Large-scale Structure: the numerical version

Lecture 2: Future surveys. Sigma(z). Cluster counts.

Note: typewritten lecture notes posted (“Lecture Notes for the whole week” in program)

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Last Time:

$$\Delta^2(k,a) = A\,\frac{4}{25}\,\frac{1}{\Omega_M^2}\left(\frac{k}{k_{\rm piv}}\right)^{n-1}\left(\frac{k}{H_0}\right)^4\,[ag(a)]^2\,T^2(k)\,T_{\rm nl}(k)$$

A is the primordial amplitude (dimensionless, $\sim 10^{-10}$)

Ω_M is the density of matter relative to critical

k_{piv} is some chosen pivot; modern convention is k_{piv}=0.05 Mpc^{-1}

H_0 is the Hubble constant

g(a) is the growth suppression factor

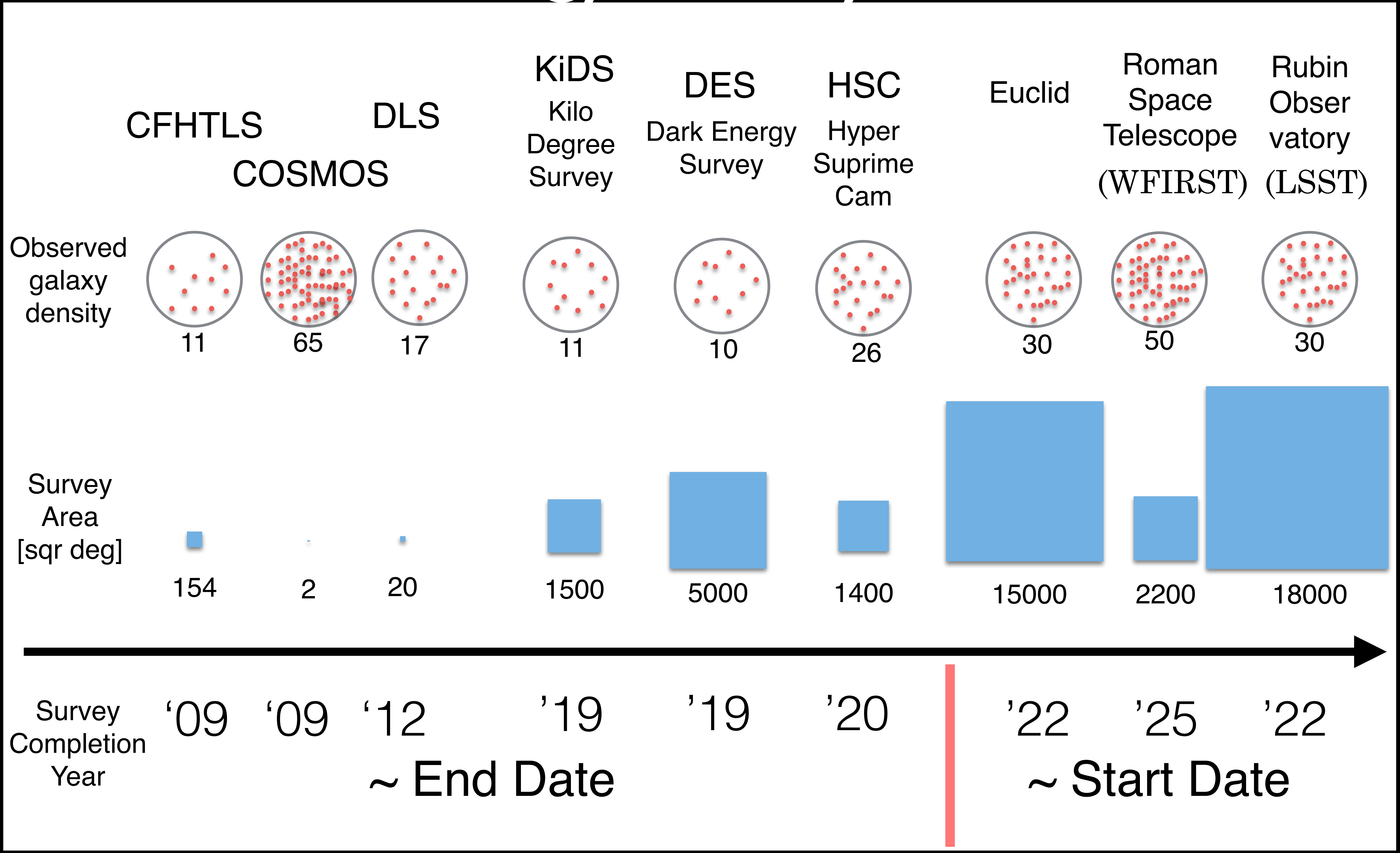
T(k) is the transfer function, accounts mainly for "turnover" in power spectrum due to radiation-matter transition.

T_{nl}(k) is the prescription for nonlinear clustering; super important on scales k $\gtrsim$ 0.1 h Mpc^{-1}

Ongoing or upcoming LSS experiments:

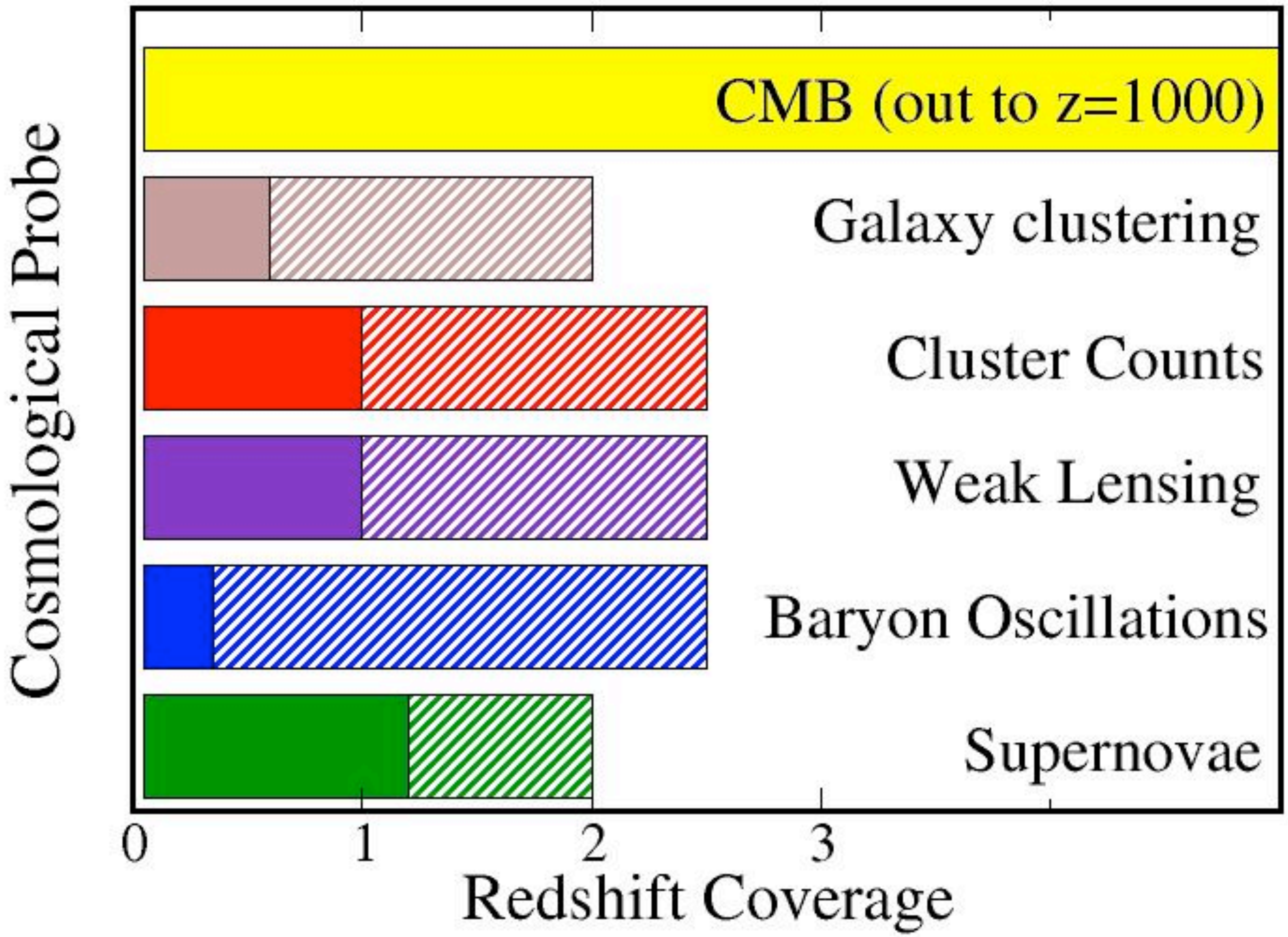
- **Ground photometric:**
  - Kilo-Degree Survey (KiDS)
  - Dark Energy Survey (DES)
  - Hyper Supreme Cam (HSC)
  - Large Synoptic Survey Telescope (LSST)
- **Ground spectroscopic:**
  - Hobby Eberly Telescope DE Experiment (HETDEX)
  - Prime Focus Spectrograph (PFS)
  - Dark Energy Spectroscopic Instrument (DESI)
- **Space:**
  - Euclid
  - Wide Field InfraRed Space Telescope (WFIRST)

(Photometric…mostly) Dark Energy Surveys



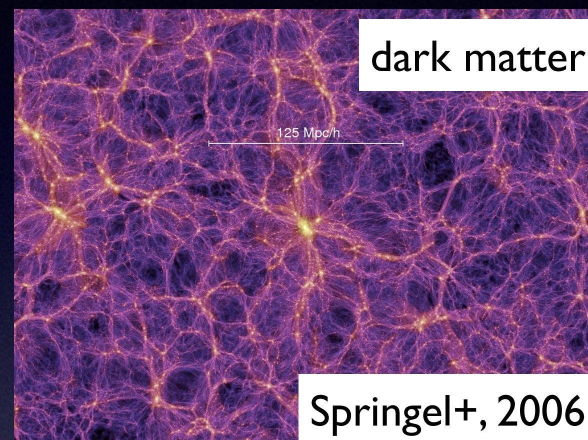
Slide from Tim Eifler

Principal probes of Dark Energy & LSS

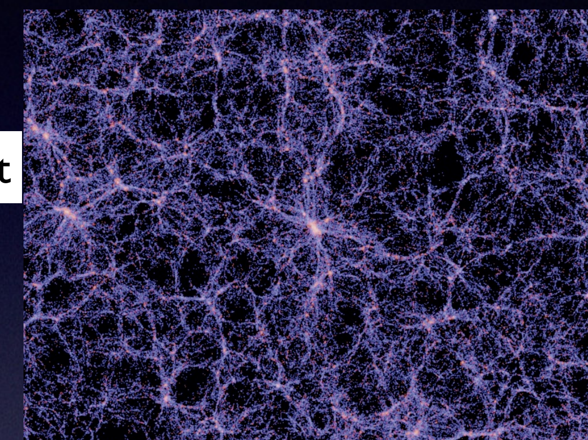


Connecting Theory and Data (photometric surveys)

cosmological model
+ parameters
↓
generate initial conditions, evolve

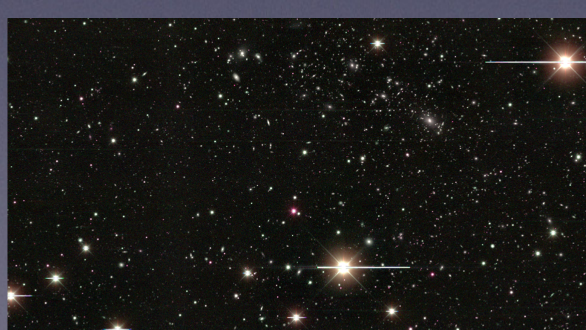


galaxies, light

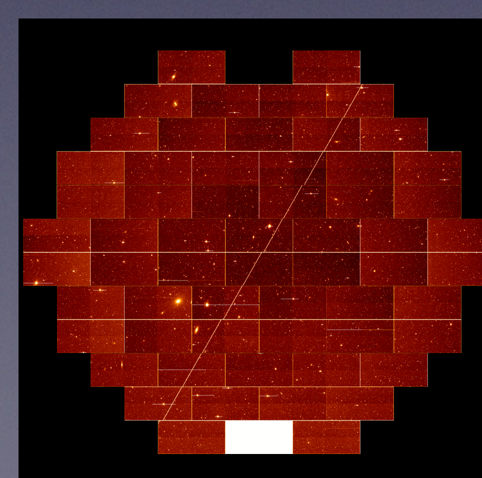


Catalog/ Summary statistics

Likelihood analysis



Instrumentation/ detector physics



Object detection / Image Processing/ Astrometry/ Photometry/Shape Measurement

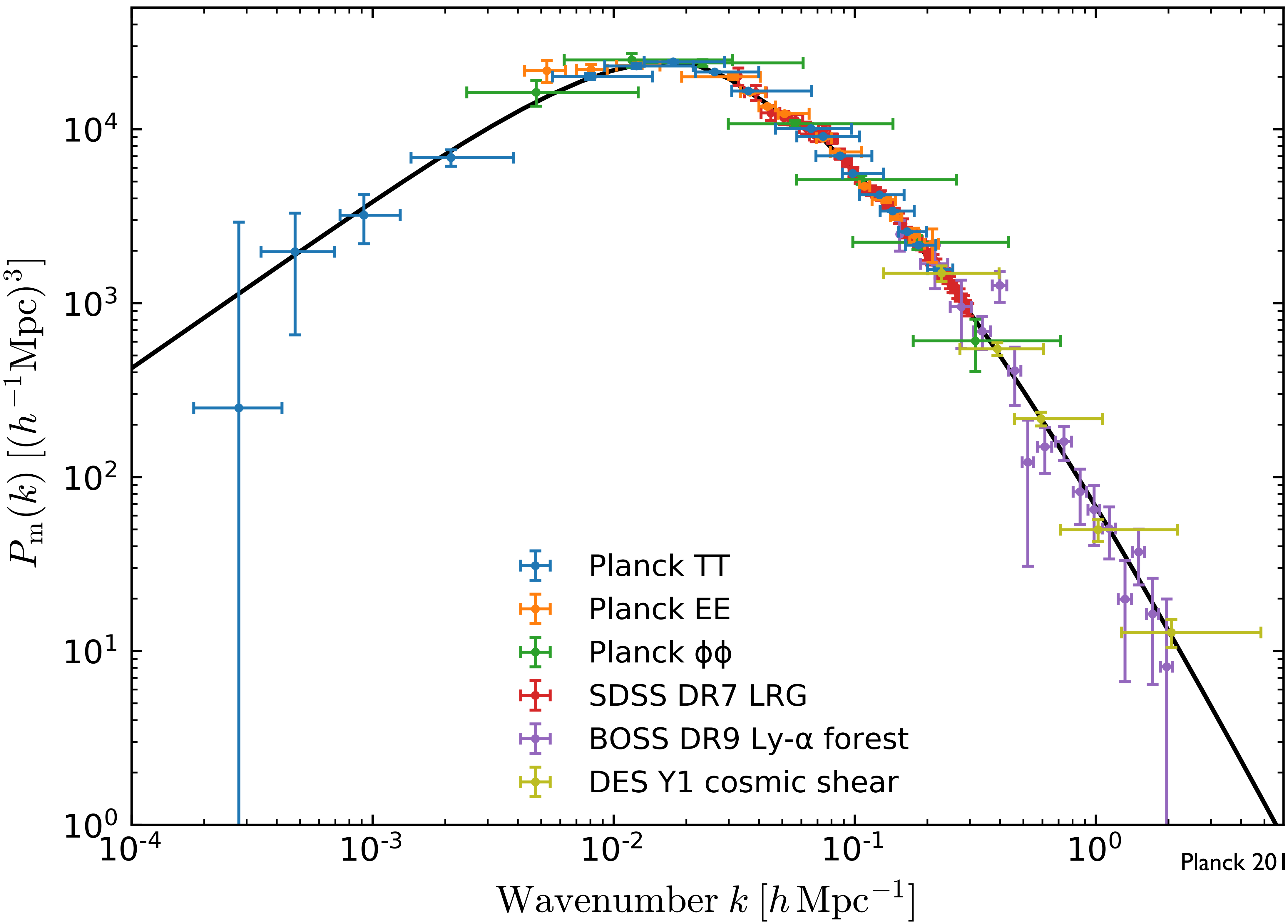
Catalog/ Summary statistics

Slide from Tim Eifler

Power spectrum is a key quantity in cosmology

- Most LSS probes effectively measure it (usually integrated with some geometrical "kernel"); see next slide….
- and Inflation predicts it, so…
- it's a great "meeting place" between theory and data!
- In the limit of Gaussian LSS, contains **all** information (but ok, LSS is not Gaussian….)

Most* key LSS probes essentially measure P(k)



Planck 2018 overview paper

*Notable exceptions: SN Ia, BAO (geometrical feature ≈ distance), cluster counts (≈ mass function)

Smoothed overdensity

An "observed" delta
is necessarily smoothed

$$\delta(\vec{r}, R) = \int W(|\vec{r} - \vec{r}'|)\delta(\vec{r}')d^3\vec{r}'$$

$$\delta_{\vec{k}}(R) = W(k, R)\delta_{\vec{k}}$$

$$W_{\rm TH}(k, R) = 3\frac{\sin(kR) - kR\cos(kR)}{(kR)^3} = \frac{3j_1(kR)}{kR}$$

Let us write the zero-lag correlation function with top-hat-smoothed field

$$\xi_{\rm TH}(0) = \int_0^\infty \Delta^2(k)|W_{\rm TH}(k, R)|^2 d\ln k$$

or, renaming it to agree with the literature, this is the
amplitude of mass fluctuations (squared) smoothed on scale R

$$\boxed{\sigma^2(R) = \int_0^\infty \Delta^2(k)\left(\frac{3j_1(kR)}{kR}\right)^2 d\ln k}$$

The famous sigma-eight

$$\sigma^2(R) = \int_0^\infty \Delta^2(k) \left(\frac{3 j_1(kR)}{kR} \right)^2 d\ln k$$

The amplitude of mass fluctuations σ(R)

is a derived quantity (power spectrum is “fundamental”), but very useful because it is a number summarizing the (square root of the) “amount of power” on a typical scale R

Can calculate it at any smoothing scale R (and any redshift z, suppressed in Eq above), but one choice is historically famous:

$$\sigma_8 \equiv \sigma(R = 8\, h^{-1}\mathrm{Mpc}, z = 0)$$

- σ_8 goes waaaay back to 1980s - was used as the measure of the overall amount of power/clustering on typical scales accessible in galaxy surveys
- In 1990s-2000s question of whether $\sigma_8 \approx 0.6$ or $\sigma_8 \approx 1.0$
- The answer is of course in the middle, $\boldsymbol{\sigma_8 \approx 0.8}$, BUT
- Tension between CMB ($\sigma_8 \approx 0.78$) and grav lensing ($\sigma_8 \approx 0.82$) - at the forefront of research in cosmology today

Big-picture summary of LSS

Say someone gives you a big box (ok, a file) with 100 million galaxy (and a 100,000 cluster) positions - from either a simulation or real sky. What can you do?

1. Count them!

⇒ fine, but this would only work for clusters, as galaxies are "too complicated" and we can't model their abundance from first principles. Cluster mass fun is dn/dlnM(z).

2. Calculate their clustering, or 2pt function, $\xi(r)$ or $P(k)$!

⇒ super. Can do that via clustering of galaxies, galaxy shears (weak lensing), clusters of galaxies, etc. This has been the workhorse of cosmology since late 1970s!

3. Calculate higher-pt statistics, like $\zeta(r_1, r_2, r_3)$ or $B(k_1, k_2, k_3)$

⇒ interesting, but hard, both to calculate and (especially!) to theoretically predict.

4. Calculate alternative measure: count peaks, measures topology, etc

⇒ promising. It does typically contain the same or partial info as 1-pt, 2-pt and higher-pt statistics, but may be easier to calculate/model in practice.

5. Study the internal structure of galaxies, star formation, etc

⇒ but now you are doing astrophysics and not cosmology, my dear friend.

So how do you estimate P(k) or ξ(r)?

The subject of estimators is science in its own right.
Many options… we discuss the simplest one.

Remember that ξ(r) is excess probability, $dP = n^2(1+\xi(r_{12}))\, dV_1 dV_2$

Then how about a Peebles-Hauser (1974) estimator

$$\hat{\xi}_{\rm PH}(r) = \left(\frac{N_{\rm rand}}{N_{\rm data}}\right)^2 \frac{{\rm DD}(r)}{{\rm RR}(r)} - 1$$

number of pairs separated by r±Δr in data (DD) and random (RR) catalogues

Better: (smaller variance): Landy-Szalay (1993) estimator

$$\hat{\xi}_{\rm LS} = \left(\frac{N_{\rm rand}}{N_{\rm data}}\right)^2 \frac{{\rm DD}(r)}{{\rm RR}(r)} - 2\frac{N_{\rm rand}}{N_{\rm data}}\frac{{\rm DR}(r)}{{\rm RR}(r)} + 1$$

Done naively, computation is N^2 process.
Can be drastically sped up by various clever algorithms.

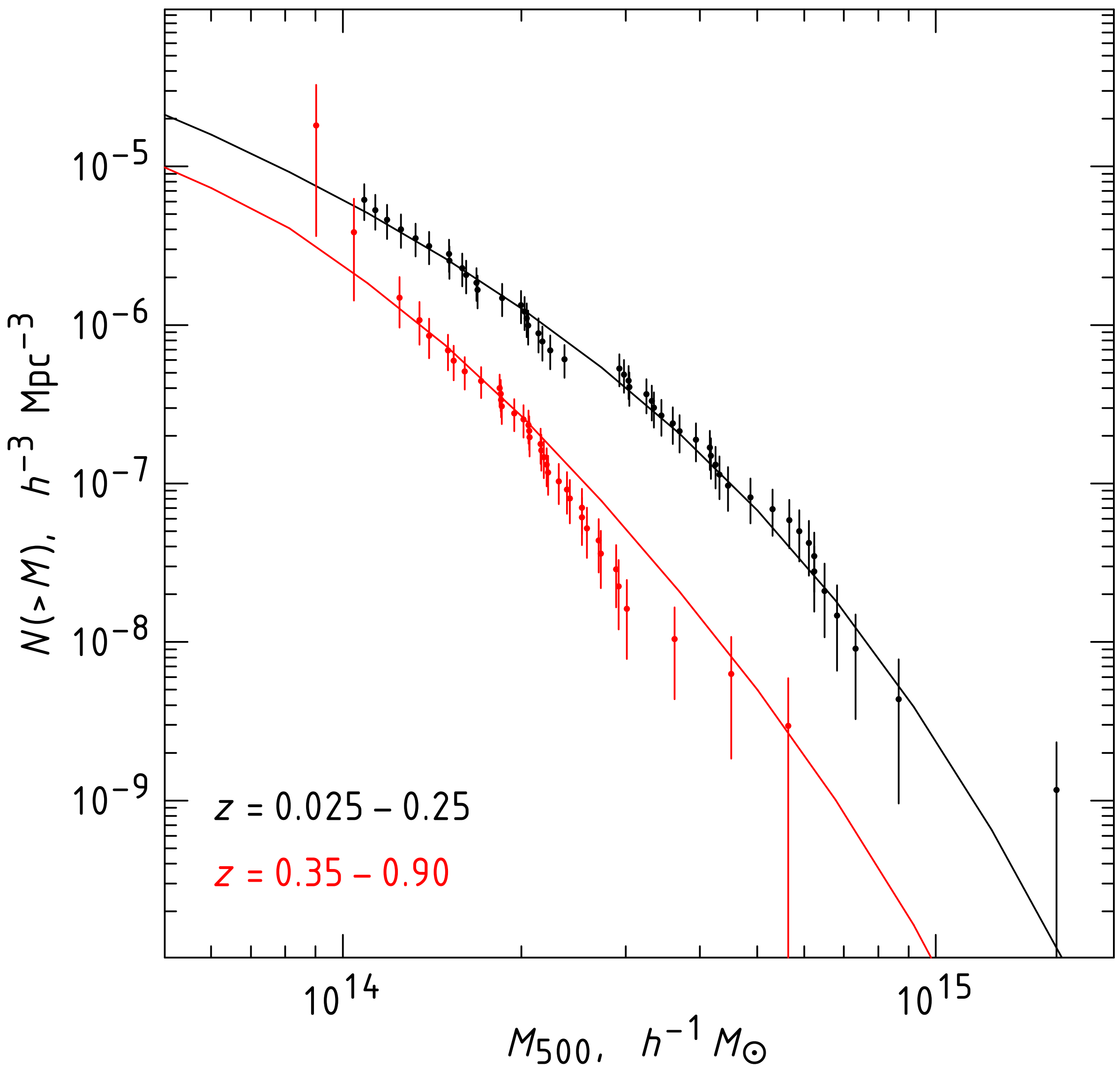
A couple of extensions

(time permitting)

1. Mass function from Press-Schechter formalism
2. Bias (of halos) from peak-background split

Cluster mass function dn/dlnM(z)

- This is the umber density (number per cubic comoving megaparsec), per unit log mass
- Hopeless to theoretically predict for galaxies, as they are nonlinear
- Clusters are "just barely linear", σ(R~5Mpc) ~ 1. Is there hope??



Measured mass function
Vikhlinin et al, ApJ 692, 1060 (2009)

Cluster mass function dn/dlnM(z)

Press & Schechter (1974) argument:

- Region of radius R has mass $M = (4\pi/3)R^3\rho_M$, where $\rho_M = \rho\Omega_M(1+z)^3$
- $\delta_c \approx 1.686$ (critical overdensity for collapse in simplest model)
- Then (assuming Gaussian fluc), fraction of collapsed objects is

$$F(M) \equiv \int_{\delta_c}^{\infty} P(\delta)d\delta = \frac{1}{\sqrt{2\pi}\sigma(M)}\int_{\delta_c}^{\infty} e^{-\delta^2/(2\sigma(M)^2)}d\delta \equiv \frac{1}{2}\mathrm{erfc}\left(\frac{\nu}{\sqrt{2}}\right)$$

where $\nu \equiv \delta_c/\sigma(R)$ is the *peak height*

The comoving number density is $$\frac{dn}{d\ln M}d\ln M = \frac{\rho_{M,0}}{M}\left|\frac{dF(M)}{d\ln M}\right| d\ln M$$

which, with F(M) given above, and multiplying by factor of 2 to account for underdensities as well (?? but !!) evaluates to:

$$\frac{dn}{d\ln M} = \sqrt{\frac{2}{\pi}}\,\frac{\rho_{M,0}}{M}\,\frac{\delta_c}{\sigma}\left|\frac{d\ln\sigma}{d\ln M}\right| e^{-\delta_c^2/(2\sigma^2)}$$

Press-Schechter mass function

$$\frac{dn}{d\ln M} = \sqrt{\frac{2}{\pi}}\,\frac{\rho_{M,0}}{M}\,\frac{\delta_c}{\sigma}\left|\frac{d\ln\sigma}{d\ln M}\right| e^{-\delta_c^2/(2\sigma^2)}$$

Basic properties

- pretty accurate (to ~10-30%) despite massive approximations
- <u>universal</u> - depends only on σ(M) and not detailed cosmological model ($\Omega_{\rm M}$, Ω_Λ, n, etc)…
- … but universality is not fundamental; departures exist at the 5% level (e.g. Tinker et al 2008)

Tour-de-force of simple analytical reasoning in cosmology (in 1974!)
Not accurate-enough for research any more, but provides insights

On our numerical exercise:

calculate the number of galaxy clusters above some mass

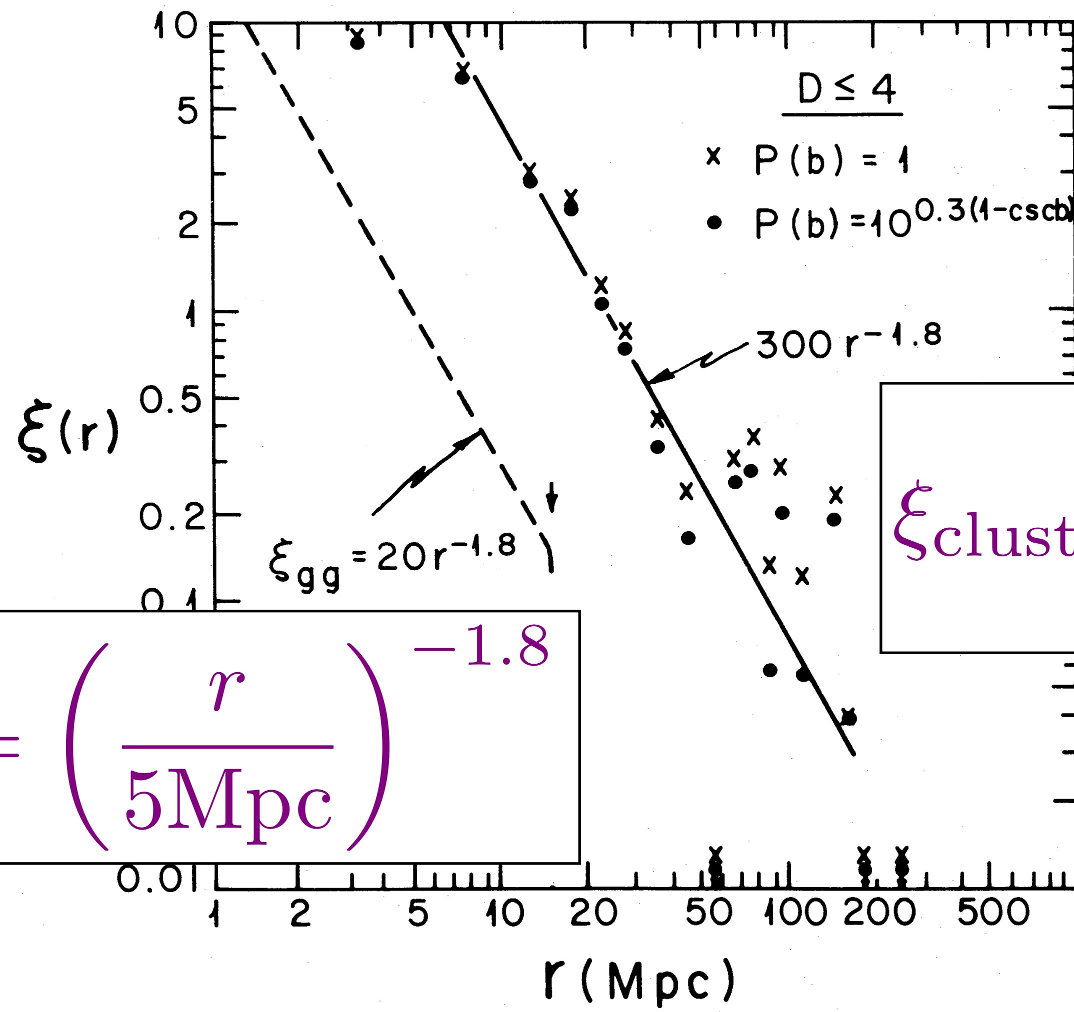
(per solid angle, per redshift)

$$\frac{dN(>M_{\rm th})}{d\Omega\, dz} = \int_{M_{\rm th}}^{\infty} \frac{dn}{d\ln M} d\ln M \frac{dV}{d\Omega\, dz}$$

$$= \int_{M_{\rm th}}^{\infty} \frac{dn}{d\ln M}(z, M)\, d\ln M \left[\frac{r^2(z)}{H(z)}\right]$$

Because $$\frac{dV}{d\Omega\, dz} = \frac{r^2 dr d\Omega}{d\Omega dz} = \frac{r^2}{H}$$

(Galaxy/halo) bias



Bahcall & Soneira 1983

$$\xi_{\text{clusters}}(r) = \left(\frac{r}{25\text{Mpc}}\right)^{-1.8}$$

$$\xi_{\text{galaxies}}(r) = \left(\frac{r}{5\text{Mpc}}\right)^{-1.8}$$

Explained by Kaiser (1984):

$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \frac{\left(\frac{\delta\rho}{\rho}\right)_{\text{halos}}}{\left(\frac{\delta\rho}{\rho}\right)_{\text{DM}}}$$

cosmologists measure

usually nuisance parameter(s)

theory predicts

Bias of galaxies (and DM halos)

$$\delta_h(k,z) = b(k,z)\delta_m(k,z) \qquad P_h(k,z) = \boxed{b^2(k,z)} P_m(k,z)$$

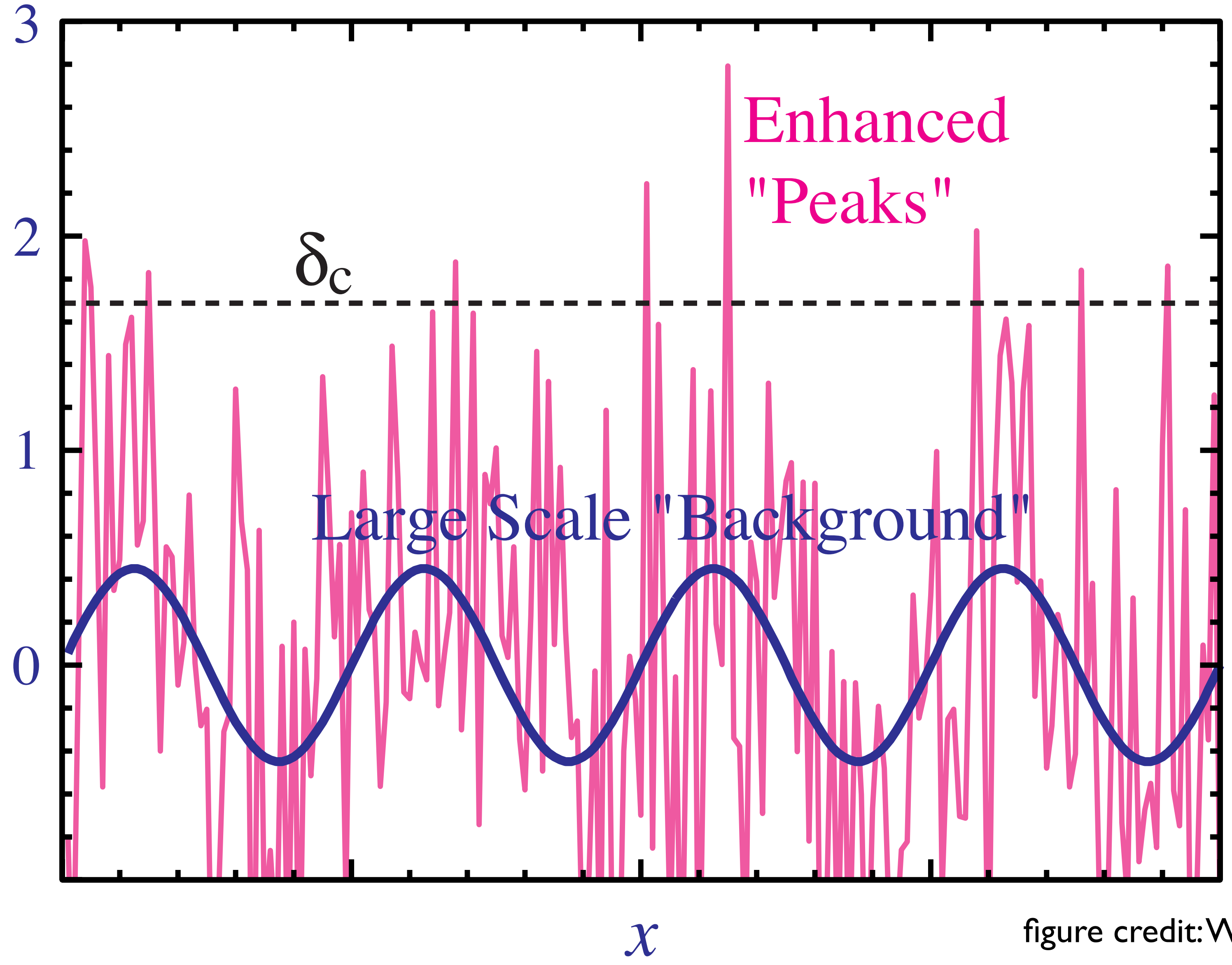


figure credit: Wayne Hu

Fact of life: the density peaks simply cluster differently than the background field, even for a Gaussian field ⇒ bias (Bardeen, Bond, Kaiser, Szalay 1987)

Peak-background formula for bias

Imagine some long-wavelength perturbation δ_b (b=background) and short-wavelength one δ_p (p=peaks) ,with

$$\delta = \delta_b + \delta_p$$

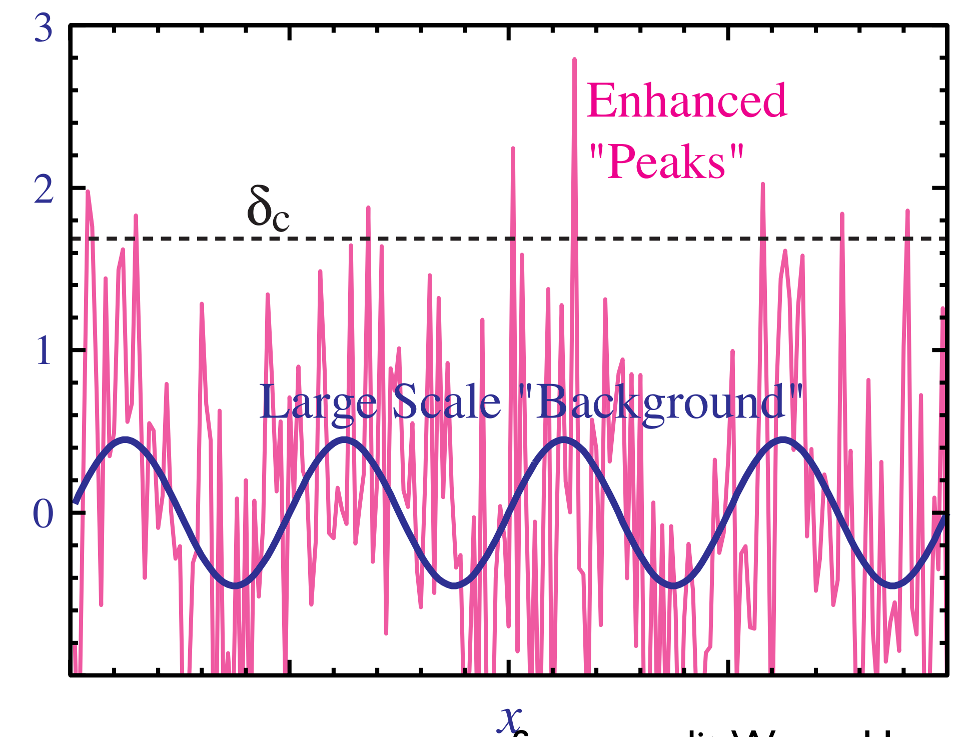


figure credit: Wayne Hu

According to the spherical collapse model, to "become a halo" the density fluctuation needs too cross some threshold (e.g. $\delta_{crit} \approx 1.686$)

But because the short perturbation is "riding" on the long perturbation, it only needs to cross the threshold of $\delta_c = \delta_{crit} - \delta_b$.

Now adopt the Press-Schechter mass function

$$n(\nu) \propto \nu \exp(-\nu^2/2) \quad \text{where, recall} \quad \nu \equiv \delta_c/\sigma$$

Peak-background formula for bias

Then:

$$n(\nu + \delta\nu) = n\left(\frac{\delta_c - \delta_b}{\sigma}\right) \approx n(\nu) + \frac{dn}{d\nu}\frac{d\nu}{d\delta}(-\delta_b)$$

$$= n(\nu) + \left(\frac{1}{\nu} - \nu\right) n(\nu) \left(-\frac{\delta_b}{\sigma}\right)$$

$$= n(\nu)\left[1 + \frac{\nu^2 - 1}{\nu\sigma}\delta_b\right]$$

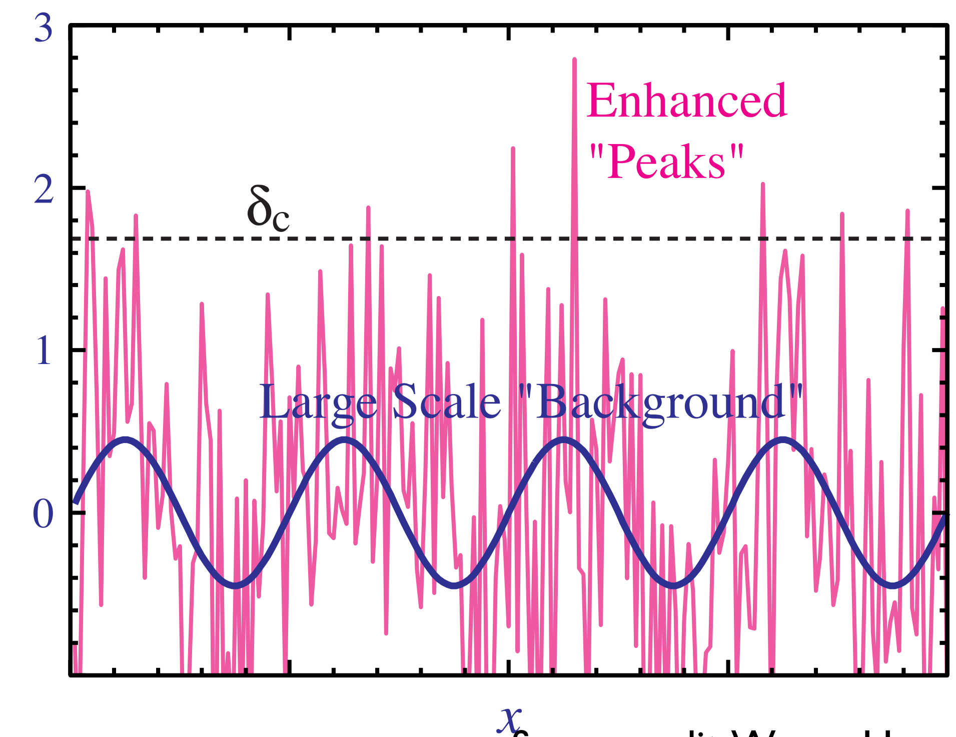


figure credit: Wayne Hu

Thus $\delta n/n = (\nu^2-1)/(\nu\delta)\ \delta b$.

Finally, switch from Lagrangian bias (above) to Eulerian, $b_E = b_L + 1$, to get

$$b(M) \simeq 1 + \frac{\nu^2 - 1}{\delta_c}$$ (bias from peak – background split)

Note: higher-peak/mass objects are more biased, as expected.

There are many extensions to the peak-background split theory, including excursion-set formalism (see Zentner 2006 review). However, one typically can't safely assume b(M, z) [or b(k, z)] is known - except arguably at largest scales where b=constant as a function of M/k (but not z!).