Beyond ΛCDM

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Focus on Gravity.

References:

P. G. Ferreira (1902.10503)
T. Baker et al (1908.03430)
M. Ishak et al (1905.09687)
C. Burrage & J. Sakstein (1709.09071)

+ ...

Preamble (history)

Gravity is the weakest force → Einstein
→ Eddington
→ "Low hanging fruit" → Schwarzschild
→ Friedman
→ Lemaitre
→ The "wilderness years" (QM, maths, ...).

Feynman: "There exists one serious difficulty and that is the lack of experiments ... the best viewpoint is to pretend that there are experiments and calculate. In this field we are not pushed by experiments but pulled by imagination"
1957 Chapel Hill

Revival → Quasars
→ CMB
→ Black holes (X-rays)
→ Singularity theorems
→ Physical Cosmology

NOW! → cosmology
→ LIGO-VIRGO
→ Sag A*
→ EHT
→ Pulsars
→ ...

What is Gravity

Newton $\vec{x}(t)$

$$\frac{d^2\vec{x}}{dt^2} = \vec{g} = -\vec{\nabla}\underset{\substack{\uparrow \\ \text{grav. pot.}}}{\Phi}$$

$$\nabla^2\Phi = 4\pi G\rho$$

Einstein

$$\frac{d^2x^\beta}{d\lambda^2} + \Gamma^\beta{}_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0$$

(geodesic equ)

$$ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$$

(metric

$$G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}Rg^{\alpha\beta}$$

$$= 8\pi G T^{\alpha\beta} - \Lambda g^{\alpha\beta}$$

$$\boxed{g_{00} = -(1 + 2\Phi)}$$

Note that: ① $\nabla_\alpha G^{\alpha\beta} = 0$ and $\nabla_\alpha T^{\alpha\beta} = 0$

② $\Gamma \sim g^{-1}\partial g$ and $R \sim \partial\Gamma + \Gamma\Gamma$ (ie 2^{nd} order)

What is Gravity

Einstein-Hilbert Action

$$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g}\,(R-2\Lambda) + \int d^4x \sqrt{-g}\,\mathcal{L}(g, \text{matter})$$

where $M_{Pl}^2 = \dfrac{1}{8\pi G}$

Vary $\delta g^{\alpha\beta}$

$$\delta S = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g}\left[(G_{\alpha\beta} + \Lambda g_{\alpha\beta})\,\delta g^{\alpha\beta} + \underbrace{(g_{\alpha\beta}\Box - \nabla_\alpha \nabla_\beta)}_{\text{surface terms.}}\right]\delta g^{\alpha\beta}$$

$$- \int d^4x \sqrt{-g}\, T_{\alpha\beta}\,\delta g^{\alpha\beta}$$

E.o.m. don't depend on boundary <u>but</u> need to impose ($\delta g + \delta \dot{g}$)

Note: can add boundary term $\quad + M_{Pl}^2 \int_{\partial M} d^4x \sqrt{-\gamma}\, K$

metric on boundary.

Extrinsic curvature

What is Gravity

Tests:

- gravitational redshift $\frac{\nu_B}{\nu_A} = \frac{d\tau_A}{d\tau_B} = \sqrt{\frac{g_{00}(A)}{g_{00}(B)}} = 1 - (\Phi_B - \Phi_A)$

  (Pound Rebka '60 – now $\sim 0.01\%$)

- orbits (mercury)

  $\Delta\theta \sim 43''$ per century

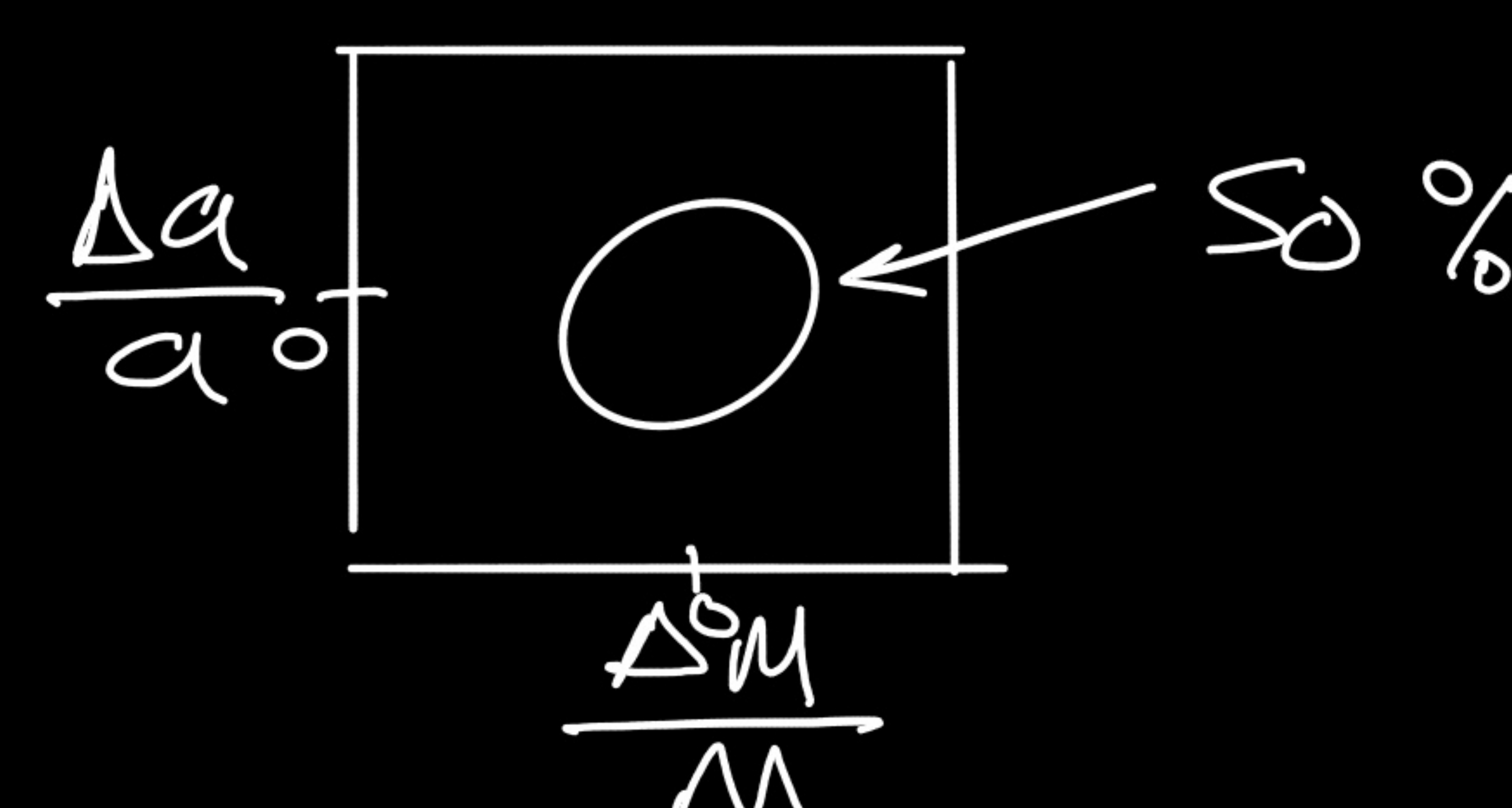
- Lensing $\Delta\Theta_{GR} = 2\Delta\Theta_N = \frac{4GM}{R}$ $\left\{ \begin{array}{l} \cdot \text{ strong lenses} \\ \cdot \text{ weak lenses} \end{array} \right.$

- orbits (Shapiro)

  $$c\Delta t \approx \frac{2GM}{c^2} \ln\left[\sqrt{\frac{r^2}{R^2} - 1} + \frac{r}{R}\right]$$

- binary pulsars, $\frac{\dot{P}}{P}$

- Gravitational Waves



Why GR?

Lovelock's Theorem (1971) — EH action is the only local, 4D, 2nd order equations of motion for metric.

Note: none of these are essential

(Lovelock, J. Math. Phys 12, 874, 1971).

Feynman / Weinberg / Deser (1960s) — EH action is the only 'consistent' theory of a spin 2 field.

Feynman, Lectures on Gravitation

Weinberg, P.Rev, 138, B988, 1965

Deser, GRG, 1, 9, 1970

But see: Padmanabhan gr-qc/0409089

Why GR?

Sketch of field theory argument

$$h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

spin 2 field

(gauge symmetry)

Where could this symmetry come from?

$$\tilde{g}_{\alpha\beta} \longrightarrow \frac{\partial x^\mu}{\partial \tilde{x}^\alpha} \frac{\partial x^\nu}{\partial \tilde{x}^\beta} g_{\mu\nu}$$

Expand:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \qquad \tilde{x}^\mu = x^\mu + \xi^\mu(\tilde{x}^\alpha)$$

Minkowski

So gauge symmetry is linearized diffeo. invariance

Why GR?

Massless Fierz-Pauli action

$$S_{FP} = \frac{M_{Pl}^2}{2} \int d^4x \left[-\frac{1}{2} \partial_\mu h_{\nu\beta} \partial^\mu h^{\nu\beta} + \partial_\mu h_{\nu\beta} \partial^\nu h^{\mu\beta} - \partial_\mu h \partial_\beta h^{\mu\beta} + \frac{1}{2} \partial_\mu h \partial^\mu h \right]$$

Vary with $\delta h^{\alpha\beta}$ we get linearized GR.

Pick a gauge $(\partial^\mu h_{\mu\nu} = \partial^\nu h_{\mu\nu} = 0)$ $\quad \Box h_{\mu\nu} = 0$

But no source...

Add $S_\phi = \frac{1}{2} \int d^4x \, \partial_\mu \phi \partial^\mu \phi$ to get $\begin{cases} \Box h_{\mu\nu} = 0 \\ \Box \phi = 0 \end{cases}$ uncoupled...

We want $\Box h_{..} \sim T^\phi_{..}$

where $T^\phi_{\mu\nu} = -\partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \eta_{\mu\nu} (\partial\phi)^2$

(Energy momentum tensor).

Why GR?

Add a coupling $\quad S_1 = S_{FP} + S_\phi + \frac{K}{2}\int d^4x\, h^{\mu\nu} T_{\mu\nu}$

Now $\quad \Box h_{\mu\nu} = K T_{\mu\nu}$

$$\Box\phi = K\,\partial_\mu\left[h^{\mu\nu}\partial_\nu\phi - \frac{1}{2}h\,\partial_\mu\phi\right]$$

Now

$$0 = \partial^\nu \Box h_{\mu\nu} = K\,\partial^\mu T_{\mu\nu}(\phi) = K\,\Box\phi\,\partial_\nu\phi \neq 0$$

inconsistent

Add graviton self energy

$$S_2 = S_1 + \frac{K}{2}\int d^4x\, h^{\mu\nu} T^h_{\mu\nu}$$

quadratic in ∂h and h.

Doesn't work : $\Box h_{\mu\nu} \neq K T^\phi_{\mu\nu} + K T^h_{\mu\nu}$

Note: keep on adding $O(3), O(4), \ldots$ only works when $O(\infty) \Longleftrightarrow$ EH action!

Why GR?

What about higher derivatives?

Sakharov 1967 : $S\,[\phi, g_{\alpha\beta}] = \int d^4x \sqrt{-g}\; g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi$

(note → no kinetic term for $g_{\alpha\beta}$ ...)

Integrate out ϕ

$$\int D\phi \; e^{\frac{i S[\phi, g]}{\hbar}} \longrightarrow e^{\frac{i S_{eff}}{\hbar}}$$

Now

$$S_{eff} \sim \sqrt{-g}\left[\Lambda + \frac{M_{pl}^2}{2} R + a_1 R_{\mu\nu} R^{\mu\nu} + a_2 R^2 + \dots\right]$$

Naively we expect corrections highly supressed

We need $R \sim M_{pl}^2$

but, typically $R << M_{pl}^2$.

E.g: $\frac{M_{pl}^2}{2} R + a_2 R^2 = \frac{M_{pl}^2}{2} R \left[1 + a_2 \frac{R}{M_{pl}^2}\right]$

Note: Ostragradsky too ...

(See Woodard astro-ph/0601672)

Why GR?

- GR leading theory for gravity
- Beautifully confirmed by experiment
- Theoretical prejudice (Lovelock's theorem)
- Theoretical prejudice (Field theory of spin-2 particle)
- Higher derivatives : supressed or unstable

But

- Lovelock gives us a way of exploring other options ...