Testing GR on large Scales : Expansion

Until 1960s cosmology was only about expansion

$$ds^2 = -dt^2 + \underline{\underline{a^2(t)}}\, d\vec{r}^{\,2}$$

Homogeneous + Isotropic:

$$G_{\alpha\beta} = \frac{1}{M_{pl}^2} T_{\alpha\beta} \;\longrightarrow\; G_{00} = 3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{M_{pl}^2}\rho$$

density

Pressure

$$G_{\alpha\beta} = \frac{1}{M_{pl}^2} T_{\alpha\beta} \;\longrightarrow\; \left(\frac{\ddot{a}}{a}\right) = -\frac{1}{6M_{pl}^2}\left(\rho + 3P\right)$$

Measure $\left\{ \begin{array}{l} z - \text{redshifts} \\ D \rightarrow \text{distances} \end{array} \right\} \longrightarrow D(z) \longrightarrow$ determine $H(t)$.

(Ferreira, ARAA, 57, 335, 2019)

Testing GR on large Scales: Expansion

Suppose GR is modified.

E.o.m now

$$\Theta_{\alpha\beta}(a,\rho,P) = 0$$

Rewrite it as

$$G_{\alpha\beta} = \frac{1}{M_{pl}^2} T_{\alpha\beta} + \underbrace{\left[G_{\alpha\beta} + \Theta_{\alpha\beta} - \frac{1}{M_{pl}^2} T_{\alpha\beta} \right]}_{\frac{1}{M_{pl}^2} U_{\alpha\beta}}$$

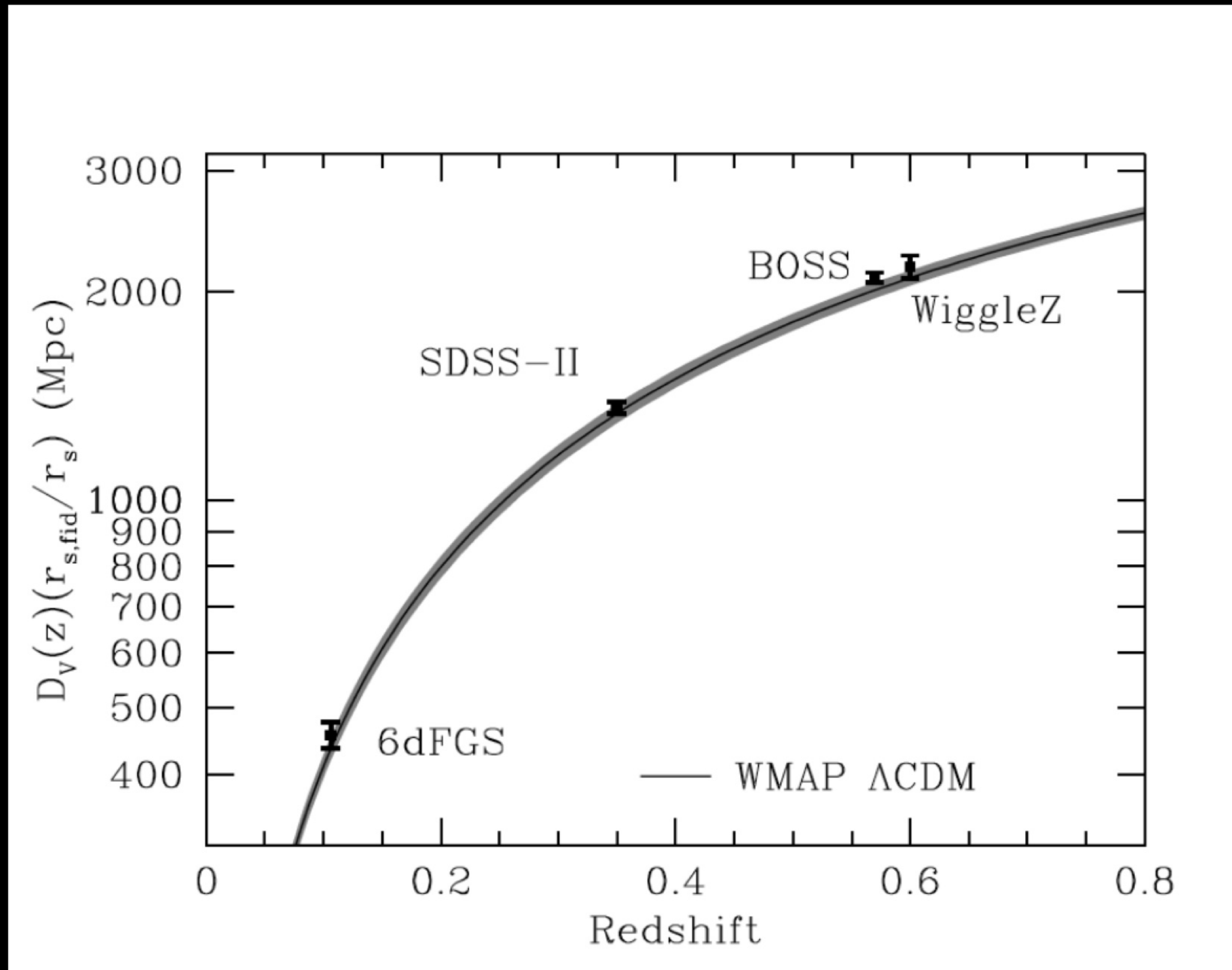
Homogeneity + isotropy mean that

$$U_{\alpha\beta} = \begin{pmatrix} X_E & \\ & Y_E\, g_{ij} \end{pmatrix} = \begin{pmatrix} \rho_{DE} & \\ & P_{DE}\, g_{ij} \end{pmatrix}$$

Indistinguishable

Testing GR on large Scales : Expansion

"Distance"



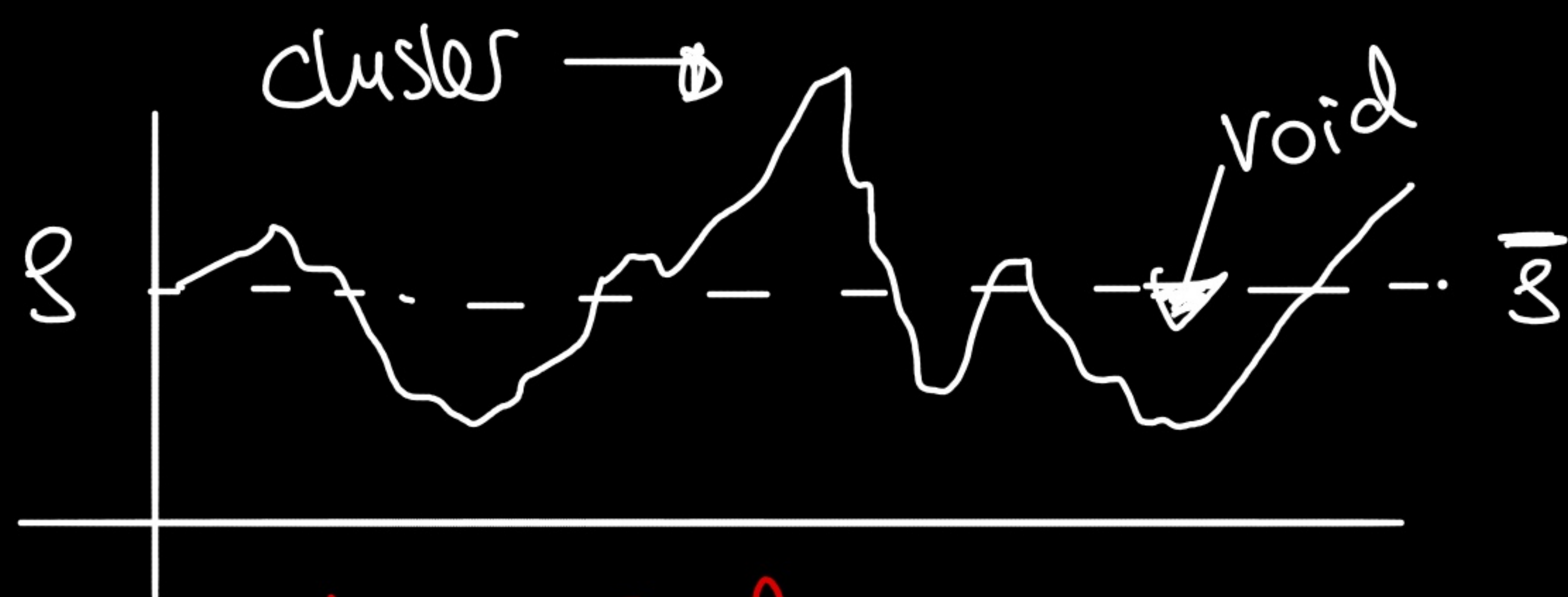
Anderson, MNRAS, 441, 1, 24 (2013)

Testing GR on large Scales : structure

Can we use perturbations?

Recap: Universe is inhomogeneous

$$\rho(\vec{x},t) = \bar{\rho}(t)\left(1+\delta(x,t)\right) \qquad \left(\text{i.e } \delta = \frac{\rho-\bar{\rho}}{\bar{\rho}}\right)$$



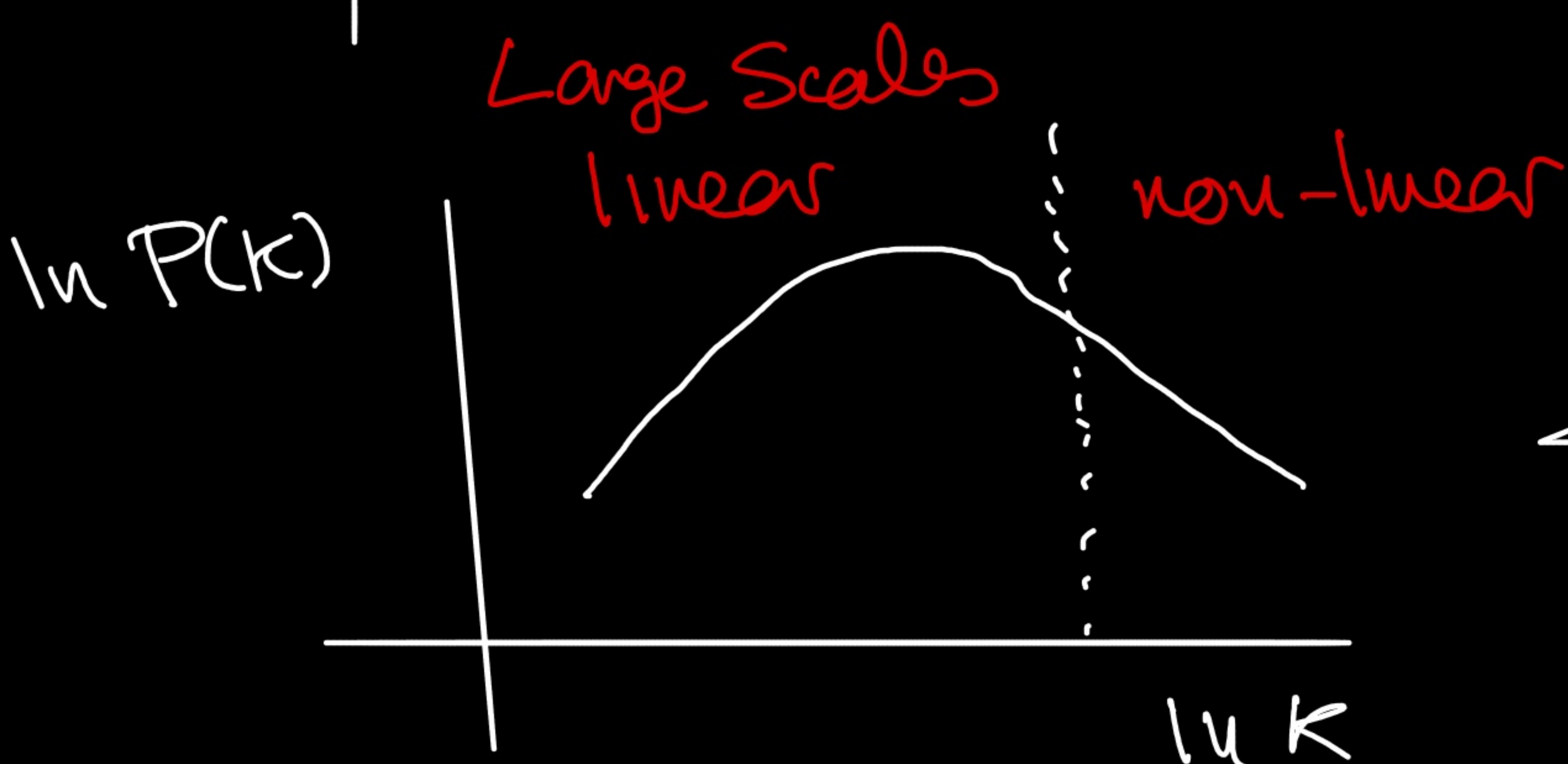
$$\delta(\vec{x}) \xrightarrow[\text{FFT}]{} \tilde{\delta}(k)$$

$$\Downarrow$$

$$\langle \tilde{\delta}(k) \rangle = 0$$

$$\langle \tilde{\delta}(k)\tilde{\delta}(k') \rangle \propto P(k)\,\delta^3(k-k')$$

Power Spectrum



Testing GR on large Scales : structure

Focus on linear theory
a) easy to calculate
b) success of modern cosmology

but statistically limited

$$N_{\text{mode}} \propto \frac{V}{\lambda^3} \sim k^3$$

Recap:

$$\rho = \bar{\rho}(1+\delta)$$

$$P = \bar{P}(1+\delta)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

$$\begin{pmatrix} -1 & \\ & a^2\delta_{ij} \end{pmatrix}$$

Φ, Ψ (Newtonian potentials)

Linearly expand Field equs:

$$\delta G_{\alpha\beta} = \frac{1}{M_{pe}^2} \delta T^{\alpha}{}_{\beta}$$

Solve:

CAMB + CLASS.

$$\text{E.g:} \begin{cases} \nabla^2 \psi = 4\pi G a^2 \rho (\delta + \dots) \\ \nabla^2(\Phi - \Psi) = \# \sigma_{\text{shear}} \dots \end{cases}$$

Testing GR on large Scales : structure

Beyond GR :

Key point: linear theory $\Longrightarrow$ action for perturbations **must** be quadratic

Consider scalar tensor theories.

Add $$\phi = \bar{\phi} + \delta\phi$$

What does quadratic action look like?

Testing GR on large Scales : structure

$$S_g = \int d^4x\, a^2 M \Big\{ \tfrac{1}{2} H^2 \left(\alpha_K - 12\,\alpha_B - 6 \right) \Phi^2 - 6H(1+\alpha_B)\Phi\dot{\Phi} + 2\Psi\partial^2\Phi$$

$$- 3\dot{\Phi}^2 - (1+\alpha_T)\Psi\partial^2\Phi - 3\left(\frac{\rho+P}{M^2} + 2\dot{H}\right)\dot{\Phi}\,\delta\phi + 6H\alpha_B\dot{\Phi}\,\delta\dot{\phi}$$

$$+ H^2\left(6\,\alpha_B - \alpha_K\right)\Phi\,\delta\dot{\phi} - 2H\left[\alpha_T - \frac{d\ln M^2}{d\ln a}\right]\Psi\partial^2\phi - 2H\,\alpha_B\,\Phi\,\partial^2\delta\phi$$

$$- \ \ldots$$

equation of state $(P = w\rho)$

Parameters: $w(t)$ +

$$\alpha_M = \frac{d\ln M^2}{d\ln a}, \quad \alpha_B, \quad \alpha_K, \quad \alpha_T$$

$$\boxed{\alpha_X = 0 \text{ for GR!}}$$

are functions of $(a(t), \bar{\phi}(t))$ i.e. functions of time only.

Bellini & Sawicki, JCAP 07, 050 (2014)

Testing GR on large scales : structure

Constraining new physics $\Longrightarrow$ measuring $(w, \alpha_M, \alpha_K, \ldots)$

Note: • What functional form for $w, \alpha_M, \ldots$

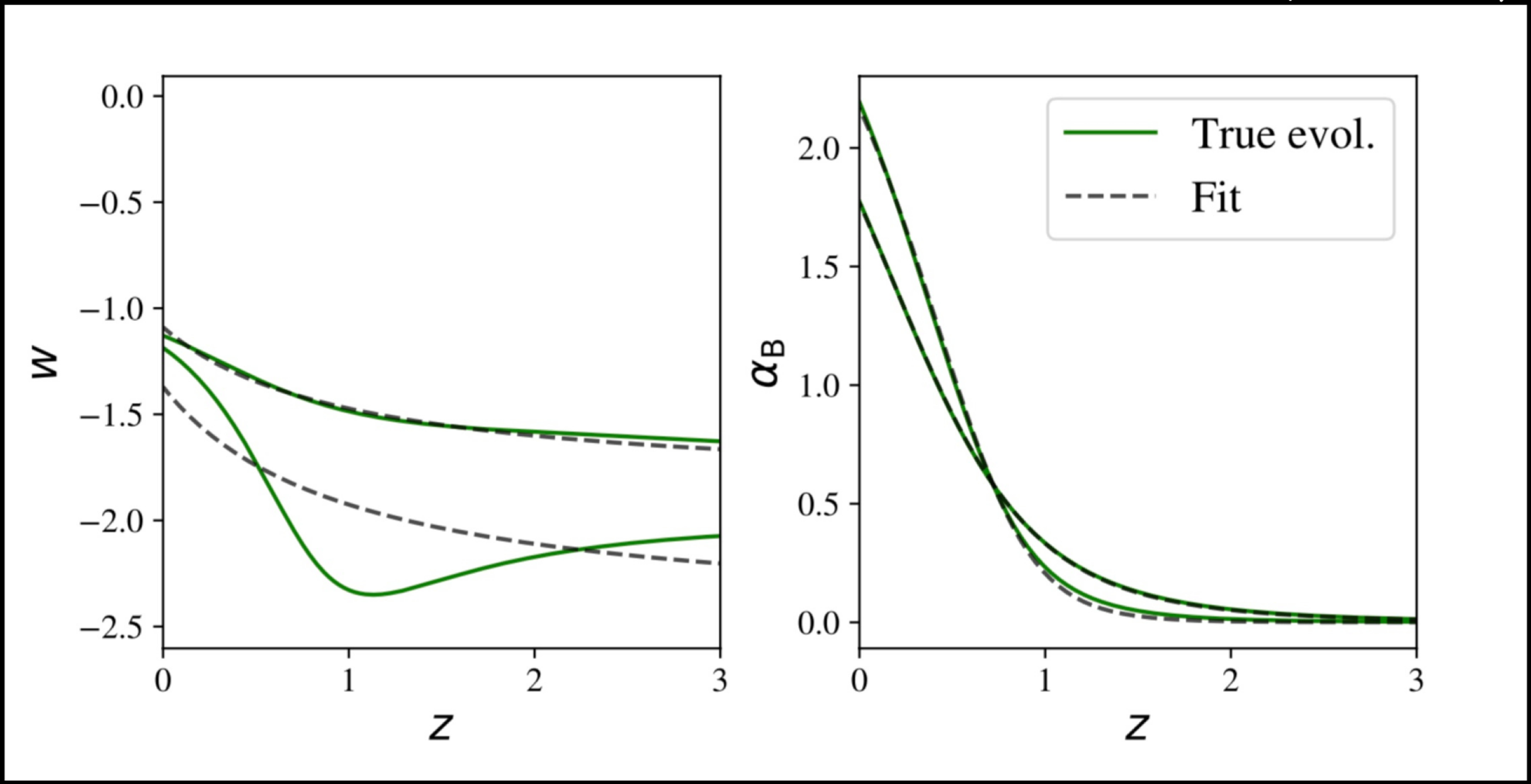
• Are they correlated?

• What range of values can they take?

Testing GR on large Scales : structure

Example : shift symmetric scalar tensor theories

(Traykova et al, 2021)



Fit: $w = w_0 + w_a(1-a)$

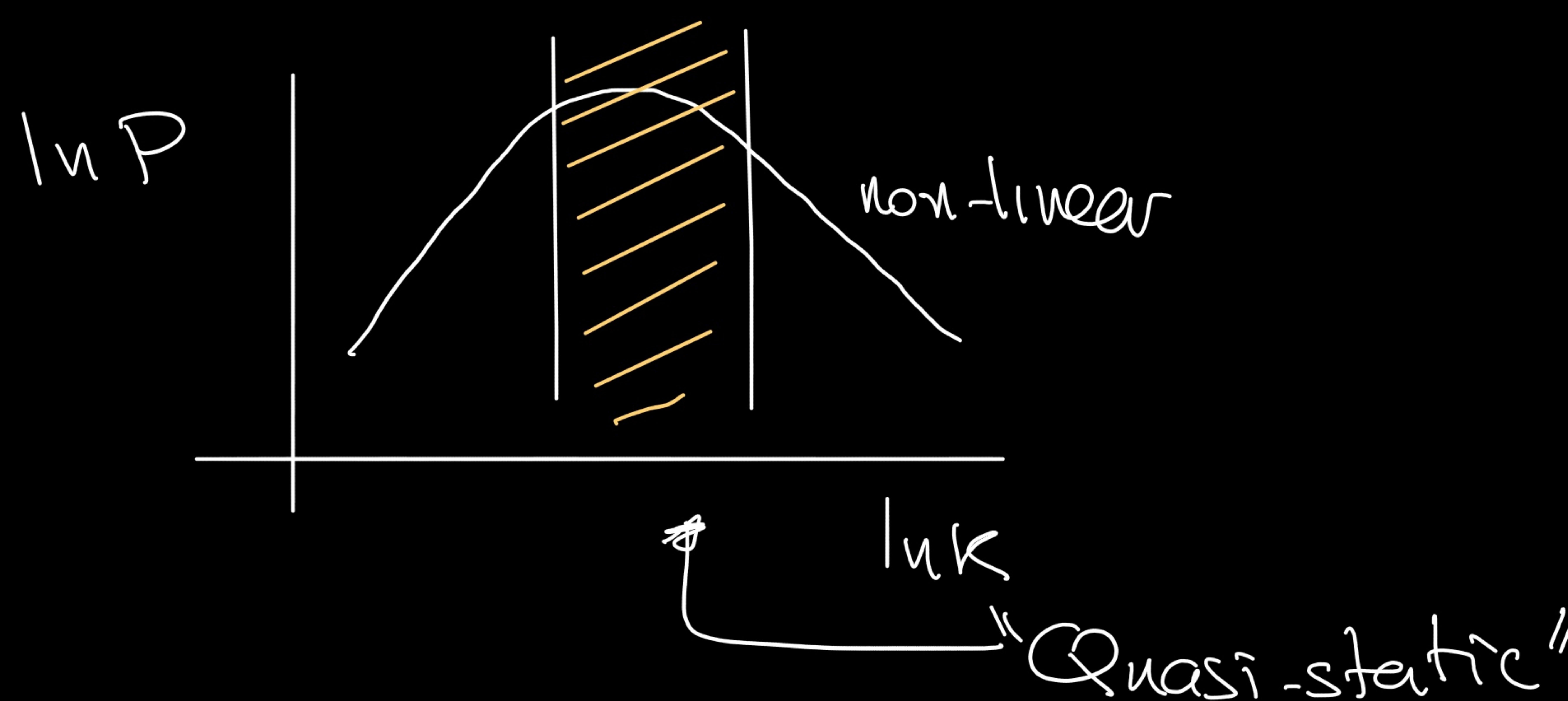
$$\alpha_B = \hat{\alpha}_B \left(\frac{H_0}{H}\right)^{4/m}$$

Testing GR on large Scales : structure

We try to observe the whole universe $\longrightarrow \sim \frac{1}{cH}$

We measure well modes on smaller scales $\frac{k}{aH} \gg 1$.

Note: not too small scales or they will be non-linear



Testing GR on large Scales : structure

Quasi-static approximation:

$\delta\varphi' \sim 0$ solve for $\delta\varphi$ to set

$\mu = \frac{G_{eff}}{G_0}$

$$\nabla^2 \Phi = 4\pi G_0 \mu a^2 \bar{\rho} \delta$$

"effective Newton's constant"

$$\Phi = \gamma \Psi$$

"gravitational slip"

Note: Generally μ and γ depend on k and t

Often assume just $\mu(t)$ and $\gamma(t)$.

Testing GR on large Scales : structure

Impact on observables : growth rate

$$\begin{cases} \dot{\delta} = -k^2 \Theta \\ \dot{\Theta} = -\mathcal{H}\Theta + \psi \end{cases}$$

$\Theta = \nabla \cdot V$

conservation of mass

conservation of momentum.

Combine equations $\left(' = \dfrac{d}{d\ln a} \right)$

$$\delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' - \frac{3}{2}\Omega_m \frac{\mu}{\gamma}\delta = 0$$

Define growth rate $\boxed{f = \dfrac{d\ln\delta}{d\ln a}}$

$q(\ln a) = \frac{1}{2}\left[1 - 3w(1-\Omega_m)\right]$

$$f' + q(\ln a) f + f^2 = \frac{3}{2}\Omega_m \xi$$

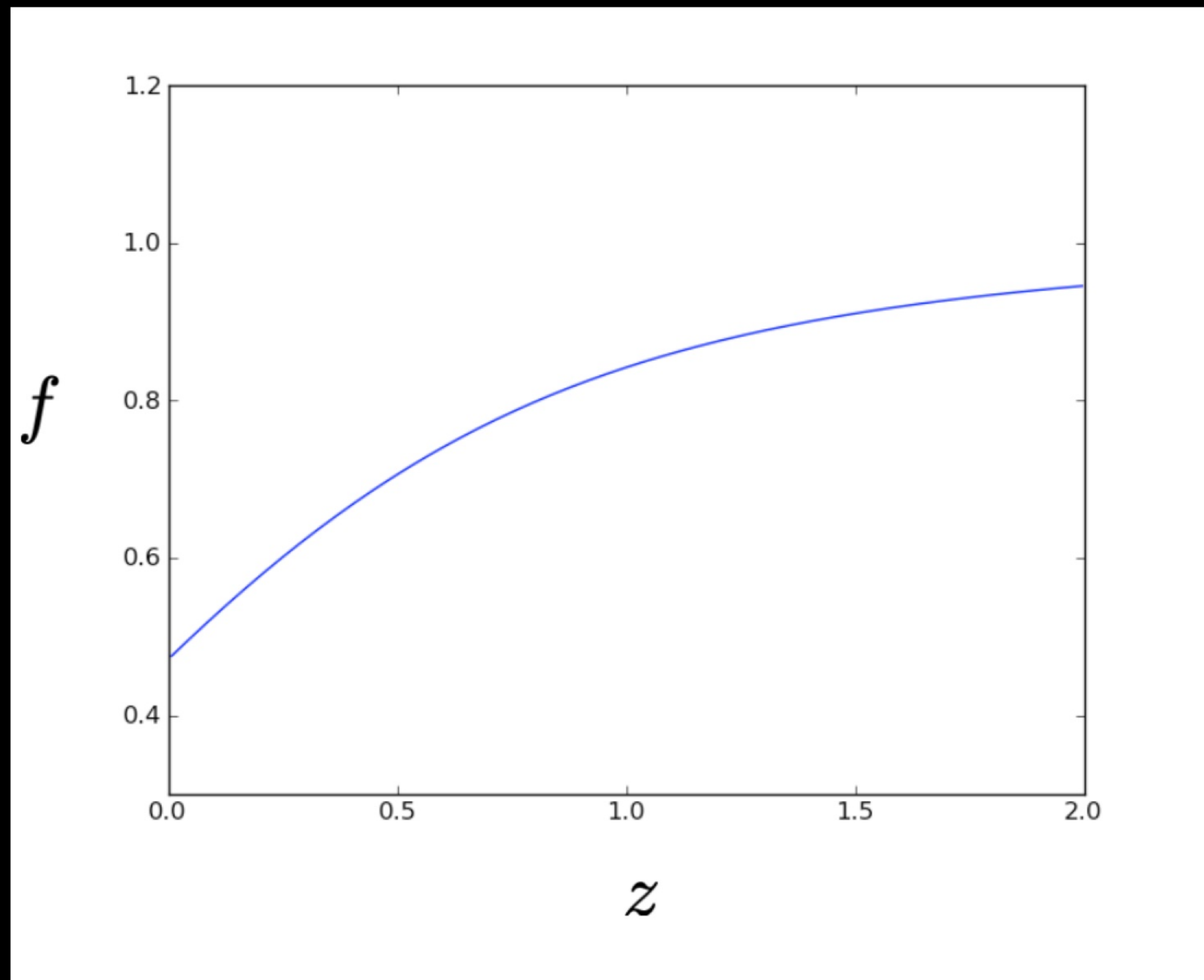
ξ affects growth.

Baker et al, PRD 89, 024026 (2014)

Testing GR on large Scales : structure

growth rate

$f = 1$ for $\Omega_m = 1$ GR



$f \sim \Omega_m^{\frac{6}{11}}$ for ΛCDM GR

$0 \leftarrow \Omega_m$

Testing GR on large Scales: structure

Lensing:

Kernel

$$\mathrm{WL} \sim \int d\eta\, K(\eta)\, \partial_i \partial_j (\Phi + \Psi)$$

$$= \int d\eta\, K(\eta)\, \partial_i \partial_j \left(1 + \frac{1}{\gamma}\right) \Phi$$

$$\sim \int d\eta\, K(\eta)\, \underbrace{\mu\left(1 + \frac{1}{\gamma}\right)}_{\Sigma} 4\pi G a^2 \delta\rho$$

So new parameter: $\Sigma = \mu\left(1 + \frac{1}{\gamma}\right)$

Summary

- Use cosmology to look for new physics
- Background evolution (expansion) $\rightarrow$ too little information
- Use large scale structure
- Linear theory very well understood but statistically limited
- General parametrization of new physics
- Impact on observables (e.g. growth rate).