Other Examples : higher dimensions

5-D spacetime $\begin{cases} \text{Coordinates} & X^A = (x^\alpha, z) \\ \text{Metric} & \gamma_{AB} \end{cases}$

Compactify z: $\quad \gamma_{AB}(x^\alpha, z) = \gamma_{AB}(x^\alpha, z + 2)$

Fourier Transform

$$\gamma_{AB}(X) = \sum_n \gamma_{AB}^{(n)}(x^\mu)\, e^{\frac{inz}{L}}$$

$\uparrow$

masses : $m = \frac{n}{L}$

See Clifton et al, Phys Rept, 513, 1 (2012)

Other Examples : higher dimensions

Metric :
$$\gamma_{AB} = \begin{pmatrix} \gamma_{\mu\nu} & \gamma_{\mu z} \\ \gamma_{z\nu} & \gamma_{zz} \end{pmatrix}$$

$\alpha = \frac{1}{2\sqrt{3}}$

$\beta = -\frac{\sqrt{3}}{2}$

Parametrize as :
$$\gamma_{\mu\nu} = e^{\frac{\phi}{\sqrt{3}}} g_{\mu\nu} + e^{-\sqrt{3}\phi} A_\mu A_\nu$$
$$\gamma_{\mu z} = e^{-\sqrt{3}\phi} A_\mu \qquad \gamma_{zz} = e^{-\sqrt{3}\phi}$$

Then

5D E.H.
$$S = \frac{1}{16\pi G_5}\int d^5x\, R_5 = \frac{1}{16\pi G_5}\int dz \int d^4x \sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4} e^{-\sqrt{3}\phi} F^2\right]$$

But $\int dz \sim L$ so
$$\frac{L}{16\pi G_5} \equiv \frac{1}{16\pi G_N}$$

$$\Phi(r) \sim \begin{cases} \frac{1}{r} & \text{large } r \\ \frac{1}{r^2} & \text{small } r \end{cases}$$

Newtonian Potential

Other Examples : higher dimensions

90s The rise of the 'Branes'

Action:

$$S = \frac{M_5^2}{2} \int_{\mathcal{M}} d^5x \sqrt{-\gamma}\,(R_5 - 2\Lambda) - \sigma \int_{\partial\mathcal{M}} d^4x \sqrt{-g}$$

tension (σ) ; Brane ($\partial\mathcal{M}$)

Define a length scale: $\ell \sim \frac{M_5^3}{\sigma}$

The metric has the form

$$ds^2 = e^{-2|z|\ell}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + dz^2$$

Brane at $z=0$, z moves away from 0 $\rightarrow$ volume reduces

As a result $\longrightarrow$ ∞ extra dimensions!

Other Examples : higher dimensions

Dvali - Gabadadze - Porratti (DGP)

add geometry to the Brane

Extrinsic curv.

$$S = \frac{M_5^2}{2} \int_{\mathcal{M}} d^5x \sqrt{-\gamma} \left(R_5 - 2\Lambda\right) + \int_{\partial \mathcal{M}} d^4x \sqrt{-g} \left(\frac{M_4^2}{2} R_4 - 2M_5^2 K - \sigma\right)$$

4D Ricci

New length scale : $\frac{1}{r_c} \sim \frac{M_5^3}{M_4^2}$

Deviations on large scales:

$$\Phi(r) \sim \begin{cases} \frac{1}{r^2} & (5\text{-D}) \quad r > r_c \\ \frac{1}{r} & (4\text{-D}) \quad r < r_c \end{cases}$$

Newtonian Potential

Other Examples : $f(R)$

Quantum correction : $S \propto \int d^4x \left[\frac{M_{pl}^2}{2} R + a_1 R_{\mu\nu} R^{\mu\nu} + a_2 R^2 \right.$

$\left. + a_3 R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \ldots \right.$

Correction kick in at

$$R \sim M_{pl}^2$$

Notable Example : Starobinsky Inflation

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{1}{6M^2} R^2 \right]$$

Becomes important at early times.

Other Examples : $f(R)$

Late times?

Toy model : $S \sim \int d^4x \sqrt{-g} \left[R - \frac{M^{2n+2}}{R^n} \right]$

Note: What is the vacuum state?

(Not Minkowski, $R=0$)

$R \to 0$ late times!

(Carroll et al astro-ph/0306438)

Generalise

$$S \sim \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, f(R)$$

Higher derivative theories! $R^2 \sim (\partial^2 g)^2$

Other Examples : $f(R)$

Not unstable. Can map onto Scalar Tensor.

$f_R = \partial_R f$

E.o.m

$$f_R R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu} \Box f_R = \frac{1}{M_{Pl}^2} T_{\mu\nu}$$

Conformal transformation.

Define $\phi = \sqrt{\frac{3}{\kappa}} \ln f_R$ and $\bar{g}_{\mu\nu} = f_R g_{\mu\nu}$

E.o.m.

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{R} = \frac{\kappa}{2} \left[\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} \nabla_\rho \phi \nabla_\sigma \phi - \bar{g}_{\mu\nu} V \right] + \frac{\kappa}{2} \bar{T}_{\mu\nu}$$

where $V = V(\phi) = \dfrac{R f_R - f}{\kappa f_R^2}$ so Scalar tensor again!

Other Examples

- Einstein-Aether, Vector-Tensors
- Massive gravity, Bigravity, Non-local
- Non-local gravity

Generally can be mapped onto scalar tensor gravity.

Summary

- Violating Lovelock's theorem can explore extensions to GR
- Scalar Tensor most studied | popular (general?)
- f(R) popular and can be mapped onto ST
- Other candidates

Main point

Extensions to GR $\Longleftrightarrow$ Extra degrees of freedom (new forces).