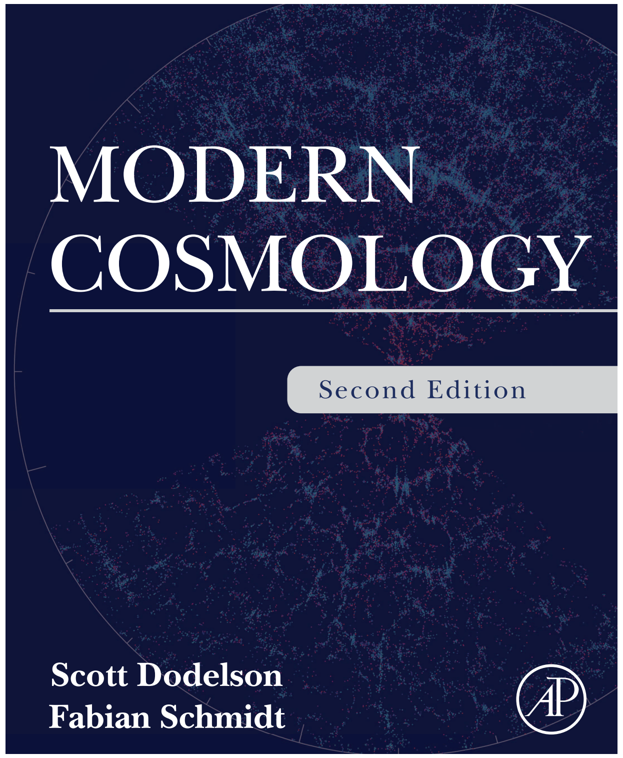
Structure Formation
Lecture 5

Fabian Schmidt
MPA

All figures taken from *Modern Cosmology, Second Edition*, unless otherwise noted



ICTP-SAIFR School on Cosmology, January 2021

Outline of lectures

1. The problem: collisionless Boltzmann equation and fluid approximation
    1. Linear evolution
2. Nonlinear evolution of matter
    1. Perturbation theory
    2. Simulations
    3. Phenomenology of nonlinear matter distribution
3. Formation and distribution of galaxies
    1. Galaxy formation in a nutshell
    2. Spherical collapse model
    3. Physical clustering of halos and galaxies; bias
    4. Observed clustering of galaxies <- HERE
4. Beyond ΛCDM

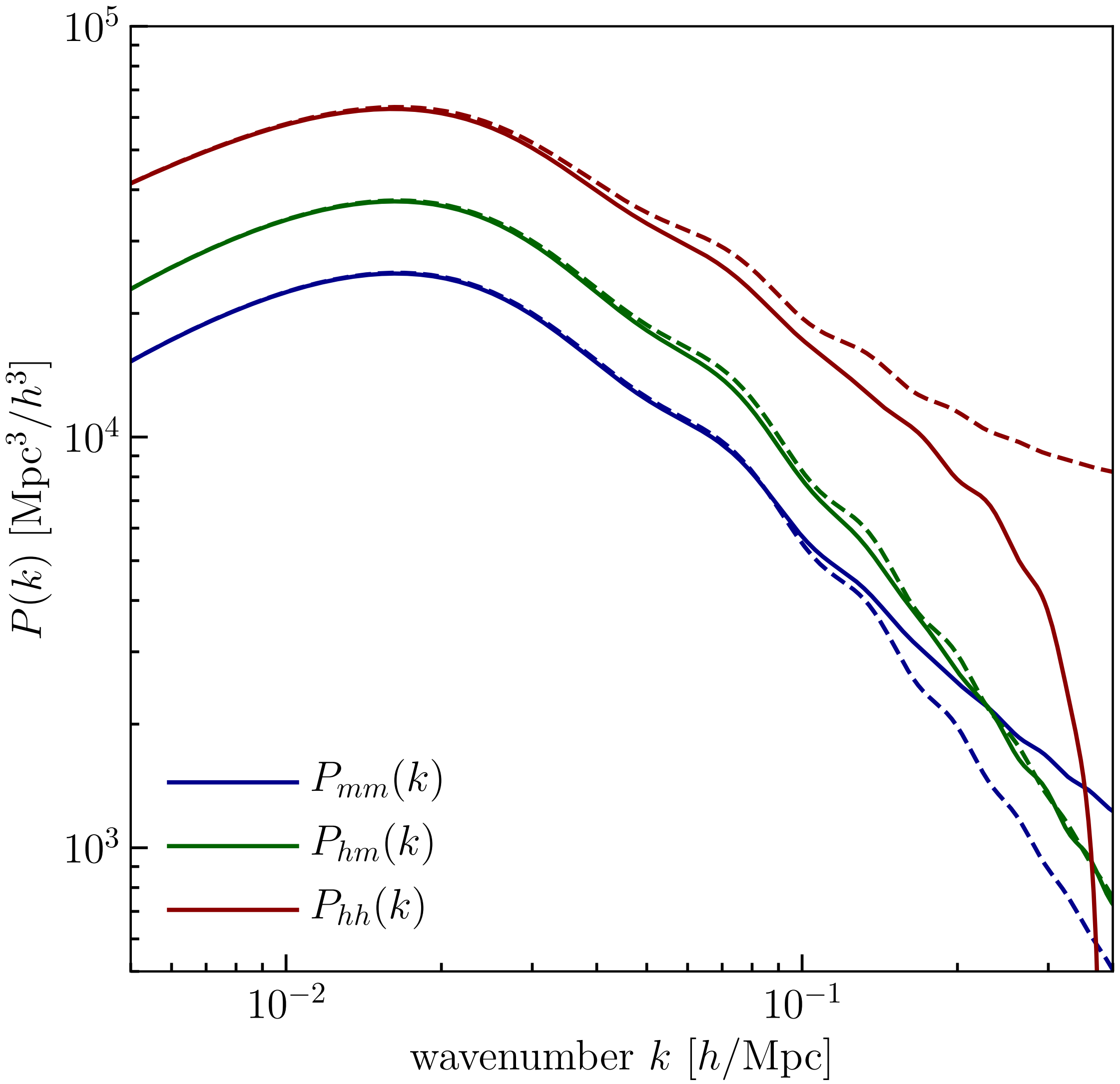
Notation

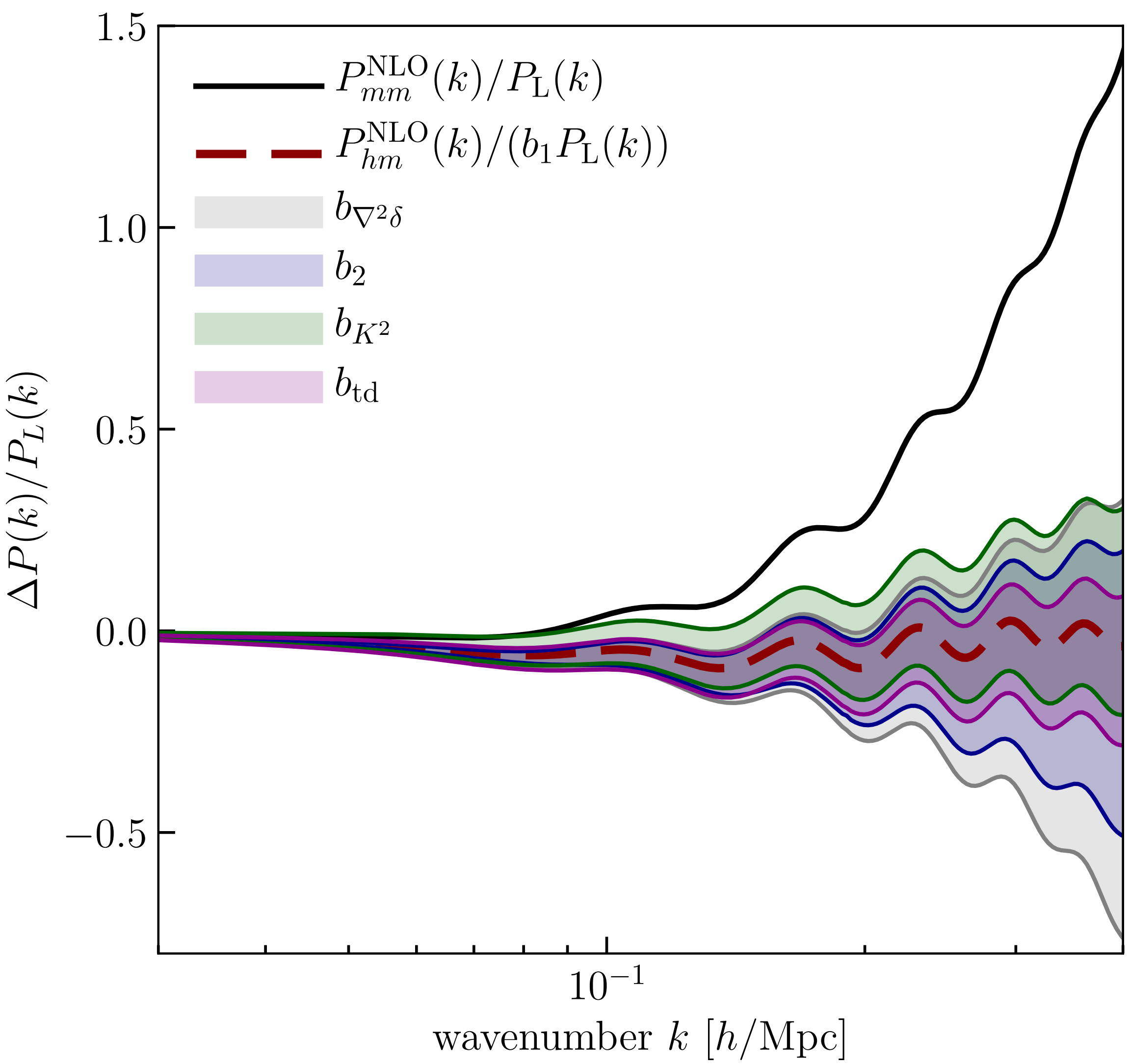
$$ds^2 = -(1 + 2\Psi(\boldsymbol{x}, t))dt^2 + a^2(t)(1 + 2\Phi(\boldsymbol{x}, t))d\boldsymbol{x}^2$$

- Comoving coordinates: $d\boldsymbol{r} = a(t)d\boldsymbol{x}$
- Conformal time: $d\eta = \frac{dt}{a(t)} = \frac{da}{a^2 H(a)} = \frac{d\ln a}{aH(a)}.$
  - Comoving distance: $d\chi = -d\eta = \frac{dz}{H(z)}$
- Particle velocity/momentum: $\boldsymbol{v} = \frac{\boldsymbol{p}}{m} = a\frac{d\boldsymbol{x}}{dt} = \boldsymbol{x}'$
- Fluid velocity; divergence: $\boldsymbol{u}; \quad \theta = \partial_i u^i$
- Gravitational potential: Ψ

Galaxy power spectrum

- Example calculation of NLO rest-frame galaxy power spectrum, using guessed, order-unity values for bias coefficients





Observed galaxy statistics

- Observed galaxy positions x_{obs} are given by position on the sky and measured redshift
- Need to connect this to “true” position x of galaxy
- Main effect: Doppler shift to redshift due to peculiar velocity of galaxy:

$$\boldsymbol{x}_{\rm obs} = \boldsymbol{x} + \frac{\boldsymbol{u}_{\rm g} \cdot \hat{\boldsymbol{n}}}{aH} \hat{\boldsymbol{n}}$$

Observed galaxy statistics

- Observed galaxy statistics are obtained from rest-frame statistics via coordinate transformation:

$$\boldsymbol{x}_{\rm obs} = \boldsymbol{x} + \frac{\boldsymbol{u}_{\rm g} \cdot \hat{\boldsymbol{n}}}{aH} \hat{\boldsymbol{n}}$$ and

$$n_{\rm g,obs} d^3 \boldsymbol{x}_{\rm obs} = n_{\rm g} d^3 \boldsymbol{x}$$ (number conservation)

so $$1 + \delta_{\rm g,obs}(\boldsymbol{x}_{\rm obs}) = [1 + \delta_{\rm g}(\boldsymbol{x}[\boldsymbol{x}_{\rm obs}])] \left| \frac{\partial^3 \boldsymbol{x}_{\rm obs}}{\partial^3 \boldsymbol{x}} \right|^{-1}$$

Observed galaxy statistics

- Observed galaxy statistics are obtained from rest-frame statistics via coordinate transformation:

$$1+\delta_{\mathrm{g,obs}}(\boldsymbol{x}_{\mathrm{obs}}) = [1+\delta_{\mathrm{g}}(\boldsymbol{x}[\boldsymbol{x}_{\mathrm{obs}}])]\left|\frac{\partial^3\boldsymbol{x}_{\mathrm{obs}}}{\partial^3\boldsymbol{x}}\right|^{-1}$$

- By combining three ingredients, we can obtain consistent theoretical description for observed galaxy statistics (e.g. power spectrum, or higher n-point functions in redshift space):
  - Perturbation theory for matter
  - Bias expansion to get δ_g
  - Velocity bias expansion to get u_g

Velocity bias

- Galaxy velocities are important probe of cosmology - but how are they related to matter velocity?
- The relative velocity between matter and galaxies is an observable, and thus cannot involve $\Psi,\ \boldsymbol{\nabla}\Psi, \boldsymbol{u}$
- Leading contribution:

$$\boldsymbol{u}_{\rm g} - \boldsymbol{u}_{\rm m} = \beta \boldsymbol{\nabla}\delta$$

- Two more derivatives: *suppressed by* k^2

Velocity bias

- $\boldsymbol{u}_{\mathrm{g}} - \boldsymbol{u}_{\mathrm{m}} = \beta \boldsymbol{\nabla} \delta$
- This is what we expect from pressure forces too:

$$\boldsymbol{F} = \boldsymbol{\nabla} \delta p \propto \boldsymbol{\nabla} \delta$$

- Summary: Galaxy velocities are unbiased on large scales
- *Observed* galaxy density at linear order:

$$\delta_{\mathrm{g,obs}}(\boldsymbol{x}_{\mathrm{obs}}, \eta) = b_1 \delta_{\mathrm{m}} + \frac{1}{aH} \partial_z u_z + \varepsilon$$

Velocity bias

- $\boldsymbol{u}_{\rm g} - \boldsymbol{u}_{\rm m} = \beta \boldsymbol{\nabla}\delta$
- This is what we expect from pressure forces too:

  $\boldsymbol{F} = \boldsymbol{\nabla}\delta p \propto \boldsymbol{\nabla}\delta$

- Summary: Galaxy velocities are unbiased on large scales
- *Observed* galaxy density at linear order:

$$\delta_{\rm g,obs}(\boldsymbol{x}_{\rm obs}, \eta) = b_1 \delta_{\rm m} + \frac{1}{aH}\partial_z u_z + \varepsilon$$

redshift-space distortions

Velocity bias

- $\boldsymbol{u}_{\rm g} - \boldsymbol{u}_{\rm m} = \beta \boldsymbol{\nabla} \delta$
- This is what we expect from pressure forces too:

  $\boldsymbol{F} = \boldsymbol{\nabla} \delta p \propto \boldsymbol{\nabla} \delta$

- Summary: Galaxy velocities are unbiased on large scales
- *Observed* galaxy density at linear order:

$$P_{\rm g,obs}(\boldsymbol{k}) = \left[b_1 + (\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}})^2 f \right]^2 P_{\rm L}(k) + P_\varepsilon$$

redshift-space distortions

$f \equiv d \ln D / d \ln a$

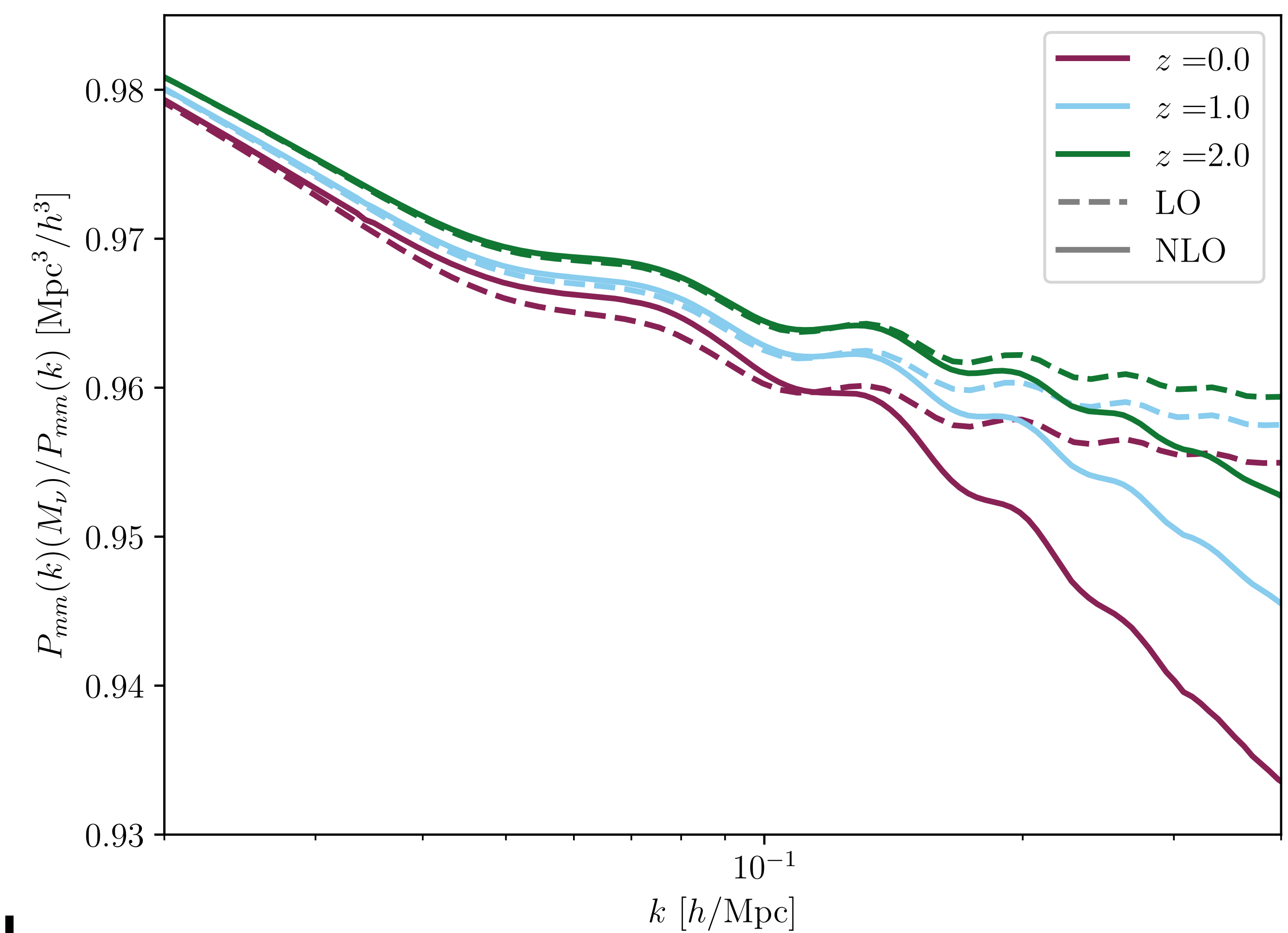
Probing cosmology with LSS

- Have already seen that *dark energy* affects growth factor
  - Probed via *gravitational lensing* (measuring entire clustering stress-energy) and by *galaxy velocities* (via redshift-space distortions)
  - Expansion history constrained by BAO feature
- *Modified Gravity*
- *Primordial non-Gaussianity*
- *Neutrinos*

Neutrinos

Effect of massive neutrinos on LSS

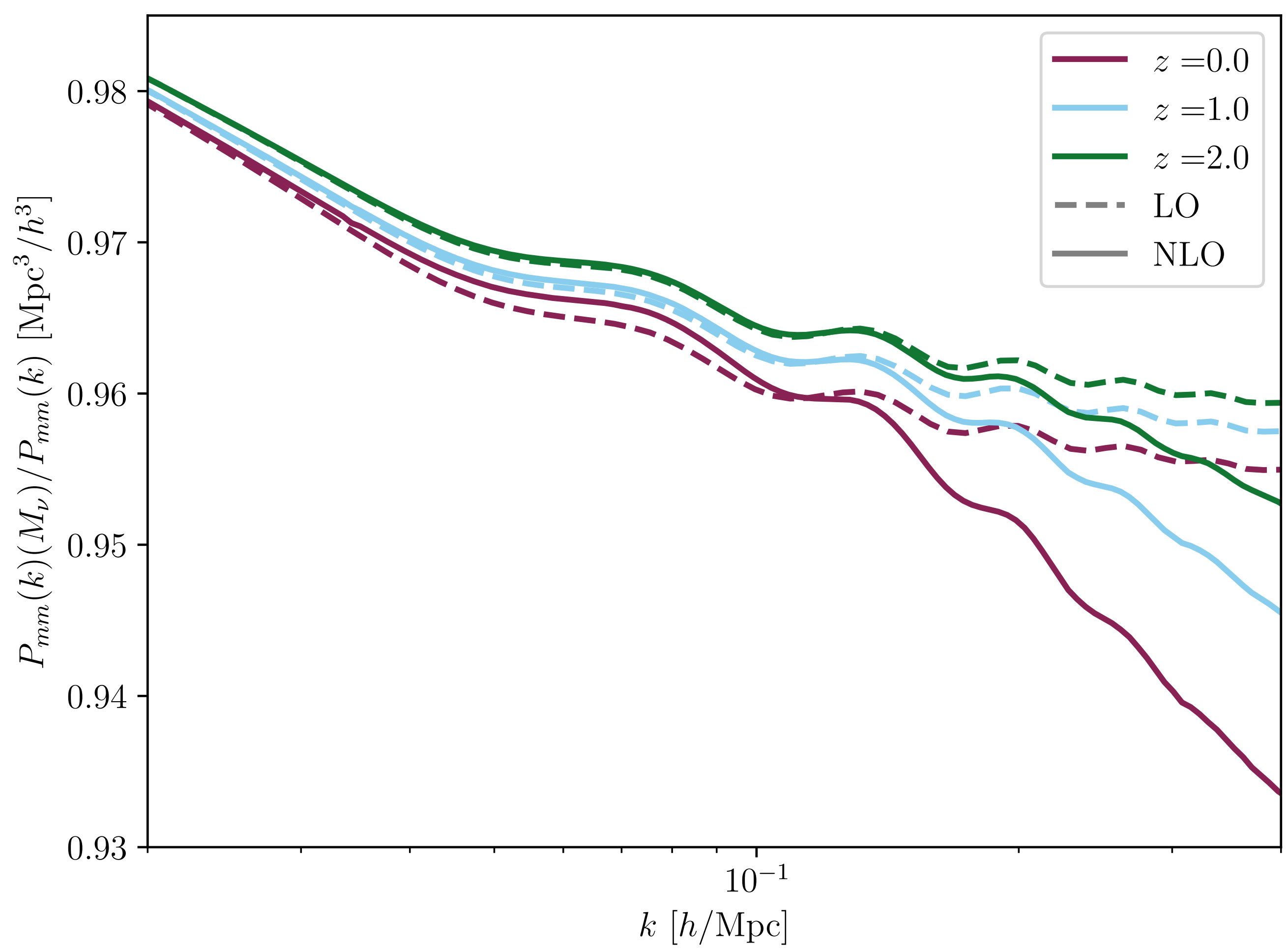
- The effect of massive neutrinos on clustering can't be neglected, because they become nonrelativistic in the late universe (unlike radiation)
- Neutrinos *change expansion history* and lead to *scale-and redshift-dependent suppression* of matter power spectrum
- Goal: measure these effects via galaxy clustering or gravitational lensing and constrain M_ν



Boyle & FS, 2020

Effect of massive neutrinos on LSS

- How to treat mildly relativistic neutrinos, and scale-dependent growth accurately?
- Various analytical and numerical approaches have been proposed
- Fortunately, a small effect, so we do not need extremely accurate treatment
- To a good approximation, *galaxy bias can be assumed to refer to CDM+baryons*



Boyle & FS, 2020

Primordial non-Gaussianity

Probing interactions during inflation

Primordial non-Gaussianity

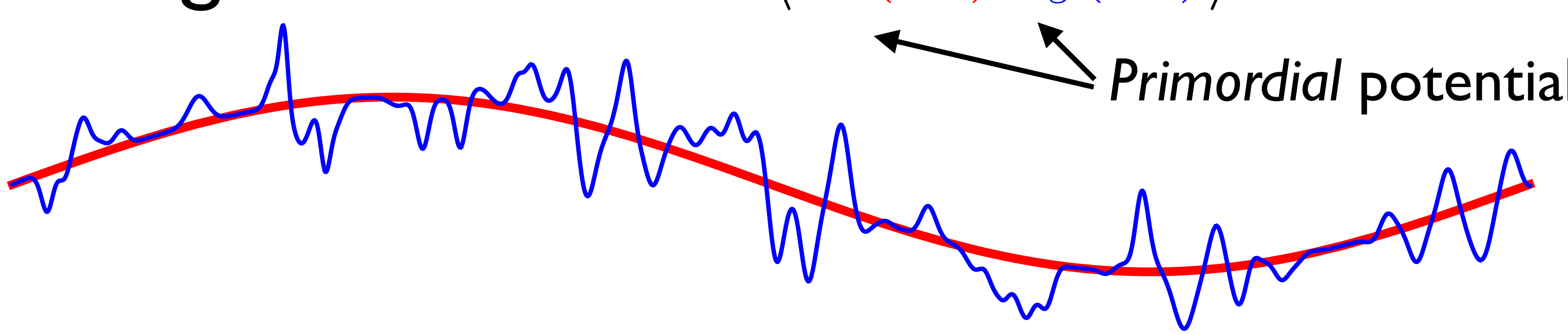
- Single-field inflation: fluctuations generated at some point know nothing about larger-scale perturbations that left the horizon long ago

Primordial non-Gaussianity

- Single-field inflation: fluctuations generated at some point know nothing about larger-scale perturbations that left the horizon long ago

- *No coupling between modes of widely different wavelengths* Φ_l, Φ_s*: $\langle \Phi_l(\mathbf{x}_1) \Phi_s^2(\mathbf{x}_2) \rangle = 0$

Primordial potential



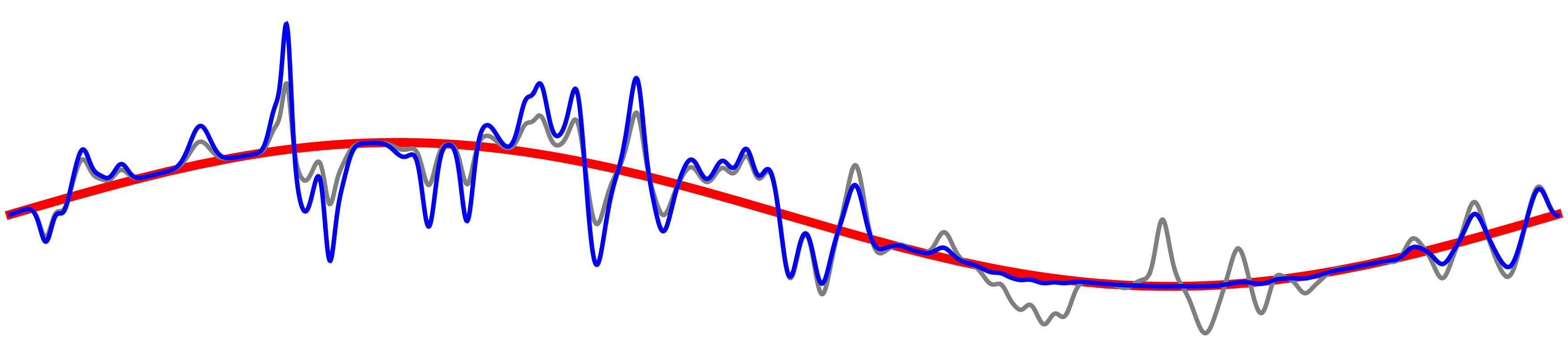
- Technically: *no non-Gaussianity of the local type generated:* $f_{\rm NL} = 0$

* $\langle \Phi_l(\mathbf{x}_1) \Phi_s(\mathbf{x}_2) \rangle = 0$ by construction

Primordial non-Gaussianity

- *A detection of local-type NG would rule out single-field inflation*

- Effect on LSS: large-scale potential perturbation rescales small-scale density field: $\delta_s(\mathbf{x}) \rightarrow [1 + 2 f_{\rm NL} \Phi(\mathbf{x})]\, \delta_s(\mathbf{x})$



LSS with non-Gaussianity

- For Gaussian initial conditions, the dependence of the tracer abundance on the small-scale fluctuations δ_s is irrelevant for clustering
  - We treated these by adding the constant noise contribution $P_\varepsilon^{\{0\}}$
- With $f_{\rm NL}$, we can no longer ignore this dependence because δ_s is correlated over long distances !

$$\delta_s(\mathbf{x}) \to [1 + 2 f_{\rm NL} \Phi(\mathbf{x})]\, \delta_s(\mathbf{x})$$

LSS with non-Gaussianity

- With local PNG, amplitude of small-scale initial fluctuations depends on value of primordial potential Φ. Thus, Φ has to appear in bias expansion
  - This effect can only be induced primordially: equivalence principle forbids galaxies from knowing about Φ,Ψ otherwise!

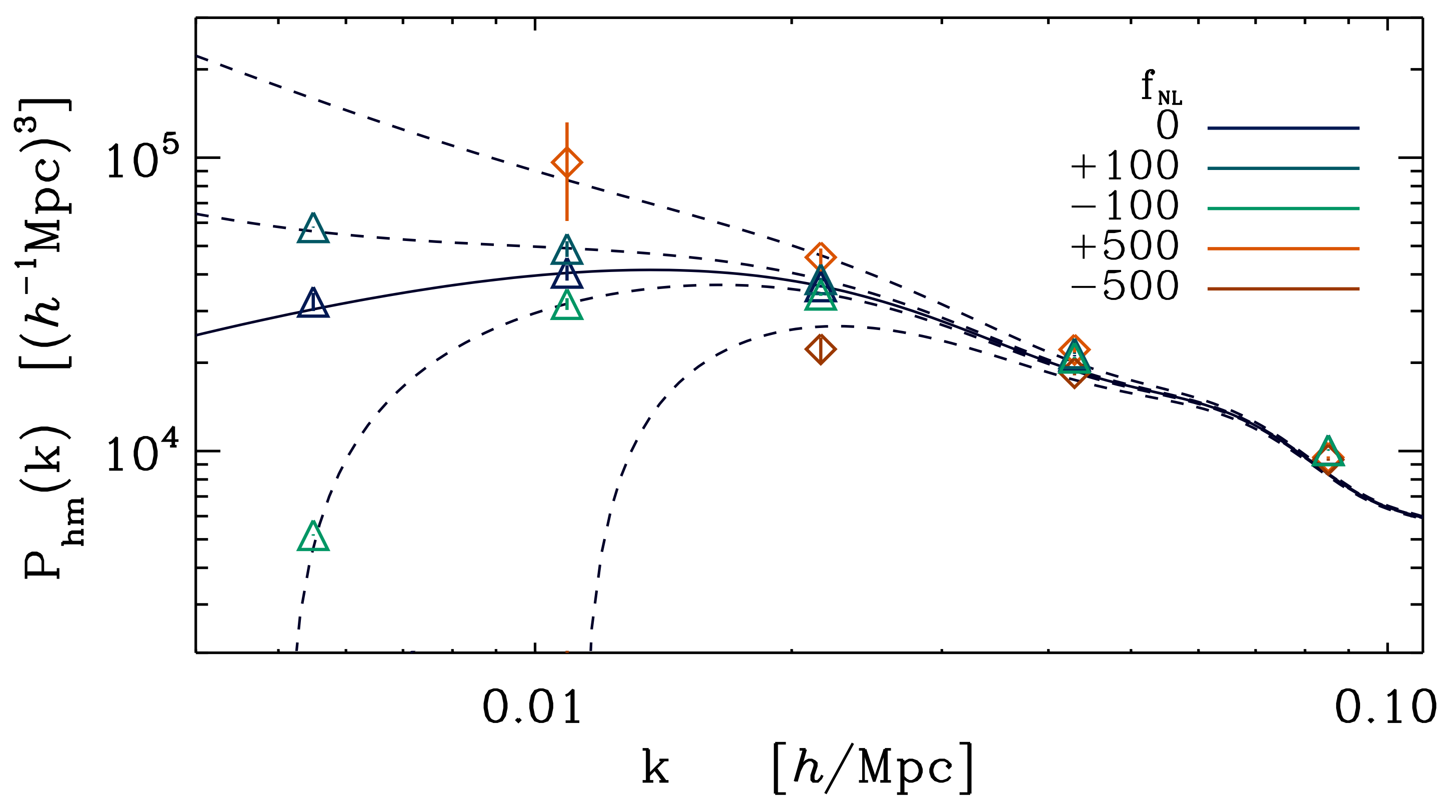
$$\delta_{\rm g}(\boldsymbol{x},\eta) = b_1(\eta)\delta_{\rm m}(\boldsymbol{x},\eta) + \ldots + b_\Phi(\eta) f_{\rm NL}\Phi(\boldsymbol{q})$$

- Leads to characteristic signature in $P_g(k)$ on large scales, since

$$\Phi(\boldsymbol{k}) \propto \frac{1}{k^2 T(k) D(\eta)}\delta_{\rm m}(\boldsymbol{k},\eta)$$

Prediction for LSS statistics

- With local NG, tracers follow the potential on large scales rather than matter
- Unique signature: probing highest energy physics with galaxies on the largest scales!
- Similar contributions to galaxy bispectrum, which we can compute as well
- Future LSS surveys should be able to improve on Planck constraints using this effect



Dalal et al., 2008

Modified Gravity

Slides from talk by FS, 2011

Viable Modified Gravity

- **Gravity well constrained on wide range of scales:**
  - *Early Universe:* BBN, CMB (z ~ 1100, L ~ 10^4 Mpc)
  - *Today:* Solar System (z=0, L ~ 10^{-11} Mpc)

- **Idea: reduce to GR in high-density / curvature regime**
  - Applies to Early Universe & Solar System
  - Some (additional) non-linear mechanism needed

General difficulty with testing GR in cosmology: we have to make assumptions about matter content ($T_{\mu\nu}$).

-> Pedro Ferreira's lecture

Scalar-Tensor Theories in Cosmology

WARNING: slightly different metric convention in following slides.

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)d\boldsymbol{x}^2$$

Scalar-Tensor Theories in Cosmology

WARNING: slightly different metric convention in following slides.

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)d\boldsymbol{x}^2$$

Einstein frame

- **Scalar-tensor theory:** $g_{\mu\nu} = \exp(-\phi)\tilde{g}_{\mu\nu}$

$$\Rightarrow \Psi = \Psi_N + \frac{1}{2}\phi$$

$$\Phi = \Phi_N - \frac{1}{2}\phi$$

$$\Psi - \Phi = \Psi_N - \Phi_N$$

$$\Psi + \Phi = \phi$$

(at late times)

Metric perturbations obtained in GR

Scalar-Tensor Theories in Cosmology

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)d\boldsymbol{x}^2$$

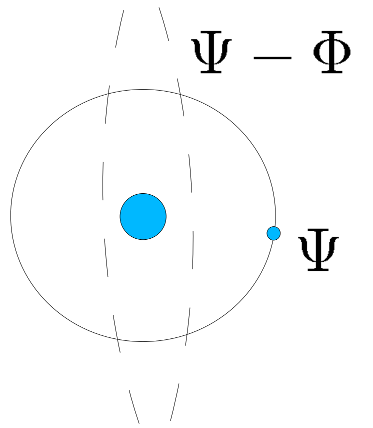
Einstein frame

- **Scalar-tensor theory:** $g_{\mu\nu} = \exp(-\phi)\tilde{g}_{\mu\nu}$

$$\Rightarrow \Psi = \Psi_N + \frac{1}{2}\phi \qquad\qquad \Psi - \Phi = \Psi_N - \Phi_N$$

Non-rel. dynamics

Lensing

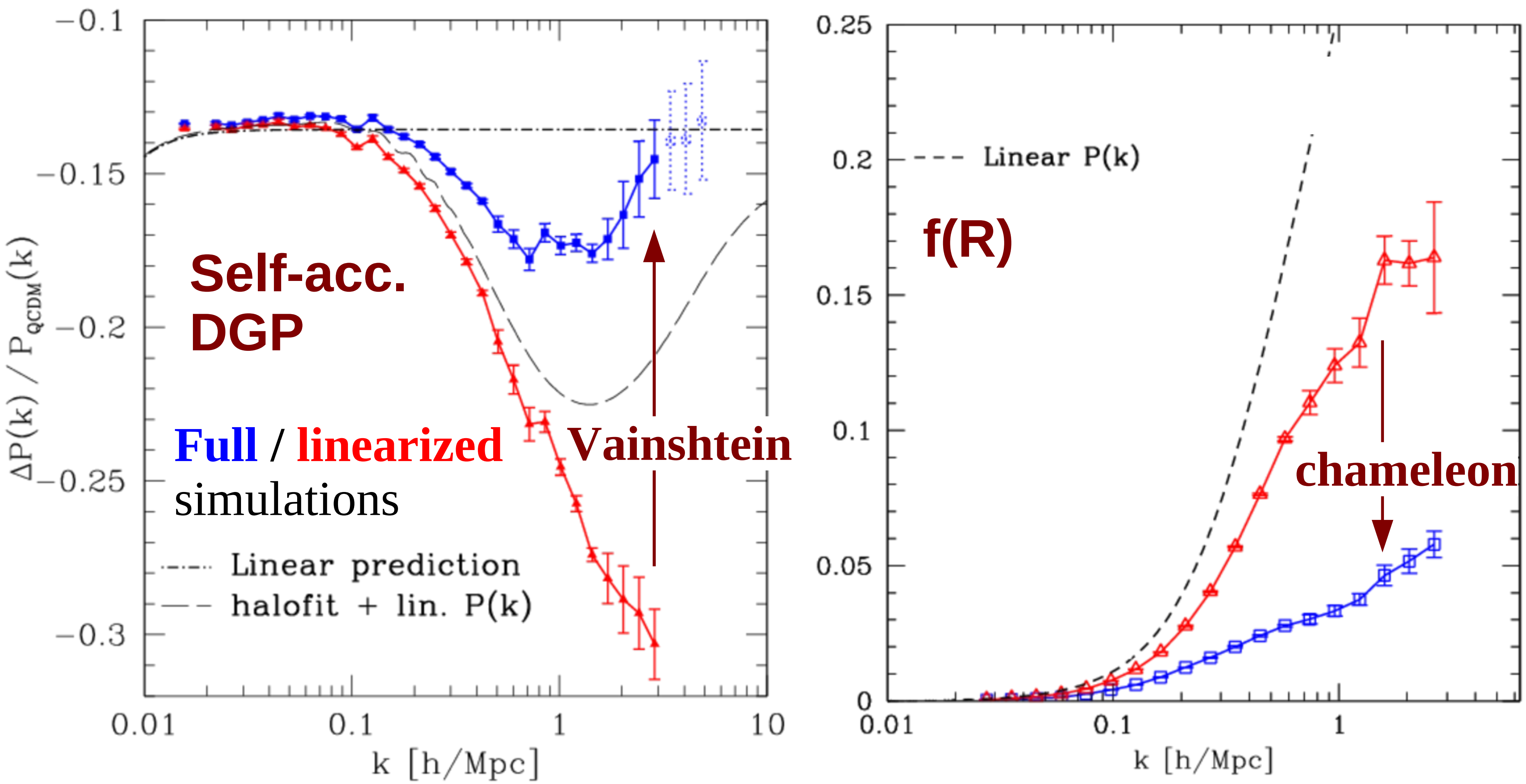


In general also: $G_N \to G_N\, f(\phi)$

In a wide class of modified gravity theories, forces on non-relativistic objects are enhanced, but lensing is unmodified!

Linear vs Non-linear Scales...

– Rel. deviation of matter P(k) from ΛCDM / smooth DE



Simulations of modified gravity are substantially more effort than those for standard GR…

Direct Tests of Gravity ... ?

- **Compare (non-rel.) dynamics with lensing:**

$$\Psi = \Psi_N + \frac{1}{2}\phi \qquad \Psi - \Phi = \Psi_N - \Phi_N$$

  - *Linear regime:* redshift distortions vs weak lensing

Zhang et al 08, Reyes et al 2010

  - *Non-linear regime:* dynamical mass vs lensing mass

Schwab et al, Smith 09
FS, 2010

X-ray, SZ, galaxy dynamics in clusters
Rotation in galaxies

Unique signature of modified gravity - hard to generate in any other way!