Why modify Gravity?

- Gravity unlike Strong, Weak, Electro $\rightarrow$ not 'UV complete'
- 'Just' an Effective Field Theory
- Low energies : $\dfrac{\rho_\Lambda}{\rho_{\text{Mpc}}} \sim 10^{-120}$ (CC problem)
- Large Numbers (Dirac's version)

$$G \sim \frac{M}{R} \begin{array}{l} \leftarrow \text{mass in Horizon} \\ \leftarrow \text{size of Horizon} \end{array}$$

... if $M \sim \rho R^3$

$$\frac{1}{R^2} G \sim \rho \xrightarrow[\text{covariantize}]{} \Box G \sim \rho$$

+ Dynamical.

How can we modify gravity?

Lovelock Revisited

- Local
- Metric
- 2nd order
- 4D

We can violate any of these assumptions!

Ur Example : Jordan-Brans-Dicke Gravity

(Brans & Dicke, PRev, 124, 3, 925, 1961)

1 free parameter

Action

$$S = \int d^4x \sqrt{-g} \left[\Phi R - \frac{\omega}{\Phi} \partial_\mu \Phi \partial^\mu \Phi + \mathcal{L}_m (g, \text{matt}) \right.$$

G is a field

One parameter (like GR)

$\omega \to \infty$ then $(\partial_\mu \phi)^2 \to 0 \to \Phi$ constant GR!

Field equations

from boundary

$$\Phi G_{\mu\nu} + \overbrace{\Box \Phi g_{\mu\nu} - \nabla_\mu \nabla_\nu \Phi}^{\text{from boundary}} + \frac{1}{2} \frac{\omega}{\Phi} (\nabla \Phi)^2 g_{\mu\nu} - \frac{\omega}{\Phi} \nabla_\mu \Phi \nabla_\nu \Phi = 8\pi T_{\mu\nu}$$

and

$$\Box \Phi = \frac{1}{2\omega + 3} T^\mu{}_\mu \leftarrow \text{Dirac!}$$

UV Example : Jordan-Brans-Dicke Gravity

Comments:

- $\Phi \sim \frac{1}{G} \sim M_{Pl}^2$
- Boundary term gives $\partial^2 \Phi$ (zero in GR)
- There is an additional source of energy-momentum T^{Φ}.
- $\Box \Phi$ sourced by $\rho + 3P$

Newtonian limit
$$G = \frac{4+2\omega}{3+2\omega} \frac{1}{\Phi_0}$$

Current constraints from Shapiro time delay

$\omega > 4 \times 10^4$.

Berti et al, CQG 32, 243001 (2015)

Conformally transforming : Jordan to Einstein

Make JBD "like GR"

Transform the metric : $g_{\mu\nu} \longrightarrow \psi g_{\mu\nu}$

so $R \longrightarrow \frac{1}{\psi^2}[R - 3\Box \ln\psi - \frac{3}{2}\partial^\alpha \ln\psi\, \partial_\alpha \ln\psi$

Action transforms:

$$S'_{JBD} = \int d^3x \sqrt{-g}\left[\frac{\Phi}{\psi}R - \frac{3}{2}\Phi\frac{(\nabla\psi)^2}{\psi^3} - \frac{\omega}{\Phi\Psi}(\partial\Phi)^2 + \frac{\mathcal{L}_m}{\psi^2}\right]$$

Choose $\frac{\Phi}{\Psi} = \frac{M_{Pl}^2}{2}$ and redefine $\Phi = e^{\frac{1}{\sqrt{\omega+3/2}}\alpha}$

Fifth Force

To get

$$S'_{JBD} = \int d^4x \left[\frac{M_{Pl}^2}{2}R - (\nabla\alpha)^2 + e^{-\frac{4}{2\omega+3}\alpha}\mathcal{L}_m\left(e^{\frac{1}{\sqrt{\omega+3/2}}\alpha} g_{,\text{mett}}\right)\right.$$

Fifth Force

Why is it a 'fifth' force?

Consider a relativistic particle

$$S_{matt} = -mc\int\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}\,d\lambda \xrightarrow[g\to\psi g]{} -mc\int\sqrt{-\psi g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}\,d\lambda$$

Geodesic equation is now $\left(u^\nu = \dfrac{dx^\nu}{d\lambda}\right)$

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} = -\frac{1}{2}\frac{1}{\sqrt{\psi}}\partial_\beta\psi\left(g^{\beta\mu}+u^\beta u^\mu\right)$$

$$= \underbrace{F^\mu}_{\text{Fifth force}}$$

Generalizing Scalar-Tensor Theories

Restrict ourselves to 2nd order e.o.m.

E.g.. K-essence (or K-inflation)

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\int d^4x\sqrt{-g}\,X \longrightarrow \int d^4x\sqrt{-g}\,K(X)$$

- More complex non-minimal coupling

$$\int d^4x\sqrt{-g}\,\Phi R \longrightarrow \int d^4x\sqrt{-g}\,f(\Phi)R$$

- Potentials!

$$V(\Phi)$$

Higher derivative action $\longrightarrow$ 2^{nd} order e.o.m.

Flat space example: $S_\lambda = \int d^4x \left[\phi^2 \Box\Box\phi - \lambda \phi \Box\phi\Box\phi\right]$

In principle, 4^{th} order e.o.m.

E.o.m.

$$\frac{\delta S_\lambda}{\delta \phi} = (4-2\lambda)\left(\phi\Box\Box\phi + 2\partial^\alpha\phi\partial_\alpha\phi\Box\phi\right)$$

4^{th} order term.

$$+ (2-3\lambda)\left(\Box\phi\Box\phi + 4\partial^\alpha\partial^\beta\phi\,\partial_\alpha\partial_\beta\phi\right)$$

Set $\lambda = 2$ to get rid of 4^{th} order part.

$$S_2 = 2\int d^4x\, \partial_\alpha\phi\partial^\alpha\phi\,\Box\phi$$

Note: Invariant under Galilean trans: $\phi \to \phi + c + c_\mu x^\mu$

$\boxed{\text{Galileon Actions}}$

Nicolis et al, PRD, 79, 064036, 2009

General Class of S.T. $\longrightarrow$ Horndeski action

$$S = \int d^3x \sqrt{-g} \sum_{i=2}^{5} \mathcal{L}_i (\phi, X, \Box\phi, g_{\mu\nu}$$

Horndeski, IJTP, 10, 363, 1974
Deffayet et al, PRD, 79, 084003, 2009

$$\mathcal{L}_2 = K \qquad \mathcal{L}_3 = - G_3 \Box\phi$$

$$\mathcal{L}_4 = G_4 R + G_{4X} \left\{ (\Box\phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi \right\}$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - G_{5X} \left\{ (\nabla\phi)^3 - 3 \nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi \Box \phi + 2 \nabla^\nu \nabla_\mu \phi \nabla^\alpha \nabla_\nu \phi \nabla^\mu \nabla_\alpha \phi \right\}$$

Note: $G_A = G_A(\phi, X)$

$$G_{AX} = \partial_X G_A$$

<u>Not</u> most general with 2^{nd} order e.o.m $\longrightarrow$ Beyond Horndeski, DHOST

Summary

- Violate Lovelock assumptions $\rightarrow$ new theories
- Archtypal theory is Jordans-Brans-Dicke
- Scalar field is new force "fifth force"
- Tightly constrained ($\omega > 40{,}000$)
- Extended to Horndeski (and beyond!)