho})$$

- Units: $\hbar = 1$, $c = 1$, $M_p = \sqrt{\frac{\hbar c}{8\pi G}} = 1$

[some length $= \ell$. convert to meters?

SI units: $c = 3 \times 10^8 \frac{m}{s}$

$\hbar = 1 \times 10^{-34} \frac{kg\, m^2}{s}$

$G = 6.7 \times 10^{-11} \frac{m^3}{kg\, s^2}$

$\rightarrow$ construct something with unit "m":

$$\left[\sqrt{\frac{\hbar \cdot 8\pi G}{c^3}}\right] = \sqrt{\frac{kg\, m^2}{s} \frac{m^3}{kg\, s^2} \frac{s^3}{m^3}} = m$$

$$\rightarrow \ell \cdot 1 = \ell \cdot \sqrt{\frac{\hbar\, 8\pi G}{c^3}} = \ell \cdot 8 \times 10^{-35}\, m$$

$\rightarrow$ SI units unambigiously reconstructed]

- time evolution

$$(F1),(F2) \rightarrow \frac{d\rho}{dt} = -3H(\rho+p)$$

$$\hookrightarrow \frac{d\rho}{da} = \frac{d\rho}{dt}\frac{1}{\dot{a}} = -\frac{3\rho}{a}\left(1+\underbrace{\frac{p}{\rho}}_{\equiv w}\right)$$

w equation of state $= \text{const}$

$$\hookrightarrow \frac{d\rho}{d\ln a} = -3(1+w) \rightarrow \rho \propto a^{-3(1+w)}$$

non-relavistic matter $\quad p=0 \rightarrow w=0 \rightarrow \rho = a^{-3}$ $\quad$ ideal gas in expanding volume

radiation $\quad p=\frac{1}{3}\rho \rightarrow w=1/3 \rightarrow \rho = a^{-4}$ $\quad$ + redshift

$$k=0 \xrightarrow[(F1)]{} a(t) \propto t^{\frac{2}{3(1+w)}}$$

$\rightarrow$ for $w > -1$ this implies signlarity, $a=0$

"Big Bang"

2) Big Bang Puzzles

• Horizon problem

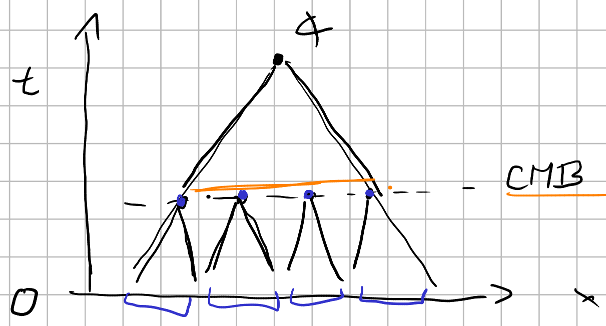
past light-cone, region of causal contact

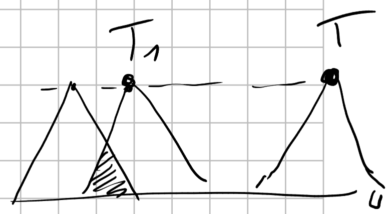
$$\tau = \int_0^t \frac{dt'}{a(t')} \underset{H=\frac{\dot a}{a}}{=} \int_0^a \frac{da}{Ha^2} = \int_{-\infty}^{a} d\ln a \, \frac{1}{aH} \quad 1/\dot a$$

(F2): $\ddot a = -\frac{1}{6} a \rho (1+3w) < 0$ for $w > -1/3$

→ $1/\dot a \nearrow t$

→ fraction of universe in causal contact increases with time





→ CMB as observed today consists of $\sim 10^5$ regions, which were never in causal contact

→ why do they al have the same temperature?

- Flatness problem

$$(F1) \to 1 - \underbrace{\overbrace{\frac{\rho(a)}{3H^2(a)}}^{\equiv \Omega(a)}}_{\substack{\Omega(a_0)\simeq 1 \\ \text{observed} \\ \text{today}}} = -k \underbrace{\frac{1}{(aH)^2}}_{\substack{\text{grows with time} \\ \text{for } \omega > -1/3}}$$

$$\Omega(a_0) \simeq 1 \to |\Omega(a_{BBN}) - 1| \leq \mathcal{O}(10^{-16}) \text{ for } k \neq 0$$

$\to$ another severe fine-tuning problem for $k \neq 0$

- Bonus: explains inhomogeneities in the CMB

- Solution

period of decreasing comoving horizon in the cosmic past

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \iff \omega < -1/3$$ "negative pressure"

$$\overset{(F2)}{\iff} \ddot{a} > 0$$ "accelerated expansion"

$$\frac{\ddot{a}}{a} = H^2\Big(1 + \underbrace{\frac{\dot{H}}{H^2}}_{\equiv -\varepsilon}\Big) \iff \varepsilon < 1$$ "slowly varying Hubble parameter"