Dark Energy from Gravitational Waves

We have focussed on <u>scalar-tensor</u> theories

$$(\phi, g_{\alpha\beta})$$

Examples of couplings:

$$(\partial\phi)^2, V(\phi), f(\phi)R, G_{\alpha\beta}\partial^\alpha\phi\,\partial^\beta\phi, \Box\phi\,((\partial\phi)^2 + \cdots)$$

General theory $\rightarrow$ Horndeski and beyond.

Dark Energy from Gravitational Waves

Scalar Sector

Examples:

- "Fifth Forces": $\phi = \bar{\phi} + \delta\phi$

$$\nabla^2 \delta\phi + m^2 \delta\phi \propto S$$

Screening, etc, ...

- Density perturbations:

$$\delta' + f^2 + (1+q) f \propto \frac{\mu}{\gamma} \Omega_m$$

$f = \frac{d\ln\delta}{d\ln a}$ ← growth rate.

due to ϕ.

Dark Energy from Gravitational Waves

Tensor Sector (spin-2, traceless, transverse).

Recall: $\Box h_{x,t} = 0$

(with expansion)

$$h''_A + 2\mathcal{H} h'_A + k^2 h_A = 0 \qquad \begin{cases} A = +, \times \\ ' = \frac{d}{d\tau} \\ \mathcal{H} = \frac{a'}{a} \end{cases}$$

In scalar tensor theories.

$$h''_A + (2 + \alpha_M) h'_A + (1 + \alpha_T) k^2 h_A = 0$$

Horndeski parameters

Dark Energy from Gravitational Waves

Tensor Sector (spin-2, traceless, transverse).

Modifications tied to observables.

$$h_A'' + (2+\alpha_M)h_A' + (1+\alpha_T)k^2 h_A = 0$$

Change in gravitational wave luminosity distance

$$d_{GW} = d_L \exp\left[-\frac{1}{2}\int_0^{\ln a} d\ln a\, \alpha_M(\ln a)\right]$$

"Light" luminosity distance

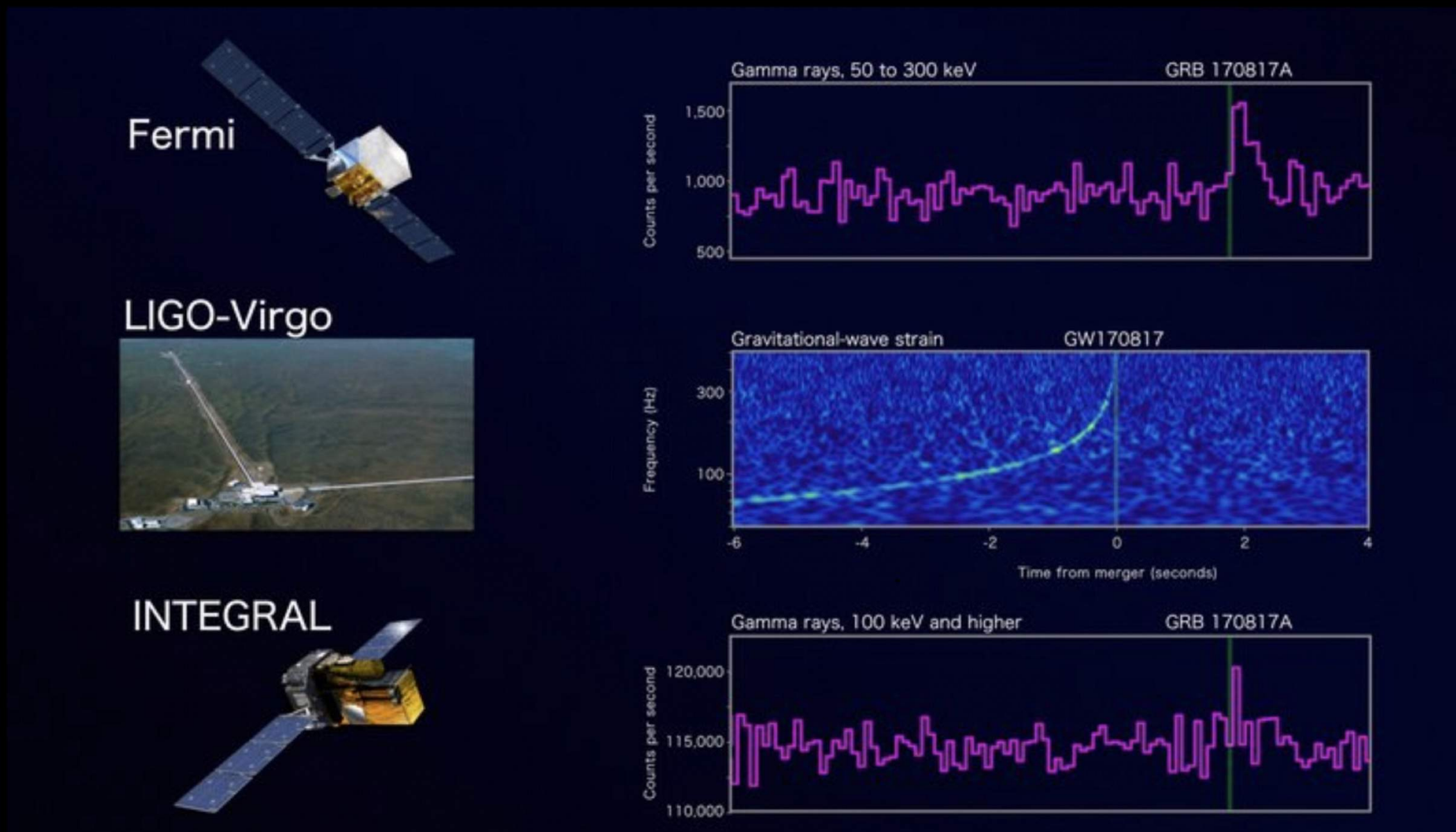
Chang in gravitational wave speed

$$c_{GW}^2 = c^2(1+\alpha_T)$$

Speed of light

Dark Energy from Gravitational Waves

GW170817 → Binary Neutron Star (BNS) merges



Dark Energy from Gravitational Waves

GW170817 $\rightarrow$ Binary Neutron Star (BNS) merges

Time between GW and light : $\underline{\underline{\Delta t \sim 1.7\,s}}$

Distance to event : $D \sim 40$ Mpc

Now : GW : $D = c_{GW} T_{GW}$

Light : $D = c\, T_{Light}$

and $c_{GW} = c\sqrt{1+\alpha_T} \sim c\left(1+\frac{1}{2}\alpha_T\right)$

Then $\left|\frac{D}{c} - \frac{D}{c_{GW}}\right| = \left|T_{GW} - T_{Light}\right| <$ few sec

$\Longrightarrow$ $|\alpha| < 10^{-15}$!

Dark Energy from Gravitational Waves

GW170817 $\rightarrow$ Binary Neutron Star (BNS) merger

Very tight constraint : $\alpha_T \sim 0$!

What does this mean (imply)?

$$\sqrt{-g}\,(\partial\phi)^2 \,,\ \sqrt{-g}\,V \,,\ f(\phi)\,R \,,\ \ldots \longrightarrow \alpha_T = 0$$

but

$$f(\chi)R,\ G_{\alpha\beta}\,\partial^\alpha\phi\,\partial^\beta\phi,\ \Box\phi\left((\partial\phi)^2 + \ldots\right),\ \ldots \longrightarrow \alpha_T \neq 0$$

Note: $\partial\phi$ works as a medium ...

Dark Energy from Gravitational Waves

GW170817 $\rightarrow$ Binary Neutron Star (BNS) merges

Example: $S = \int d^4x \sqrt{-g}\, \frac{M^2(X)}{2} R$ $\qquad X = \frac{1}{2}(\partial\phi)^2$

We then have

$$\alpha_T = \frac{\frac{\partial M^2}{\partial \ln X}}{M^2 - 2\frac{dM^2}{d\ln X}}$$

Then: $\alpha_T = 0$ if

a) $\frac{dM^2}{d\ln X} \sim 0$ (GWaves don't couple to medium)

b) $\frac{dM^2}{d\ln X} \ll M \Rightarrow \Omega_\phi \ll 1$ (medium is negligable)

Dark Energy from Gravitational Waves

Recall Horndeski

$$S = \int d^3x \sqrt{-g} \sum_{i=2}^{5} \mathcal{L}_i (\phi, X, \Box\phi, g_{\mu\nu}$$

$$\mathcal{L}_2 = K \qquad \mathcal{L}_3 = - G_3 \Box\phi$$

$$\mathcal{L}_4 = G_4 R + G_{4X} \left\{ (\Box\phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi \right\}$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - G_{5X} \left\{ (\nabla\phi)^3 - 3 \nabla^\mu \nabla^\nu \phi \nabla_\mu \nabla_\nu \phi \Box \phi \right.$$
$$\left. + 2 \nabla^\nu \nabla_\mu \phi \nabla^\alpha \nabla_\nu \phi \nabla^\mu \nabla_\alpha \phi \right\}$$

Note: $G_A = G_A(\phi, X)$

$$G_{AX} = \partial_X G_A$$

Dark Energy from Gravitational Waves

But now:

$$S_{ST} = \int d^4x \left\{ f(\phi) R + K(\phi, X) + G_3(\phi, X) \Box \phi \right\}$$

Bakes et al 2017
Creminelli et al 2017
Sakstein et al 2017
Ezquiaga et al 2017.

Huge reduction in theory space!

Dark Energy from Gravitational Waves

Examples of specific acceptable models

- Extended JBD : $\int d^4x \sqrt{-g} \left[\phi R + \frac{\omega(\phi)}{\Phi} (\partial\Phi)^2 + V(\Phi) \right]$
- Cubic Galileon : $\int d^4x \sqrt{-g} \left[R + \frac{1}{2}(\partial\phi)^2 + \frac{c_3}{\Lambda^3} \Box\phi (\partial\phi)^2 \right]$
- Shift Symmetric : $\int d^4x \sqrt{-g} \left[R + f(X) + \Box\phi\, g(X) \right]$
  (Kinetic Braiding).

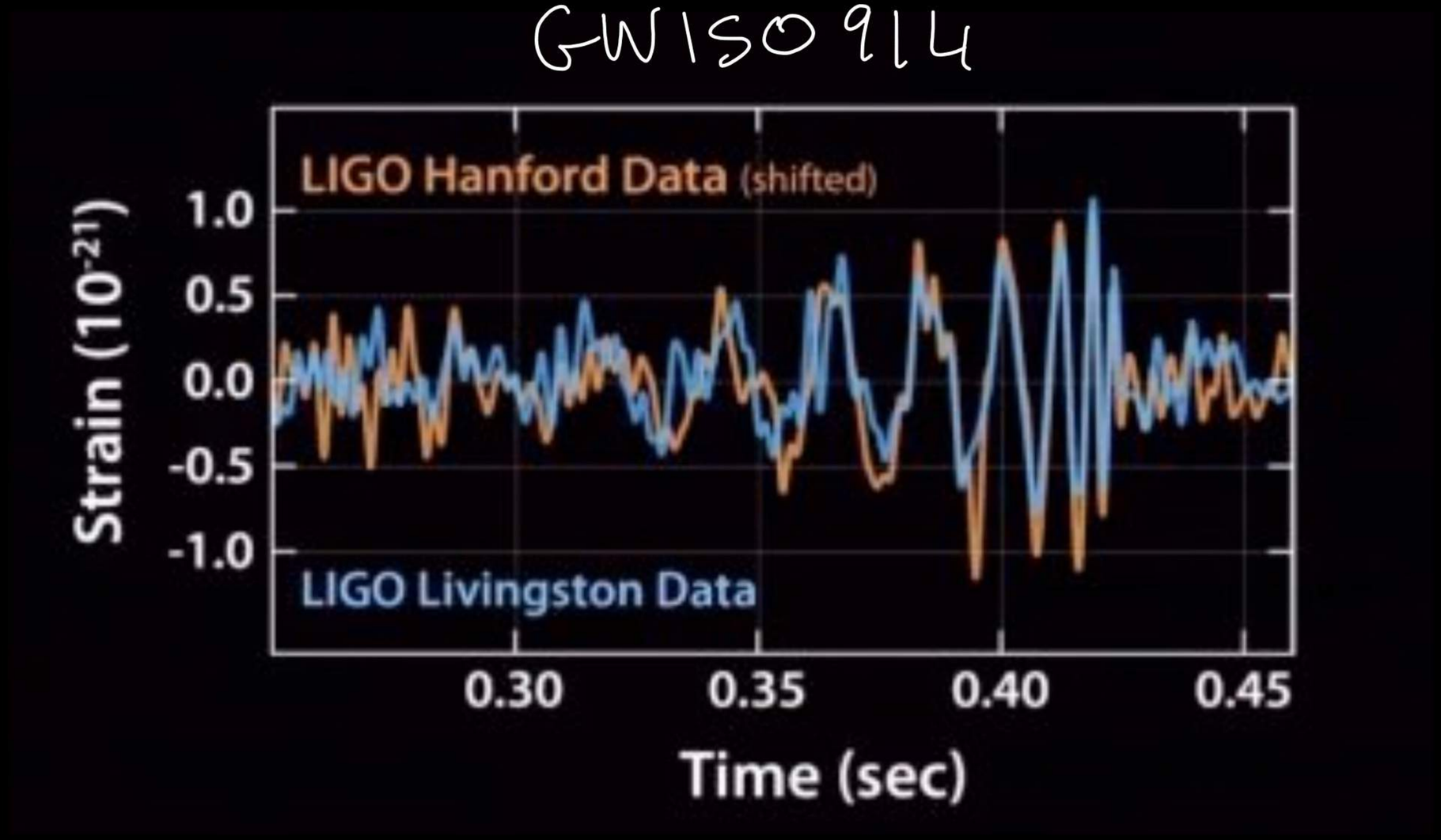
Caveats :

- $\Omega_\phi \sim O(1) \longrightarrow$ the scalar field has to have cosmological relevance
- Degenerate, higher derivative theories (DHOST)
- Cutoff dependence and EFT Regime

Dark Energy from Gravitational Waves

Can we go further with Gravitational Waves?

GW150914



Dark Energy from Gravitational Waves

Use gravitational collapse events

- Black hole formation
- Black hole mergers
- Neutron star mergers
- Exotica ...

Dark Energy from Gravitational Waves

Black Hole Hair

In GR: Black holes uniquely described in terms of Schwarzschid or Kerr

$$(M, J, Q)$$

Beyond GR: not necessarily the case.

E.g. Scalar tensor : $\phi = \phi(r)$

"Fifth" force

$$g_{\alpha\beta} \neq g^{Sch}_{\alpha\beta}$$

But not always the case!

Dark Energy from Gravitational Waves

Black Hole Hair

Brans Dicke Gravity

$$S = \int d^4x \sqrt{-g} \left\{ \Phi R + \frac{\omega}{\Phi} (\partial\phi)^2 + \cdots \right\}$$

No Hair ! $\quad ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r^2}\right)^{-1} dr^2 + \cdots$

$$\phi(r) = \phi_0$$

Black Holes in the Brans-Dicke

Theory of Gravitation

S. W. Hawking

Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England

Received October 15, 1971

Dark Energy from Gravitational Waves

Black Hole Hair

But : Einstein - Scalar - Gauss - Bonnet

$$S_{\text{Brans Dicke}} + \int d^4x \sqrt{-g}\, \phi \left\{ (R_{\alpha\beta\mu\nu})^2 - 4(R_{\mu\nu})^2 + R^2 \right\}$$

Hair!

$$g_{00} = -\left(1 - \frac{2GM}{r} + \frac{GM Q_s^2}{6r^3} + \cdots\right)$$

$$\phi(r) = \frac{P}{r} + \frac{GMP}{r^2} + \cdots$$

Fifth force or Hair

where Q_s is scalar charge

Dark Energy from gravitational waves

GW170817 and Black hole hair

Recall: if $\Omega_\phi \sim O(1)$

then $S_{ST} = \int d^4x \{ f(\phi) R + K(\phi, X) + G_3(\phi, X) \Box \phi \}$

$$\boxed{\text{Circumstantial evidence that all BH have no hair, if } \Omega_\phi \sim O(1), \text{ in ST.}}$$

Note: EDGB + Chern Simons do not have $\Omega_\phi \sim O(1)$

"Higher derivative" $\rightarrow$ DHOST $\rightarrow$ may have hair.

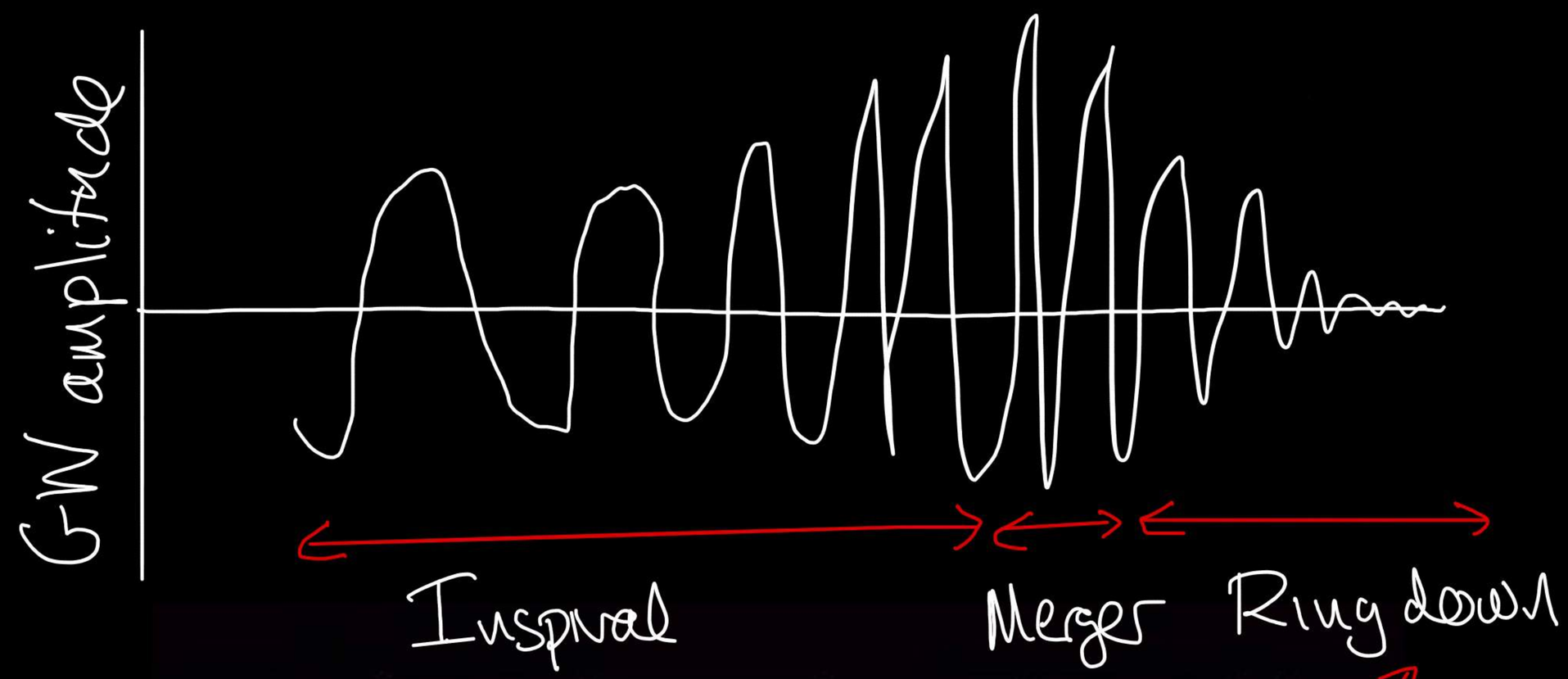
Dark Energy from gravitational waves

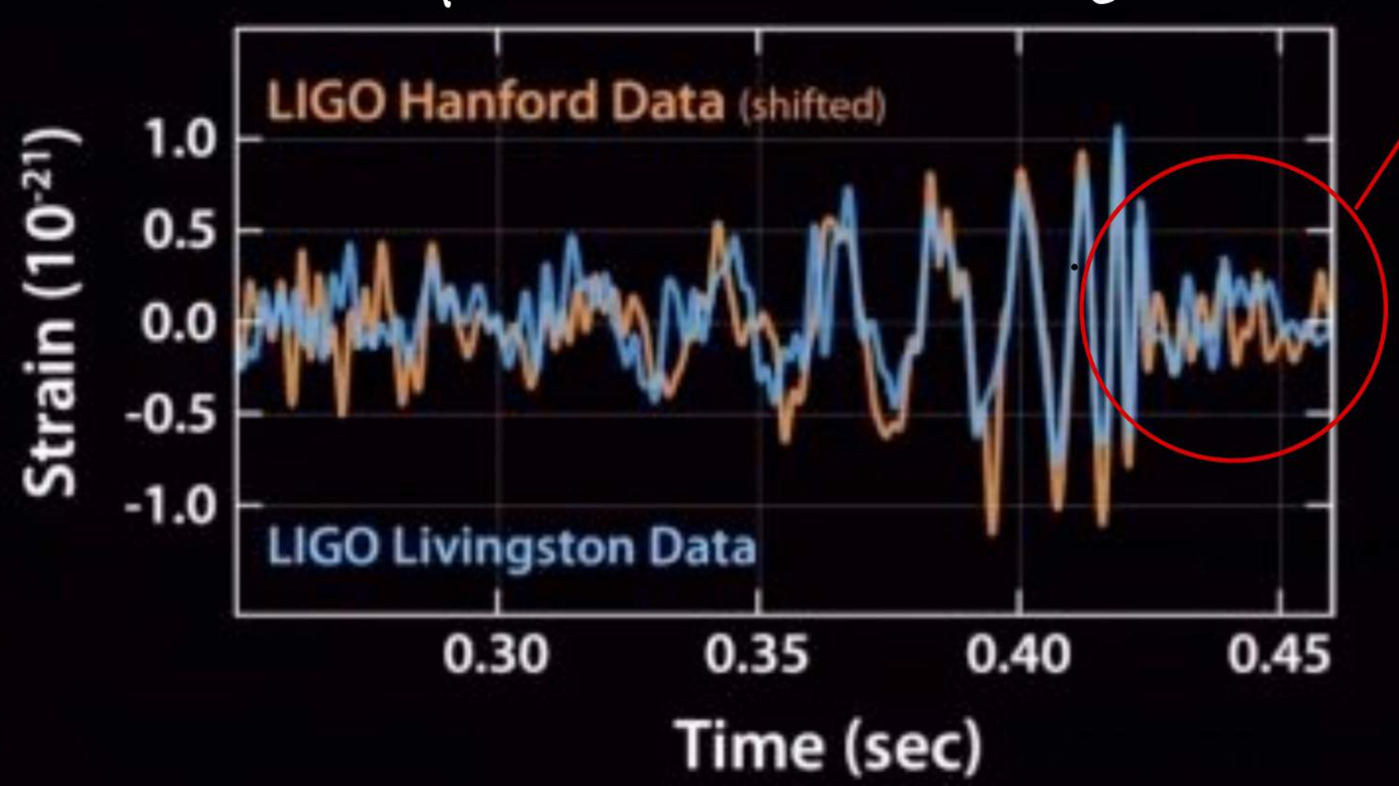
How, then, can we use BH to test for scalar tensor theories?

- No fifth forces — no PPN parameters
- No effect on orbits
- No effect on mergers and GWaves

Nothing?

Dark Energy from gravitational waves

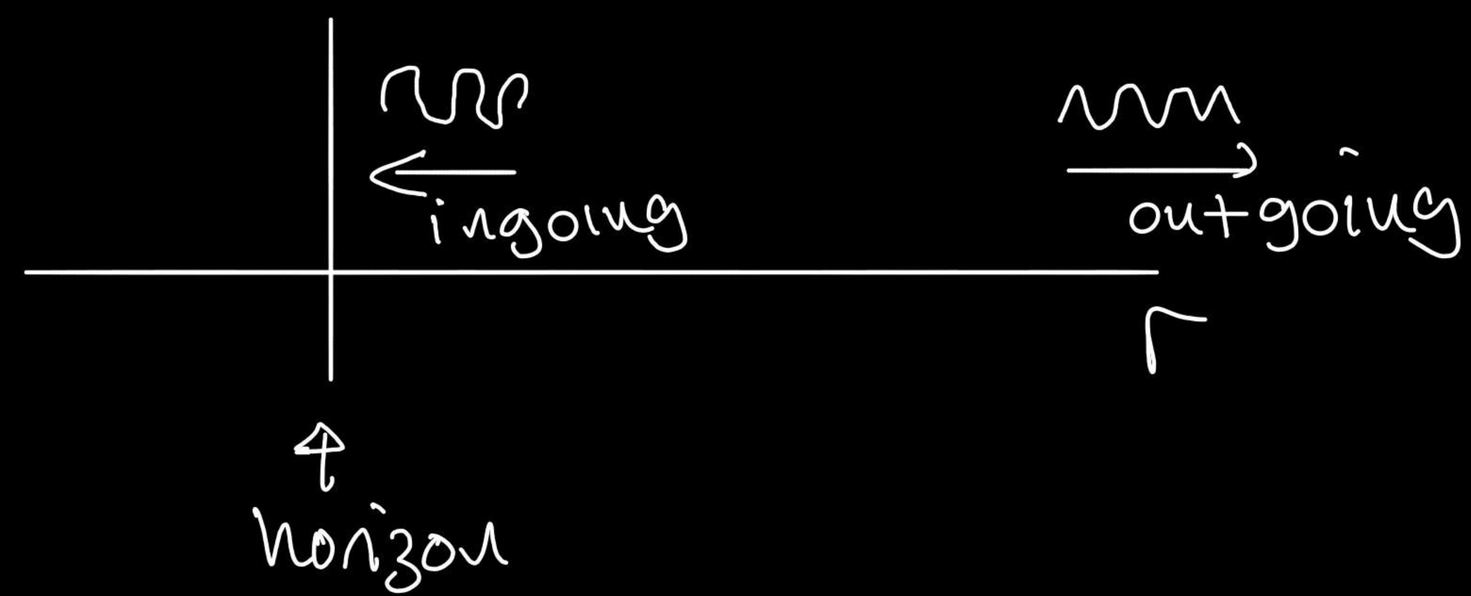




Dark Energy from gravitational Waves

Quasi Normal Modes (QNM) and Ringdown

Solve $g_{\mu\nu}$, ϕ wave equations with special boundary conditions



Falling into BH horizon + outwards to $r \to +\infty$.

Frequencies are complex!

Quasi-Normal-Modes (QNMs)

Dark Energy from gravitational waves

QNM, a primer

Take $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

BH metric

$h_{\mu\nu} = h_{\mu\nu}(\omega)\, e^{i\omega t}$

Decompose in spherical basis and odd (Ψ) and even (Z) modes.

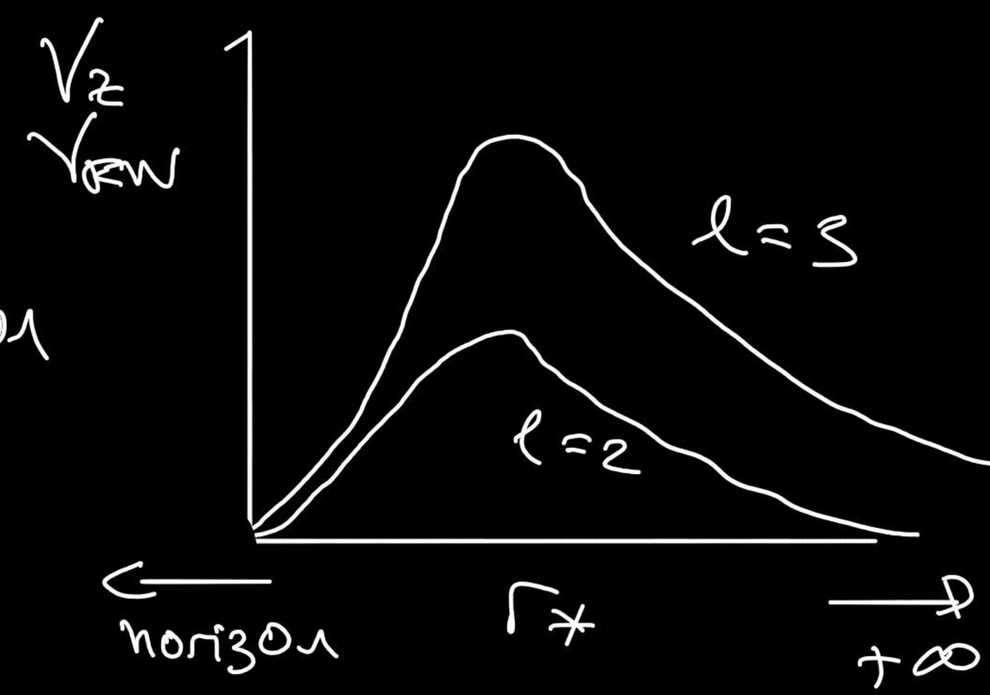
$$\Box h_{\mu\nu} = 0$$

Even modes $\rightarrow$ Zerilli equation

$$\frac{d^2}{dr_*^2} Z_\ell + \left[\omega^2 - V_Z(r)\right] Z_\ell = 0$$

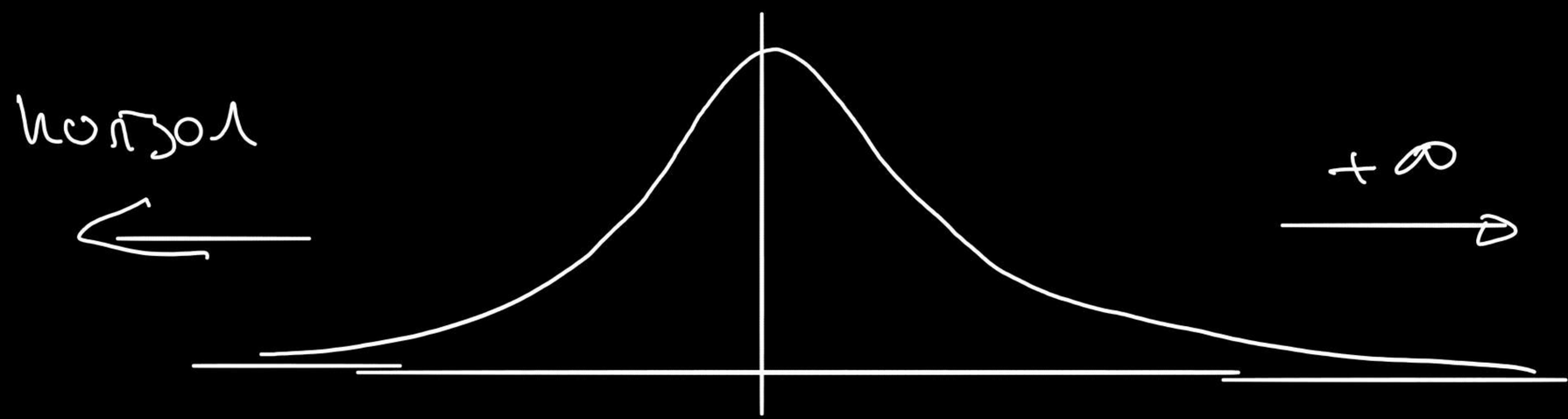
Odd modes $\rightarrow$ Regge-Wheeler equation

$$\frac{d^2}{dr_*^2} \Psi_\ell + \left[\omega^2 - V_{RW}(r)\right] \Psi_\ell = 0$$



Dark Energy from gravitational Waves

Complex eigenvalues



Select solutions that are

- ingoing in the horizon
- outgoing at $+\infty$

$\Rightarrow$ dissapative $\quad \omega = \omega_R + i\omega_I$ — Decay rate.

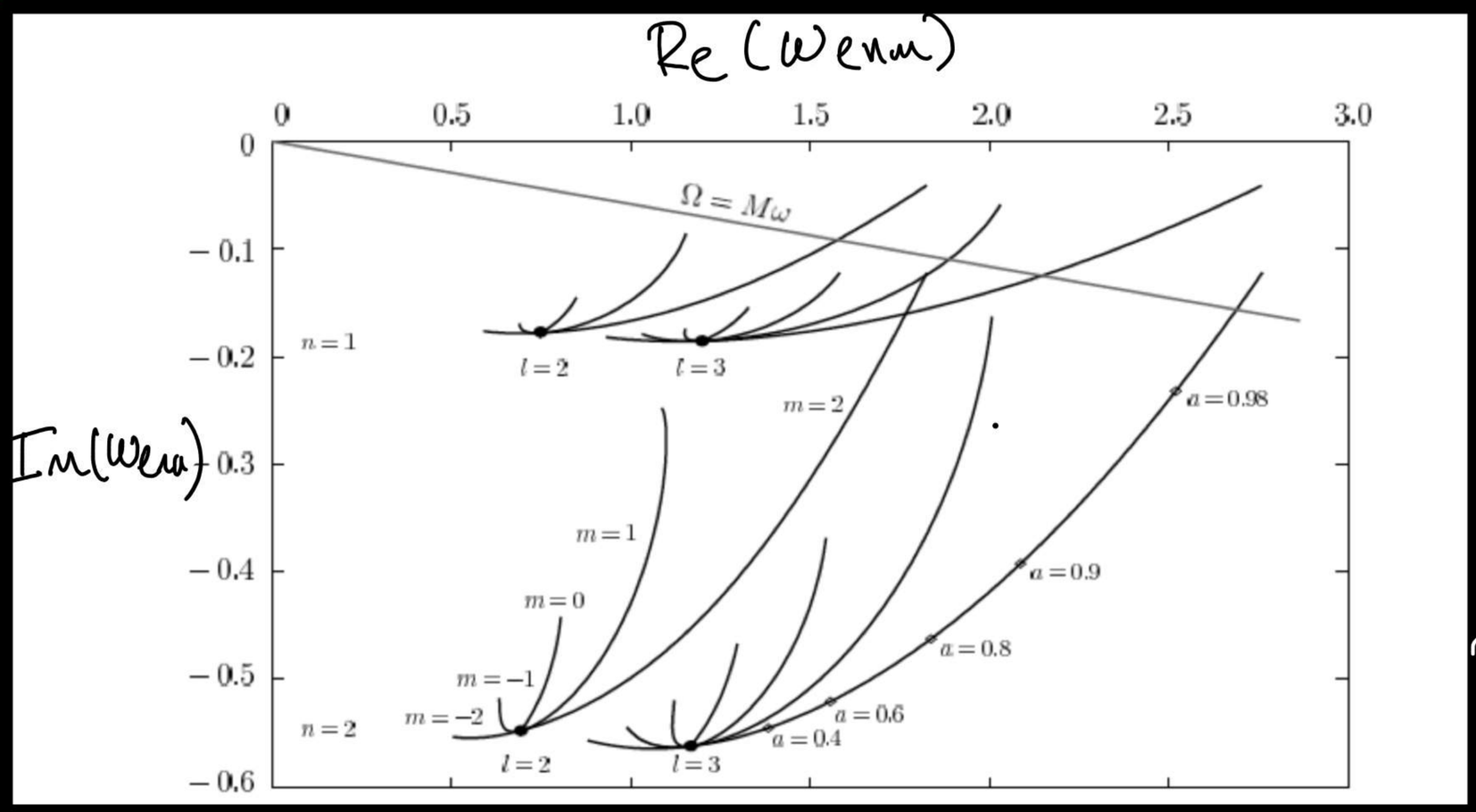
Dark Energy from gravitational waves

Folklore: QNM no-hair theorem

ω_{nlm} are uniquely determined by

QNM frequencies

(M, J, Q)



Dreyer et al 2003

Dark Energy from gravitational Waves

But, with Scalar Tensor : mixing of metric and scalar perturbations

Even parity gravitational wave (Zerilli equation)

$$\begin{cases} \dfrac{d^2 \Psi}{d r_*^2} + \left[\omega^2 - V_Z(r)\right] \Psi = \text{Source}(\varphi) \\[2ex] \dfrac{d^2 \varphi}{d r_*^2} + \left[\omega^2 - V_S(r, \mu^2)\right] \varphi = 0 \end{cases}$$

Non-minimal coupling

Effective mass of scalar field perturbation.

Note : $\delta\phi = e^{i\omega t} \varphi$

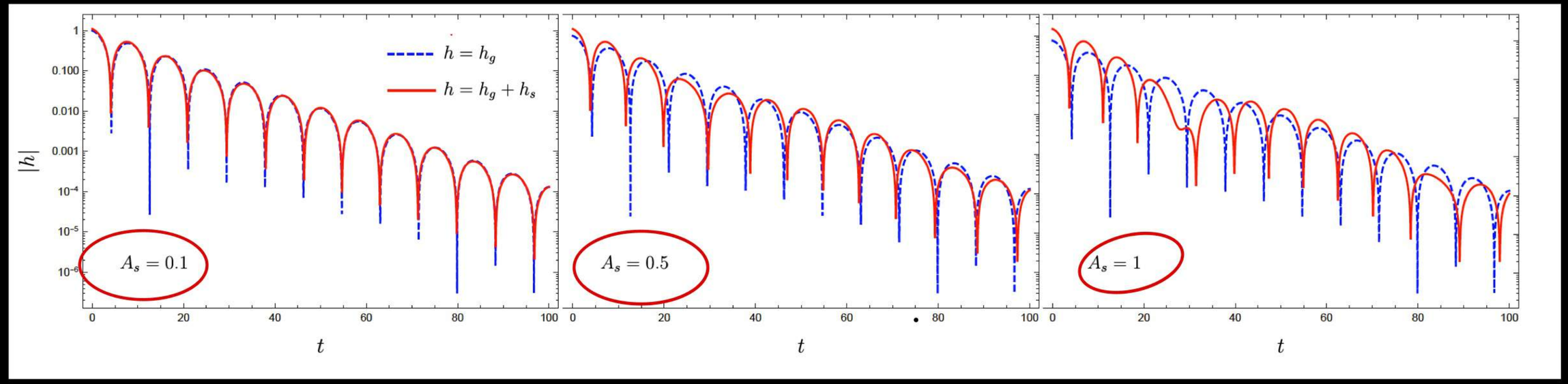
$h = e^{i\omega t} \Psi$

Tattersall & Ferreira, PRD 97, 10407 (2018)

Dark Energy from gravitational waves

Assume that somehow this scalar perturbation is excited with amplitude A_s

(normal GW has amplitude $= 1$)

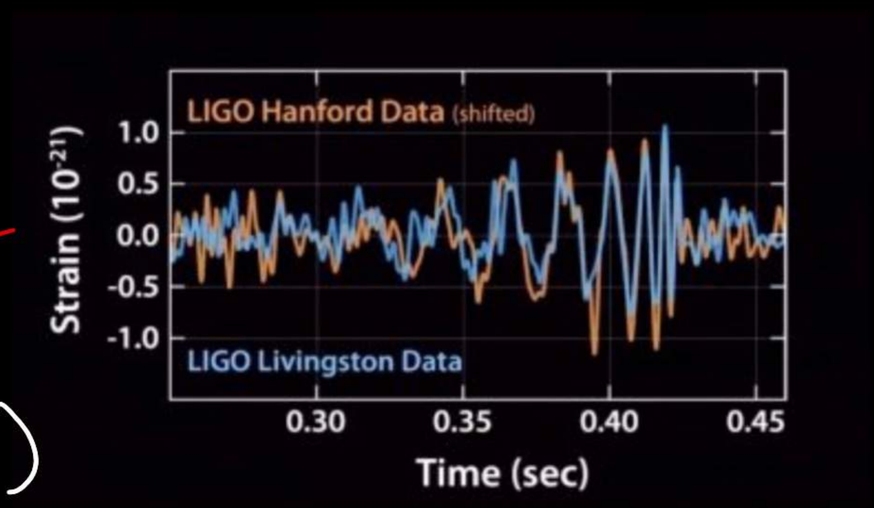


Can we pick up h_s (scalar part of metric)?

Dark Energy from gravitational waves

Detectability

→ how easily/well can we detect these extra "dark energy" modes?



Useful definitions:

Strain: $h(t,\vec{x}) \longrightarrow \tilde{h}(f)$

Fourier transform

Define signal to noise:

$$\rho = 4\int_0^\infty \frac{\tilde{h}^*(f)\,\tilde{h}(f)}{S_h(f)}\,df$$

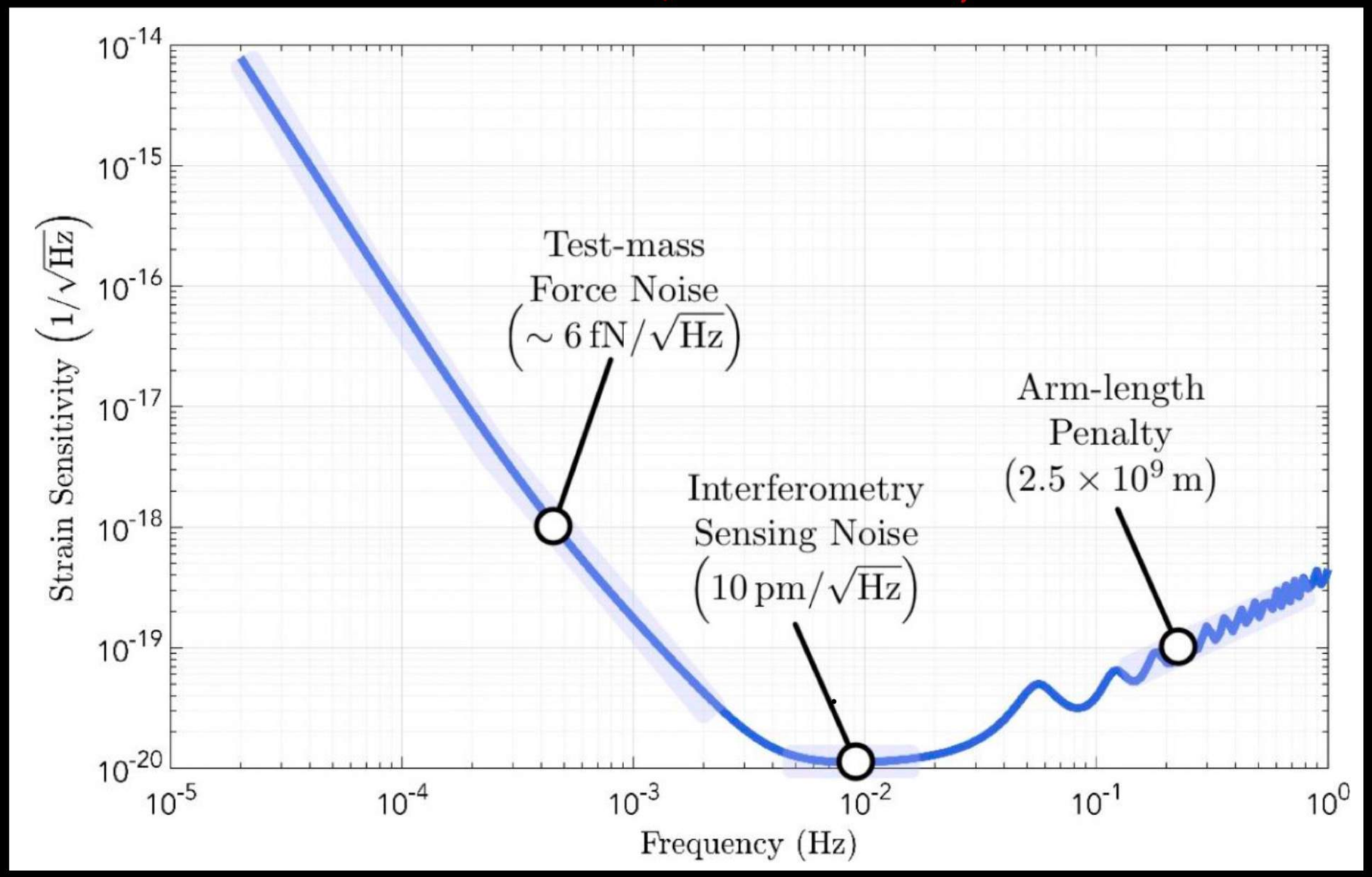
Noise

Dark Energy from gravitational waves

Detectability

detectable range

$S_h(f)$



(www.elisascience.org) LISA sensitivity curve

Dark Energy from gravitational waves

Detectability

Recall "surviving action"

$$S_{ST} = \int d^4x \sqrt{-g} \left\{ f(\phi) R + K(\phi, X) + G_3(\phi, X) \Box \phi \right\}$$

Define effective mass

$$m_s \sim \frac{h}{c} \frac{K_{\phi\phi}}{K_X - 2 G_{3\phi} + 3 \frac{f_{\phi\phi}^2}{f}}$$

$V_{\phi\phi}$

Recall $A_\phi = \frac{\partial}{\partial \phi} A$ and $A_X = \frac{\partial}{\partial X} A$

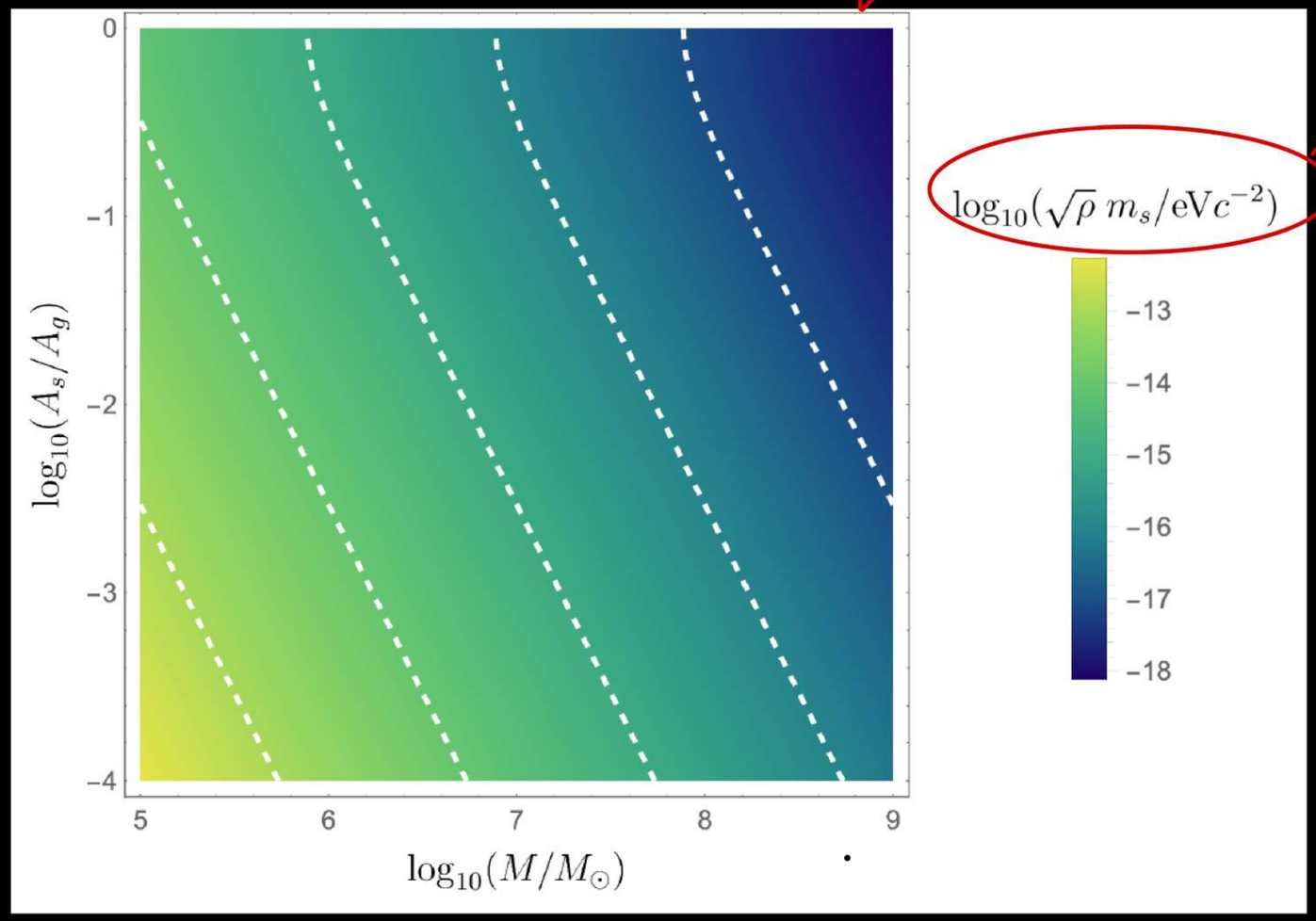
Dark Energy from gravitational waves

Detectability

lower masses detectable

higher signal to noise
⇓
lower mass can be detected

Amplitude of scalar modes relative to tensor



Mass of the final black hole

Tattersall & Ferreira, PRD 99, 104082, 2019

Dark Energy from gravitational waves

Where do we get As?

Assumptions behind no-hair theorems very restrictive:

- stationary
- asymptotic flatness
- ...

The Real Universe is different.

Counterexamples:

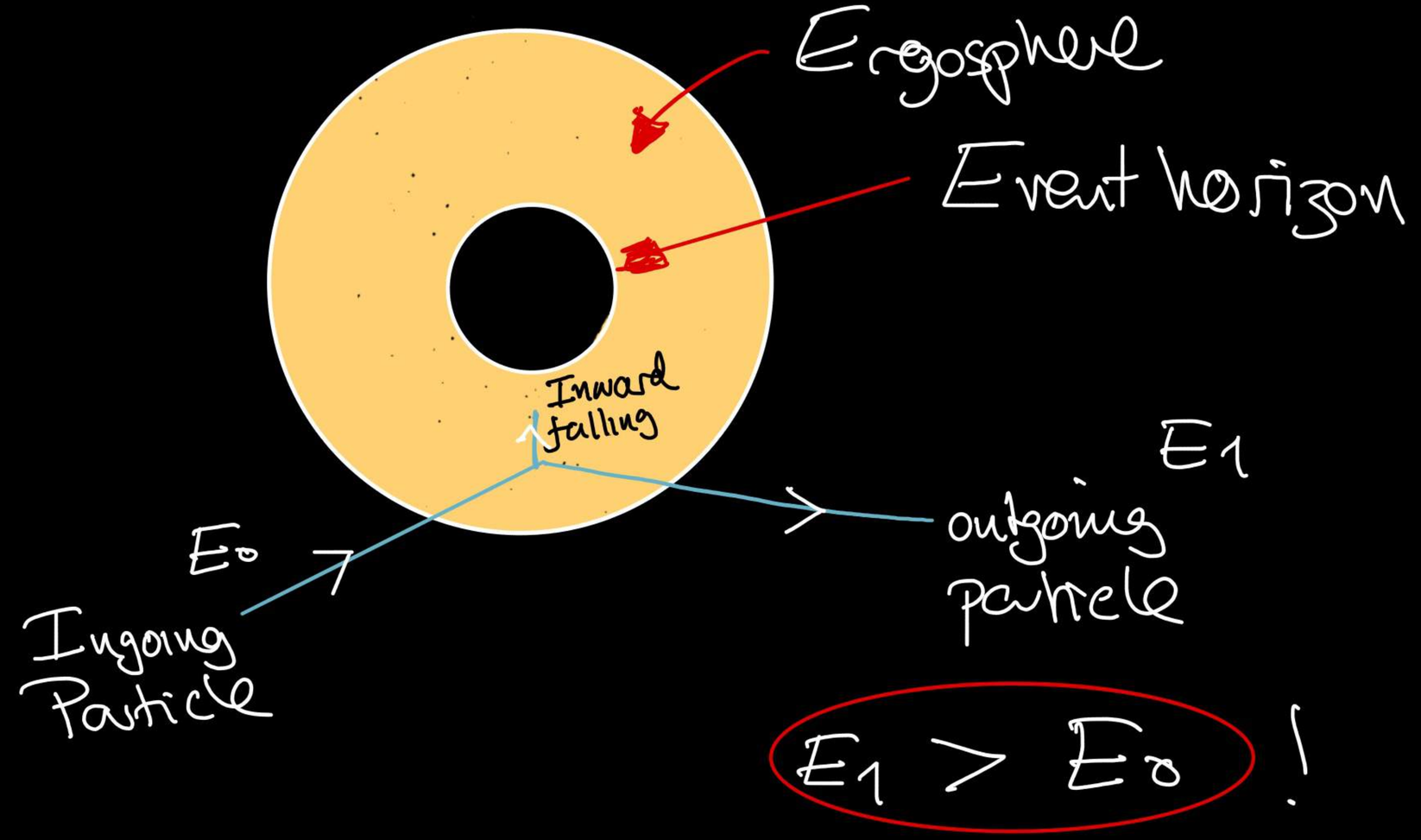
- Complex scalar fields (Radu & Herdeiro 2015)
- Einstein-Skyrme (Droz-Heusler-Straumann 1991)
- Horndeski (but $\Omega_\phi \ll 1$) (Sotiriou-Zhou 2014)

Dark Energy from gravitational waves

Where do we get As?

Superradiance → Long lived hair.

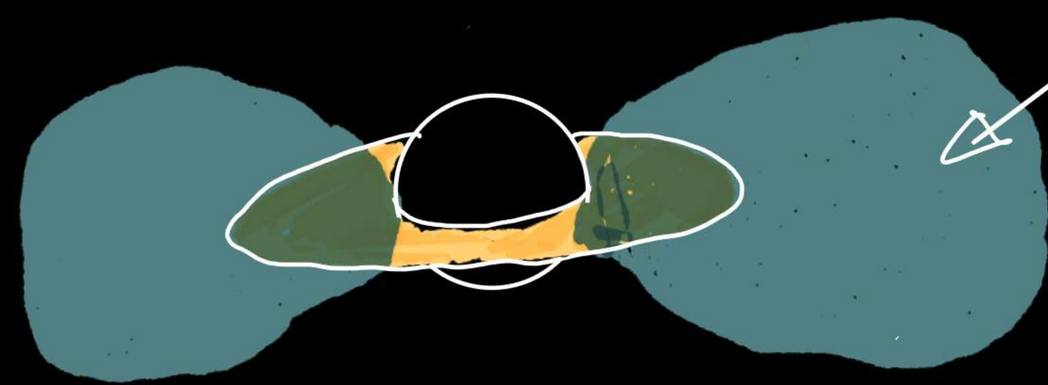
Penrose process → extract energy and angular momentum from Kerr BH



$E_1 > E_0$!

Dark Energy from gravitational waves

Superradiance



Superradiant cloud of scalar field

Fluctuations in scalar field amplified

$\Downarrow$

Scalar field clouds

$\Swarrow$ BH hair

$\Downarrow$ BH Bombs

$\Rightarrow$ ...

(Brito, Cardoso, Pani 2020)

Dark Energy from gravitational waves

Where do we get As?

Scalar field accretion (the Jacobson effect)

$$\Box \phi + m^2 \phi = 0 \xrightarrow[r \to \infty]{\text{asymptotically}} \phi \sim \cos(mt)$$

$$\Downarrow$$

$$\delta\phi = \frac{\delta_0}{a^3} \neq 0$$

(e.g. axions)

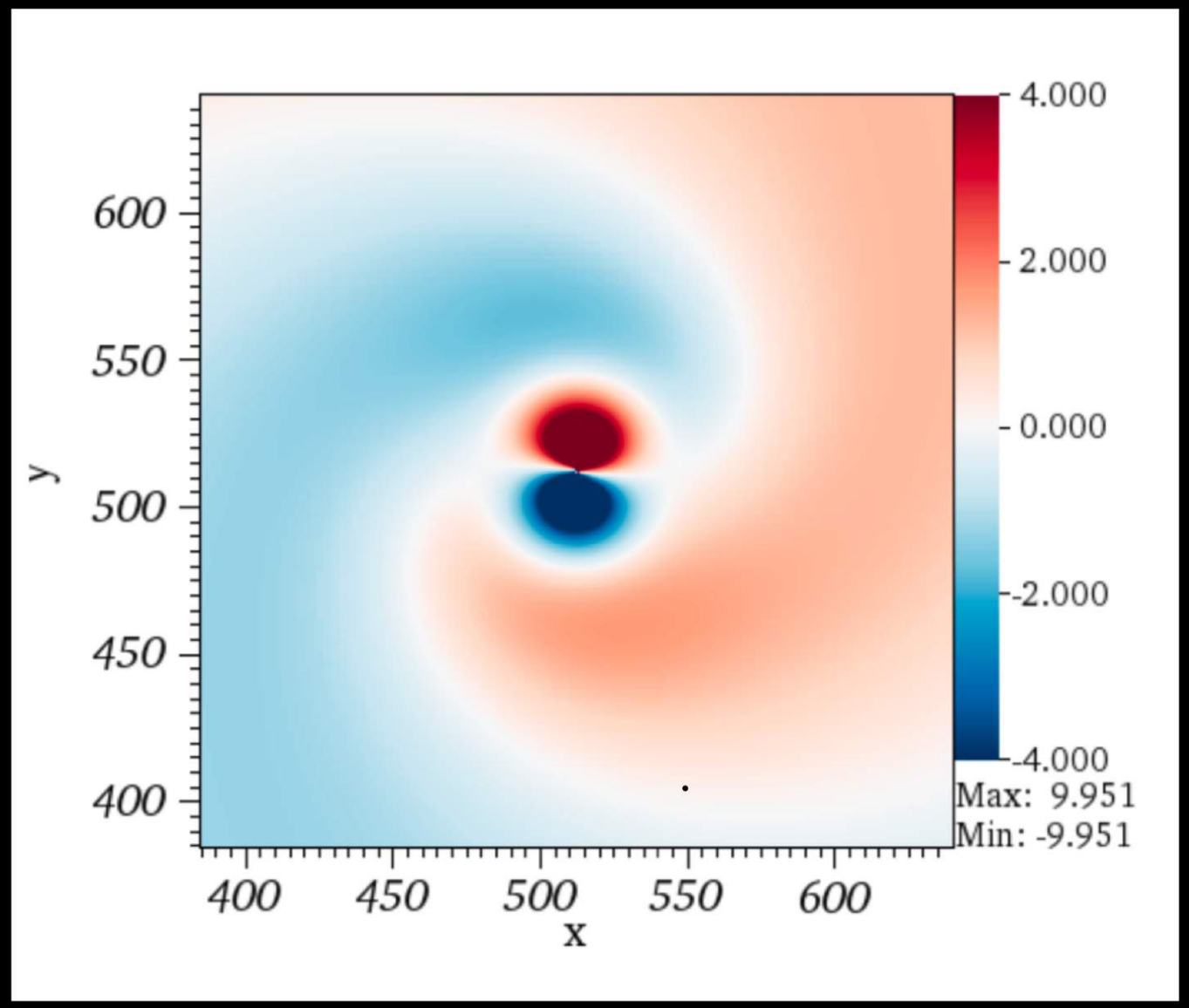
Thus: non-trivial boundary conditions fuels growth of hair by accretion

- Clough et al 1904.12783
- Hui et al 1904.12803
- Baumann et al 2011.07870

Dark Energy from gravitational waves

Where do we get As?

Scalar accretion around Kerr



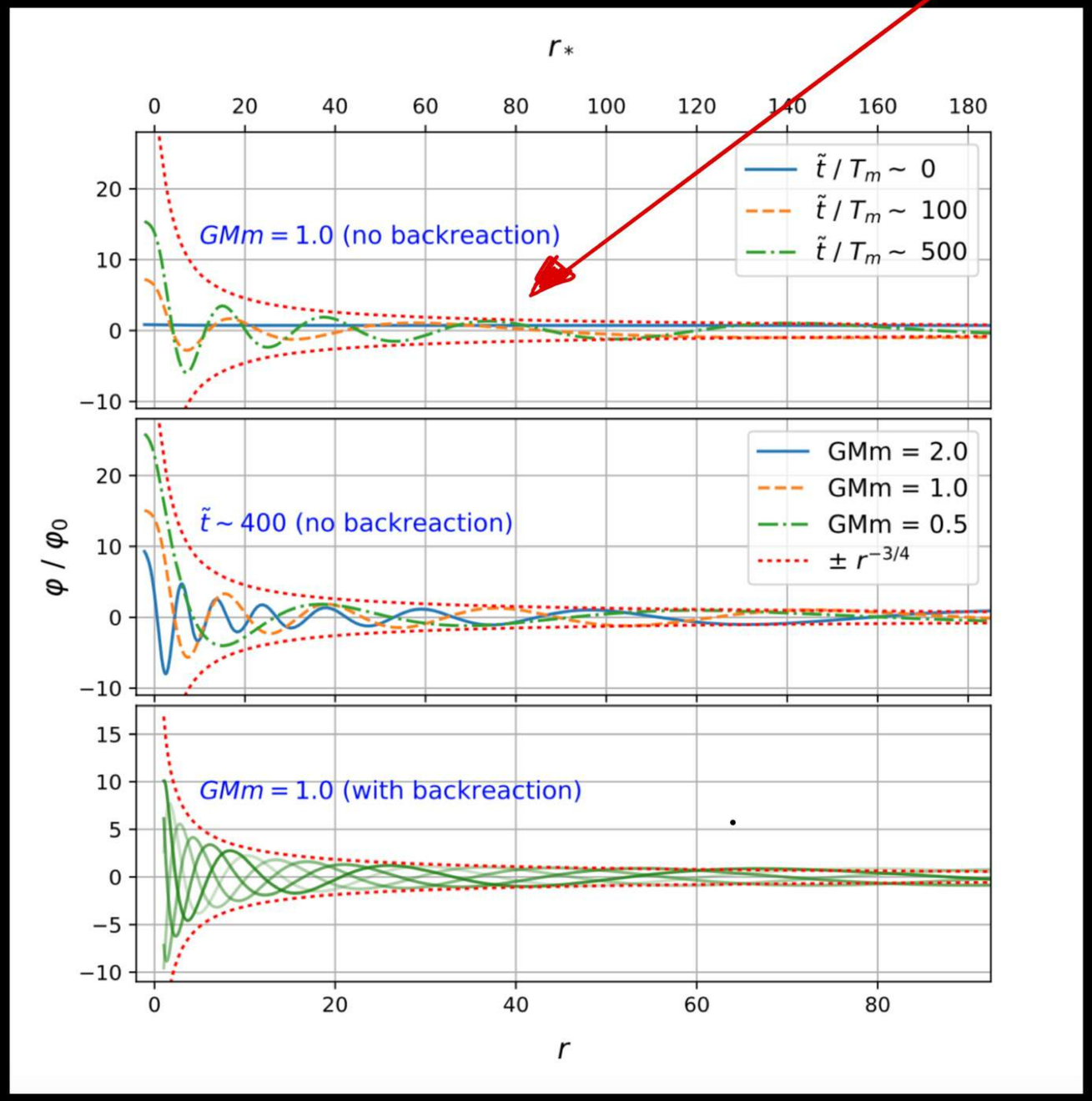
Baker et al
2011.07870

Dark Energy from gravitational Waves

Where do we get As?

Gradual Pile up of scalar hair

Amplitude of Scalar field



Dark Energy from gravitational Waves

Where do we get As?

Don't focus only on BH-BH mergers.

- NS - NS
- BH - NS

} Neutron stars will support fifth forces (i.e "hair")

- Exotica
  - Boson stars
  - Topological Defects
  - ...

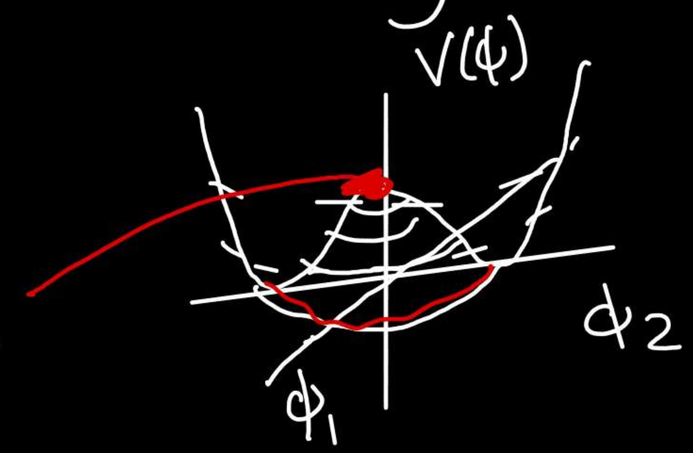
Dark Energy from gravitational Waves

Cosmic Strings

(U(1) or SO(2) symmetry)

$$S_{cs} = \int d^4x \sqrt{-g} \left\{ \frac{M_{pl}^2}{2} R + \frac{1}{2}(\partial\vec{\phi}) + V(\vec{\phi}) \right\}$$

e.g.



Topologically stable lines of field

(mexican hat)

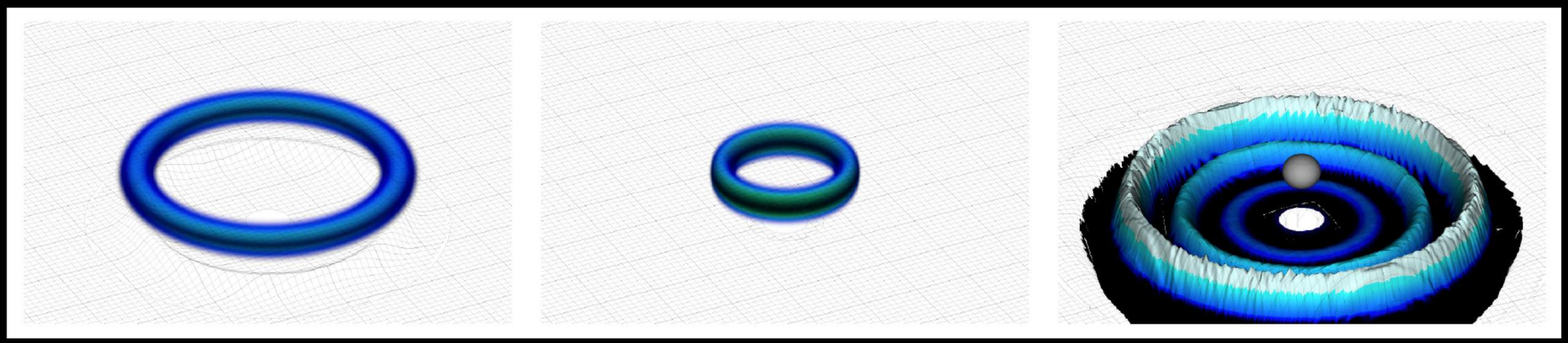
Add Non-minimal coupling:

$$S_{NMCS} = S_{cs} + \int d^4x \sqrt{-g} \left(f(\vec{\phi}) R \right)$$

$\Rightarrow$ Mix gravity and scalar sector.

Dark Energy from gravitational waves

Cosmic string collapse to form black hole
(Numerical relativity simulation)



J. Aurrekoetxea, GRCHOMBO 2020
(Youtube)

Dark Energy from gravitational waves

Final Wave form

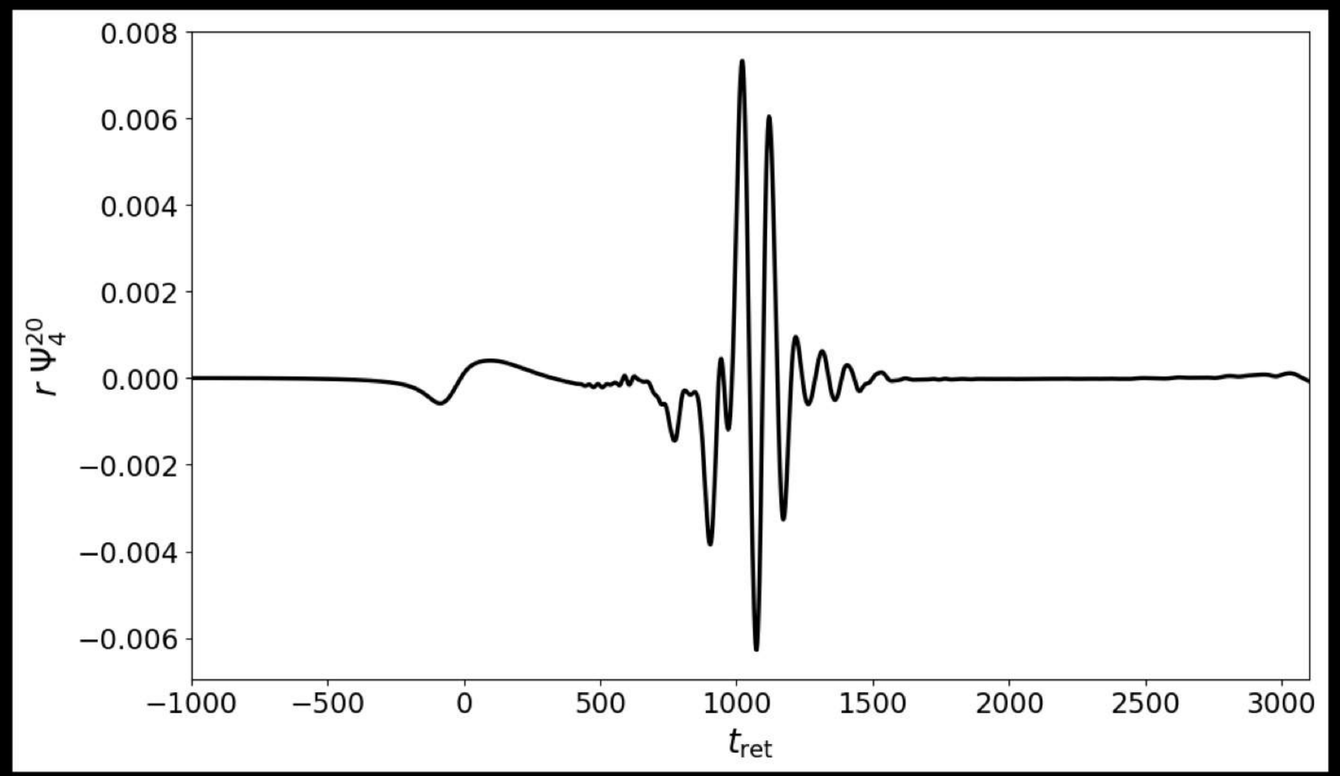
"

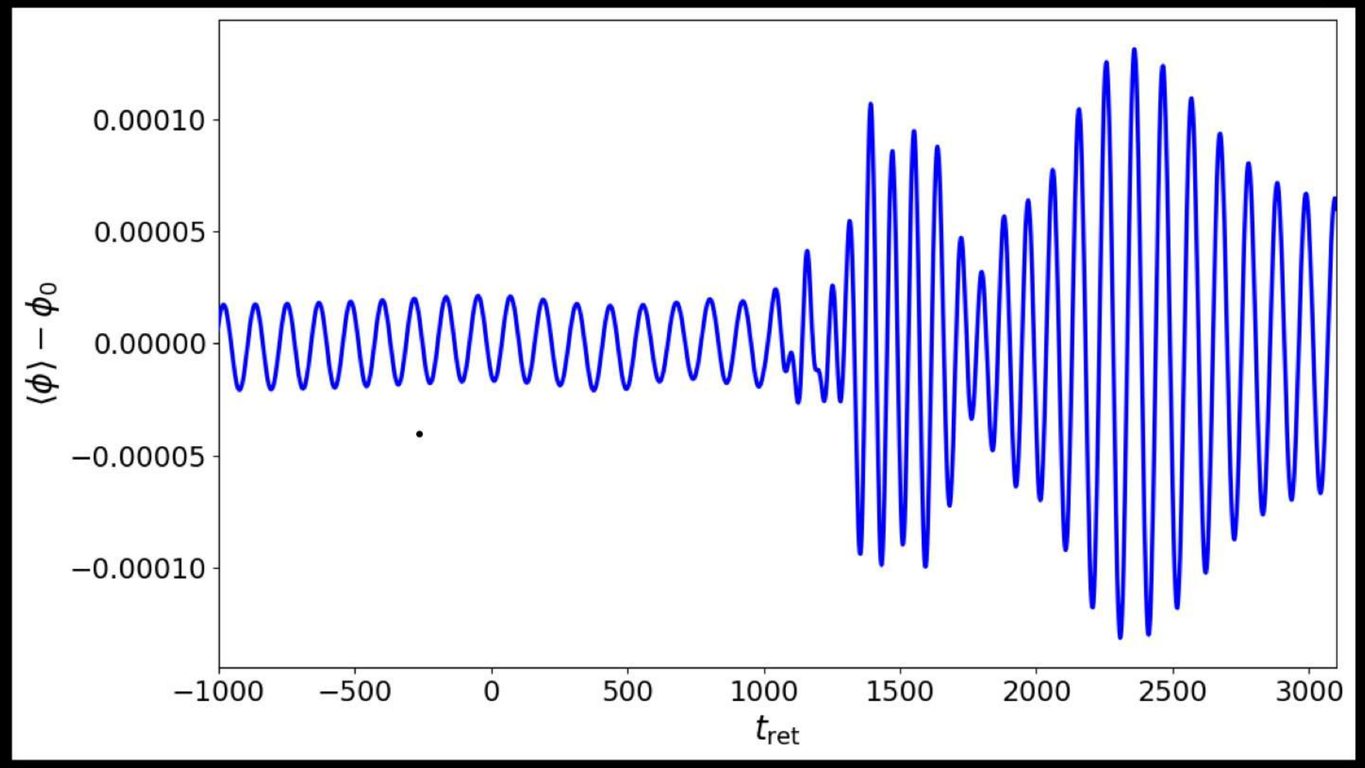
Tensor mode

+

Scalar mode

Challenge: can we detect this and pick out scalar mode?





Aurrekoetxea et al (2021)

Dark Energy from gravitational waves

Summary

- Gravitational wave can be used to place constraints on theories of 'dark energy' (i.e cosmology beyond ΛCDM)
- GW170817 was crucial
- limits theories → No-hair black holes ...
- ... but look at ringdown.
- Challenge: find 'dynamical sources' of scalar hair | dark energy around black holes