Testing GR on large scales: Expansion

Until 1960s cosmology was only about expansion

\[ ds^2 = -dt^2 + a^2(t) \, dr^2 \]

Homogeneous + Isotropic:

\[ G_{\alpha\beta} = \frac{1}{M_p^2} T_{\alpha\beta} \]

\[ G_{00} = 3 \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{M_p^2} \frac{8}{3} \]

\[ \left(\frac{\dot{a}}{a}\right) = -\frac{1}{6M_p^2} (8 + 3P) \]

Measure \( z - \text{redshifts} \)

\( D = \text{distance} \)

\( D(z) \rightarrow \text{determine } H(t) \).

(Ferreira, ARAA, 57, 335, 2019)
Testing GR on large scales: Expansion

Suppose GR is modified.

E.o.m now

\[ \Theta_{\alpha\beta}(a, s, \eta) = 0 \]

Rewrite it as

\[
G_{\alpha\beta} = \frac{1}{M_p^2} T_{\alpha\beta} + \left[ G_{\alpha\beta} + \Theta_{\alpha\beta} - \frac{1}{M_p^2} T_{\alpha\beta} \right] \]

Homogeneity & isotropy mean that

\[
\frac{1}{M_p^2} U_{\alpha\beta} = \begin{pmatrix} \chi_E & Y_E g_{ij} \end{pmatrix} = \begin{pmatrix} 8 \delta E & P_{DE} g_{ij} \end{pmatrix}
\]

Indistinguishable
Testing GR on large scales: Expansion

Distance

\[ D_{\nu}(z) = \frac{r_{\text{eff}}(z)}{r_0} \] (Mpc)

SDSS-II

BOSS

WiggleZ

6dFGS

WMAP \( \Lambda \)CDM

\[ \text{Redshift} \]

Testing GR on large scales: structure

Can we use perturbations?

Recap: Universe is inhomogeneous

$$g(\mathbf{x},t) = \bar{g}(t) \left(1 + \delta(\mathbf{x},t)\right) \quad \text{(i.e. } \delta = \frac{g - \bar{g}}{\bar{g}}\text{)}$$

$$\delta(\mathbf{x}) \rightarrow \hat{\delta}(k)$$

$$\langle \hat{\delta}(k) \rangle = 0$$

$$\langle \hat{\delta}(k) \hat{\delta}(k') \rangle \propto \mathcal{P}(k) \delta^2(k-k')$$

Power Spectrum
Testing GR on large scales: structure

Focus on linear theory

a) easy to calculate  
b) success of modern cosmology  
but statistically limited

\[ N_{\text{mode}} \propto \frac{K}{X^3} \sim k^3 \]

Recap:

\[ S = \overline{S}(1 + \delta) \]

\[ P = \overline{P}(1 + \delta) \]

\[ g_{\mu\nu} = \overline{g}_{\mu\nu} + \delta g_{\mu\nu} \]

\[ (^{-1} \delta \delta_{ij}) \]

\( \Phi, \overline{\Phi} \) (Newtonian potentials)

Linearly expand Field eqs:

\[ \delta G_{\mu\nu} = \frac{1}{2} \delta T^\mu_{\nu \beta} \]

Eg:

\[ \Delta^2 \Phi = 4\pi G d^2 \delta \]

\[ \nabla^2 (\Phi - \overline{\Phi}) = \# \text{ shear} \ldots \]
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Beyond GR:

Key point: linear theory $\rightarrow$ action for perturbations must be quadratic

Consider scalar tensor theories.

Add $\phi = \Phi + \delta \phi$

What does quadratic action look like?
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\[ S_g = \int d^4x a^2 M \left\{ \frac{1}{2} \dot{H}^2 \left( \alpha_K - 12 \alpha_B - 6 \right) \phi^2 + 6H \left( 1 + \alpha_B \right) \phi \dot{\phi} + 2M^2 \phi^2 - 3 \phi^2 - \left( 1 + \alpha_T \right) \phi \dot{\phi} \phi - 3 \left( \frac{3+P}{M^2} + 2H \right) \phi \ddot{\phi} + 6H \alpha_B \phi \dot{\phi} \phi \right\} + H^2 \left( 6 \alpha_B - \alpha_K \right) \phi \ddot{\phi} - 2H \left[ \alpha_T - \frac{d\ln M^2}{d\ln a} \right] \phi \dot{\phi} \phi - 2H \alpha_B \phi \dot{\phi} \phi \]

- ... equation of state \( P = wS \)

Parameters: \( W(t) \) +

\[ \alpha_M = \frac{d\ln M^2}{d\ln a}, \quad \alpha_B, \quad \alpha_K, \quad \alpha_T \]

are functions of \( (a(t), \phi(t)) \) i.e. functions of time only.

Bellini & Sawicki, JCAP 07, 050 (2014)
Testing GR on large scales: structure

Constraining new physics $\Rightarrow$ measuring $(\omega, \chi_m, \alpha_k, \ldots)$

Note: What functional form for $\omega, \chi_m, \ldots$?

- Are they correlated?
- What range of values can they take?
Testing GR on large scales: structure

Example: shift symmetric scalar tensor theories

(Tayehvand et al., 2021)

Fit: \[ W = \omega_0 + \omega_a (1 - a) \]

\[ \alpha_B = \hat{\alpha}_B \left( \frac{H_0}{H} \right)^{4/M} \]
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We try to observe the whole universe \( \rightarrow \sim \frac{1}{cH} \)

We measure well modes on smaller scales \( \frac{k}{aH} \gg 1 \).

Note: not too small scale or they will be non-linear.
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Quasi-static approximation:

\[ \delta \Phi' \sim 0 \] solve for \( \delta \Phi \) to get

\[ \nabla^2 \Phi = 4\pi G_0 M a^2 \delta \]

\[ \Phi = \gamma \Phi \]

\[ M = \frac{GM}{G_0} \]

"Effective Newton's constant"

"Gravitational slip"

Note: Generally \( M \) and \( \gamma \) depend on \( \kappa \) and \( t \).

Often assume just \( M(t) \) and \( \gamma(t) \).
Testing GR on large scales: structure

Impact on observables: growth rate

\[ \begin{cases} 
\dot{\delta} = -k^2 \Theta \\
\dot{\Theta} = -H \Theta + \Psi 
\end{cases} \]

Conservation of mass

Conservation of momentum

Combine equations \( \dot{a} = \frac{d}{dn} \)

\[ \delta'' + \left( 1 + \frac{H^2}{H} \right) \delta' - \frac{3}{2} \Sigma m \frac{\mu}{8} \delta = 0 \]

Define growth rate \[ f = \frac{d \ln \delta}{d \ln a} \]

\[ \frac{d}{dn} = \frac{1}{2} \left( 1 - 3W(1-2m) \right) \]

\[ f^3 + 9 (\ln a) f + f^2 = \frac{3}{2} \Sigma m \delta \]

Baker et al., PRD 89, 024026 (2014)
Testing GR on large scales: structure growth rate

\[ f = 1 \quad \text{for} \quad \Lambda \Omega_m = 1 \quad \text{GR} \]

\[ f \propto \Omega_m^{\frac{6}{11}} \quad \text{for} \quad \Lambda \Omega_m \quad \text{GR} \]

\[ \Omega_m \geq \Omega_m \]
Testing GR on large scales: structure

Lensing:

\[ \mathcal{W}_L \sim \int d\nu K(\nu) \Xi_i \Xi_j (\Phi + \bar{\Psi}) \]

\[ = \int d\nu K(\nu) \Xi_i \Xi_j (1 + \frac{1}{8}) \hat{\Phi} \]

\[ \sim \int d\nu K(\nu) m (1 + \frac{1}{8}) \ell \pi \sigma_a^2 \delta^8 \]

\[ \leq \]

So new parameter:

\[ \Xi = m (1 + \frac{1}{8}) \]
Summarize

- Use cosmology to look for new physics
- Background evolution (expansion) - too little information
- Use large scale structure
- Linear theory very well understood but statistically limited
- General parameterization of new physics
- Impact on observables (e.g., growth rate).