Structure Formation
Lecture 4

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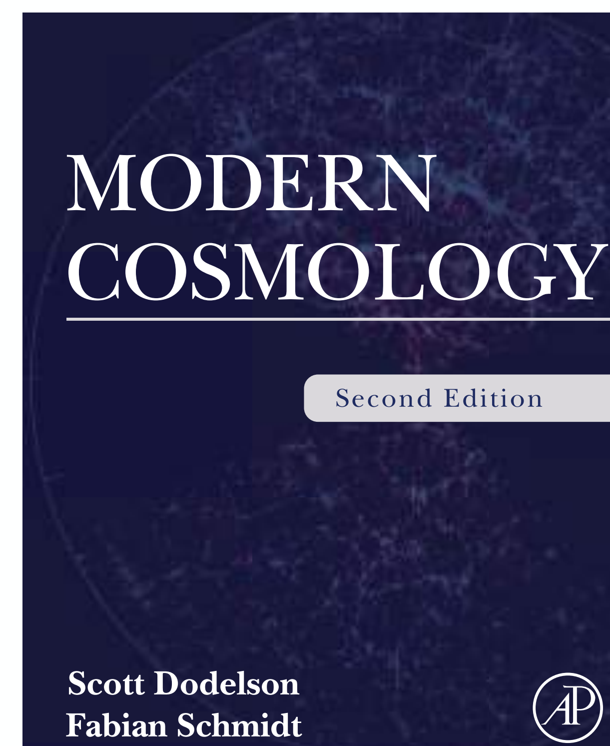
MPA

All figures taken from *Modern Cosmology, Second Edition*, unless otherwise noted



ICTP-SAIFR School on Cosmology, January 2021

Outline of lectures

1. The problem: collisionless Boltzmann equation and fluid approximation
    1. Linear evolution
2. Nonlinear evolution of matter
    1. Perturbation theory
    2. Simulations
    3. Phenomenology of nonlinear matter distribution
3. Formation and distribution of galaxies
    1. Galaxy formation in a nutshell <- HERE
    2. Spherical collapse model
    3. Physical clustering of halos and galaxies; bias
    4. Observed clustering of galaxies
4. Beyond ΛCDM

Notation

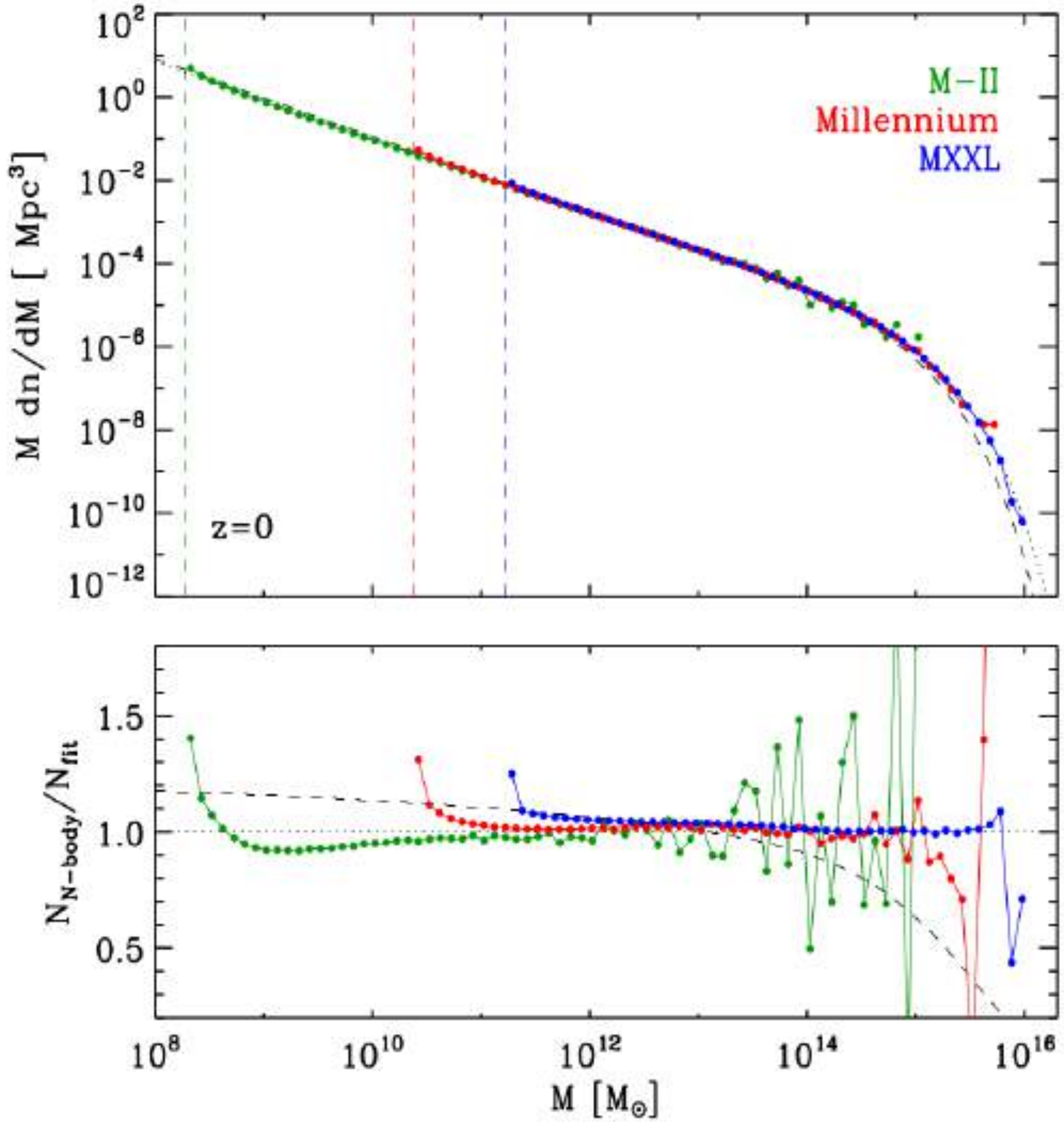
$$ds^2 = -(1 + 2\Psi(\boldsymbol{x}, t))dt^2 + a^2(t)(1 + 2\Phi(\boldsymbol{x}, t))d\boldsymbol{x}^2$$

- Comoving coordinates: $d\boldsymbol{r} = a(t)d\boldsymbol{x}$
- Conformal time: $d\eta = \frac{dt}{a(t)} = \frac{da}{a^2 H(a)} = \frac{d\ln a}{aH(a)}.$
  - Comoving distance: $d\chi = -d\eta = \frac{dz}{H(z)}$
- Particle velocity/momentum: $\boldsymbol{v} = \frac{\boldsymbol{p}}{m} = a\frac{d\boldsymbol{x}}{dt} = \boldsymbol{x}'$
- Fluid velocity; divergence: $\boldsymbol{u}; \quad \theta = \partial_i u^i$
- Gravitational potential: Ψ

Halo abundance

- Mean number density of halos in logarithmic mass bins
- Power-law at small masses
- Exponential cutoff at high masses - reflecting Gaussian statistics of initial density field

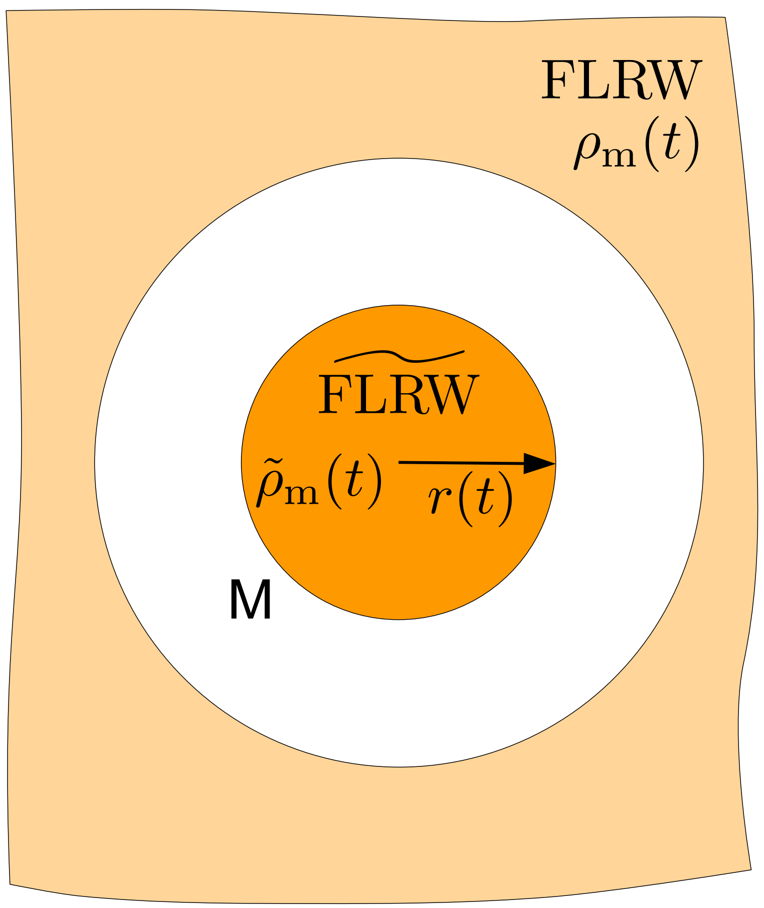
dn/dM ~ $M^{-1.9}$ (at small masses)



Angulo et al 2012

Spherical collapse picture

- Consider isolated, uniform spherically symmetric overdense region (i.e. embedded in patch of unperturbed background)
- Can solve for evolution of this region exactly up until collapse!



$$\ddot{r}(t) = -\frac{GM}{r^2(t)} + \frac{8\pi G}{3}\rho_\Lambda r(t)$$ Eq (12.67)

Newtonian equation (plus Λ term) for physical (not comoving) radius r(t)

Spherical collapse picture

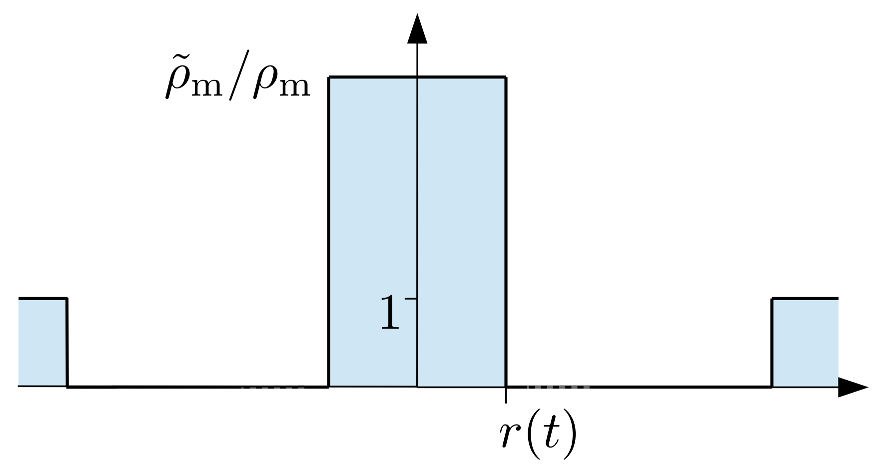
$$\ddot{r}(t) = -\frac{GM}{r^2(t)} + \frac{8\pi G}{3}\rho_\Lambda r(t) \quad \text{Eq (12.67)}$$

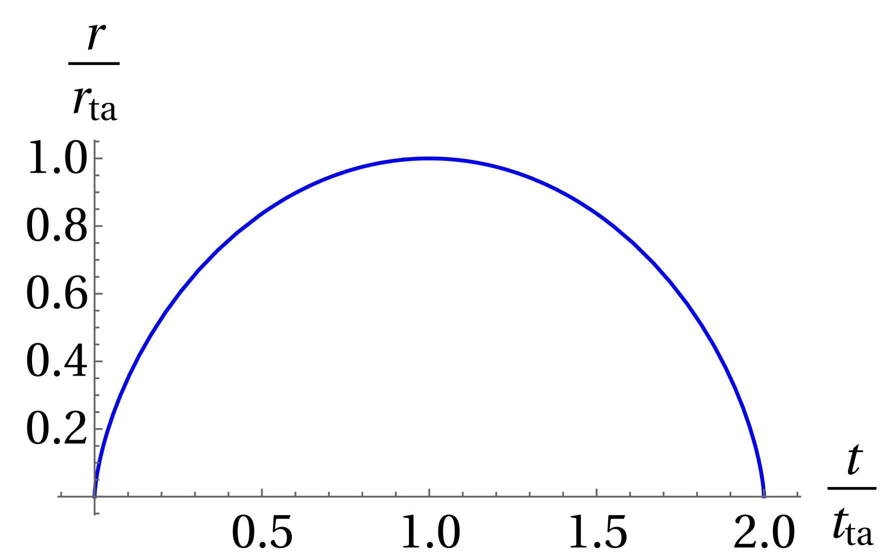
- Parametric solution possible if we neglect Λ term:

$$r(t) = \frac{r_{\rm ta}}{2}(1-\cos\theta), \qquad t = \frac{t_{\rm ta}}{\pi}(\theta - \sin\theta). \quad \text{Eq (12.68)}$$

- Can show that region *of any size* that collapses at a given time has linearly-extrapolated initial over density of

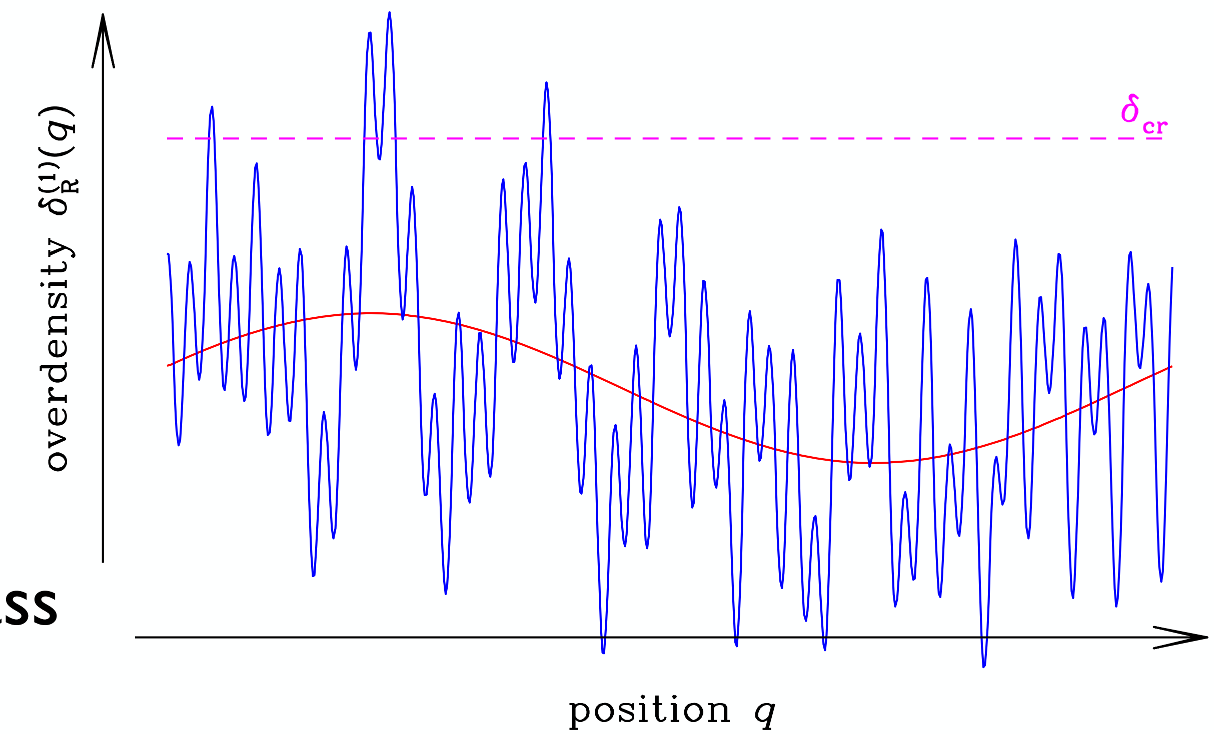
$$\delta^{(1)}(\boldsymbol{x},\eta) = \delta_{\rm cr} = 1.686$$





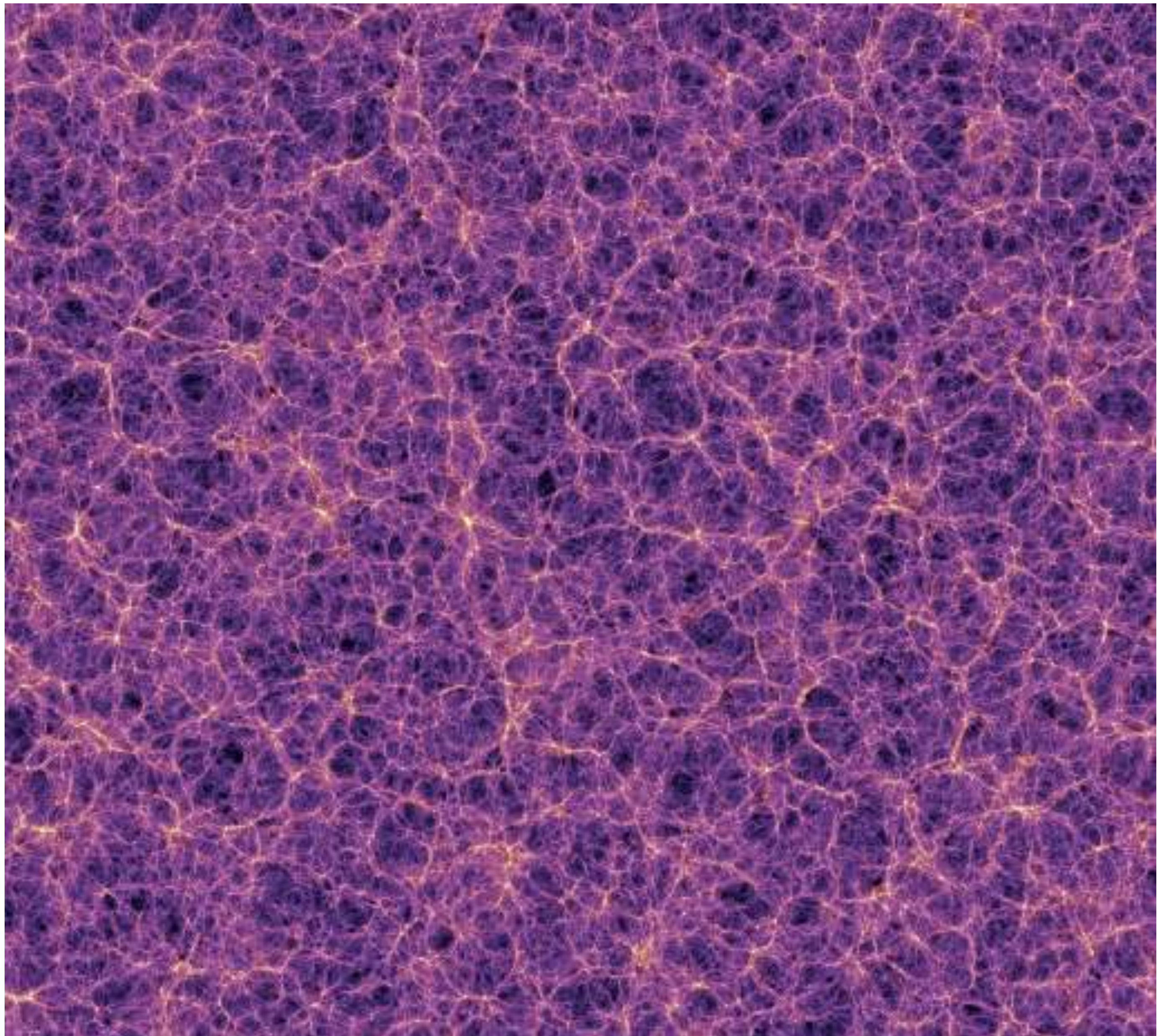
Spherical collapse and excursion set

- Region *of any size* that collapses at a given time has linearly-extrapolated initial over density of $\delta^{(1)}(\boldsymbol{x}, \eta) = \delta_{\rm cr} = 1.686$
- Basis for semi-analytic approach to halos:
  1. Compute linear density field $\delta^{(1)}$
  2. Smooth on a scale *R* and identify which points lie above δ_{cr}
  3. Identify those with future halos with mass $R_L(M) = R$
- With some refinements to avoid double-counting, known as *excursion set* approach
- Very rough, but useful to have in mind



From matter to galaxies

- So far, studied what happens to cold collisionless matter - but what about the gas and stars we actually observe?



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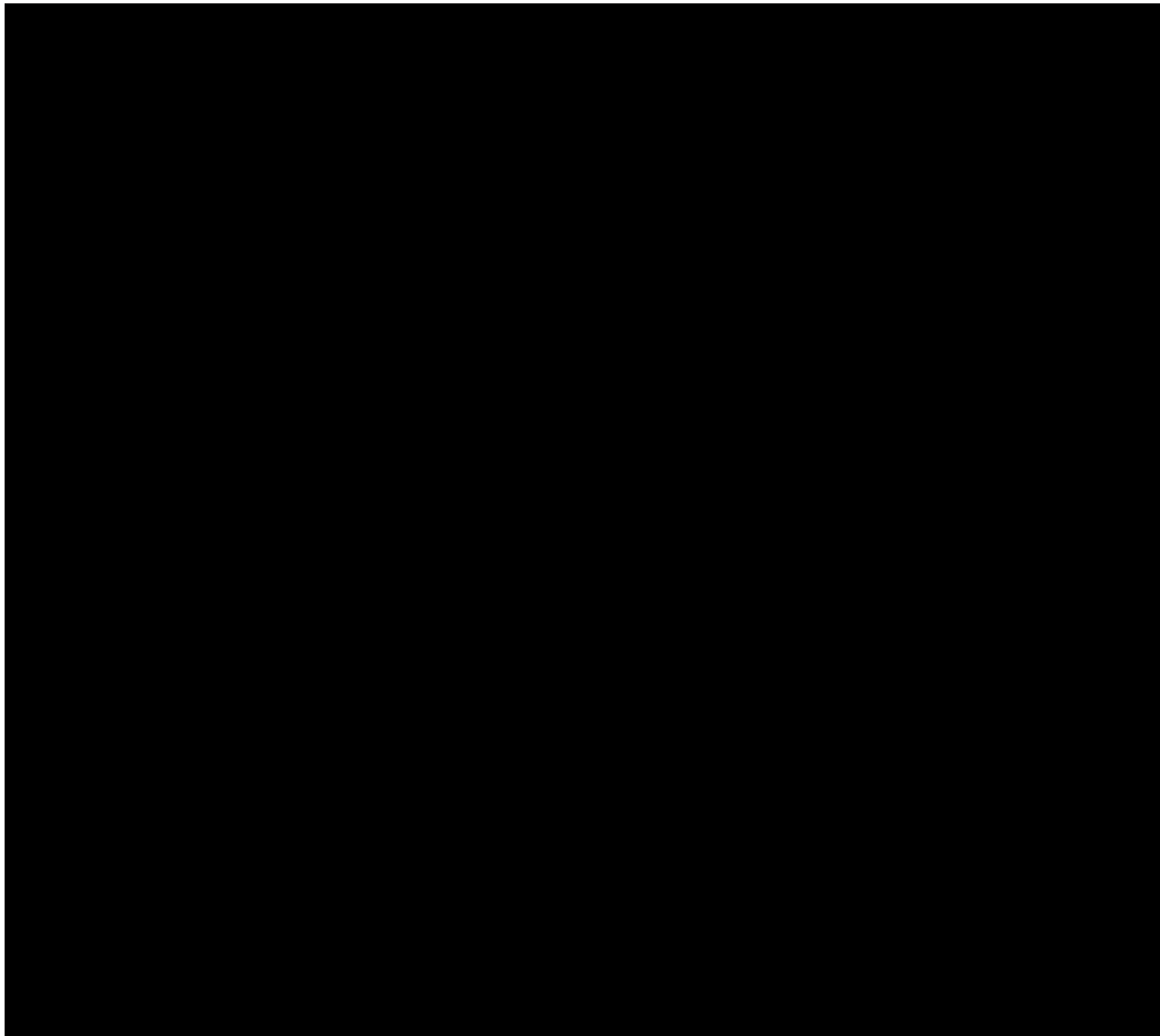

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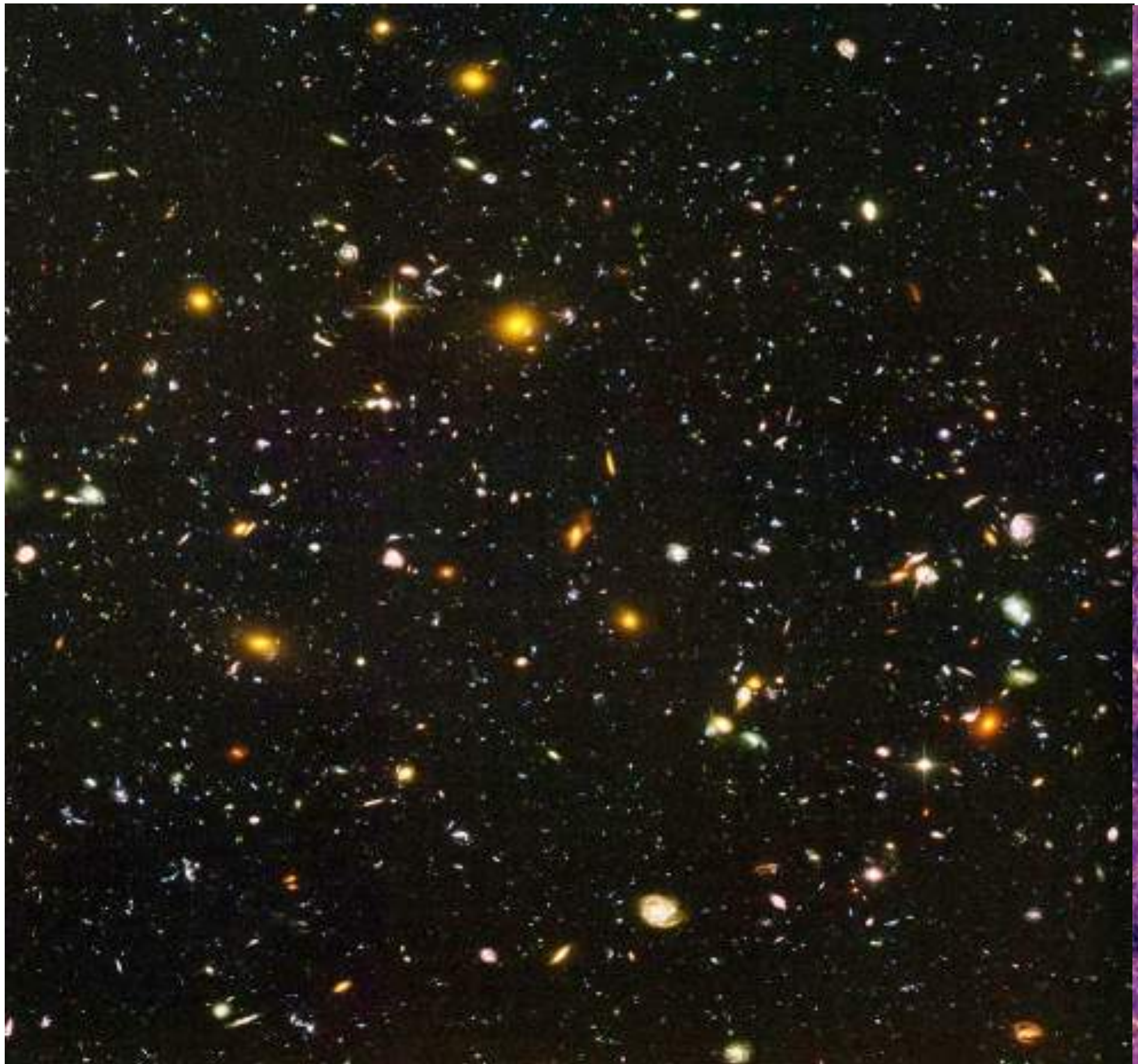
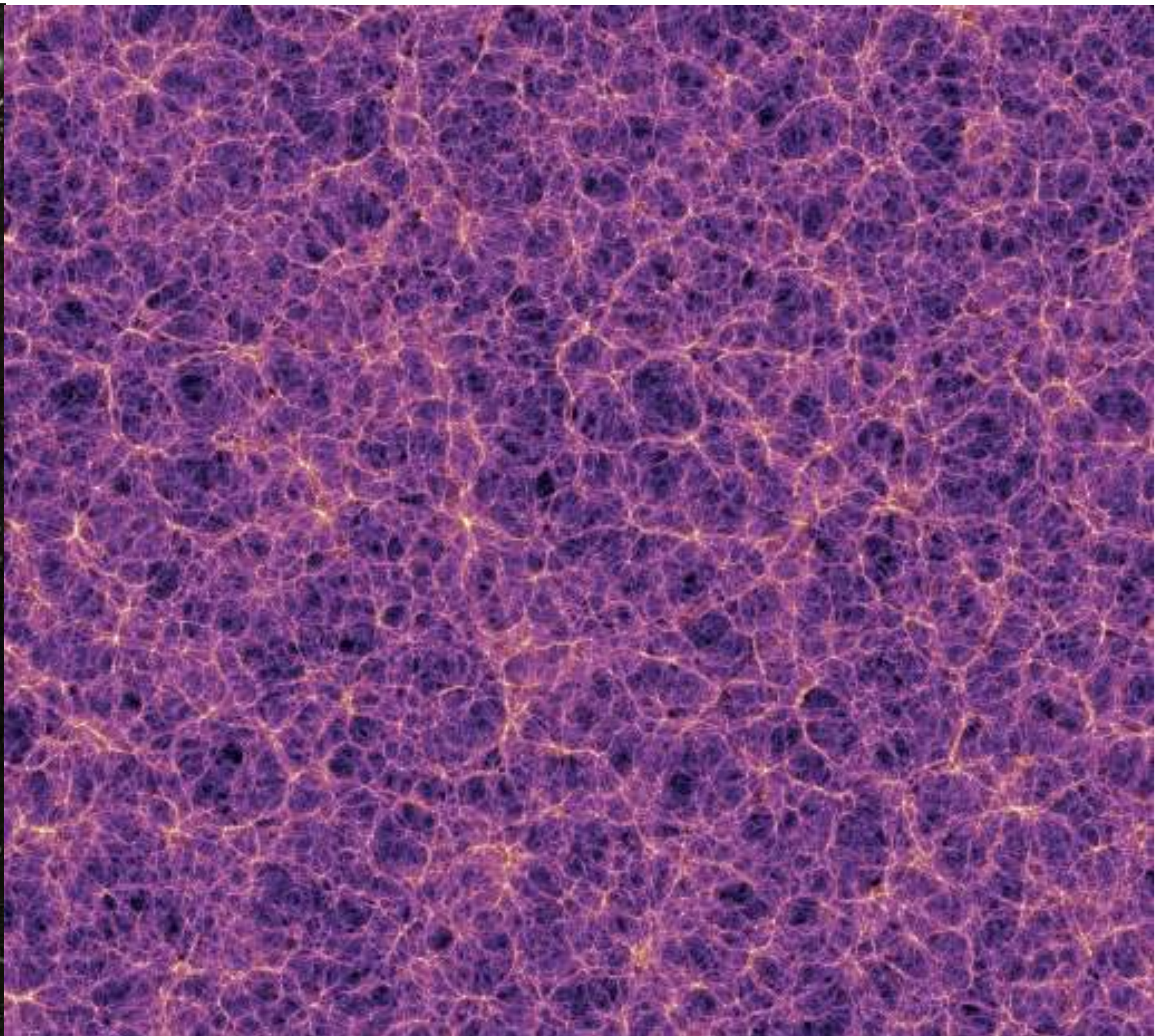
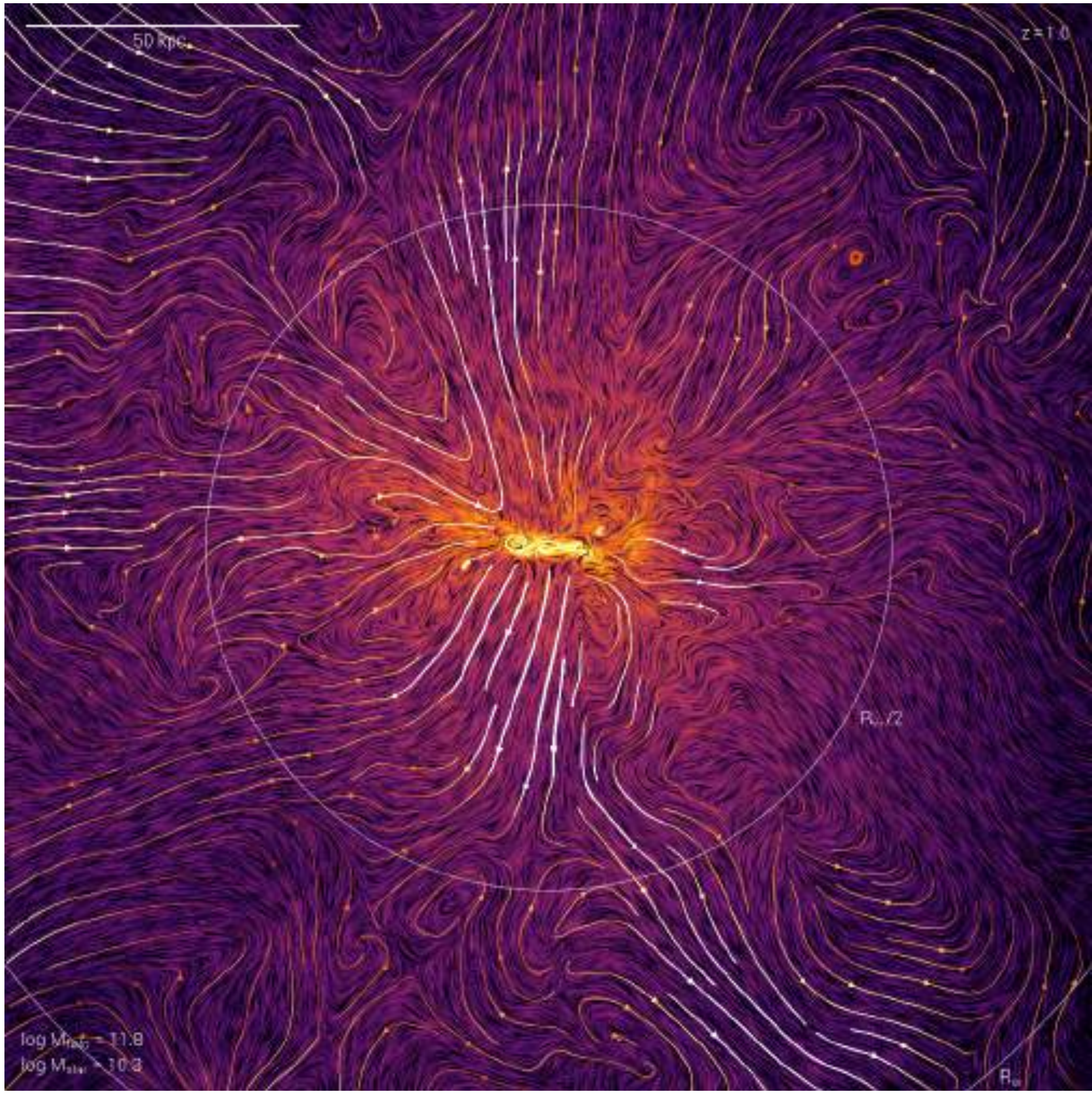




Galaxy formation in a nutshell

- Generally, gas follows the DM component in its collapse until stopped by pressure
- Minimum collapsing object set by Jeans scale, which is function of temperature
- Gas begins to cool once it has collapsed to sufficient density so it can radiate away energy via collisional excitations - reducing Jeans scale and allowing further collapse
- Eventually stars form, as proto-galaxy continues to accrete gas
- Star formation regulated by balance of gravitational growth and *feedback*, due to radiation, heating, or ejected gas (Supernovae, massive black holes)



Illustris-TNG

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Illustris-TNG

Models for galaxy clustering

- We cannot simulate galaxy formation realistically yet (and certainly not over cosmological volumes)
- One approach: attempt to populate halos with galaxies
  - *Halo occupation distribution (HOD):* N_{gal}(halo) parametrized as function of halo mass
  - *Subhalo abundance matching (SHAM):* populate mass-ordered halo substructure with galaxies
  - Physically motivated, but difficult to quantify the error we are making with these simplifications
- Alternative: parametrize our ignorance and make minimal assumptions: *EFT approach*
  - Minimal assumptions and controlled error - but restricted to large scales

EFT approach to galaxy clustering

- Idea: follow treatment of perturbations to matter, as far as possible
- But we need to take into account that galaxies form out of baryons, and their number isn't conserved!
- Start from perturbative expansion of fluctuations in galaxy number counts:

$$\frac{n_{\mathrm{g}}(\boldsymbol{x},\eta)-\bar{n}_{\mathrm{g}}(\eta)}{\bar{n}_{\mathrm{g}}(\eta)} = \delta_{\mathrm{g}}(\boldsymbol{x},\eta) = \delta_{\mathrm{g}}^{(1)}(\boldsymbol{x},\eta)+\delta_{\mathrm{g}}^{(2)}(\boldsymbol{x},\eta)+\cdots+\delta_{\mathrm{g}}^{(n)}(\boldsymbol{x},\eta)$$

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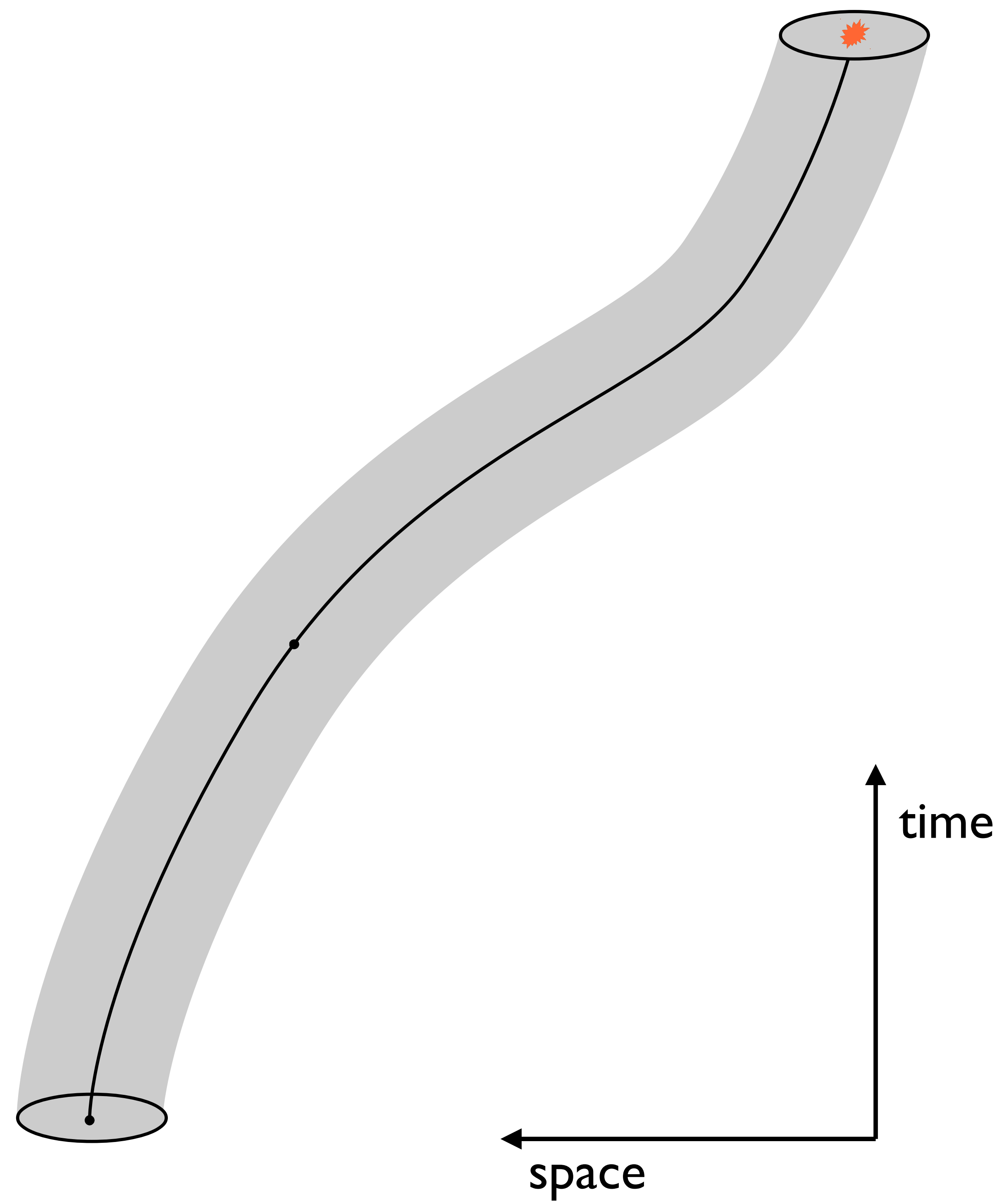
- Goal: write galaxy density as a sum of observables (or *operators*; -> later) *O* multiplied by free *bias coefficients*:

$$\delta_g(\boldsymbol{x},\eta) = \sum_O b_O(\eta) O(\boldsymbol{x},\eta)$$

At fixed order in perturbation theory, there should only be a finite number of these…

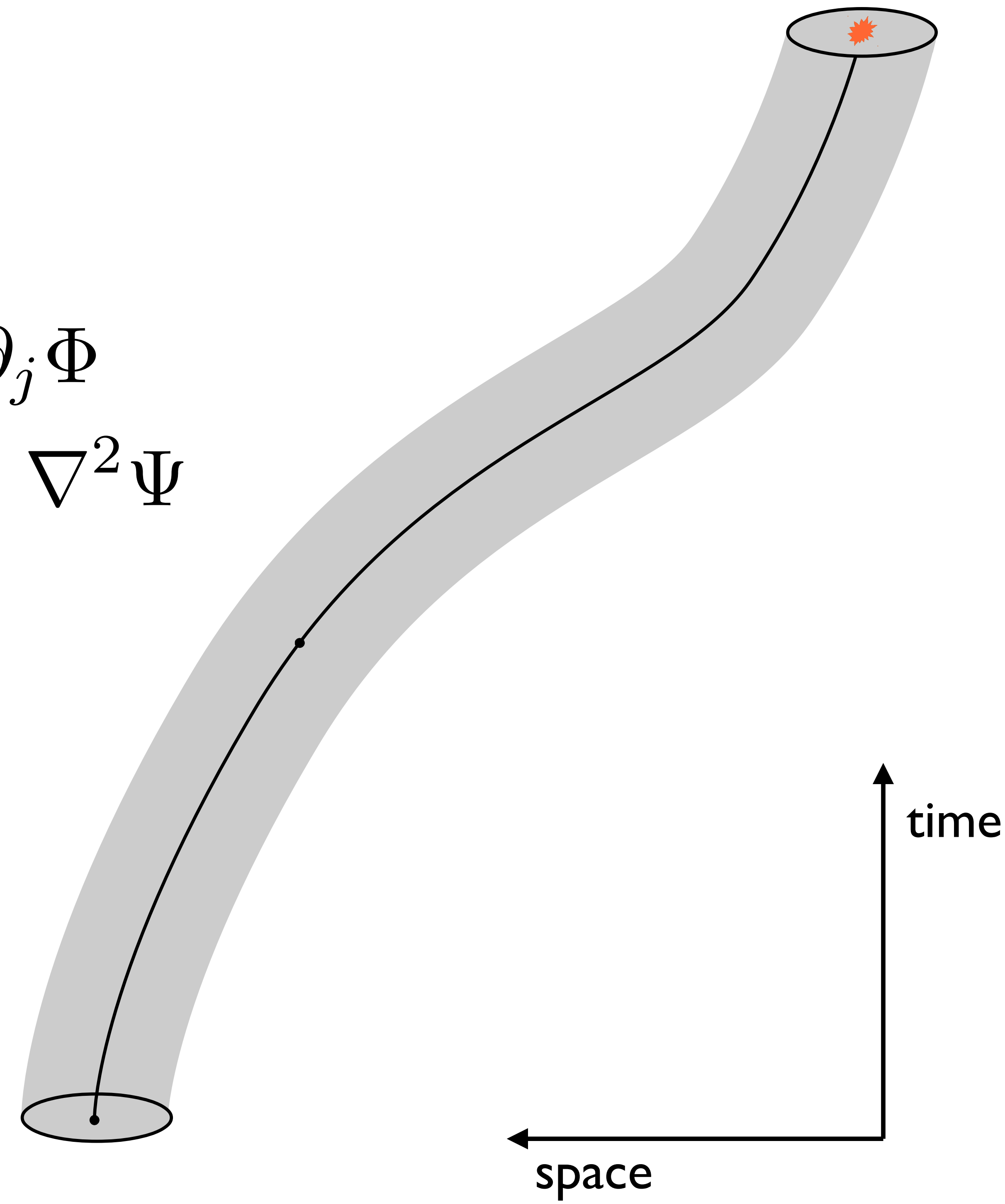
Spacetime view of galaxy formation

- Consider coarse-grained (large scale) view of region that forms a galaxy at conformal time τ
- Formation happens over *long time scale*, but *small spatial scale* R_*
  - For halos, expect $R_* \lesssim R_L$
- Approximate galaxy formation as *spatially local* (on large scales)



Spacetime view of galaxy formation

- Leading gravitational observable is tidal field $\partial_i \partial_j \Phi$ which includes density $\delta \propto \nabla^2 \Psi$
- Along *entire trajectory* of forming galaxy



Galaxy bias expansion

- Ignore time evolution for now
- Then, we have a *local bias relation:*

$$\delta_g(\boldsymbol{x},\eta) = F_g(\partial_i\partial_j\Psi(\boldsymbol{x},\eta),\eta)$$

- Then, it is easy to write down bias expansion, at first, second, … order:

$$\delta_g = \sum_O b_O O$$

$$O \in \{\delta,\ \delta^2,\ (K_{ij})^2,\ \ldots\}$$

$$K_{ij}(\boldsymbol{x},\eta) \equiv \left[\frac{\partial_i\partial_j}{\nabla^2} - \frac{1}{3}\delta_{ij}\right]\delta(\boldsymbol{x},\eta)$$

$$\propto \left[\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right]\partial_i\partial_j\Psi(\boldsymbol{x},\eta)$$

scaled tidal field (more practical)

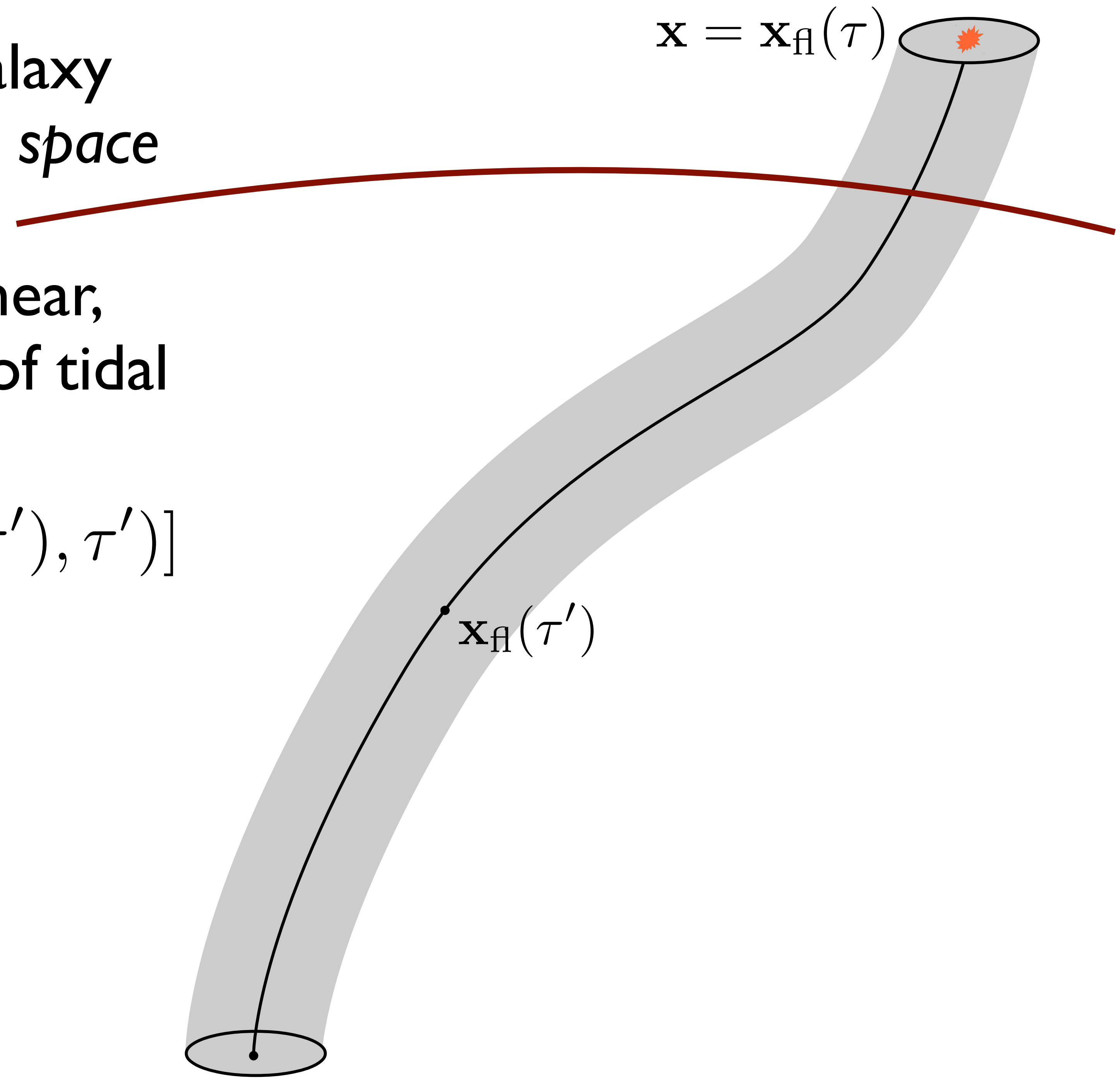
$\delta \equiv \delta_{\mathrm{m}}$ evolved matter density field

$$\delta_{\mathrm{g}}(\boldsymbol{x},\eta) = \delta_{\mathrm{g}}^{(1)}(\boldsymbol{x},\eta) + \delta_{\mathrm{g}}^{(2)}(\boldsymbol{x},\eta) + \cdots + \delta_{\mathrm{g}}^{(n)}(\boldsymbol{x},\eta)$$

Non-locality in time

- Continue to approximate galaxy density as a local function *in space*
- We are then left with nonlinear, nonlocal-in-time functional of tidal tensor:

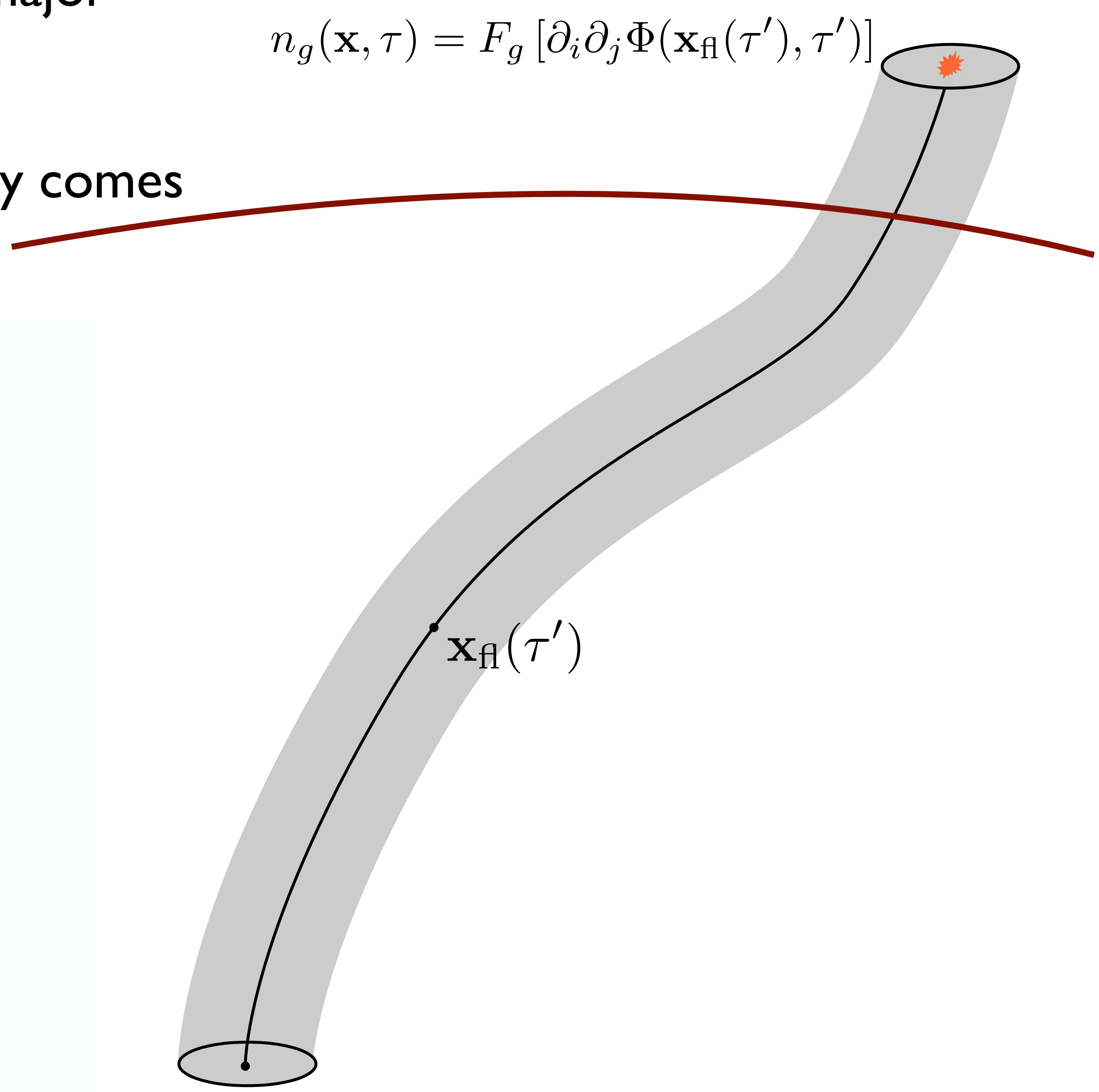
$$n_g(\mathbf{x},\tau) = F_g\left[\partial_i\partial_j\Phi(\mathbf{x}_{\rm fl}(\tau'),\tau')\right]$$



$\tau \equiv \eta$ in following slides...

Non-locality in time

- Nonlocality in time seems like a major problem!
- But the scale-free nature of gravity comes to the rescue



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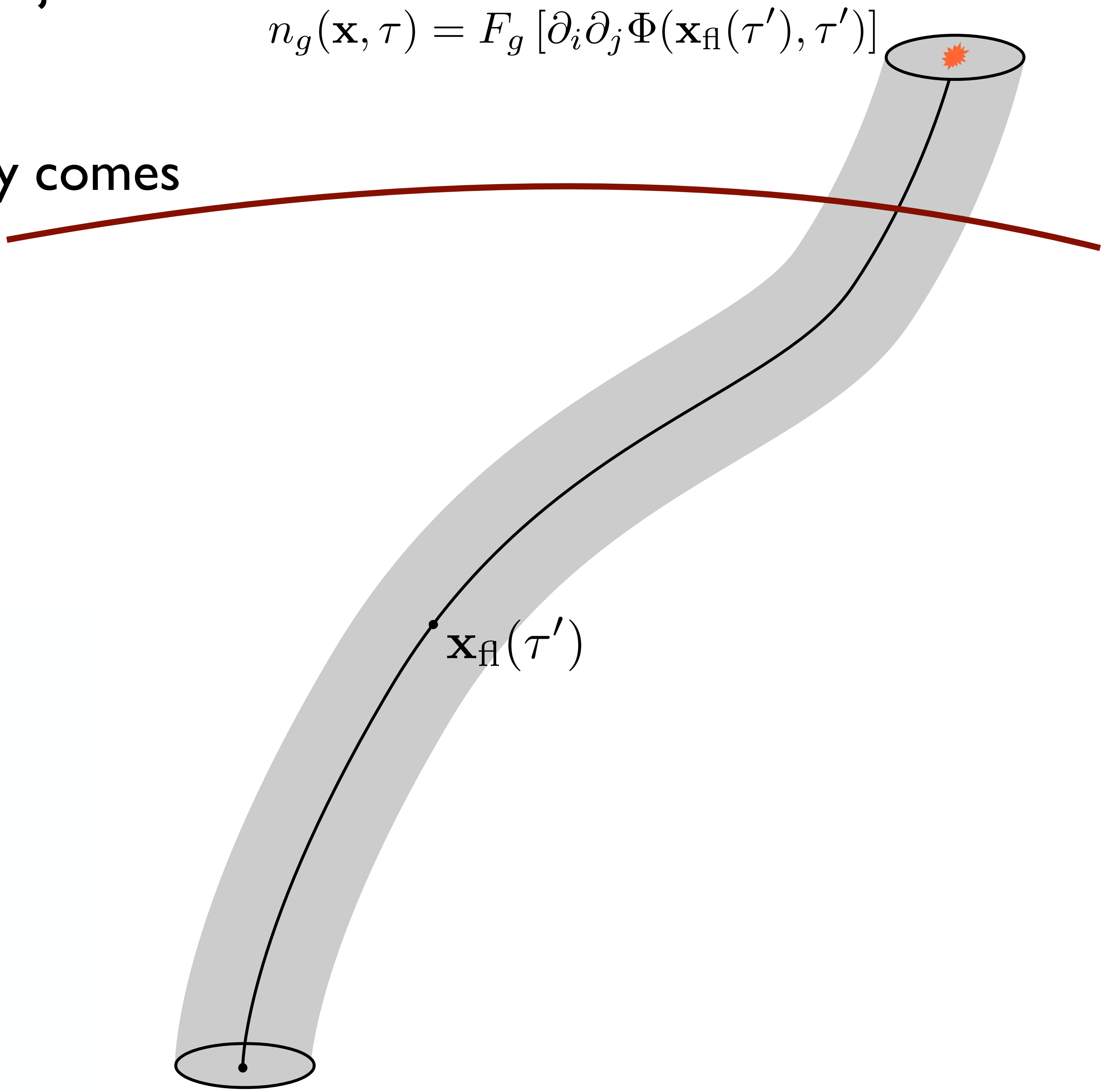
- Consider linear density term:

$$\int_0^\tau d\tau'\, f_{g,\delta}(\tau,\tau')\delta(\boldsymbol{x}_{\rm fl}(\tau'),\tau')$$



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- Consider linear density term:

$$\int_0^{\tau} d\tau'\, f_{g,\delta}(\tau,\tau')\delta(\boldsymbol{x}_{\mathrm{fl}}(\tau'),\tau')$$

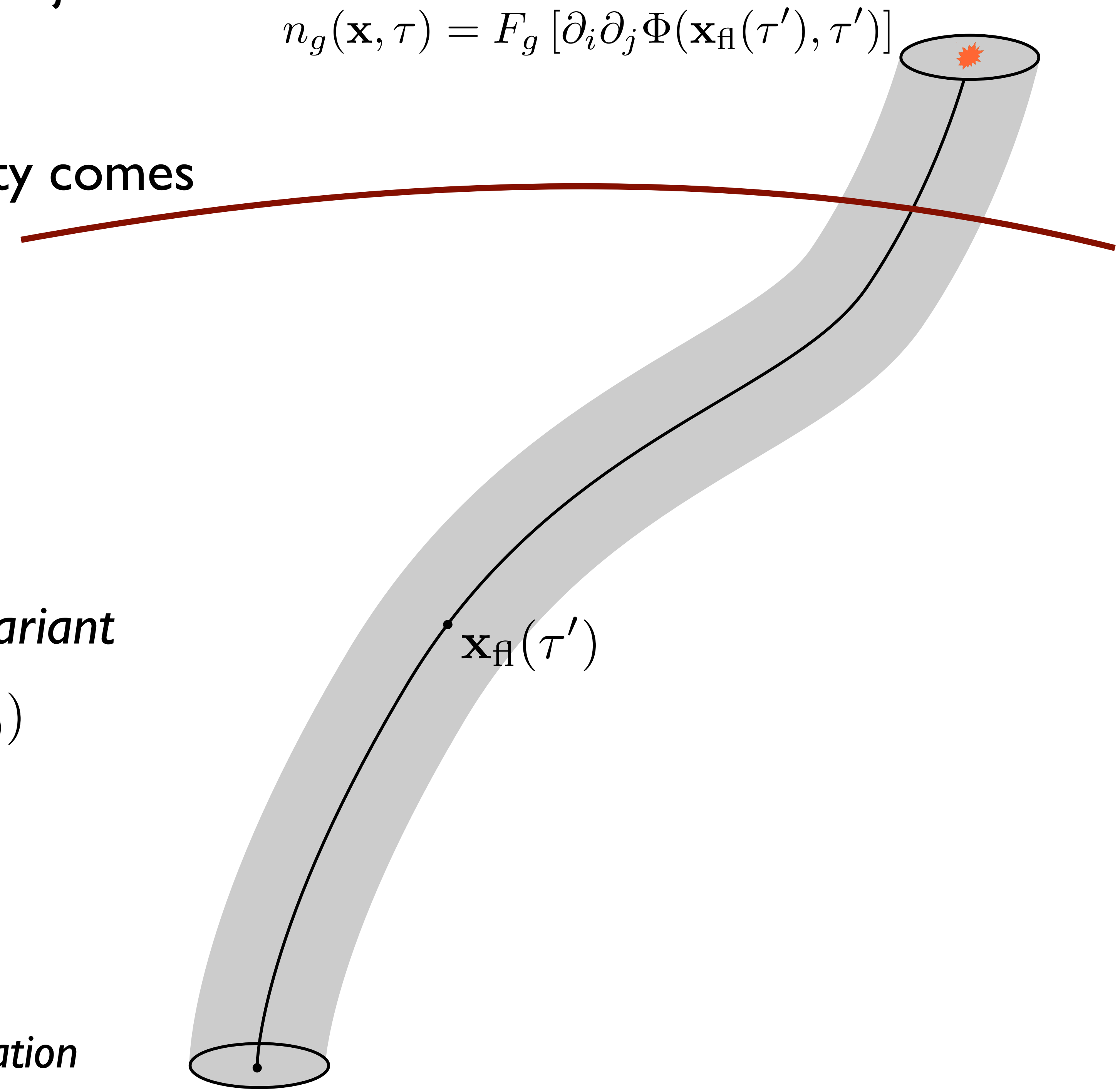
- At linear order: *growth is scale-invariant*

$$\delta(\boldsymbol{x}_{\mathrm{fl}}(\tau'),\tau') = D(\tau')\delta^{(1)}(\boldsymbol{x},\tau_0)$$

- Integral simply becomes

$$b_1(\tau)\delta^{(1)}(\boldsymbol{x},\tau)$$

Linear local bias relation



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Non-locality in time

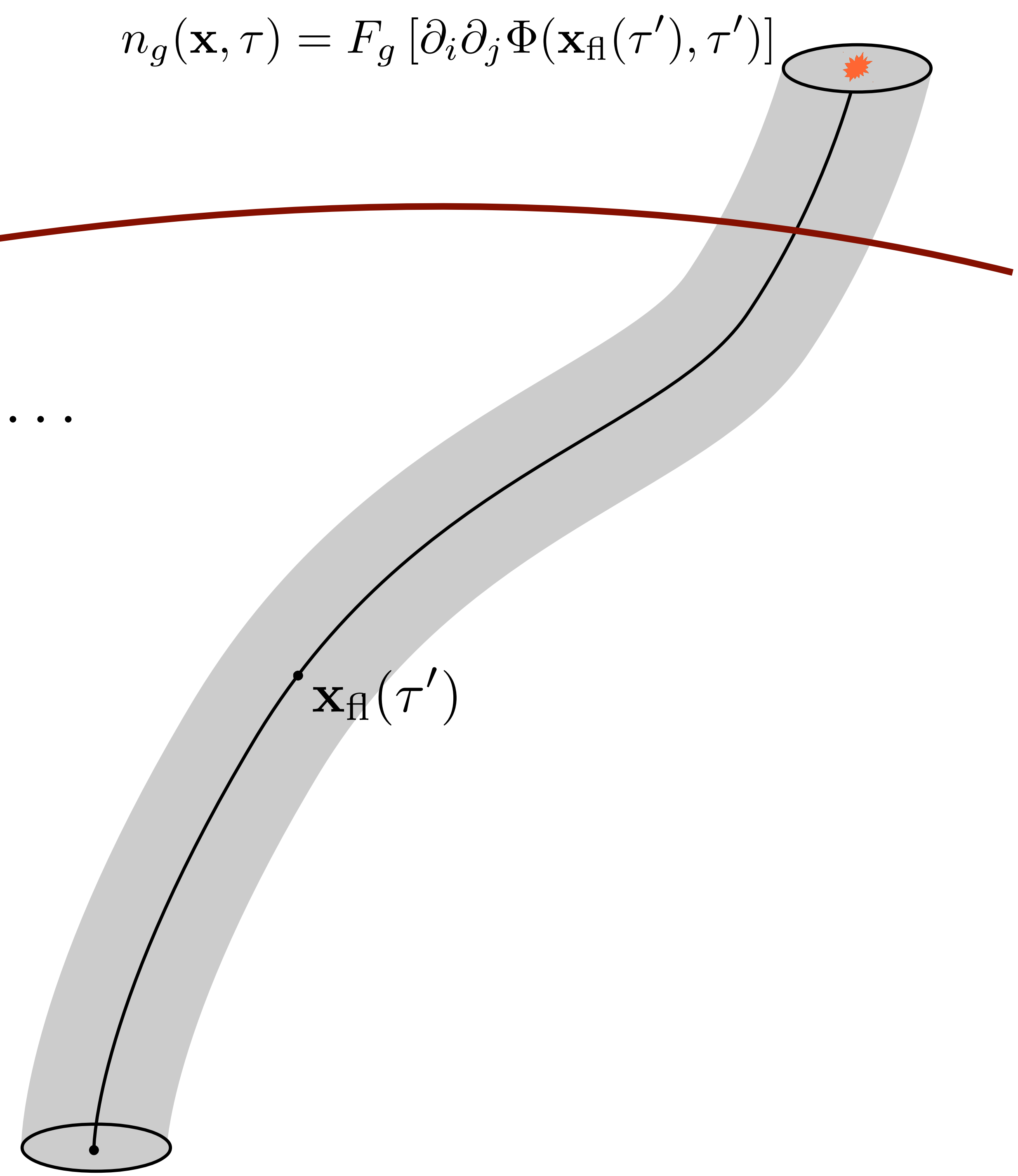
- We can similarly deal with non locality in time at higher order, since expansion continues to factorize:

$$\delta(\mathbf{x},\tau) = D(\tau)\delta^{(1)}(\mathbf{x}) + D^2(\tau)\delta^{(2)}(\mathbf{x}) + \cdots$$

- Allows us to obtain a complete expansion of galaxy density field:

$$n_g(\boldsymbol{x},\tau) = \bar{n}_g(\tau)\left[1 + \sum_O b_O(\tau)O(\boldsymbol{x},\tau)\right]$$

up to given desired order in perturbations



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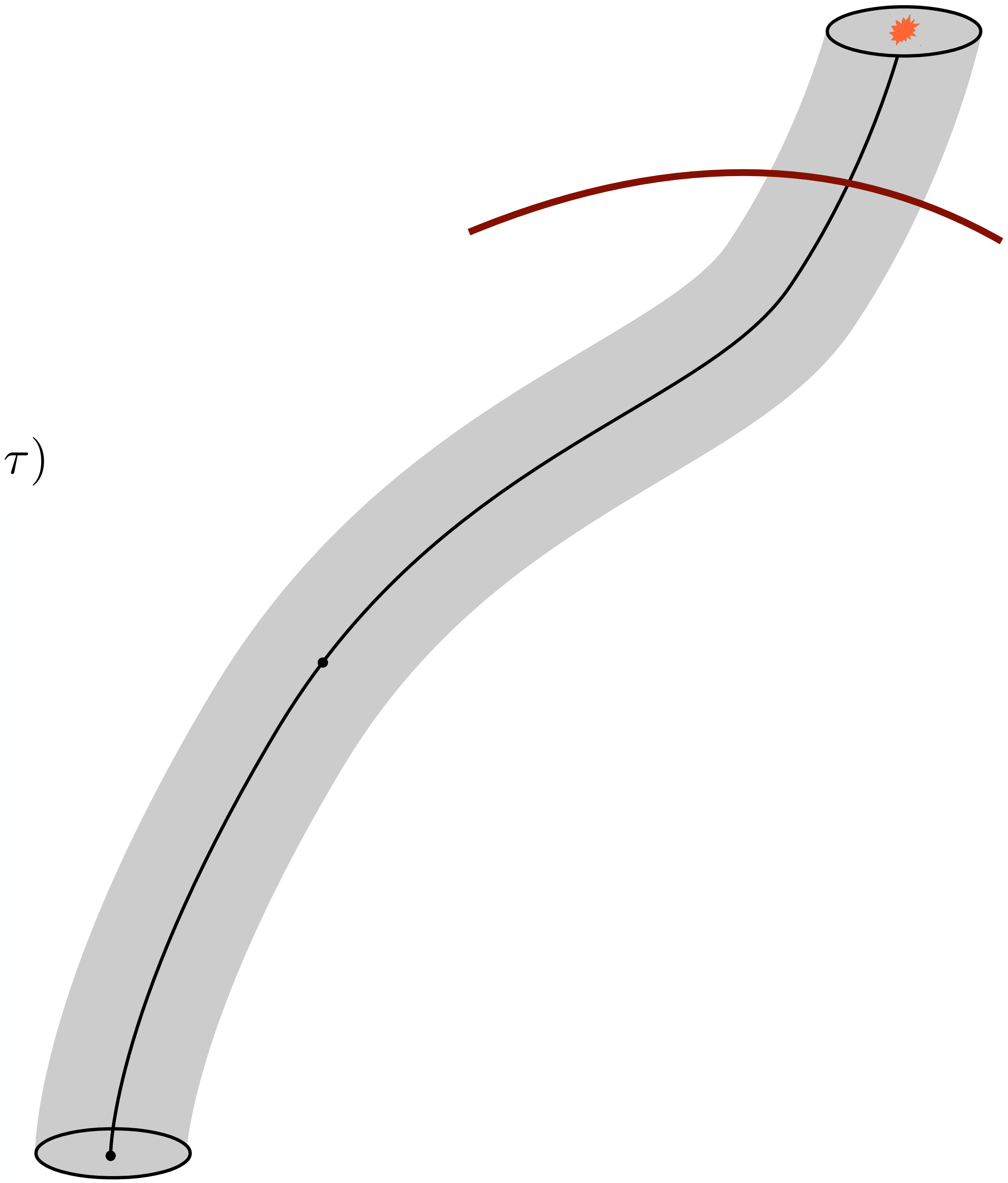
Spatial nonlocality and scale-dependent bias

- Beyond large-scale limit: need to expand *spatial nonlocality* of galaxy formation
- Higher derivative biases are suppressed with scale R_*
- E.g., $R_*^2 \nabla^2 \delta \longrightarrow \delta_g(\mathbf{k}, \tau) = \left(b_1 + b_{\nabla^2\delta} k^2 R_*^2\right) \delta(\mathbf{k}, \tau)$



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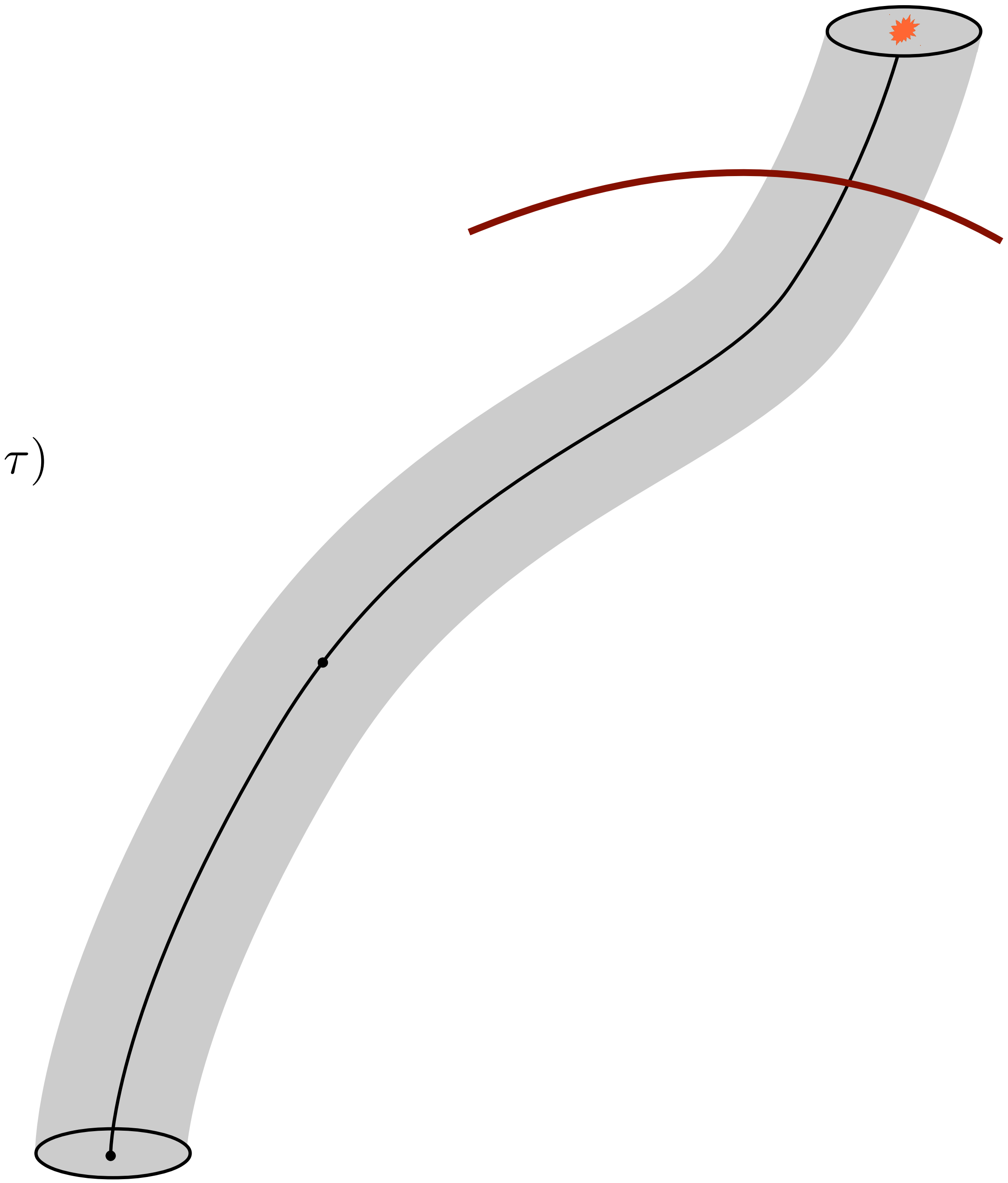
- This also allows for baryonic physics, which *has to come with additional derivatives*
  - Example: pressure perturbations $\delta p = c_s^2 \delta\rho$
  - Pressure force: $\boldsymbol{F} = \boldsymbol{\nabla}\delta p \propto \boldsymbol{\nabla}\delta$
- Identical in form to effective sound speed in matter we encountered before



EFT approach in LSS

- Effective field theory: write down all terms (in Lagrangian or equations of motion) that are consistent with symmetries
  - Gravity: general covariance
  - Galaxy density: 0-component of 4-vector (momentum density)

EFT approach in LSS

- Effective field theory: write down all terms (in Lagrangian or equations of motion) that are consistent with symmetries
  - Gravity: general covariance
  - Galaxy density: 0-component of 4-vector (momentum density)
- Order contributions by perturbative order, and number of spatial derivatives (gradient expansion)

EFT approach in LSS

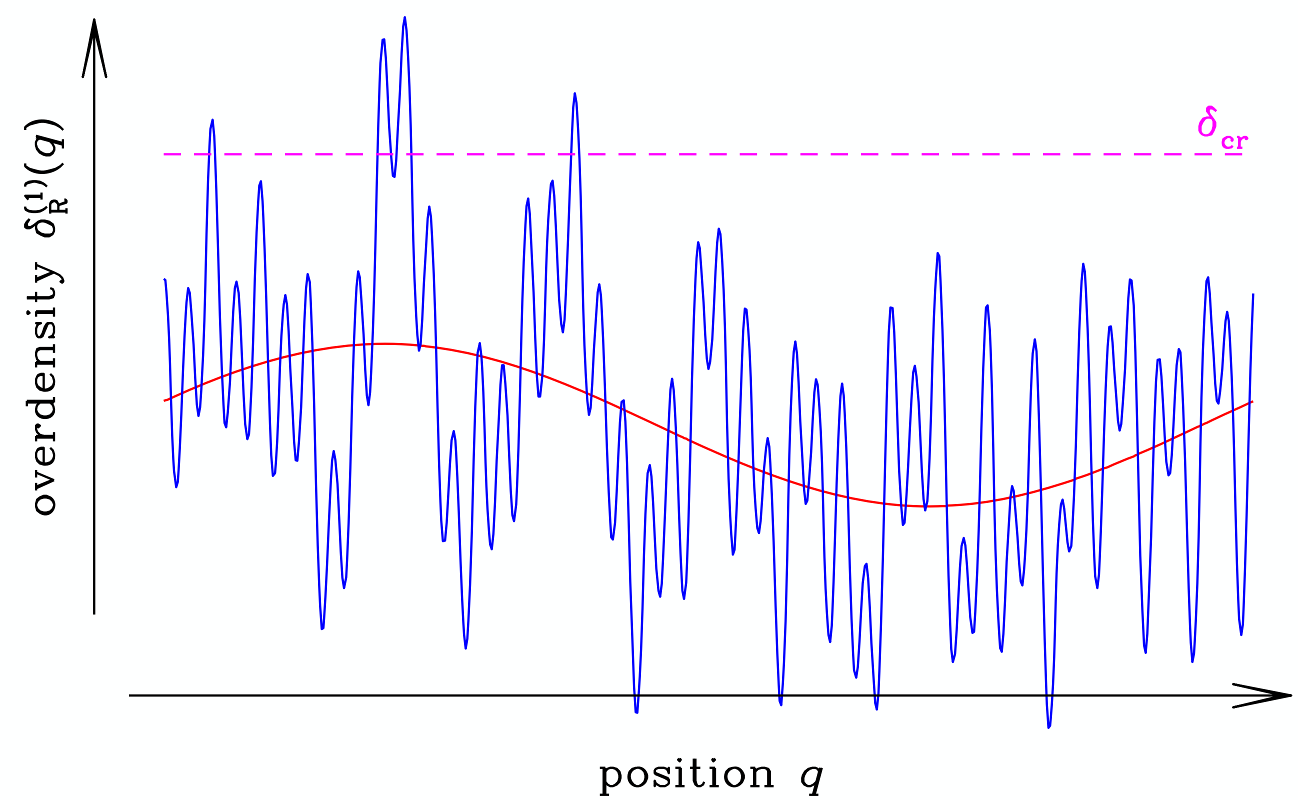
- For large-scale structure (LSS), general covariance boils down to the statement that $\Psi,\ \nabla\Psi$ and v *cannot appear in bias expansion*
  - In other words, leading gravitational observable is tidal field including density, like we did above
  - Since we take into account entire evolution, $\theta,\ \partial_i u^j$ are already incorporated as they can be obtained from Euler equation

Physical picture of bias

- Spherical collapse model: halos form in regions of smoothed initial density field that are above collapse threshold

- High excursions of Gaussian random field are more clustered than field itself

- Can be calculated by considering definition of correlation function (homework):



$$\xi_{\rm thr}(r) = \frac{p\left(\delta_R^{(1)}(\boldsymbol{x}+\boldsymbol{r}) > \delta_{\rm cr}, \delta_R^{(1)}(\boldsymbol{x}) > \delta_{\rm cr}\right)}{[p(\delta_R^{(1)}(\boldsymbol{x}) > \delta_{\rm cr})]^2} - 1$$

Physical picture of bias

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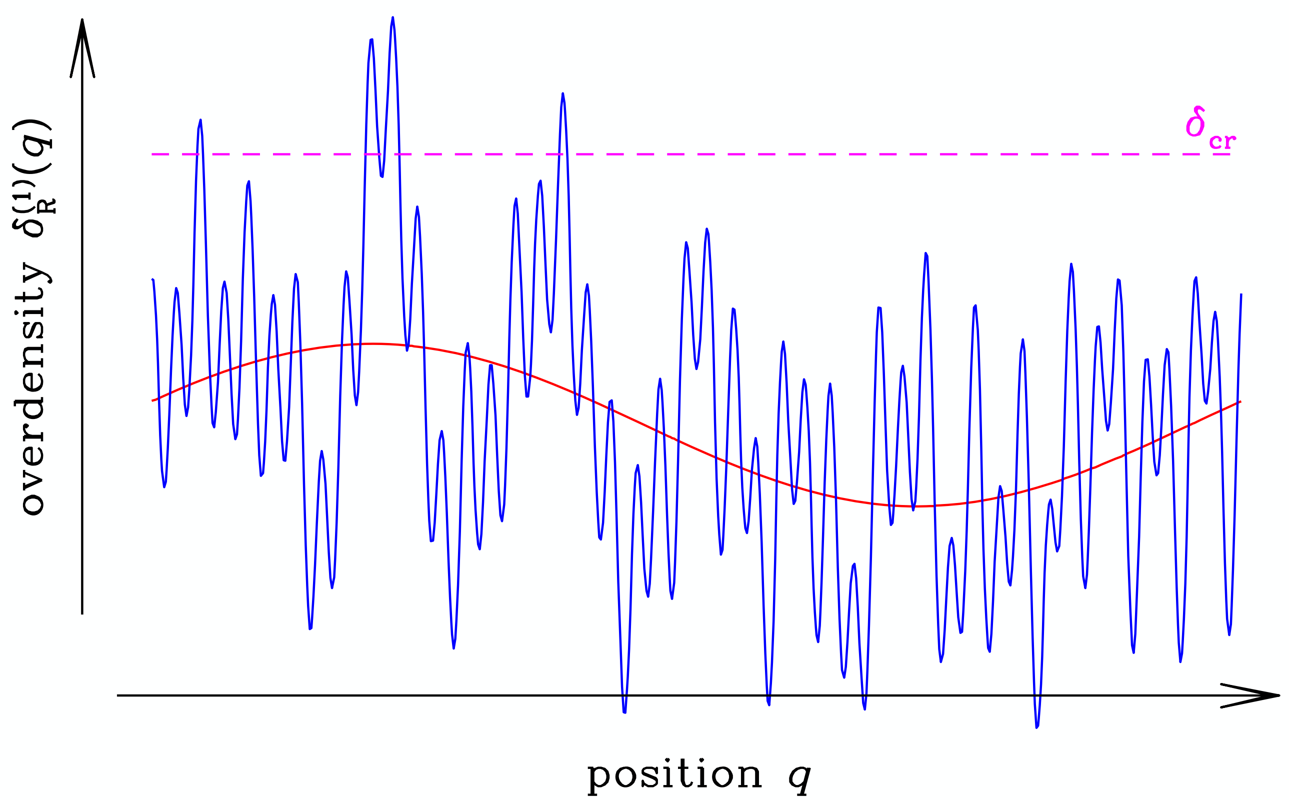

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- Clustering on large scales (r >> smoothing scale R):

$$\xi_{\rm thr}(r) = (b_1^{\rm thr})^2 \xi_R^{(1)}(r) + \frac{1}{2}(b_2^{\rm thr})^2 [\xi_R^{(1)}(r)]^2 + \cdots$$

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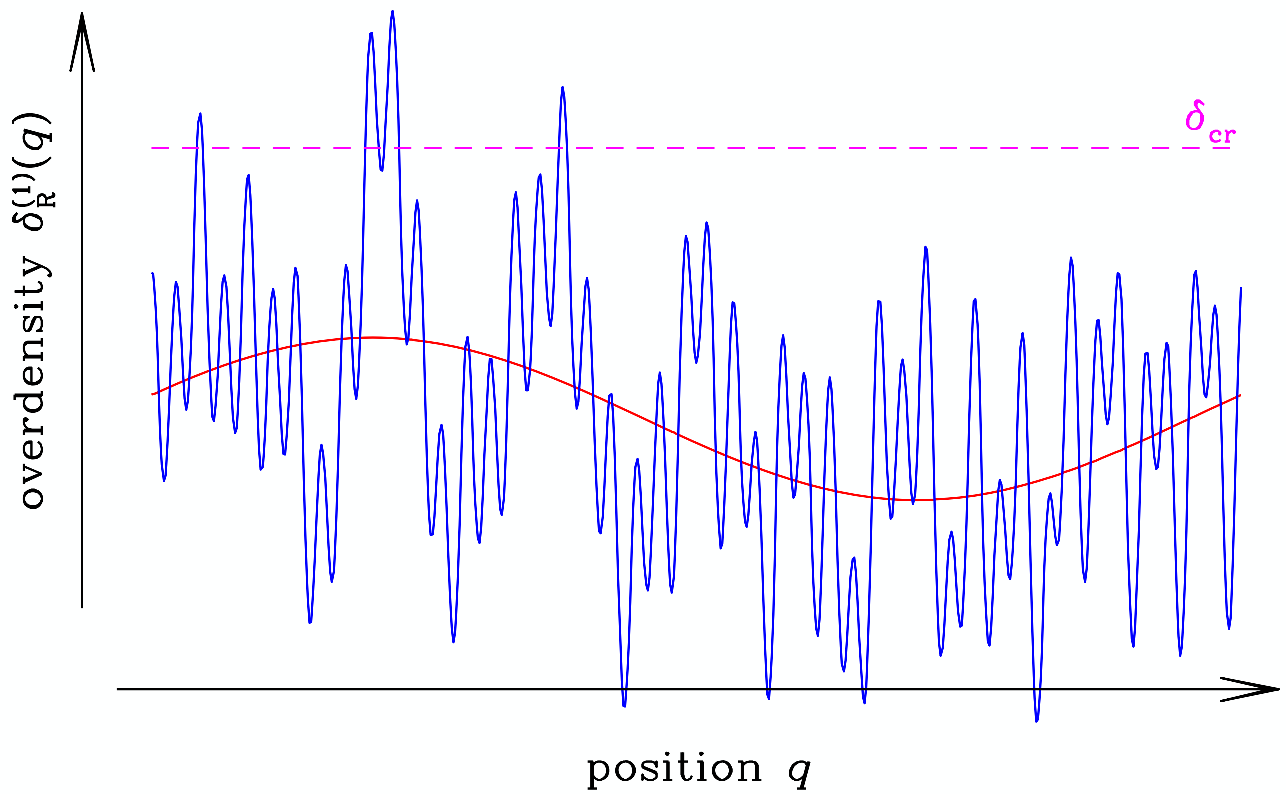

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Complete bias expansion

$$n_g(\boldsymbol{x},\tau) = \bar{n}_g(\tau)\left[1 + \sum_O b_O(\tau) O(\boldsymbol{x},\tau)\right]$$

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- The picture is not complete yet, since this relation can only hold in a "mean-field" sense

Complete bias expansion

$$n_g(\boldsymbol{x},\tau) = \bar{n}_g(\tau)\left[1 + \sum_O b_O(\tau) O(\boldsymbol{x},\tau) + \varepsilon(\boldsymbol{x},\tau) + \varepsilon_\delta(\boldsymbol{x},\tau)\delta(\boldsymbol{x},\tau)\cdots\right]$$

- The picture is not complete yet, since this relation can only hold in a "mean-field" sense
- Small-scale perturbations introduce stochasticity ε (and higher-order terms)
- Cannot predict ε as field, but know the *form of statistics:*

$$\langle \varepsilon(\boldsymbol{k})\varepsilon^*(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_D(\boldsymbol{k}-\boldsymbol{k}')\left[P_\varepsilon + k^2 P_\varepsilon^{\{2\}} + \cdots\right]$$

- In the end, stochasticity reduces to fixed number of additional free parameters

From bias expansion to statistics

- Once we have bias expansion, we can derive PT kernels for galaxies, and hence galaxy statistics, as a function of the bias parameters:

$$\delta_{\rm g}(\boldsymbol{x},\eta) = \delta_{\rm g}^{(1)}(\boldsymbol{x},\eta) + \delta_{\rm g}^{(2)}(\boldsymbol{x},\eta) + \cdots + \delta_{\rm g}^{(n)}(\boldsymbol{x},\eta)$$

-> Lecture 2

$$\delta_{\rm g}^{(n)}(\boldsymbol{k},\eta) = D_+^n(\eta)\left[\prod_{i=1}^{n}\int\frac{d^3k_i}{(2\pi)^3}\right](2\pi)^3\delta_{\rm D}^{(3)}\left(\boldsymbol{k}-\sum_{i=1}^{n}\boldsymbol{k}_i\right)$$
$$\times F_{{\rm g},n}(\boldsymbol{k}_1,\cdots,\boldsymbol{k}_n;\eta)\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n).$$

For example (see homework):

$$F_{{\rm g},2}(\boldsymbol{k}_1,\boldsymbol{k}_2;\eta) = b_1(\eta)F_2(\boldsymbol{k}_1,\boldsymbol{k}_2) + \frac{1}{2}b_2(\eta) + b_{K^2}(\eta)\left[\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2}{k_1^2k_2^2}-\frac{1}{3}\right].$$

Eq (12.87)

Application: galaxy power spectrum

- Assume we can measure rest-frame galaxy density
    - That is, neglect redshift-space distortions and other projection effects
- Leading-order (galaxy) power spectrum at fixed time:

$$P_{gg}(k) = b_1^2 P_{\mathrm{L}}(k) + P_\varepsilon^{\{0\}}$$

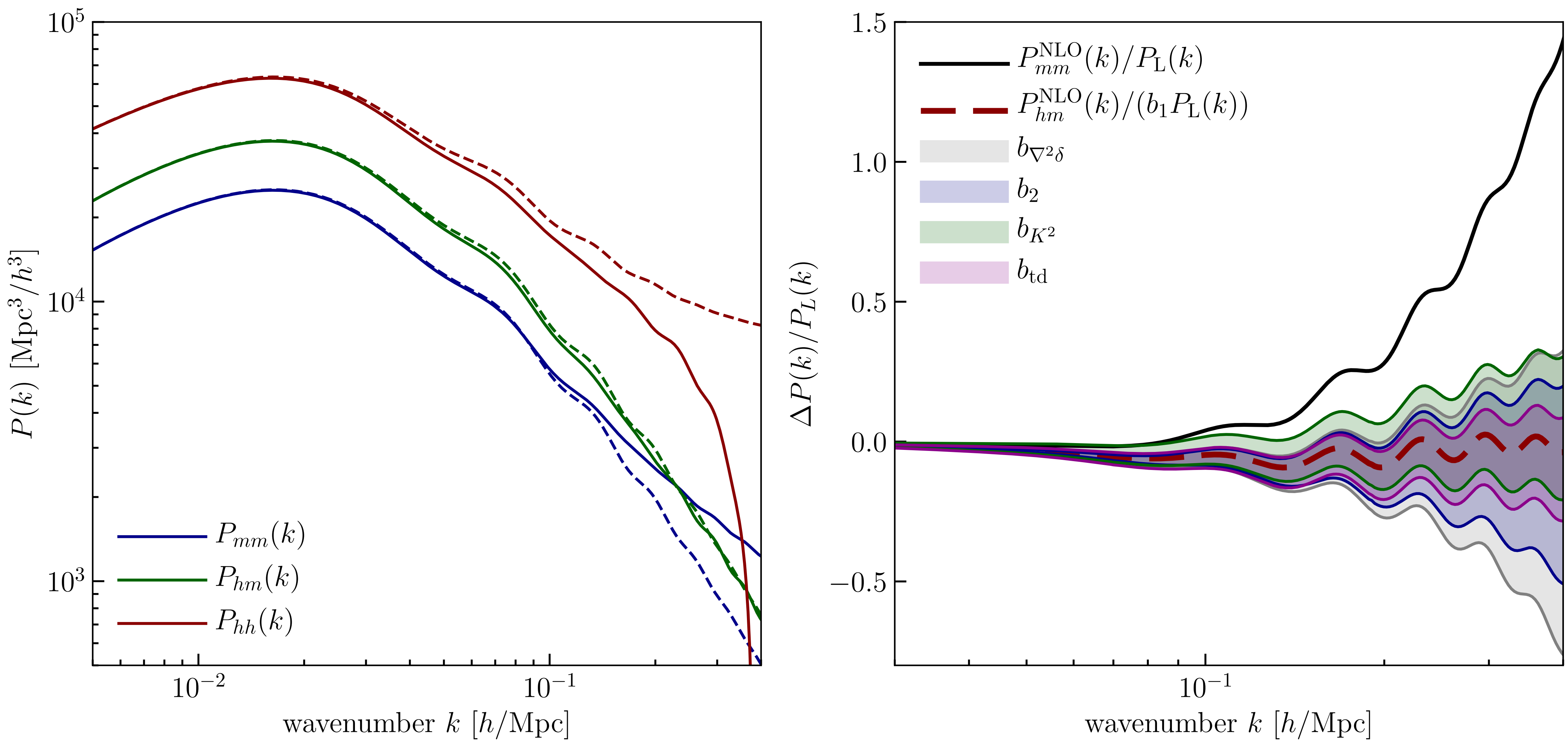
- 2 free parameters
- Noise term is often approximated as Poisson, but not accurate in general. $P_\varepsilon^{\{0\}} \overset{\text{Poisson}}{=} \frac{1}{\bar{n}_g}$

Application: galaxy power spectrum

- Next-to-leading order (NLO): involve 2 additional quadratic, 1 cubic, and 2 higher-derivative parameters

Application: galaxy power spectrum

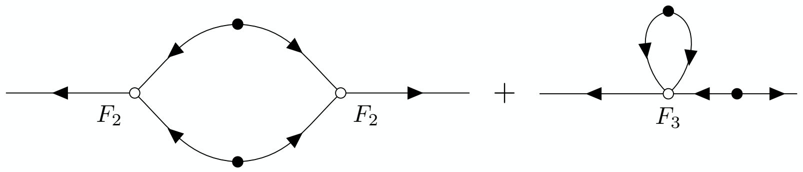
- Example calculation of NLO galaxy power spectrum, using guessed, order-unity values for bias coefficients



Application: galaxy power spectrum

- Next-to-leading order (NLO): involve 2 additional quadratic, 1 cubic, and 2 higher-derivative parameters
  - Quadratic and cubic terms scale like

$$\epsilon_{\text{loop}} \equiv \left(\frac{k}{k_{\text{NL}}}\right)^{3+n} \approx \left(\frac{k}{0.25\,h\,\text{Mpc}^{-1}}\right)^{1.3}$$



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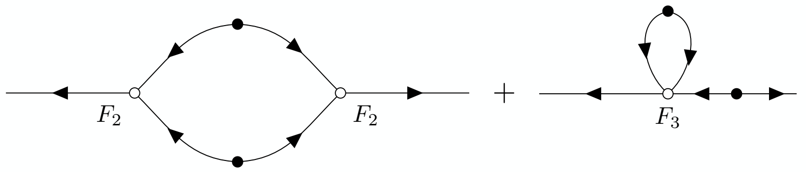

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Application: galaxy power spectrum

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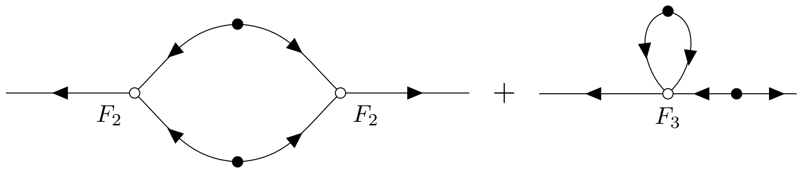
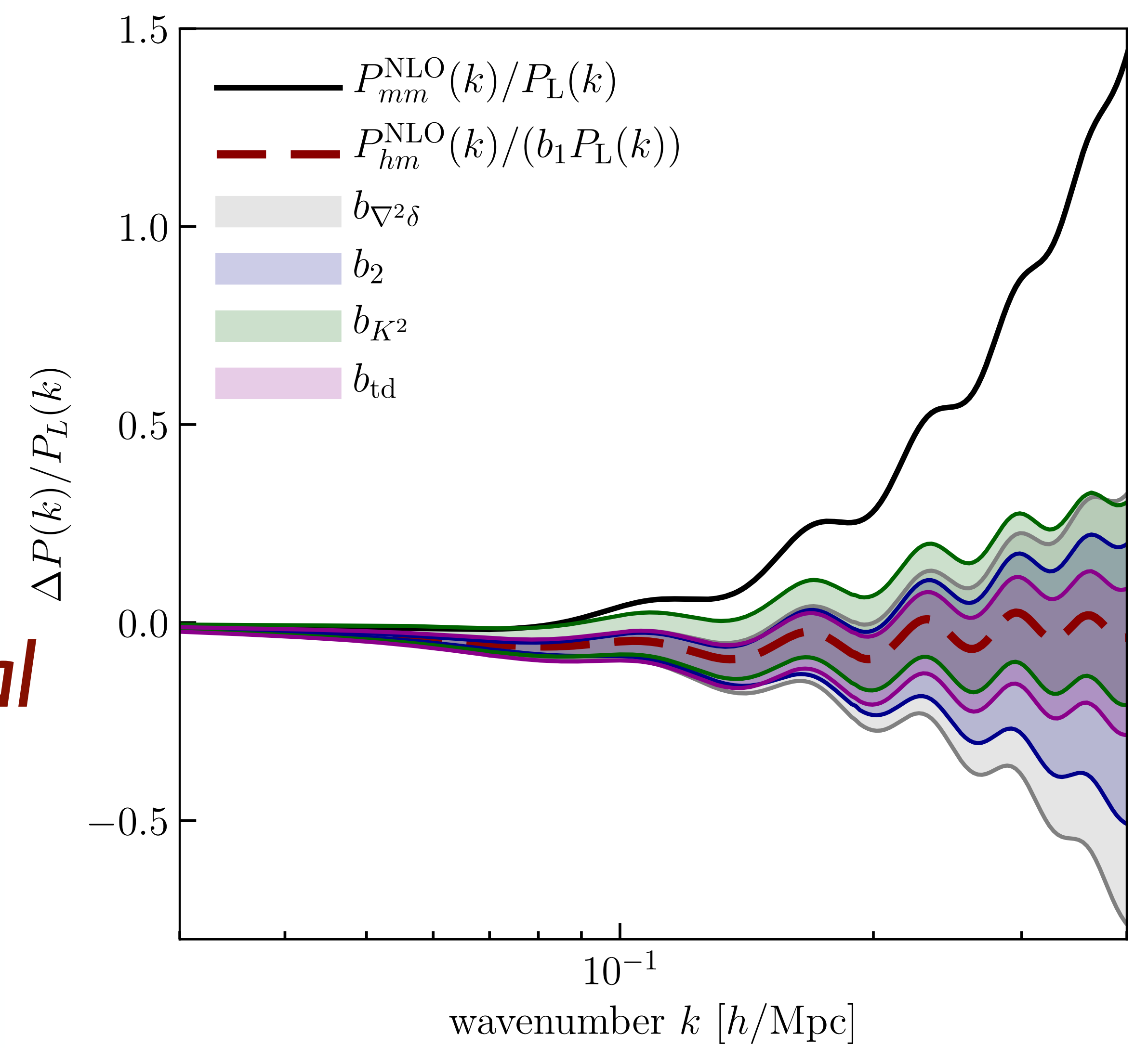


  - Controlled by shape of P(k) and nonlinear scale
  - Higher-derivative contributions scale as $\epsilon_{\rm deriv.} \equiv k^2 R_*^2$
- Obviously, NLO corrections become important toward smaller scales (higher k)
- Importantly: Two independent expansion parameters!

Application: galaxy power spectrum

- Beyond leading order: *5 additional parameters*
- Many contributions have very similar shape
- Free parameters *limit cosmological information that is available in power spectrum by itself*



Beyond the galaxy power spectrum

- There are significant degeneracies between bias and cosmology in galaxy power spectrum (e.g., $b_1(\eta)$ and $D(\eta)$)
- Currently, a lot of interest in looking at more general galaxy statistics to gain more cosmological information
  - Bispectrum (three-point function)
  - Position-dependent power spectrum
  - Voids
  - …
- Solid understanding of power spectrum is always the basis, however.

Observed galaxy clustering

- We are missing just one ingredient now: how to go from intrinsic (rest-frame) clustering of galaxies to observations
  - Lecture 5
- Then we'll also look at how galaxy clustering and other LSS probes can tell us about new cosmological physics