
Scientific Report 2020

Title: Pattern formation in coral reefs
Postdoc: Vivian de Araujo Dornelas Nunes
Supervisor: Nathan Jacob Berkovits
Fapesp process: 2020/04751-4
Period: 01/05/2020 a 30/11/2021
Reporting period: 01/05/2020 a 30/11/2020

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Project progress

The organization of biological populations that diffuse, interact, and compete for resources is an important question in ecology. In particular, it is relevant to understanding how populations behave in different environmental contexts. During the first months of the project, two different scenarios have been studied.

In the first, in collaboration with Celia Anteneodo (PUC-RIO), Maike A. F. dos Santos (PUC-Rio), and Eduardo H. Colombo (Princeton University), we analyzed how heterogeneous diffusion can reduce the critical patch size (i.e., the minimum patch size that allows population persistence) of a single-species population living in a patchy landscape.

In the second, in collaboration with Ricardo Martinez-Garcia (IFT-UNESP/ICTP-SAIFR), we are analyzing the reproductive and movement behavior of fluid-thriving microbial populations in the presence of open chaotic flows.

- **Role of heterogeneous diffusion on a population survival**

Some species live in irregular landscapes in which only certain regions provide individuals with resources, shelter and other key ingredients for survival. In addition, habitat fragmentation and degradation, accelerated by human activities, has been producing significant impacts on ecosystems, leading many species to extinction. Therefore, it is important to understand how the spatial characteristics of the habitat influence the survival of species. Focusing on a single patch, an important problem in ecology is determining the critical patch size for species survival.

In this work, we investigated the impact of heterogeneous diffusion, using Stratonovich's calculus, on the minimum patch size that allows the population to survive. Combining numerical simulations and analytical calculations, we showed that, in general, this critical size is smaller than that of a homogeneous medium with the same average diffusivity. Numerically, it is also possible to compare this result with that using Itô calculus (which promotes the opposite effect), and to the Hänggi-Klimontovich interpretation of the dynamics (which reinforces this effect).

This work was published in PHYSICAL REVIEW E.

- **Microbial population dynamics in turbulent environments**

Public goods (PG) are molecules produced by certain microbes that are released into the environment and provide a reproductive benefit not only for the producers themselves, but also for other individuals (free-riders). An example of PGs are the digestive enzymes secreted by certain bacteria to break down complex polymers into simpler carbon sources that can be imported and catabolized more easily by the cells.

In this project, we are interested in studying the ecology and evolution of a microbial community living in an aqueous environment (including turbulent flows) and made of a PG producer and a non-producer species. Aqueous environments provide a very interesting

environmental context to study PG production, because the external flow propagates both microorganisms and the substances they release and may generate very complex and unexplored dynamics.

Currently, we are exploring these questions using a mathematical model that uses reaction-advection-diffusion equations for the concentration PGs and a stochastic birth-death process for the population dynamics.

The final goal is to understand how the key environmental and physiological parameters of the model affect the evolution and growth of producers and free-riders.

Next steps

After understanding numerically how each process (biophysical, environmental, ecological and evolutionary) influences the behavior of the community, we will develop analytical approximations to our results. Finally, we expect to apply the model to real ecosystems.

Published article

DOS SANTOS, M. A. F. ; DORNELAS, V. ; COLOMBO, E. H. ; ANTENEODO, C. *Critical patch size reduction by heterogeneous diffusion*. PHYSICAL REVIEW E, v. **102**, p. 042139, (2020).

Scientific Report

Title: Quantum measurement simulability and applications to Bell nonlocality

Postdoc: Leonardo Guerini

Supervisor: Nathan Jacob Berkovits

Period: 01/Dec/2019 - 31/Mar/2020

Fapesp project: 2018/04208-9



Nathan Jacob Berkovits



Leonardo Guerini



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1 Project and previous work

As reported previously, the project was modified to address the topic of quantum computation. More specifically, we are developing a technique for optimally make use of classical manipulations within the run of a quantum circuit, in the current paradigm of NISQ (noisy, intermediate scale quantum) devices [1], where error correction is still unavailable. The main idea is to substitute the noisiest operations by more accessible and better controlled ones, making use of a classical stochastic process represented given by a quasiprobability representation [2].

Such technique is quite general, but it comes with a cost in terms of sampling complexity. I.e., one would need to run the classically-extended quantum circuit a greater number of times than the original one in order to obtain a better precision on the final outcome. Together with the development of computational programmes, the analysis of the trade-off between sampling complexity and outcome precision is the central object of our current research.

2 Research activities

In these last months we have been writing a programme to simulate quantum circuits in three distinct scenarios: (i) with an ideal quantum computer, (ii) with a realistic, noisy quantum computer as the ones currently available, and (iii) with a noisy quantum computer assisted by classical interventions, meaning that instead of performing a target quantum operation we would measure the systems, perform a stochastic classical protocol based on the quasiprobability representation of such operation in terms of Pauli operators, and reprepare the systems accordingly. Every quantum computation aims at generating a certain expectation value, obtained from a large number of implementations of the circuit. Hence, our aim is to compare the error generated by scenarios (ii) and (iii) when compared with the ideal case (i). Asymptotically, scenario (iii) will produce more accurate statistics than scenario (ii), since the quasiprobability representation yields the precise expectation value as long as a large number of samples is considered. However, we risk to demand an exponential sampling complexity, depending on which operation we choose to simulate.

The main application we have in mind is the simulation of the swap operator. This is a classical operation that simply interchanges two systems, as in $|\psi\rangle \otimes |\varphi\rangle \rightarrow |\varphi\rangle \otimes |\psi\rangle$. Hence, it presents a cheap cost, arising simply from its decomposition into Pauli operators. Nevertheless, it is an essential operation specially due to the restricted connectivity of the systems in a quantum circuit: operations between distant qubits imply the performance of a series of swaps, usually performed quantumly, in terms of controlled-not gates.

Specific applications are the swap test [3], which can be used to calculate the fidelity between two unknown quantum systems, and the quantum Fourier transform, that appears as an fundamental ingredient in more complex circuits such as Shor algorithm. Regarding the swap test, our results for systems of two

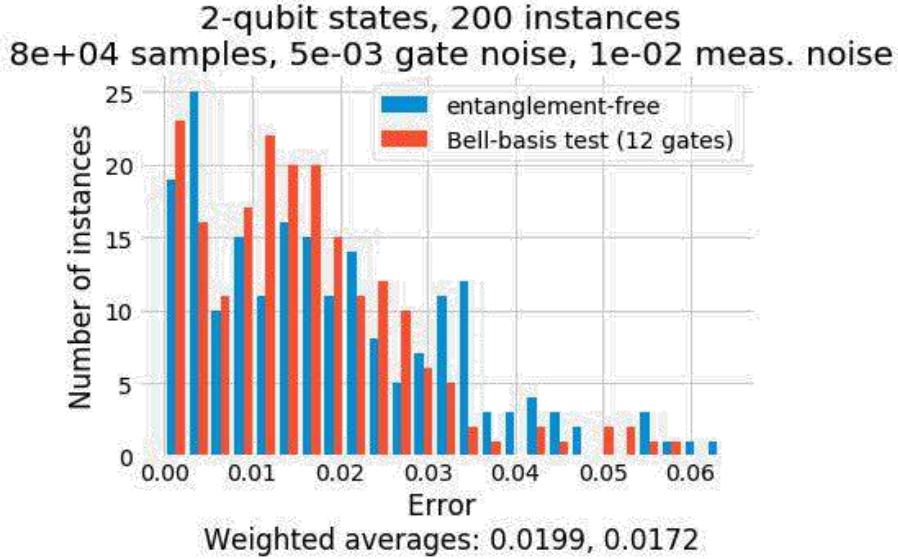


Figure 1: Comparison between the Bell-basis method for implementing the swap test and our entanglement-free method. The histogram displays the error in the estimation of the fidelity between each of 200 pairs of 2-qubits states, where 80,000 samples were taken in the presence of depolarising noise of 0.005 on each gate and 0.01 on the measurements, following up-to-date parameters [4].

and three qubits are promising. We find that a reasonable number of samples already provide a precision comparable to the best known algorithm [3], which is experimentally demanding for requiring a Bell-basis measurement of all involved qubits. Our method, in turn, requires no entanglement. An illustration of our preliminary results is showed in Fig. 1.

However, the number of gates in the circuit grows exponentially with the number of qubits in the systems under study. Hence, a standard computer already runs out of computational power when performing the swap test for 4-qubit systems. This is partially due to the fact that we are choosing uniformly random system to study; we then realised that switching to the tensor networks paradigm [5] we would largely improve our efficiency. This is an approach to many-body systems that allows one to efficiently represent a restricted yet physically more relevant class of systems, whose entanglement is more localised. The improvement and translation of our computational tools to this paradigm is the main task we are working on currently.

As a side project, in a recent visit to Universidade Estadual de Camp-inas (Unicamp), together with Prof. Rafael Rabelo (Unicamp) and in collaboration with Prof. Rafael Chaves (Universidade Federal do Rio Grande do Norte/International Institute of Physics) we started a project to study local-hidden-variable models for the prepare-and-measure scenario. This would allow

us to certify non-entanglement breaking channels that can be simulated by classical channels in this scenario, in analogy with entangled quantum states that admit a local-hidden-variable model and, hence, are useless in the Bell scenario [6].

3 List of publications and ongoing projects

We now summarise the finished and ongoing projects developed during the whole postdoctoral scholarship.

- In collaboration with Dr. Marco Quintino (Univ. of Tokyo) and Prof. Leandro Aolita (UFRJ), we proved an equivalence between restricted compatibility of quantum measurements and the certification of quantum channels [7]. More specifically, we showed that the statistics of a quantum experiment can be sampled in a distributed fashion without quantum communication if and only if the measurements implemented are compatible when restricted to the quantum states being measured. This provides a strategy for attesting the quantumness of a communication channel in a practical experiment, given that the preparation of the system is trustworthy. In other words, we presented semi-device independent quantum communication witnesses based on incompatibility. When compared to recent channel-certification experimental works [8], our technique requires less assumptions and simpler set-ups..
- In collaboration with Prof. Alexandre Baraviera (UFRGS), we showed a generalisation of Birkhoff-von Neumann theorem addressing the mathematical description of quantum measurements, i.e. positive-operator-valued measures [9]. Perhaps surprisingly, we find that measurement compatibility is a necessary and sufficient condition for obtaining a characterisation similar to Birkhoff-von Neumann's. The paper was submitted to Journal of Mathematical Physics and is currently under review.
- In a joint work with Prof. Leandro Aolita (UFRJ), Prof. Rafael Chaves (UFRN/IIP), Dr. Daniel Cavalcanti (ICFO, Spain), and the experimental group led by Prof. Fabio Sciarrino (Sapienza Univ. di Roma) we investigated the certification of private randomness in the instrumental scenario, exploiting a classical-quantum gap recently reported in this framework [10]. By verifying that the Entropy Accumulation Theorem can be safely extended to the instrumental scenario, we proved the security of our strategy against the most general adversary (that could be quantum-correlated to the system). The paper containing these theoretical and experimental results is currently under review on Communications Physics.
- A follow-up from our work on certification of quantum channels is currently being finished, together with Dr. Marco Quintino (Univ. of Tokyo). There, we use semi-definite programmes to quantify the classical and quantum communication cost of distributively sampling quantum statistics, by

means of measuring the minimum number of classical bits and the entanglement of the quantum system used as resource, respectively.

- Our latest main project was already described in this report, regarding the usage of quasiprobability representations to mitigate the noise of quantum circuits with a measure-and-reprepare classical protocol. This is an ongoing collaboration with Prof. Leandro Aolita (UFRJ) and Juan Carrasquilla (Vector Institute, Canada).
- Finally, our most recently started project, together with Prof. Rafael Rabelo (Unicamp) and Prof. Rafael Chaves (UFRN/IIP) investigates a notion of local-hidden-variable model for the prepare-and-measure scenario, as already depicted in this report.

4 Scientific meetings

- Universidade Estadual de Campinas. Scientific visit to Prof. Marcelo Terra Cunha and Prof. Rafael Rabelo. Presentation of a talk entitled “Simulation of quantum circuits via quasiprobability representations”. December, 2019.
- Universidade Federal do Rio de Janeiro. Scientific visit to Prof. Leandro Aolita. December, 2019.
- Universidade Federal do Rio de Janeiro. Scientific visit to Prof. Leandro Aolita. February, 2020.

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SCIENTIFIC REPORT

BOOTSTRAP NÃO-PERTURBATIVA E A MATRIZ S

Posdoc: Andrea Leonardo Guerrieri

Supervisor: Nathan Berkovits

Fapesp process: 2017/02303-1

Period: 01/12/2019 to 31/08/2020

Andrea L. Guerrieri

Nathan Berkovits

1 Introduction

In the early days of particle physics, the self-consistent Bootstrap approach based on sound physical principles as causality, crossing and unitarity seemed to be the only way to describe the myriads of resonances produced in nuclear collisions. Those first attempts, culminated with the rigorous proof of the Froissart-Martin bound and with the Veneziano amplitude, have given a crucial boost both to the birth of string theory and to provide a solid framework to fit experimental data. Despite the many non-perturbative methods developed to tackle them, determining scattering amplitudes in strongly coupled theories, such as QCD, or Ising Field Theory still remains a challenge for theoretical physics.

2 The S -matrix bootstrap program

During the period that goes from the 01/12/2019 to the 31/08/2020 I have devoted my efforts to the development of the numerical S -matrix Bootstrap approach. Inspired by the success of the Conformal Bootstrap program, this approach aims at bounding the space of possible S -matrices compatible with basic QFT assumptions using new efficient numerical algorithms for convex optimization. [1] The program was initiated by the study of $2 \rightarrow 2$ boson scattering in $1 + 1$ -dimensions [2], and then extended to the generic D -dimensional case [3].

Since the start of my Postdoc, I have mostly worked on applying the existing numerical setup to some interesting physical theories such as the scattering of low energy QCD mesons [4], for instance pions, and the scattering of flux-tube branons [5]. In this last year I have extended the numerical setup to include the scattering of higher dimensional massless particles, such as Goldstones or SUGRA scalar particles, and multi-particle processes in $1 + 1$ dimensions. Moreover, I have developed a dual description of the Bootstrap approach that leads to rigorous bounds on couplings in $1 + 1$ dimensional QFTs.

Below I explain in detail the results achieved.

2.1 Bootstrapping Massless Effective Field Theories [6]

The scattering of massless particles at low energy is usually described by an Effective Field Theory (EFT). Of particular interest is the Chiral Effective Field Theory, which describes the scattering of the QCD pions in the massless limit when they behave as Goldstone Bosons of the $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ symmetry breaking pattern.

Beyond the leading universal behavior, the scattering of Goldstones depends on a number of unknown low energy constants that one should measure from experiments. I have bounded the first non-universal low energy constants (also called Wilson coefficients) appearing in the Chiral Lagrangian using the numerical non-perturbative Bootstrap.

The same numerical techniques can be applied to the scattering of particles belonging to the gravity super-multiplet in SUGRA theories in $d = 10, 11$. Preliminary numerical results suggest that the coefficient of the first irrelevant operator in graviton scattering in this theory is bounded from below, however it is still not possible to com-

pare this bound to any prediction from string theory since it is far from converging. I hope I will get more precise results in the near future.

2.2 Dual S -matrix Bootstrap [7]

The S -matrix Bootstrap in general is formulated as a convex constrained optimization problem. Such problems by their nature admit a dual formulation: if, for instance, in the primal version one maximizes an observable given some physical constraints, in the dual version one minimizes a dual observable in many cases in an unconstrained space of functions. The absence or the trivialization of the constraints makes the problem much easier to solve numerically.

In [7] I have started developing the dual formulation of the S -matrix bootstrap in $1+1$ dimensions for the scattering of particles of different masses. This will serve as a starting point to efficiently bootstrap efficiently popular physical theories such as Ising Field Theory or Thri-critical Ising.

3 Organization of events

- I am organizer of the S -matrix Bootstrap Workshop 2020 (which will be moved to 2021 due to the Covid-19 pandemic).

It is the fourth workshop of this kind focused on S -matrix related topics. Its main purpose is to gather experts on several different research fields related to the S -matrix and trigger their discussion. The topics covered span from String Theory S -matrix, to QCD phenomenology. I plan to invite top researchers in many related fields.

4 Conferences and Seminars

4.1 Conference Talks

Invited Talk: “Flux-Tube S -matrix Bootstrap”

Large N theories and strings, Princeton University, NJ, US

19 Feb. 2020

5 Publications

- A. L. Guerrieri, A. Homrich and P. Vieira, “Dual S -matrix Bootstrap I: 2D Theory,” [arXiv:2008.02770 \[hep-th\]](https://arxiv.org/abs/2008.02770).

6 Ongoing projects

At the moment I am also involved in two projects that are related to the work I have done during the whole Postdoc period.

6.1 Bootstrapping multi-branon amplitudes

The first project examines the scattering of multiple (more than four) flux-tube branons in $D = 3$ target space. This system exhibits the simplest non-trivial spontaneous symmetry breaking pattern. For this reason, it seems the perfect ground to explore the possibility of bootstrapping a system of S -matrices, including for the first time not only $2 \rightarrow 2$, but also $2 \rightarrow 4$ and $3 \rightarrow 3$ processes. Given this premise, it is clear that this is the most ambitious project I am pursuing. It is challenging both from theoretical and technological aspects, and a positive outcome is not guaranteed. However, the recent history of the Conformal Bootstrap shows that including multiple correlators is crucial to turn bounds into “islands”. As anticipated in [8], it is possible that the theory of large- N_c QCD flux-tube might be single out using the S -matrix Bootstrap including multiple correlators or refining the analytic properties we assume. I believe that such possibility is worth the effort and I plan to push forward this program in the near future.

6.2 Exploring complex spin constraints

It is a well established fact in theoretical physics that the analytic continuation of correlation functions or scattering amplitudes allows to relate the t -channel physics to the asymptotic s -channel behavior. I am currently exploring the analytic continuation of the unitarity constraints into the unphysical region projecting the amplitude in complex spin partial waves in a project in collaboration with A. Homrich, J. Penedones and P. Vieira. The aim of this project is two-fold: on one hand we are trying to understand the effect of complex spin constraints on the existing numerical bounds, on the other hand, since complex spin singularities are related to the high energy behavior of the amplitude, we hope to be able to relate better the IR physics to its UV completion.

7 Progress

As mentioned above, in the year 2019/2020 I put my efforts in developing new directions for the S -matrix Bootstrap field of research. To achieve that goal I needed to develop a plethora of different skills: analytical, numerical, physical and informatics.

I have run the bootstrap numerics in $d > 2$ on the cluster “symmetry” at the Perimeter Institute. The code I have used is SDPB-Elemental [9], the parallelized version of SDPB: a semidefinite programming solver using the interior-point method. In the process I have learnt how to prepare scripts that could run on several nodes and how to perform large scale computations.

The target theories we have tackled, namely chiral pions and SUGRA particles allowed me to deepen my knowledge of those effective field theories. I have also performed perturbative computations to understand at some level the singularity structure arising in the scattering of massless particles.

Moreover, I have studied in detail the dual theory of general optimization problems as summarized in [7] and the mathematics I have learned will be crucial in the future to investigate the dual formulation of higher dimensional $d > 2$ bootstrap setups.

8 Future direction

I truly believe the numerical S -matrix bootstrap explorations have something to teach us about the UV completion of effective field theories that might call for more sophisticated analytical methods. For this reason, I plan to study other phenomenologically relevant effective field theories and to develop more efficient numerical techniques. Here I report a list of possible future projects I plan to start.

- The most exciting novelty in this field is given by dual lagrangian techniques. The dual problem often turns out to be orders of magnitudes faster than the primal, therefore allowing to tackle more difficult numerics. In the near future I plan to develop more the dual formulation, especially in higher dimensions and for multi-particle problems where the primal numerics are already challenging for the simpler setups. Moreover the dual bounds are genuinely rigorous bounds, and I hope this will prompt a more general interest in this field of research.
- As my research has highlighted, one of the most promising applications to test the effectiveness of the novel bootstrap approach is low energy QCD. In my first paper I have explored somehow qualitative features of the space of low energy constants in π - π scattering. [4] However, it is certainly possible to bound the low energy constants quantitatively matching the low energy expansion of the bootstrap ansatz with the chiral lagrangian prediction. Moreover, it is easy to input the information on the physical spectrum and fix the values of some of the better known low energy constants to put even more strict bounds on the ones that are known less. In this way I may show how the S -matrix bootstrap can be of immediate help to a better determination of fundamental QCD parameters.
- As a last possible application I would like to mention the coupled system of a flux-tube and a glueball. This system would mix the S -matrix of branons with the form factor of the glueball coupled to the brane. This problem has been already studied in the context of the scattering of closed strings off a branon. It would represent a first attempt to generalize the positivity ideas on a mixed system of correlators and explore the possible phenomenological implications for the flux tube dynamics.

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Dual S-matrix Bootstrap I: 2D Theory

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Abstract

Using duality in optimization theory we formulate a dual approach to the S-matrix bootstrap that provides rigorous bounds to 2D QFT observables as a consequence of unitarity, crossing symmetry and analyticity of the scattering matrix. We then explain how to optimize such bounds numerically, and prove that they provide the same bounds obtained from the usual primal formulation of the S-matrix Bootstrap, at least once convergence is attained from both perspectives. These techniques are then applied to the study of a gapped system with two stable particles of different masses, which serves as a toy model for bootstrapping popular physical systems.

arXiv:2008.02770v1 [hep-th] 6 Aug 2020

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1 Introduction

Figure 1 is extracted from [1] and [2].

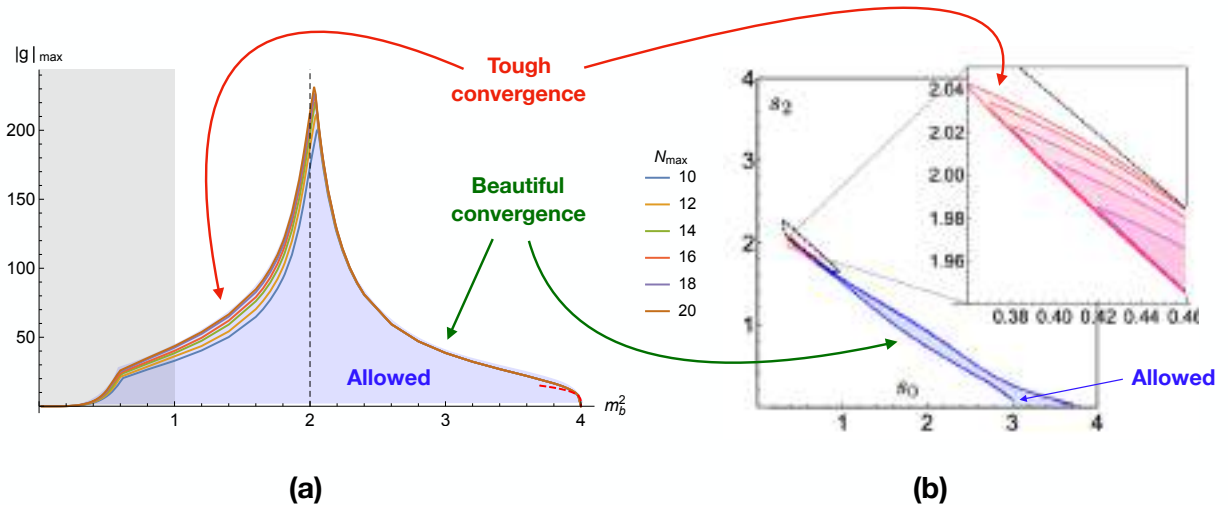


Figure 1: **a)** Maximal cubic coupling showing up in the scattering of the lightest particle in a gapped theory with a single bound-state (in this channel at least) [1]. Convergence is perfect when the bound-state mass (measured in units of the lightest mass) is bigger than $\sqrt{2}$ and quite painful otherwise. **b)** The allowed chiral zeroes space of putative pion S-matrices associated to an $SU(2)$ chiral symmetry breaking patterns draws a beautiful peninsula like object with a sharp tip [2].¹ Convergence is great almost everywhere except close to the tip where numerics struggle. In those cases where the primal problem struggles, having a dual rigorous bound would be a blessing. This paper is about such dual bounds.

These works explore the allowed space of physical 4D S-matrices. One parametrizes a vast family of S-matrices compatible with given physical and mathematical assumptions and maximize or minimize quantities within this ansatz to find the boundaries of what is possible. The more parameters the ansatz has, the better is the exploration. As the number of parameters become very large, one hopes that these boundaries converge towards the true boundaries of the S-matrix space.

Sometimes this works beautifully as illustrated in the figure; sometimes convergence is painful, to say the least, as also illustrated in the figure. In those cases where convergence is a struggle, what can we do? Sometimes, it is a simple matter of improving the ansatz; sometimes it is not clear what exactly is missing. And in either case, how can we ever tell how close to converging are we anyways?

A solution would be to develop a dual numerical procedure – called the dual problem – where instead of constructing viable S-matrices we would instead rule out unphysical S-

¹There are, at least, other two structures would benefit a dual description. One is the “pion lake” [2], found imposing the presence of the physical ρ resonance only. Another interesting and recent structure is the “pion river” [3], found imposing additional constraints on the scattering lengths arising from χ PT and monotonicity of the relative entropy. The dual formulation would allow to rigorously define these structures excluding theories not compatible with the assumed low energy QCD behavior.



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Scientific Report 2020

Movement strategies and population dynamics in heterogeneous environments

Postdoctoral Fellow: Gabriel Andrechetto Maciel

Supervisor: Nathan Jacob Berkovits

Fapesp process: 2019/21227-0

Period: 01/Jan/2020 - 30/Nov/2020

Nathan Berkovits

Gabriel Andrechetto Maciel

How spatial self-organization enables coexistence of two competing species

Competition is a widespread phenomenon with critical consequences for the composition of ecosystems. A common conclusion of early mathematical models, though, is that the coexistence between competing species is impossible or restricted to weak interactions. In order to understand how these apparently highly unstable systems are so ubiquitous in nature, in the past few decades a vast wealth of studies have looked for mechanisms present in real populations that could facilitate the coexistence of competing species. In this work we have explored in detail a coexistence mechanism based on the spatial self-organization of the populations.

We studied the dynamics of competition between two species that can interact locally and non-locally. Population dynamics were studied through kernel-based models, which are known to induce non-uniform distribution patterns of single species. Thus, denoting population densities of the two competitors at time $t > 0$ and position $x \in \mathbb{R}$ by $\rho_1(x, t)$ and $\rho_2(x, t)$, population dynamics are governed by the integrodifferential equations:

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} = & D_1 \frac{\partial^2 \rho_1}{\partial x^2} + b_1 \rho_1 \left(1 - \frac{\rho_1 + a_{12} \rho_2}{K_1} \right) \\ & - \left(\delta_1 \int G_{11}(|x - x'|) \rho_1(x', t) dx' + h_1 \int G_{12}(|x - x'|) \rho_2(x', t) dx' \right) \rho_1 \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial \rho_2}{\partial t} = & D_2 \frac{\partial^2 \rho_2}{\partial x^2} + b_2 \rho_2 \left(1 - \frac{\rho_2 + a_{21} \rho_1}{K_2} \right) \\ & - \left(\delta_2 \int G_{22}(|x - x'|) \rho_2(x', t) dx' + h_2 \int G_{21}(|x - x'|) \rho_1(x', t) dx' \right) \rho_2, \end{aligned} \quad (2)$$

where b_i and K_i are, respectively, the growth rate and carrying capacity of species $i = \{1, 2\}$. a_{12} and a_{21} are the coefficients of local interspecific competition as in the classical Lotka-Volterra model. The coefficients δ_i and h_i control the effects of non-local intra and interspecific competition, respectively. Finally, D_1 and D_2 are the diffusivities and the kernel function G_{ij} weighs the impact of species j on species i at different distances from central locations.

We have combined analytical and numerical techniques in the analysis of the above equations. Our results show that nonlocal interactions can lead to a rich variety of nonuniform distributions in two species competition systems. The self-organized patterns can exhibit, for example, in phase oscillations of coexisting competitors when local competition is weak and anti-phase oscillations otherwise. Most notably, the species coexist under conditions where competitive exclusion occurs in the usual local model. Coexistence here is manifested through a mechanism that involves the weaker competitor being able to explore areas where the stronger competitor population is sparse.

We are finalizing an article about this project and we might have it submitted to a scientific journal soon. I have also presented this work in a short talk on the annual

meeting of the Society of Mathematical Biology.

Jaguar movement in complex landscapes

We have also started a second project in which we are studying the movement of jaguars. We are analysing a GPS-based movement dataset for jaguars in different regions of Brazil (Morato, R. G., et al, 2018). We aim to build movement models to extract information on the movement behavior of this species. There are several interesting questions that can be explored in this context. For example, we are planning to investigate the effect of unsuitable regions on the movement behavior of jaguars. As we have data for different terrestrial biomes it would be interesting to compare the results for the different regions. Another feature of movement that we are planning to explore in the future is site fidelity of jaguars. For example, is there a single, or multiple, preferred locations to where jaguars are attracted to? Attraction towards the center of a home range have been modelled through Ornstein-Uhlenbeck (OU) processes (Fleming et al., 2014). We are planning to study models that include this type of processes and use, in addition, information about the landscapes. This project has been developed in collaboration with Roberto Morato from the “Instituto Chico Mendes de Conservação da Biodiversidade.”

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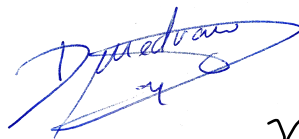
Modern Methods for Scattering Amplitudes in Field (Gauge Theories and Gravity) and String Theory

Postdoc: Diego Medrano Jiménez

Supervisor: Nathan Berkovits

FAPESP process: [2019/07286-3](#)

Reporting period: 1/12/2019 to 30/11/2020



Scientific Report 2020

(December 1, 2019 – November 30, 2020)

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1 Research

During the last year, my research has been developed in the context of the S-matrix bootstrap and integrability. In particular, these are the topics I have been working on related to the FAPESP position project:

1.1 Elliptic deformation (ED) of supersymmetric sine-Gordon (SSG)

The *S-matrix bootstrap* explores the space of possible consistent 2-dimensional QFTs with the presence of specific symmetries and delimits the boundaries for which the corresponding two-to-two S-matrices are *analytic*, *crossing symmetric* and *unitary*. Normally, the most interesting theories are those that saturate unitarity, which are located at the boundary of the allowed region, since most of them come with *integrability* as an extra feature.

In particular, within the space of \mathbb{Z}_2 -symmetric trilinear couplings studied in [1], an unknown theory was discovered and characterized as an integrable continuous deformation of SSG. It does not preserve supersymmetry but comes with an interesting periodic energy behavior described by some Jacobi elliptic functions and parametrized by the deformation parameter κ . The theory was called *elliptic deformation* (ED) of SSG and, although the exact S-matrix can be checked in [2], a lagrangian description is still missing. Part of my research project aims at obtaining the complete lagrangian.

First of all, and after some attempts, we managed to perform a perturbative expansion in κ of the S-matrix, allowing for a direct connection of the theory with its tree-level scattering amplitudes. The idea was to obtain the new couplings of the deformation and guess the corresponding interactions giving rise to those amplitudes. Crosscheck can be done as well to loop order. However the procedure is quite ambiguous since many vertices have the same contribution, and exploiting integrability turns out to be more effective in obtaining a lagrangian. See next sections.

1.2 Integrable deformations of SG and SSG: no-particle production

A theory is said to be integrable whenever its n -point S-matrix can be factorized into a product of two-to-two processes and those smaller matrices fulfill the Yang-Baxter equation. A direct consequence is that these theories have *no-particle production* at all orders. Therefore, imposing no-particle production from scratch allows in many cases to reconstruct the interactions and lagrangians of existing integrable theories. Examples of the identification of pure bosonic as well as some supersymmetric extensions can be found in [3,4] respectively.

In order to identify ED, we started looking at all possible Lorentz invariant structures (vertices) out of the field content of the undeformed theory SSG, and constructing all possible tree-level two-to-four amplitudes —i.e. imposing $M_{2\rightarrow 4} = 0$ —. This is just the first contribution of the perturbative computation, and we managed to match one of the solutions with the leading order in κ of the ED S-matrix. Next subleading terms in the effective lagrangian would come by imposing $M_{2\rightarrow 6} = 0$, however the problem of finding all possible 8-point interactions and generating the corresponding two-to-six tree-level diagrams becomes rapidly laborious.

Additionally, we also found a different solution to be consistent with the $T\bar{T}$ deformation of SSG. In general, $T\bar{T}$ is an integrable irrelevant deformation that can be computed for any theory from its stress-energy tensor $T_{\mu\nu}$ [5,6], that shifts every element of the S-matrix by a constant phase —i.e. CDD-factors—. It belongs to a broader family of integrable deformations known as X_s deformations [7], for which a lagrangian description is also still missing.

Thus, proceeding in an analogous way in the bosonic sector from no-particle production, we tried to identify integrable deformations of sine-Gordon (SG), finding the leading terms of the effective lagrangians for $T\bar{T}$ and X_3 .

In summary, we obtained expressions for the effective lagrangians of various SG ($T\bar{T}$ and X_3) and SSG ($T\bar{T}$ and ED) deformations. The next step is to go beyond perturbation theory and obtain whenever possible the lagrangians in closed form —i.e. resummed to all orders.

1.3 Integrable deformations of SG and SSG: Lax pairs

In the case of $T\bar{T}$, the closed form of the lagrangian can be obtained directly by solving the corresponding flow equation of the deformation [8]. We computed both $T\bar{T}$ -SG and $T\bar{T}$ -SSG, and compared them with the results from no-particle production.

In order to obtain the exact lagrangians of X_3 -SG and ED-SSG, we are now exploring integrability from a different point of view: the existence of a *Lax pair* [9]. If we start from the Lax pair of SG and SSG, the idea is to find all possible deformations that keep the flatness condition invariant [10]. This would allow us to write directly the equations of motion of the deformed theories and from there to guess the exact expressions for the lagrangians. Computations on this section are still ongoing work.

2 Seminars and conferences

- Zoomplitudes 2020. May 11-15, 2020.
- Bootstrap 2020. June 1-26, 2020.
- Bootstrap School 2020. June 8-12, 2020.
- Strings 2020. June 29 - July 3, 2020.
- Snowmass Theory Frontier meeting. July 30, 2020.
- Integrability in Gauge and String Theory IGST 2020. August 24-28, 2020.

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Scientific Report 2020

Title Machine Learning techniques applied to Cosmological Problems
Postdoc Martin Emilio de los Rios
Supervisor Nathan Berkovits
FAPESP process 2019/08852-2
Reporting period 1/12/2019 to 30/11/2020

Nathan Berkovits



INTERNATIONAL CENTER FOR THEORETICAL PHYSICS
SOUTH AMERICAN INSTITUTE FOR FUNDAMENTAL RESEARCH

1 Introduction.

Machine learning techniques represents a new way of analysing big datasets in an agnostic and homogeneous way. These methods are very useful and powerful tolls to find patterns and relations between the variables that are involved in an specific problem. These techniques have been applied with a lot of success in technological problems and in other different areas of science, including astronomy and physics.

2 Research.

- **Reconstruction of cosmological parameters through machine learning techniques:**

The main goal of this project is to analyze the cosmic microwave background with machine learning techniques. Specifically I developed an auto-encoder (neural network with a bottle-neck layer) with a custom loss function that ensure the physical interpretation of the latent-space variables. The advantage of such method is two-fold, on one hand using the encoder we can estimate the best-fit cosmological parameters from the cosmic microwave power spectra and on the other hand we can use the decoder as a forward model to estimate the full-posterior probability of the parameters in a much faster way than traditional methods. We are planning to submit the results of this work to its evaluation on an international journal in the first months of 2021.

- **Back-splash galaxies classification:**

In this project I am working in collaboration with colleagues from Argentina and Chile. The main goal is to identify back-splash galaxies in phase-space (distance to the cluster center vs relative line of sight velocity between the galaxy and the cluster). Specifically we build a data-set of simulated galaxies using the semi-analytical model developed by *Cora et al.* [1] applied to the Multi-Dark cosmological simulations [4]. Using this data-set we trained different machine learning methods finding better results that the obtained using the traditional methods for the classification of this kind of galaxies. This project give place to a paper that it is accepted for its publication on the international journal MNRAS, and to a public code that it is available in my [github repository](#).

- **Estimation of dark matter in spiral galaxies:**

The main goal of this project is to estimate the dark matter distribution in spiral galaxies using deep learning techniques applied to the photometric image of the galaxies and the velocity field map of the galaxy gas. During the first part of this project we build a data-set of simulated photometric images and velocity fields of spiral galaxies and trained deep learning methods to estimate the dark matter distribution achieving $\approx 20\%$ of precision in al the relevant radii. In the next stage of the project we will add different observational effects (resolution effects of the telescope, shot noise, spread in the inclinations and distances of the galaxies, etc.) to the data-set and analyze the uncertainties that these effects may introduce. We will also plan to compare the results of deep learning methods with the traditional rotation curves techniques. Is important to remark that this work is in collaboration with Dr. Fabio Iocco, Dra. Francesca Calore, Dr. Bryan Zaldivar and Dr. Mihael Petac.

- **Deep learning analysis of strong lensing images.** The standard cosmological model with a cold dark matter component predicts several dark substructures with masses bellow $10^8 M_\odot$ that are not heavy enough to maintain the necessary gas to form stars, and hence they can only be detected through their gravitational effects. Strong lensing images are one of the most

interesting observables where to look for the signal of such subhalo population. In this project we will create a synthetic dataset of strong lensing images, varying the lower-mass cut-off of the subhalo population, and then we will explore the possibility of measuring this cut-off with deep learning techniques.

3 Conferences and seminars.

- 62° Annual Meeting of the Argentinian Astronomy Society. **Oral Contribution.** From 13/10/2020 to 16/10/2020. Rosario-Argentina (by Videoconference).
- Deep Learning for Science School. from 07/2020 to 09/2020. Lawrence Berkeley National Laboratory (by Videoconference).
- II Joint Trieste/SAIFR School on Particle Physics. From 22/06/2019 to 02/07/2019. ICTP-SAIFR. Sao Paulo-Brazil (by Videoconference).
- First School on Data Science and Machine Learning. From 16/12/2019 to 20/12/2019. ICTP-SAIFR (Sao Paulo-Brazil).
- 2nd Latin American School on Parallel Programming for High Performance Computing. From 02/12/2019 to 13/12/2019. ICTP-SAIFR (Sao Paulo-Brazil).

4 Other Activities.

During this year Dr. Antonino Troja and myself organized the weekly Journal Club on Cosmology and Astrophysics of the ICTP-SAIFR/IFT-UNESP.

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Research Report

Antonino Troja

Academic activities

Summary

Observations of the Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) of the Universe allow us to better understand the physics that drives the evolution of the primordial perturbation, generated after the Big Bang, all the way to the large-scale distribution of galaxies we see today, leaving, however, important open problems. The latest Planck results show that the Universe is composed only for the $\approx 4\%$ of the total energy density by ordinary baryonic matter. The remaining 96% is unknown, composed by non-baryonic matter (the so-called dark matter $\approx 26\%$) and the mysterious dark energy ($\approx 70\%$).

It is impossible to directly observe dark matter, since it interacts only gravitationally. However, it is possible to relate the distribution of the visible matter (galaxies) with the distribution of the total matter content by writing

$$\delta_m \approx b_1 \delta_g + \frac{1}{2} b_2 \delta_g^2,$$

where δ_m is the total matter density distribution, δ_g is the galaxy density distribution, b_1 and b_2 are the linear and quadratic bias parameters respectively. This relation is local and it is satisfied by the galaxy distribution only on average, since there is a significant scatter among different regions of the Universe.

Since the galaxy distribution is the only observable in photometric and spectroscopic surveys, we need to know as precisely as possible the relation between dark matter and galaxies. To this end, it is crucial to understand the statistical properties of the galaxy distribution. At first order, the galaxy distribution is analyzed by taking its power spectrum, i.e. the Fourier transform of the 2-point correlation function, which parametrizes the excess probability of finding two galaxies at a certain distance. Unfortunately, the amplitude of the power spectrum (parametrized by the amplitude of dark matter fluctuations at $8h^{-1}\text{Mpc}$, σ_8) is degenerate with the bias parameters, leading to weak constraints. For this reason, the bispectrum is now one of the main tools in constraining cosmological parameters. The bispectrum is the Fourier counterpart of 3-points correlation function, which parametrizes the excess probability of finding three galaxies in a given triangular configuration. The dependence of the cosmological parameters on a particular triangular configuration (equilateral, isosceles, squeezed, etc...) allows to constrain different parameters by taking different bispectrum configuration, thus removing the degeneracy.

Activity performed

Galaxy Clustering in Harmonic Space from the Dark Energy Survey Year 1 Data

In collaboration with Prof. Rogerio Rosenfeld, Dr. Felipe Andrade-Oliveira and Dr. Hugo Camacho

We perform an analysis in harmonic space of DES-Y1 galaxy clustering data using products obtained for the real space analysis. We test our pipeline with a suite of lognormal simulations, which are used to validate scale cuts in harmonic space as well as to provide a covariance matrix that takes into account a realistic DES-Y1 mask. We then apply this pipeline to DES-Y1 data taking into account survey property maps derived for the real space analysis. We compare with real-space DES-Y1 results obtained from a similar pipeline. We show that in hindsight the harmonic space analysis we develop yields results that are compatible with the real space analysis. This verification will pave the way to performing an independent harmonic space analysis for the upcoming DES-Y3 data.

State of the art: Draft ready to be submitted to Dark Energy Survey (DES) Collaboration internal review (step to be done before actual submission to a scientific journal)

Dark Energy Survey Year 3 Results: Covariance Methods

In collaboration with Dark Energy Survey (DES) collaboration

A key ingredient in the estimation of cosmological parameters in a given model using data is the covariance matrix that describes the correlated uncertainties in the data.

There are several methods to estimate covariance matrices that can be divided in three main categories: from data itself (in methods such as subsampling, jackknife and bootstrap), from simulations and from theory. In this paper we will follow the methodology used in the analysis of the first year of DES data and adopt a theoretical covariance for the Y3 analyses that combine data from galaxy clustering and weak gravitational lensing in real space in three different two-point correlation functions, the so-called 3x2pt data vector. Several improvements were implemented with respect to the Y1 methods and will be discussed below.

We test and validate different theoretical covariance matrices against lognormal simulations using as a measure of performance the χ^2 distribution from the simulations. We also run Markov Chain Monte Carlo (MCMC) to check biases in the estimation of the cosmological parameters.

State of the art: Draft structure written and under development, test on covariance are still on-going.

Non-Gaussian likelihood in harmonic space

In collaboration with Prof. Rogerio Rosenfeld and Vinicius Terra

Usually, the large-scale structures analysis is performed using Bayesian Inference assuming a Gaussian likelihood, requiring a robust and precise Covariance Matrix. Recently some

papers showed the necessity of Non-Gaussian Likelihoods in order to improve the strength of power spectrum statistics. Let's consider the angular power spectrum C_ℓ :

$$\hat{C}_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2, \quad (0.1)$$

where $a_{\ell m}$ are the harmonic coefficients, described by a Gaussian Probability Distribution Function (PDF) with zero mean and variance C_ℓ .

Since the power spectrum is the product of two Gaussian variables, its PDF is described by the so called Gamma distribution:

$$\forall \ell : \hat{C}_\ell \sim \text{Gamma} \left[\frac{\nu(\ell)}{2}, \frac{2C_\ell}{\nu(\ell)} \right] \quad (0.2)$$

where C_ℓ and ν are the theoretical angular power spectrum and its number of degrees of freedom respectively. For a full-sky observation, we have $\nu(\ell) = 2\ell + 1$. But, for a fractional area of a full-sphere, the degrees of freedom are affected by the geometry of the un-observed sky and the discretization of the measured field. In this case, we can write the number of degrees of freedom as

$$\nu(\ell) = f_{sky} \frac{(2\ell + 1)}{\ell_{pix}} g_{eff}(\ell), \quad (0.3)$$

where f_{sky} is the observed fraction of the sky, ℓ_{pix} is the parameter due to the pixelization of the field and g_{eff} parametrizes the quality of our measurements.

We calibrate $g_{eff}(\ell)$ using 150 FLASK mocks for a full-sky observation with 5 redshift bins equally spaced between 0.15 and 0.90, with resolution $N_{side} = 1024$.

State of the art: First draft written and first results obtained. The next step will involve the writing of Vinicius Terra's Master thesis and then the submission to a scientific journal.

Additional Remarks

Due to the outbreak of coronavirus and the following/subsequent lockdowns implemented in almost all the countries of the World, the scientific activities suffered a significant slow down. Most of my work as well as my collaborators' work was aimed to the accomplishment of essential papers, relegating most of the projects to the role of ancillary activities.

Thus, two ready-to-submit papers plus two project were halted:

- Exploring the Needlet Trispectrum, In collaboration with Prof. Domenico Marinucci (Università Roma Tor Vergata)
During an internal review, one of the collaborators objected on the validity of the unbiasedness of the estimator, since on the null test the central value was not consistent with the input value by a few percent. We propose to perform a new analysis using a bayesian approach, but we haven't started it yet.
- Modeling the Angular Bispectrum in Photometric Galaxy Surveys, In collaboration with Dr. Emiliano Sefusatti (INAF) and Dr. Martin Crocce (ICEE)
Part of the paper was discussed first with and then reproduced by some DES collaborator who, shamelessly, repeated the analysis and submitted it first. The paper was staged for a revisitation that, unfortunately, is still ongoing.

- Optimal Multi-Tracer Bispectrum Estimators for the Large-Scale Structure and Analysis of LSS on the Disc, In collaboration with Prof. Raul Abramo (USP) and PhD students from USP
Got halted because of the students PhD duties.

Should one or more of the above project be eventually submitted to be published in a scientific journal, the FAPESP contribution will be acknowledged.

Future Activity

My idea is to analyze the photometric distribution of the Large Scale Structure of the galaxies (LSS), in order to constrain its *angular spectrum* and *bispectrum*. We can indeed treat the photometric galaxy distribution as a spherical field, allowing us to use angular statistics. Due to its power in removing degeneracy, in view of present and forthcoming photometric surveys like Dark Energy Survey (DES), Large Synoptic Survey Telescope (LSST) and Euclid, the angular bispectrum is likely to become one of the main tools in the analysis of photometric of datasets.

Activities within institute

I organized and run:

- AC/JC Journal Club, in collaboration with Fabio Iocco and Ekaterina Karukes, at IFT;
- Weekly meeting group, within the cosmology group here at IFT.

Conferences, workshops and meeting attended

20-23/07/2020 - DESC July 2020 Virtual Meeting

Scientific Report 2020

Title	Correlations in many-body systems from the perspective of DFT, QIT and RG
Postoc Supervisor	Krissia de Zawadzki Nathan Jacob Berkovits
Fapesp process	2020/13115-4
Period	01/Nov/2020- 30/Nov/2020

Nathan Berkovits

Krissia de Zawadzki



INTERNATIONAL CENTER FOR THEORETICAL PHYSICS
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Abstract

This report summarizes the activities developed by Krissia de Zawadzki as a visiting postdoctoral fellow at ICTP-SAIFR from October to November 2020. The main goal of her research project is to develop analytical and computational tools to study quantum matter systems in and out-of-equilibrium with particular interest in strongly correlated regimes. Her overall fellowship proposal aims to explore concepts of Quantum Information Theory (QIT) and integrated them with two well established methods for many-body problems, namely Density Functional Theory (DFT) and Renormalization-Group (RG). I report the progress on a research line focused on the quantification of the entanglement entropy and briefly introduce proposals for two projects focused on systems at finite temperature: one concerning measuring and tracing adiabaticity and another investigating the validity of density functional approximations for systems at temperatures close to critical regimes. Other projects to be developed during the next year are discussed. Other academic activities in which the researcher is involved are presented.

1 Research

1.1 Entanglement entropy in strongly correlated systems

One of the proposed lines presented in the postdoctoral research project has the ultimate goal of improving density functionals with non-local corrections to the exchange-correlation functional based on the concept of entanglement, which is at the core of Quantum Information Theory. Among the different ways to measure entanglement in quantum systems, I proposed to study the *concurrence* and *two-site fidelity*, which quantifies the entanglement between two-particles embedded in a larger system [1]. Initially, in order to understand how non-local correlations behave in systems in the ground-state, I proposed to start investigating the two-site entanglement presented by the ground-state of the extended Hubbard Hamiltonian, which is one of the most used model to study strong correlated phases of matter ranging from metals to Mott insulators and even superconductors.

Over the past month, I devoted my attention to the entanglement measurements in the Hubbard model. I compiled results from other works available in the literature and started implementing numerical calculations to compute the single-site and two-site entanglement entropies in a variety of regimes of the extended Hubbard Hamiltonian in one dimension. I already started developing codes to obtain the entanglement entropies in the ground-state using exact diagonalization (ED) and Density Matrix Renormalization-Group (DMRG). Our immediate next step is now compile these results and investigate the map between these quantities and the ground-state energy. I will then implement a DFT code to compute the concurrence from the Kohn-Sham system as in Pittalis [2] in the 1D Hubbard model. I will resort to the approximation for the exchange-correlation potential known as the Bethe Ansatz Local Density Approximation (BA-LDA) [3] and compare with the aforementioned exact calculations. With that, I expect to be able to understand qualitatively what are the contributions of non-local entanglement enabling to correct the LDA functional and quantifying it to build a parametrized form for the improved functional. Additionally, I aim to extend the previous ideas to the realm of quantum thermodynamics, to deal with systems at finite temperature and study approximation protocols to reproduce the quantum work [4–7].

1.2 Density Functional Theory at finite temperature and critical systems

Density Functional Theory foundations and theorems hold for systems in the ground-state, $T = 0$. At finite temperatures, approximations for the exchange-correlation functional have to account for contributions of excited states, and hence would involve a time-dependent formulation. Recently, Rehr [4] discussed the validity of ground-state functionals at finite temperatures and shown that up to the characteristic Fermi temperature T_F , in principle, such approximations would produce satisfactory results. An interesting problem related to this work is the accuracy of ground-state approximations below T_F in systems for system in a phase transition at a critical temperature $T_c \leq T_F$. This is a project I intend to explore with the help of a masters student for who I will design and supervise a related project.

1.3 Measuring and tracing adiabaticity in quantum systems at finite temperature

Quantum systems undergoing an evolution that is carried out slowly in time are expected to perform a dynamic known as quantum adiabatic evolution. From the practical point of view, accessible and reliable criteria to determine the level of adiabaticity of the dynamics are lacking. Recently, Skelt [8] have proposed a natural metric for the local particle density to track the level of adiabaticity of closed interacting systems. With collaboration with Prof. Irene D'Amico, we intend to extend this formalism to open quantum systems, that is, systems in contact with the environment. To adapt the metric, instead of the local particle density, we will work with the reduced density matrix. Our goal is to inspect the behavior of this metric with respect to the coupling to the environment. We plan to work on this project over the next semester.

1.4 Transport in correlated nanostructures

As a part of the research project, I have also proposed a second line focused on the study of transport in quantum devices (quantum dots and molecular nanostructures) using a hybrid approach combining ab-initio and renormalization-group calculations, an extension of the researcher's PhD thesis [9]. While in this work, an analytical formalism and a numerical self-consistent cycle were proposed for the single-electron transistor, I would like to explore more complex devices, such as a molecular transistor. This a problem of interest of the condensed matter group at ICTP, lead by Prof. Alexandre Reily, so I am planning to collaborate with him in this project.

2 Papers published and submitted in 2020

- 1 K. Zawadzki, R. M. Serra, I. D'Amico, "Work-distribution quantumness and irreversibility when crossing a quantum phase transition in finite time" *Physical Review Research* **2** (3), 033167
- 2 K. Zawadzki, A. Nocera, A. E. Feiguin, "A time-dependent scattering approach to core-level spectroscopies" [arXiv:2002.04142](https://arxiv.org/abs/2002.04142) [cond-mat.str-el]
- 3 K. Zawadzki, L. Yang, A. E. Feiguin, "Beyond perturbation theory: A time-dependent approach to inelastic scattering spectroscopies in-and away from equilibrium"

[arXiv:2008.09709](https://arxiv.org/abs/2008.09709) [cond-mat.str-el]

- 4 A. H. Skelt, K. Zawadzki, I. D’Amico, “Approximating quantum thermodynamic properties using DFT” (in preparation)

3 Organization of events

Before joining ICTP, I have been involved with the organization of a series of online seminars, called [Quantum Matter Seminars](#), conceived by Prof. Adrian Feiguin (Northeastern University) in June 2020.

I have been collaborating as a volunteer in the group [PUB-Boston](#), which organize monthly meetings to disseminate the research developed by the Brazilian community of researchers in Boston (USA).

In the next year, I plan to submit a proposal for a joint School and Workshop on numerical methods for strongly correlated systems to be hosted by ICTP-SAIFR. This event would bring experts in numerical calculations to give lectures on exact diagonalization, Density Matrix Renormalization-Group and Tensor Networks. It would be followed by invited and contributed talks about the latest progress in the development of these techniques and, also, their applications.

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