Physics of the Cosmic Microwave Background

Lecture I: setting the stage

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Basic definitions [note: $c = 1$]

CMB = **electromagnetic radiation** permeating the Universe.

It can be described by its **specific intensity**

$$I(\nu, \hat{n}) \qquad [\mathrm{erg/s/Hz/sr/cm^2}]$$

frequency

direction of propagation

…or equivalently by its **photon occupation number *f***
[number of photons per quantum state]

$$I(\nu, \hat{n}) = 2h_{\mathrm{P}}\nu^3 f(\nu, \hat{n})$$

…or equivalently by its **phase-space density = $(2/h^3)f$**
[number of photons per $d^3x\, d^3p$]

Basic definitions [note: $k_B = 1$, $E_\gamma = h_P \nu$]

For a perfect blackbody at temperature *T*:

$$f = \frac{1}{\exp(h_P \nu / T) - 1} = \frac{1}{\exp(E_\gamma / T) - 1}$$

For **generic radiation** (non-blackbody), can always ***define T***:

$$T(E_\gamma, \hat{n}) = \frac{E_\gamma}{\ln(1/f(E_\gamma, \hat{n}) + 1)}$$

$$T(\nu, n) = T_0 \quad + \Delta T(\nu) \quad + \Delta T(n) \quad + \delta T(\nu, n)$$

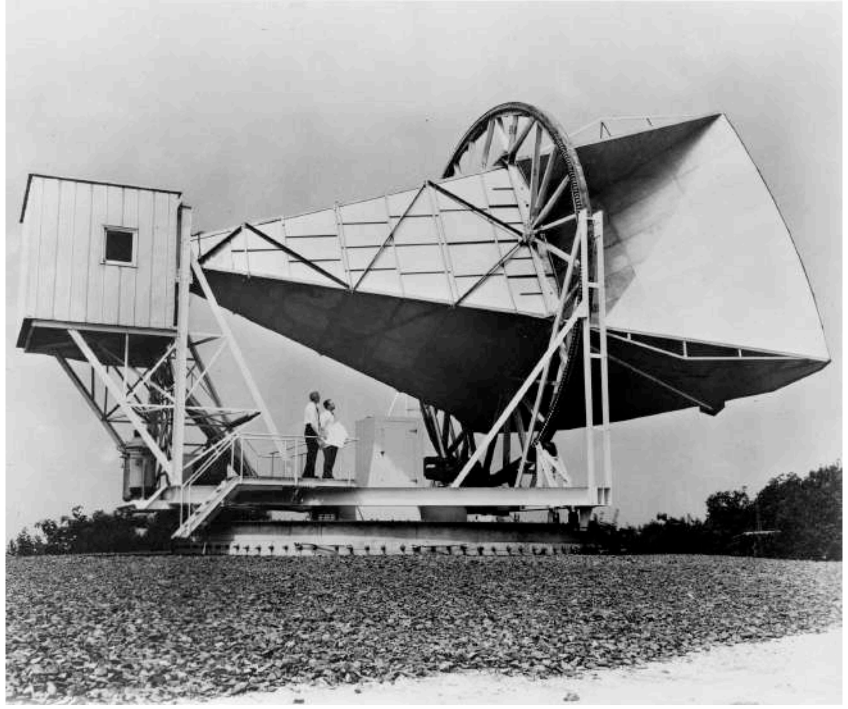
spectral distortions | **anisotropies** | spectral-spatial distortions (e.g. SZ)

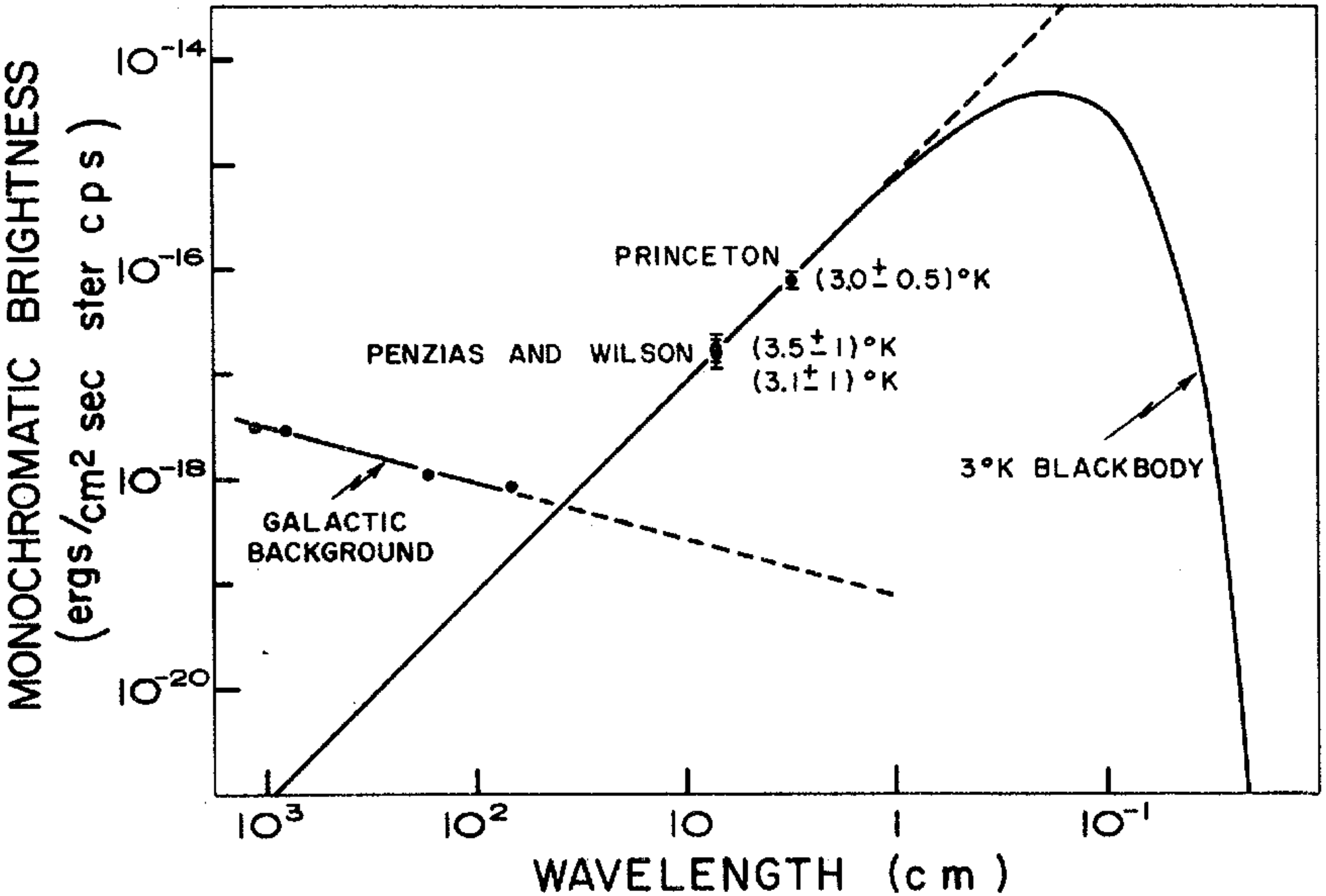
1965: Discovery of the CMB

Penzias & Wilson detect "excess radiation" at $\nu = 4$ GHz

Corresponds to **$T \approx 3$ K radiation** interpreted as the CMB by

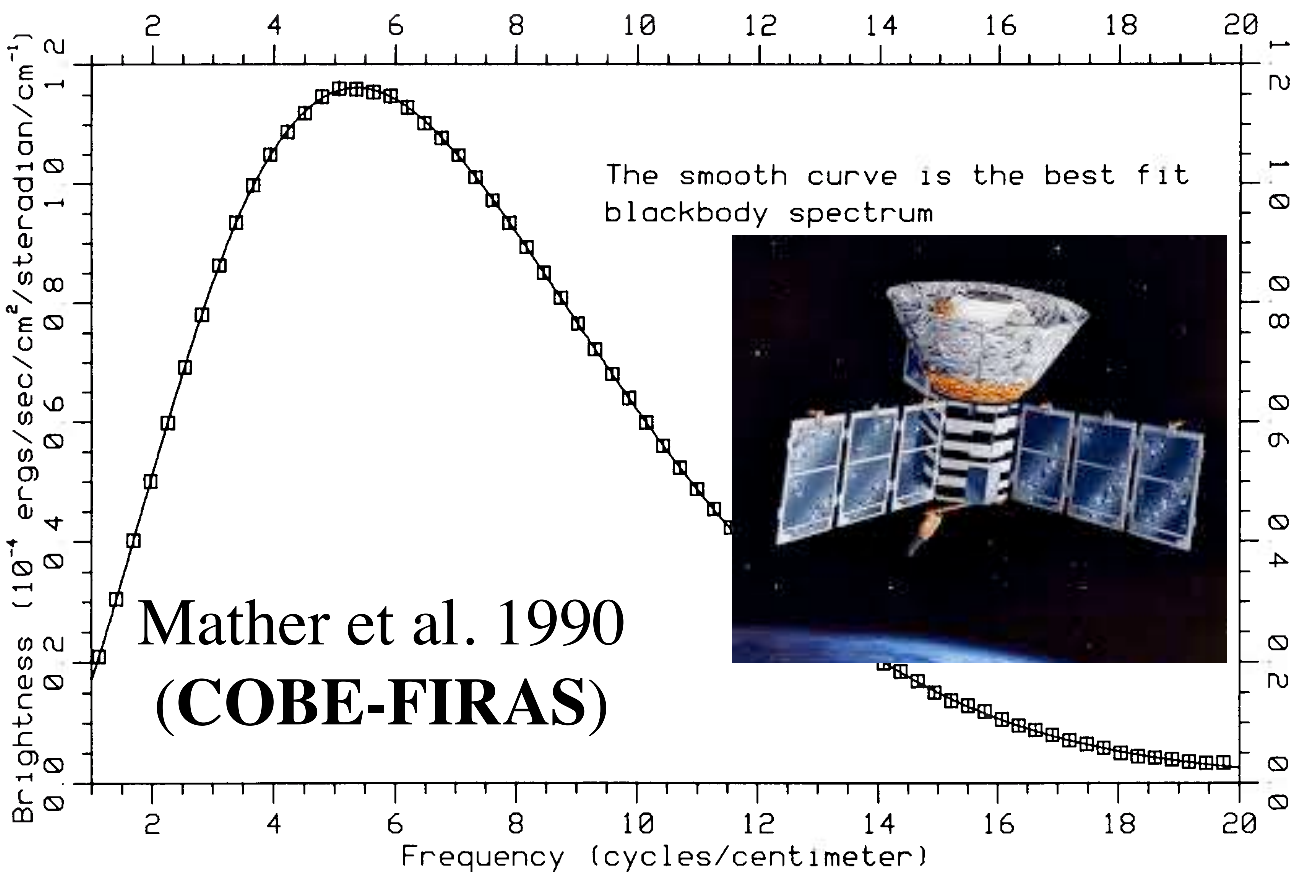
Dicke, Peebles, Roll & Wilkinson

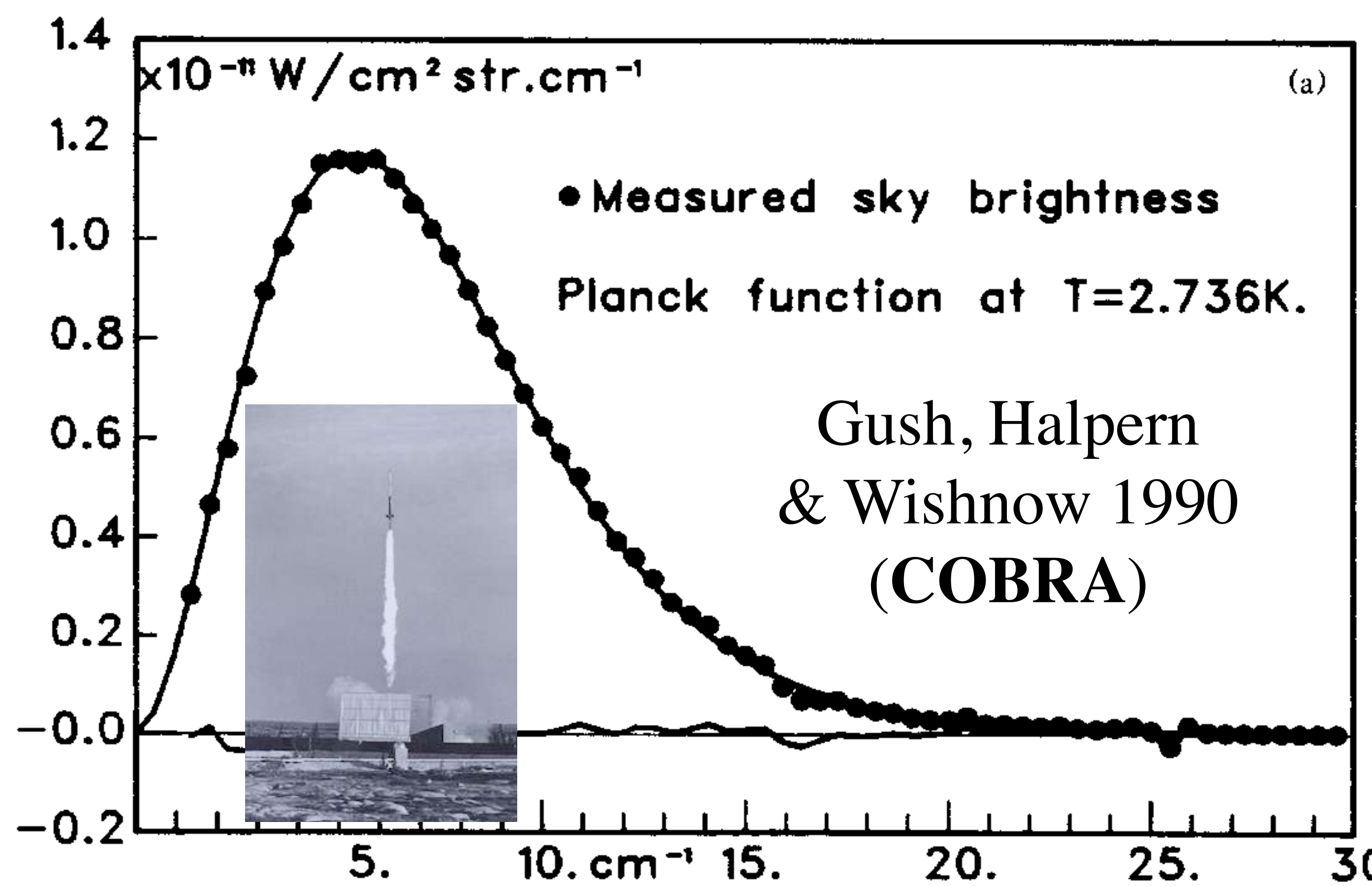




1990: The CMB frequency spectrum

COBE measures the CMB spectrum, consistent with blackbody spectrum (i.e. no detected spectral distortions) (best limit: spectral distortions < 0.01%, Mather et al. 1999)





COBE-FIRAS + other data (Fixsen 2009):

$T_0 = 2.7255 \pm 0.0006$ K

Aside on CMB spectral distortions

- Suppose photons start with a blackbody spectrum.

- Absent injection of additional energy, phase-space density is conserved, photons retain a blackbody spectrum ***$T_\gamma \propto 1/a$***

- **Suppose some process injects energy** into the photons (either fresh photons, or heats up photons).

- If energy injection is at ***$t \lesssim$* 2 months**, it gets **fully thermalized**

Free-free (Bremsstrahlung) $e^- + p \leftrightarrow e^- + p + \gamma$

Double-Compton scattering $e^- + \gamma \leftrightarrow e^- + \gamma' + \gamma''$

Efficiently change photon number and energy; recover perfect blackbody at different *T*

Aside on CMB spectral distortions

- If energy injection is at **2 months ≲ *t* ≲ 300 yrs**, photons can no longer be efficiently created or destroyed, but **Thomson scattering efficiently changes photon energy** (not number)

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

➡ Photons acquire a Bose-Einstein distribution with non-zero **chemical potential *μ***

$$f_\gamma(E_\gamma) = \frac{1}{\exp\left(E_\gamma/T_\gamma + \mu\right) - 1}$$

$$\mu \sim \frac{1}{\rho_\gamma} \int_{2\ \mathrm{mo}}^{300\ \mathrm{yr}} dt\ \dot{\rho}_{\mathrm{inj}}$$

Aside on CMB spectral distortions

- If energy injection is **after 300 yrs**, it cannot be thermalized.

➡ Photon spectrum gets **distorted from blackbody**. Specific shape of distortion depends on energy injection process.

$$f_\gamma(E_\gamma) = f_\gamma^{\mathrm{BB}}(E_\gamma)\left[1 + \Delta(E_\gamma)\right]$$

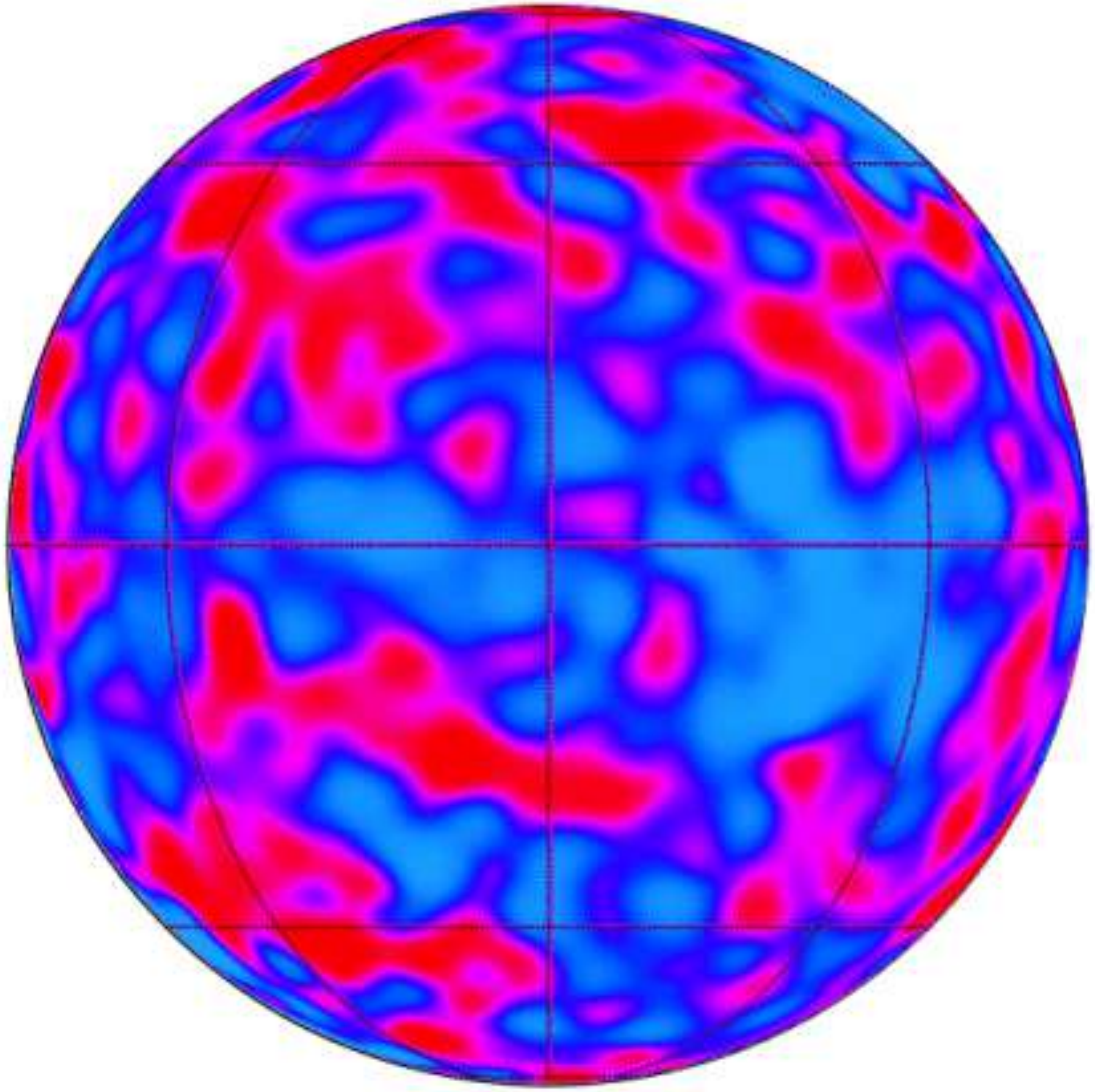
$$\Delta \sim \frac{1}{\rho_\gamma}\int_{300\ \mathrm{yr}}^{t_0} dt\ \dot{\rho}_{\mathrm{inj}}$$

➡ Upper limits on CMB spectral distortions imply stringent **constraints on exotic energy injection** at ***t*** ≳ **2 months**

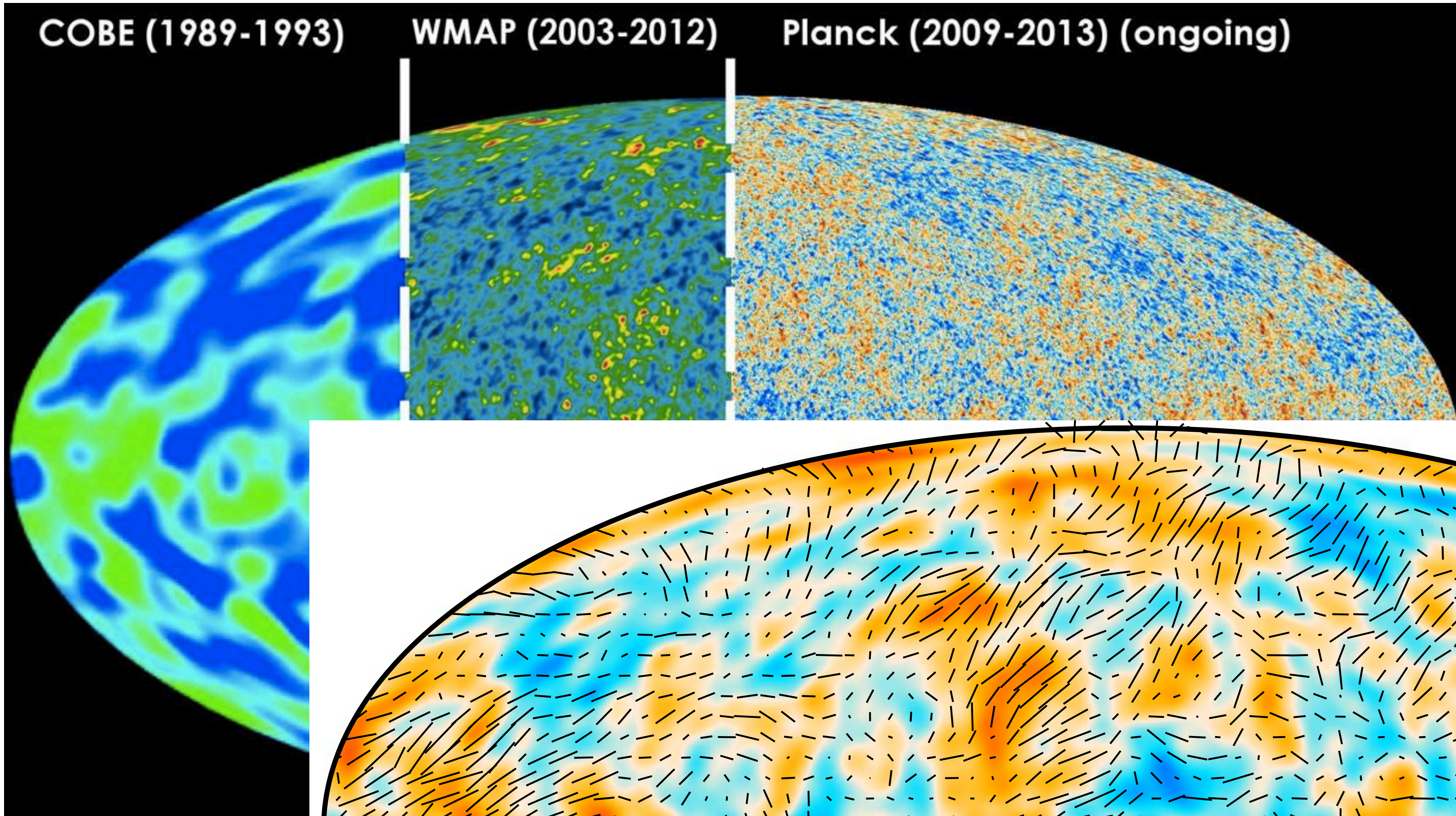
CMB temperature anisotropies

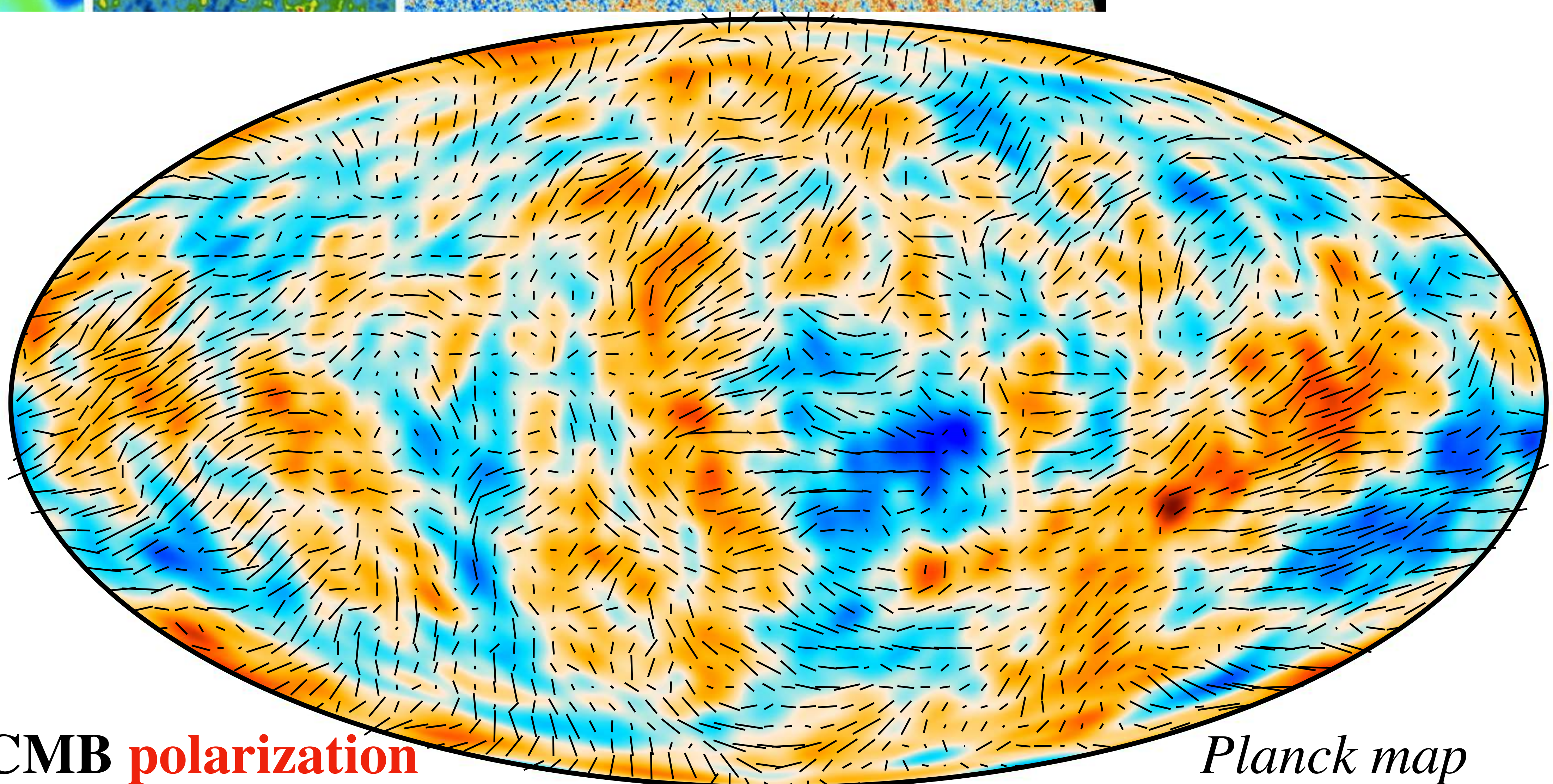
For now we will assume that CMB has a perfect blackbody spectrum and focus on CMB anisotropies

COBE-DMR 1992



CMB temperature anisotropies



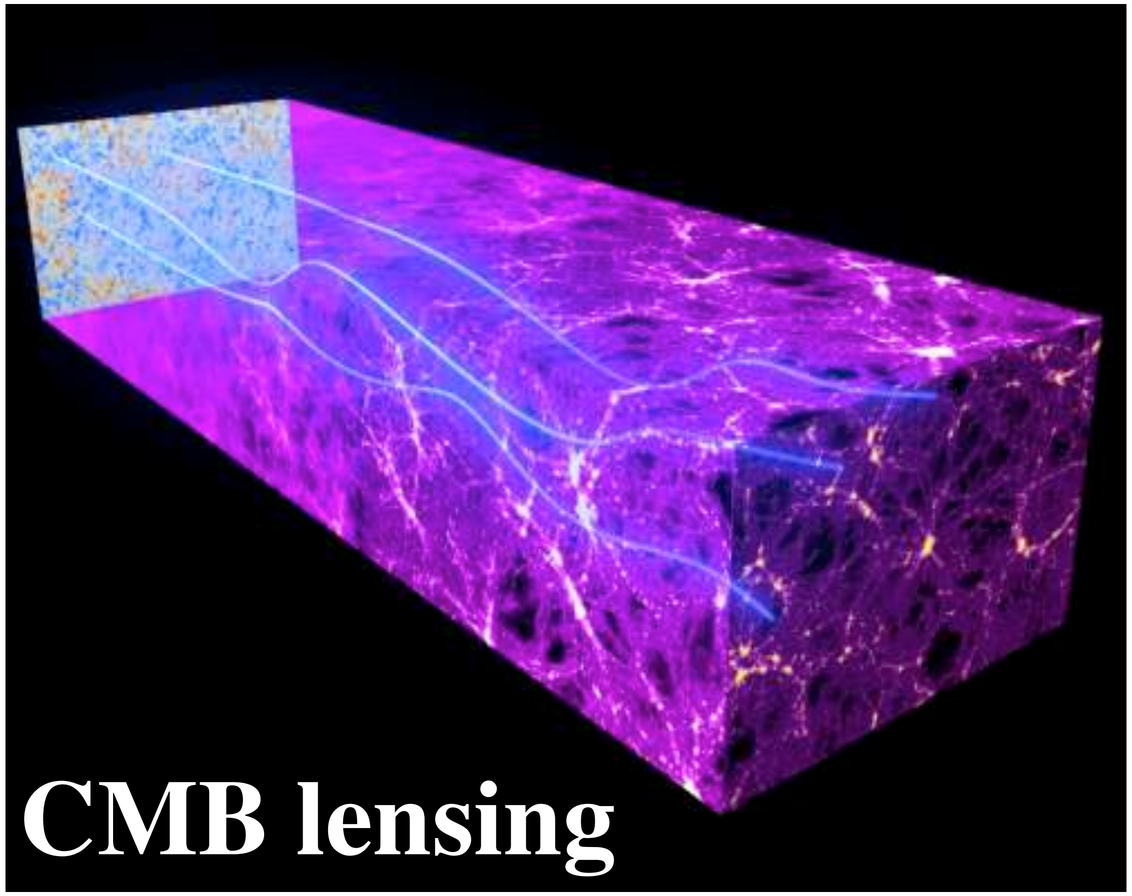


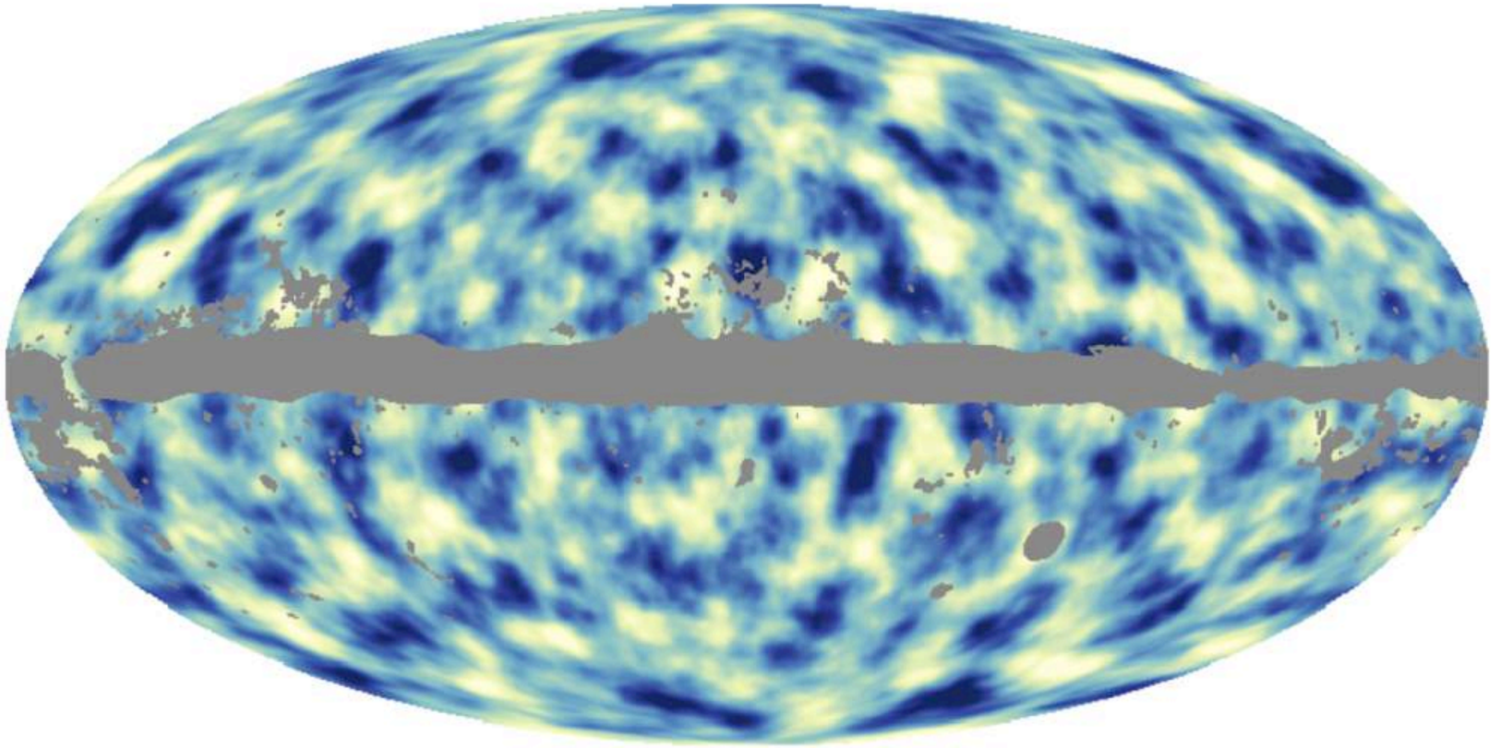
CMB **polarization**

Planck map

CMB lensing

CMB photons gravitationally lensed by structure between and now => probes structure formation (+ neutrino masses).

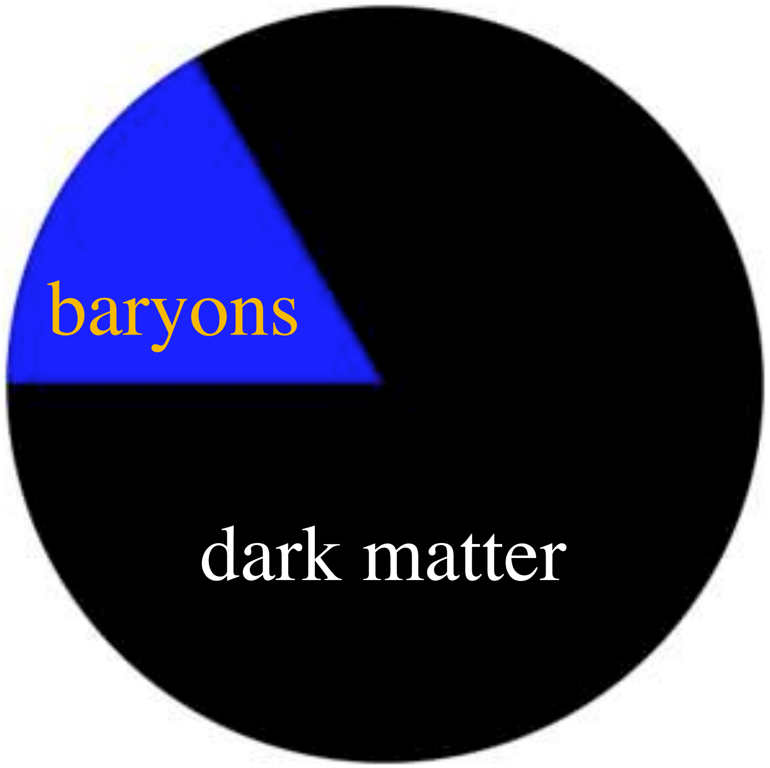


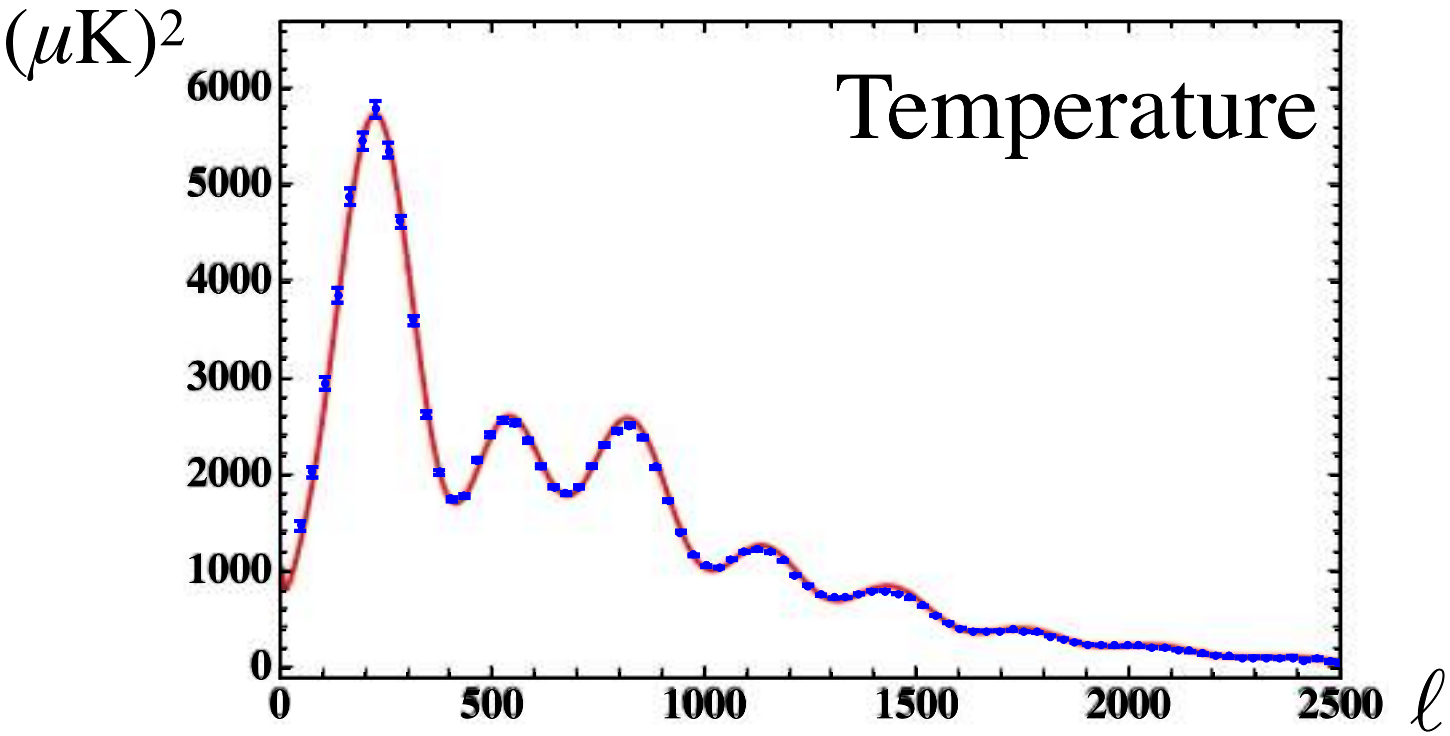


Planck map of the lensing potential

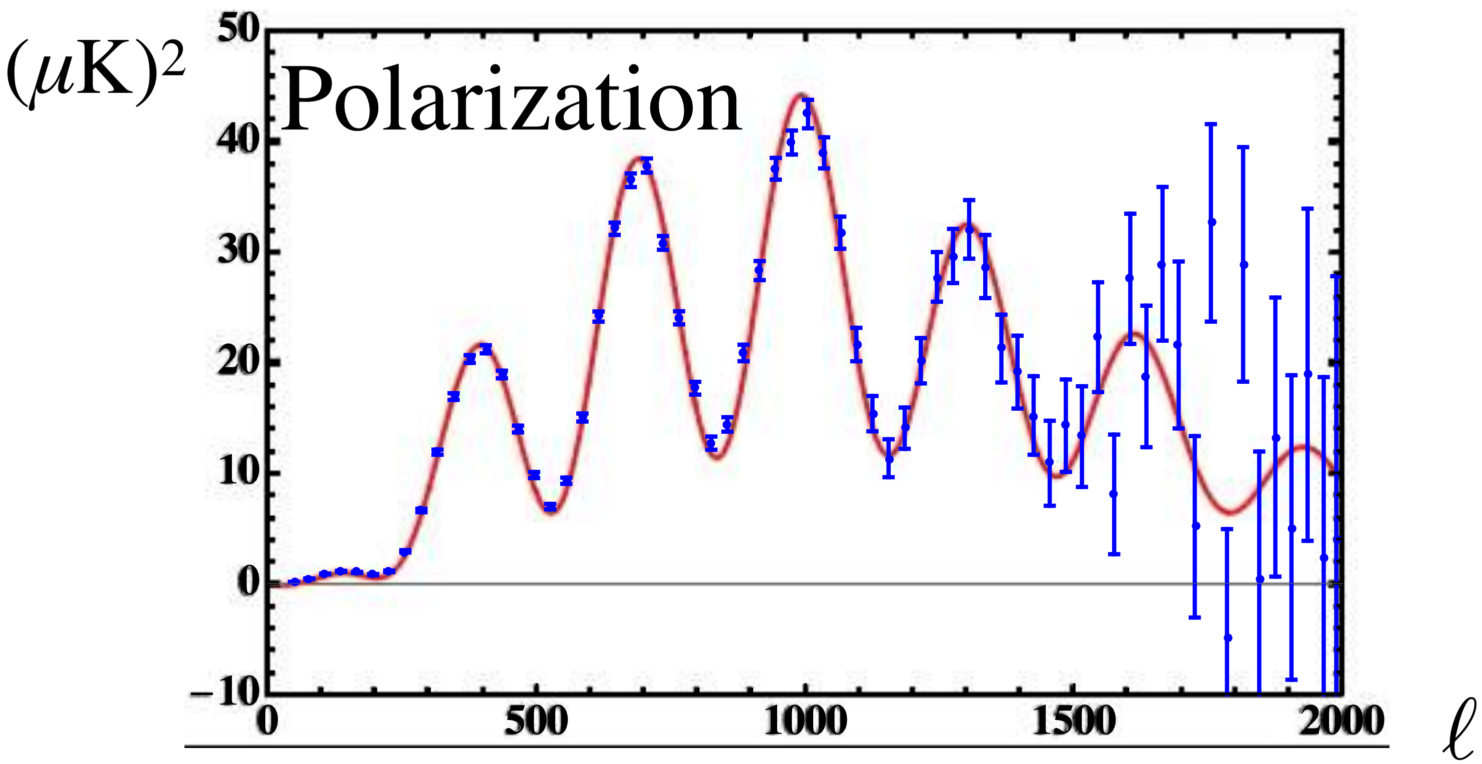
Ongoing and future measurements: SPT, ACT, Simons Observatory, CMB Stage IV

The CMB: a pillar of high-precision cosmology



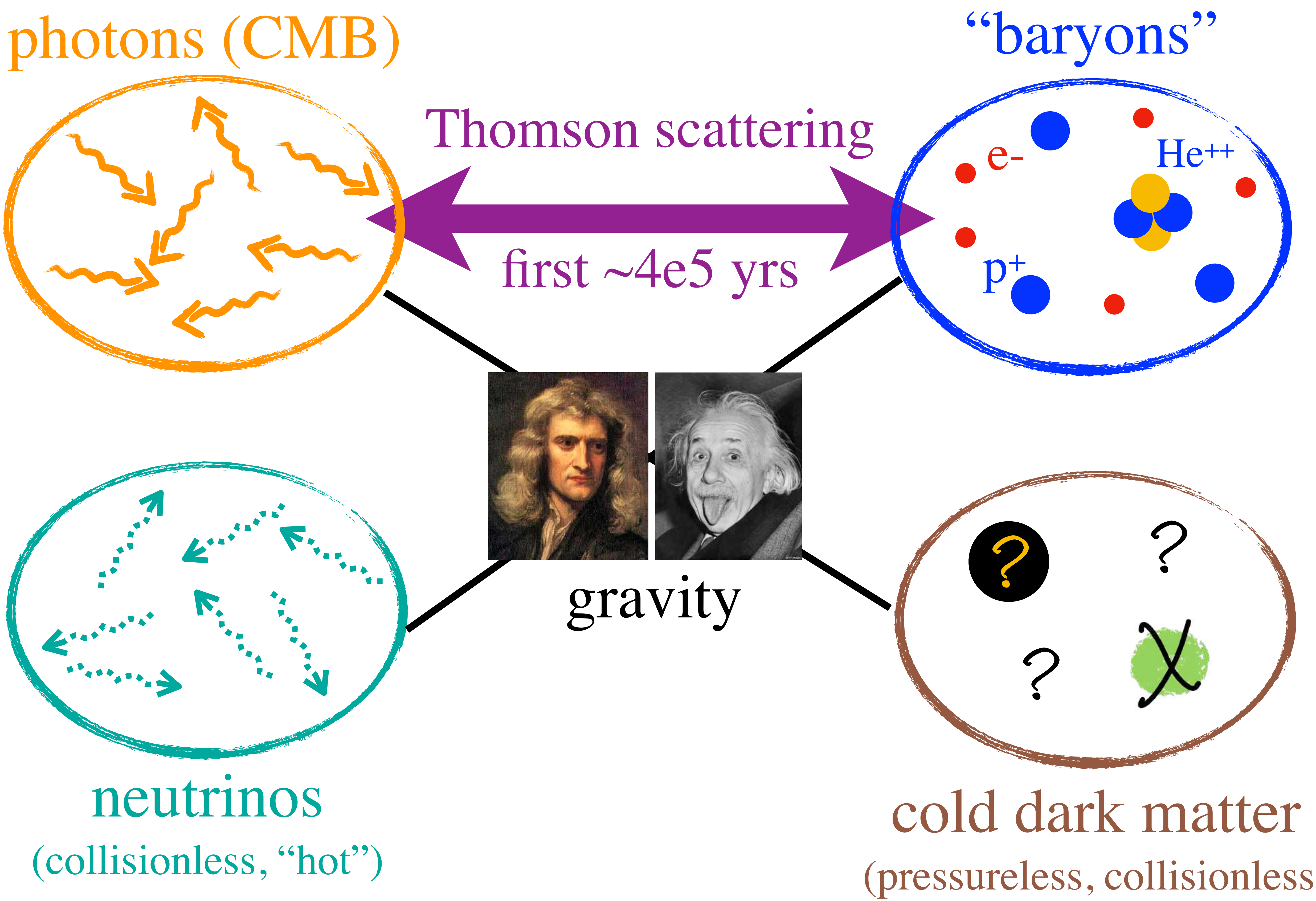


Parameter	Plik [1]
$\Omega_b h^2$	0.02237 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012
$100\theta_{MC}$	1.04092 ± 0.00031
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042



Planck collaboration 2018

Protagonists and stage



The stage: FLRW spacetime

Friedmann-Lemaître-Robertson-Walker (FLRW) metric for a homogeneous and isotropic (and spatially flat) Universe:

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)d\vec{x}^2 \\ &= a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right] \end{aligned}$$

a: **scale factor** (*a*=1 today)

x **: comoving coordinate**

η**: conformal time**

In the absence of interactions, particle momenta (as observed by comoving observers) "redshift" as

$$p_{\rm obs} \propto 1/a \equiv (1+z)$$

Cosmological **redshift:** $1+z = 1/a$

The stage: FLRW spacetime

Hubble expansion rate: $$H = \frac{d\ln a}{dt}$$

Hubble "constant" $\boldsymbol{H_0} = H(a = 1) = H(\text{today})$

$$H_0 = (67.4 \pm 0.5)\text{km/s/Mpc} \qquad \text{[Planck 2018]}$$

$$H_0 = (73.5 \pm 1.4)\text{km/s/Mpc} \qquad \text{[local measurements]}$$

The famous "Hubble tension".

Friedman's equation (in a **spatially flat** Universe):

$$H^2 = \frac{8\pi G}{3}\overline{\rho}_{\rm tot} = \frac{8\pi G}{3}\left(\overline{\rho}_\gamma + \overline{\rho}_\nu + \overline{\rho}_{\rm c} + \overline{\rho}_b + \overline{\rho}_\Lambda\right)$$

The stage: FLRW spacetime

- Mean **number** densities: $$\overline{N} \propto a^{-3}$$

- Non-relativistic species (cold dark matter, baryons): $$\bar{\rho}_X = \overline{N}_X\, m_X \propto a^{-3}$$

- For radiation: $$\bar{\rho}_\gamma = N_\gamma \langle p_\gamma \rangle \propto a^{-4}$$

- For massive neutrinos:

$$\bar{\rho}_\nu \propto a^{-4} \quad \text{while} \quad T_\nu \gg m_\nu,$$

$$\bar{\rho}_\nu \propto a^{-3} \quad \text{once} \quad T_\nu \ll m_\nu,$$

The stage: FLRW spacetime

Dimensionless density parameters:

$$\Omega_X = \frac{8\pi G\, \overline{\rho}_{X,0}}{3H_0^2} \qquad \sum_X \Omega_X = 1$$

$$H^2(a) = H_0^2 \left[\Omega_\Lambda + (\Omega_c + \Omega_b)a^{-3} + \Omega_\gamma a^{-4} + \Omega_\nu \frac{\overline{\rho}_\nu(a)}{\overline{\rho}_{\nu,0}}\right]$$

Equivalently:

$$H_0 = 100\ h\ \mathrm{km/s/Mpc}$$

$$\omega_X \equiv \Omega_X h^2$$

Perturbed FLRW metric:

$$ds^2 = a^2(\eta)\left[-(1+2\psi)d\eta^2 + (1-2\phi)d\vec{x}^2\right] + a^2(\eta)h_{ij}^{\mathrm{GW}}dx^i dx^j$$

Cold dark matter

Approximated as a **collisionless** and **pressureless** ideal fluid entirely described by its density and velocity fields

$$\rho_c(\eta,\vec{x}) = (1+\delta_c(\eta,\vec{x}))\ \bar{\rho}_c(\eta) \qquad\qquad \vec{v}_c(\eta,\vec{x})$$

Fluid equations: **relativistic generalization of**

$$\frac{\partial \rho_c}{\partial t} + \vec{\nabla}\cdot(\rho_c \vec{v}_c) = 0$$

continuity (mass conservation)

$$\frac{\partial \vec{v}_c}{\partial t} + (\vec{v}_c\cdot\vec{\nabla})\vec{v}_c = -\vec{\nabla}\phi_{\mathrm{Newt}}$$

momentum equation

See Fabian Schmidt's lectures for the rich physics of CDM gravitational collapse and structure formation.

Massive neutrinos

- Three neutrino (+ antineutrino) flavors ν_e, ν_μ, ν_τ

- Three mass eigenstates $\nu_1, \nu_2, \nu_3 \neq \nu_e, \nu_\mu, \nu_\tau$

- Individual neutrino masses are still unknown, but sum of neutrino masses constrained to **$\Sigma\, m_\nu < 0.12$ eV** (Planck 2018)

- Neutrinos decouple from the plasma at $T \sim$ MeV, while ultra-relativistic. They have a (perturbed) **relativistic Fermi-Dirac occupation number**

$$\overline{f}_\nu(p) = \frac{1}{\exp(p/T_\nu) + 1} \qquad T_\nu \approx 0.71\, T_\gamma$$

Massive neutrinos

- Epochs relevant to observable CMB properties correspond to T << MeV, when neutrinos are fully decoupled from rest of the plasma, i.e. **collisionless**

Liouville's theorem: in the absence of collisions **phase-space density is conserved** along trajectories:

Collisionless Boltzmann equation:

$$\frac{df_\nu}{dt}\Big|_{\mathrm{traj}} = 0 = \frac{\partial f_\nu}{\partial t} + \frac{d\vec{x}}{dt}\Big|_{\mathrm{traj}} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{d\vec{p}}{dt}\Big|_{\mathrm{traj}} \cdot \frac{\partial f_\nu}{\partial \vec{p}}$$

Baryons

- "baryons" in cosmology= ions, electrons and neutral atoms.

- At T << MeV, all the ``baryons" are **non-relativistic**

- Big Bang Nucleosynthesis produces **Helium** (+ trace amounts of heavier elements, irrelevant for standard CMB physics).

Mass fractions: 76% Hydrogen, 24% Helium [$Y_{\mathrm{He}} = 0.24$]

Exercise: show that the ratio of helium to hydrogen **by number** is

$$f_{\mathrm{He}} \equiv \frac{N_{\mathrm{He}}}{N_{\mathrm{H}}} \approx 0.08$$

Baryons

- Baryons are an **ideal fluid**, fully described by their density, velocity and temperature (same temperature for *H, He, e^-*)

$$\rho_b(\eta, \vec{x}) = (1 + \delta_b(\eta, \vec{x}))\ \bar{\rho}_b(\eta) \qquad \vec{v}_b(\eta, \vec{x}) \qquad T_b(\eta, \vec{x})$$

- Fluid equations: **relativistic generalization of**

$$\frac{\partial \rho_b}{\partial t} + \vec{\nabla} \cdot (\rho_b \vec{v}_b) = 0$$

continuity (mass conservation)

$$\frac{d\vec{v}_b}{\partial t} = -\vec{\nabla}\phi_{\mathrm{Newt}} \ + \ \Gamma_{\mathrm{Thomson}}\,(\vec{v}_\gamma - \vec{v}_b)$$

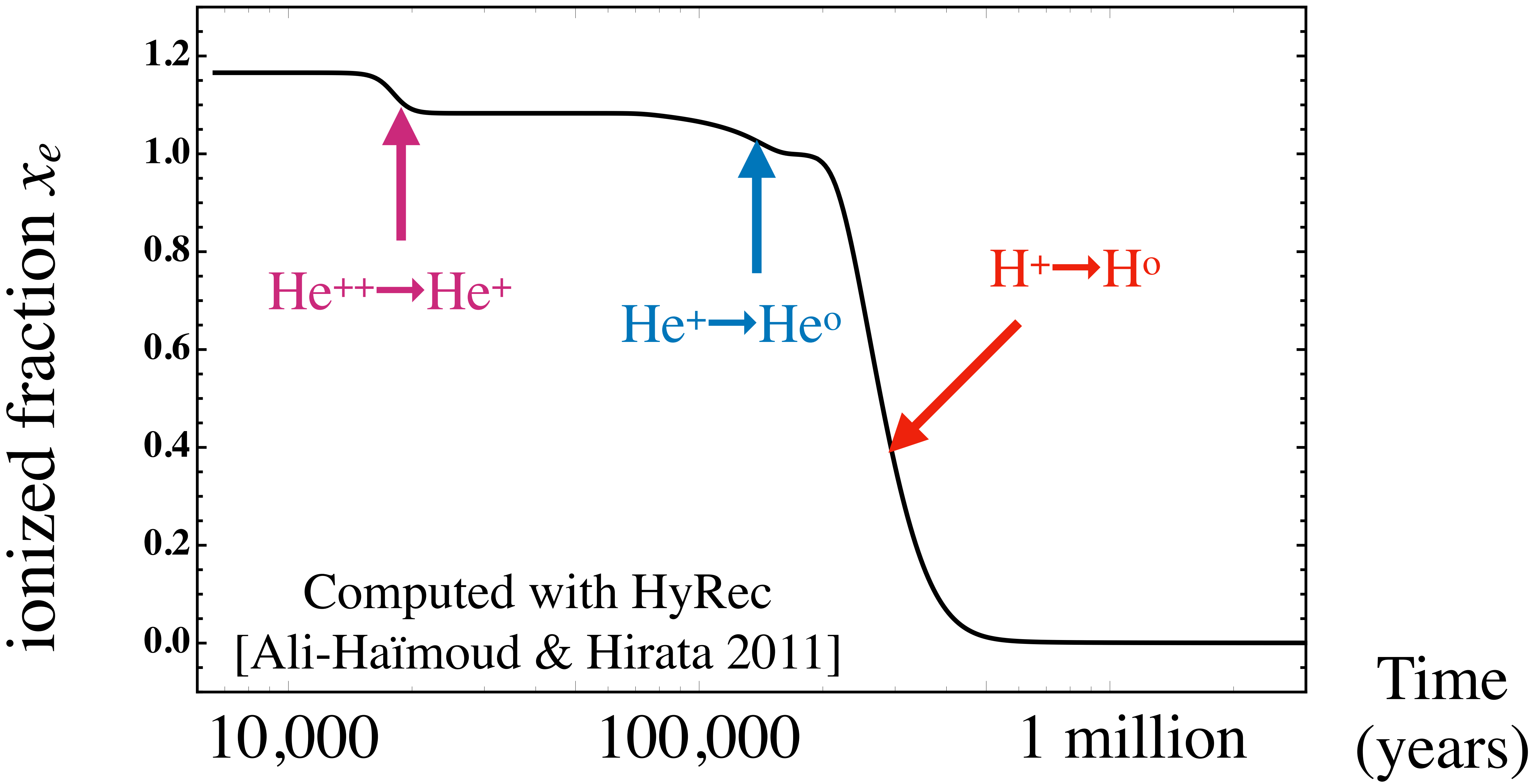
momentum equation

$$\frac{dT_b}{\partial t} = \tilde{\Gamma}_{\mathrm{Thomson}}\,(T_\gamma - T_b)$$

heat equation

Recombination

H and *He* start fully ionized, and eventually "recombine" with electrons, to form neutral atoms. **Recombination** is a crucial piece of CMB physics, we'll study it in **lecture 2.**



Photons (i.e. the CMB)

- Described by **photon occupation number f_γ**

- **Not** an ideal fluid and **not** collisionless in general — in fact, they are either one or the other

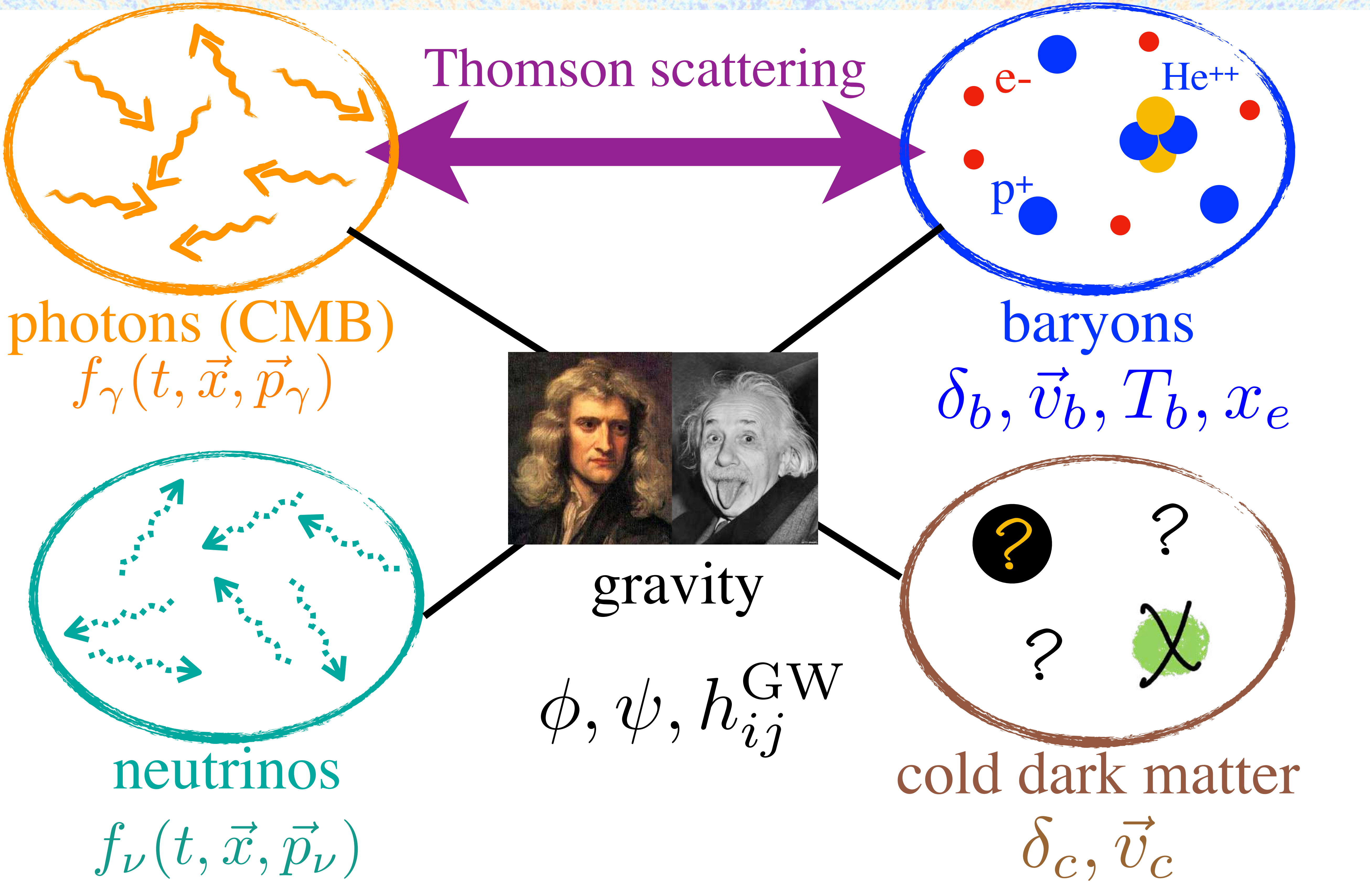
Described by the **collisional Boltzmann equation**

$$\left.\frac{df_\gamma}{dt}\right|_{\mathtt{traj}} = C[f_\gamma]$$

Thomson collision operator

We will spend **lectures 3-4** deriving this equation and discussing it solutions in some limiting regimes.

Task at hand: solve linear coupled differential equations



Initial conditions

- Observations are consistent with **adiabatic** initial conditions: all species have equal relative fluctuations in their **number density:**

$$\frac{\delta N_b}{\overline{N_b}} = \frac{\delta N_c}{\overline{N_c}} = \frac{\delta N_\nu}{\overline{N_\nu}} = \frac{\delta N_\gamma}{\overline{N_\gamma}}$$

- non-relativistic species (CDM and baryons): $\frac{\delta N}{\overline{N}} = \frac{\delta \rho}{\overline{\rho}} \equiv \delta$

- relativistic species (neutrinos and photons):

$$N \propto T^3, \quad \rho \propto T^4 \qquad \Rightarrow \frac{\delta N}{\overline{N}} = \frac{3}{4}\frac{\delta \rho}{\overline{\rho}} = \frac{3}{4}\delta$$

- On very large scales (>> Hubble radius), peculiar velocities initially vanish

Initial conditions

- Metric perturbations can be decomposed into ``scalar” [ϕ, ψ], “vector” and “tensor” modes (see **Valerie Domcke’s lecture 4**)

- “vector” modes decay and are neglected

- we will **assume scalar modes** (and will briefly touch on tensor modes [i.e. gravitational waves] in lectures 4/5).

$$\left.\frac{\delta N}{\overline{N}}\right|_i = \zeta, \qquad \phi_i = \psi_i = -\frac{2}{3}\zeta$$

primordial curvature perturbation

- Initial conditions are only described **statistically**

Initial conditions

- Initial conditions are observed to be consistent with **Gaussian**

$$\langle \zeta(\vec{k})\zeta^*(\vec{k}')\rangle = (2\pi)^3\delta_{\rm D}(\vec{k}' - \vec{k})P_\zeta(k)$$

primordial curvature power spectrum

comoving Fourier wavenumber

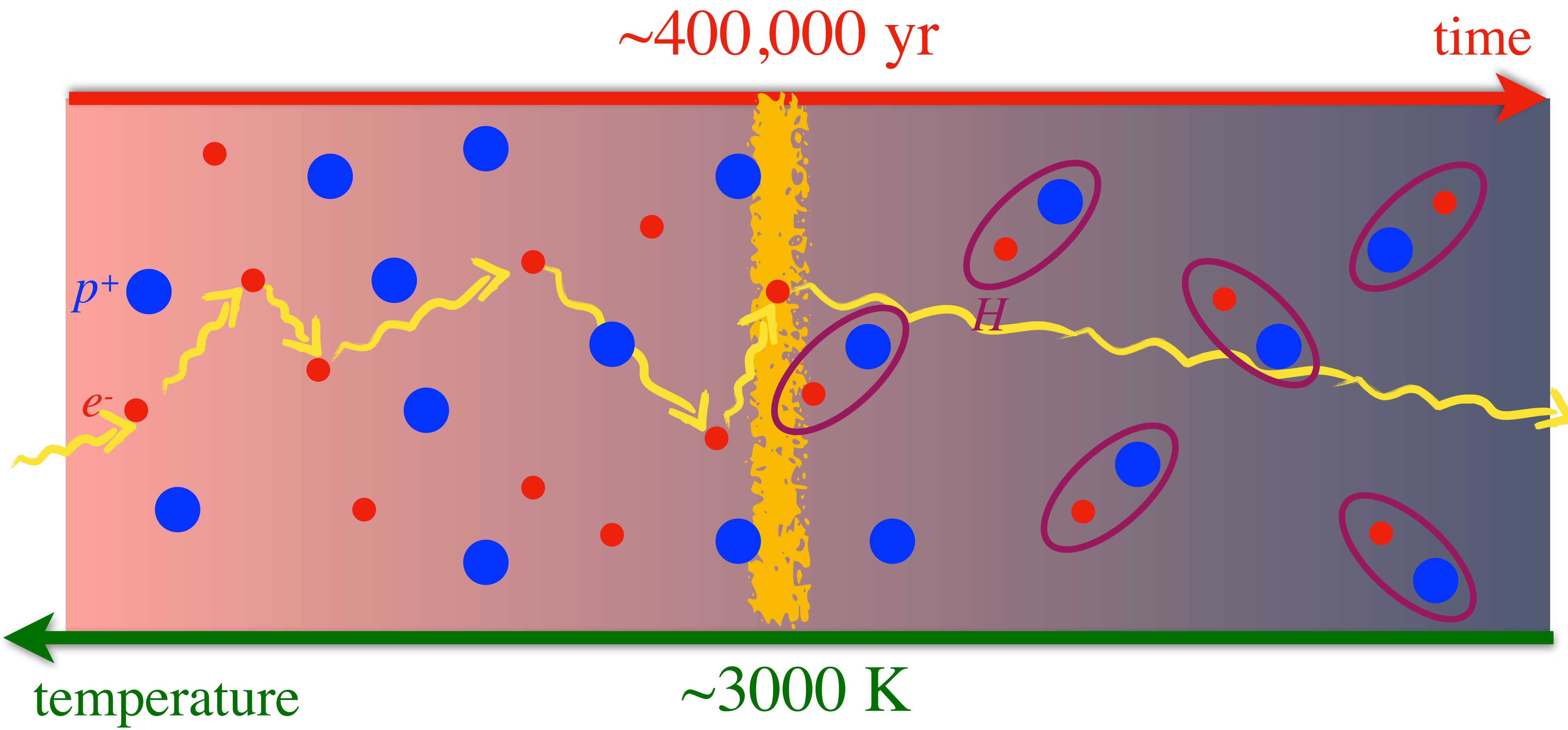
- **Variance** of primordial curvature perturbations:

$$\langle[\zeta(\vec{x})]^2\rangle = \int \frac{d^3k}{(2\pi)^3}P_\zeta(k) = \int d\ln k\, \frac{k^3}{2\pi^2}P_\zeta(k)$$

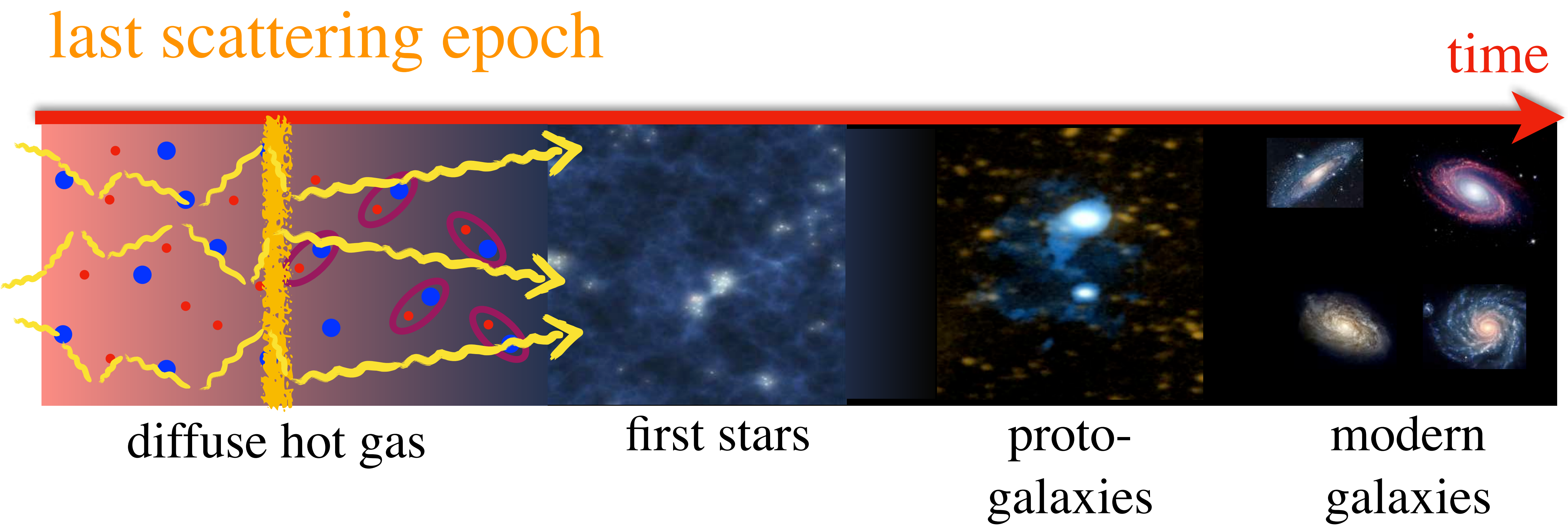
- Slow-roll inflation predicts **quasi scale-invariant** power spectrum:
(see Valerie Domcke's lectures)

$$\frac{k^3}{2\pi^2}P_\zeta(k) = A_s(k/k_*)^{n_s-1}, \quad n_s \approx 1$$

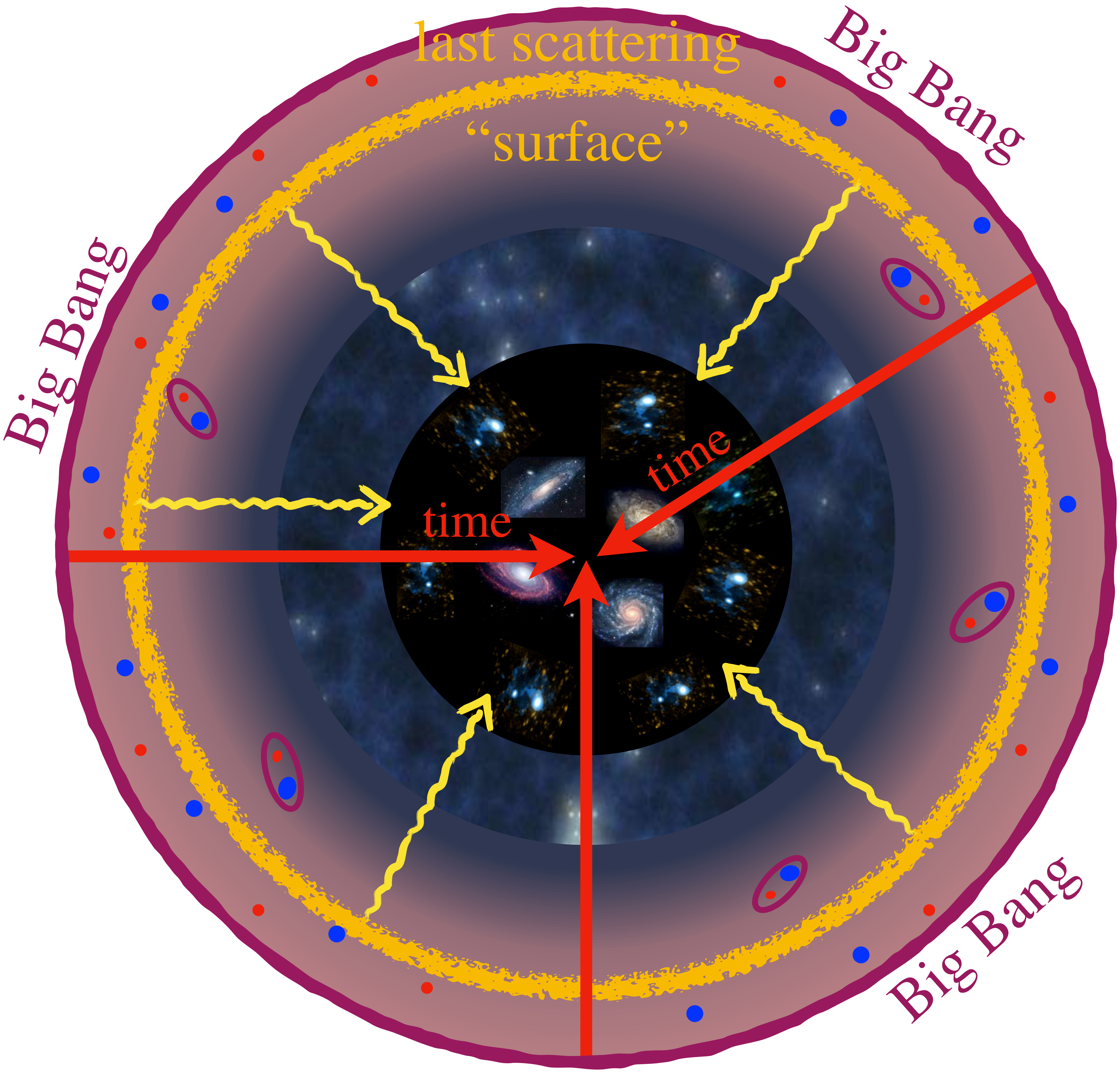
Qualitative description of what's next



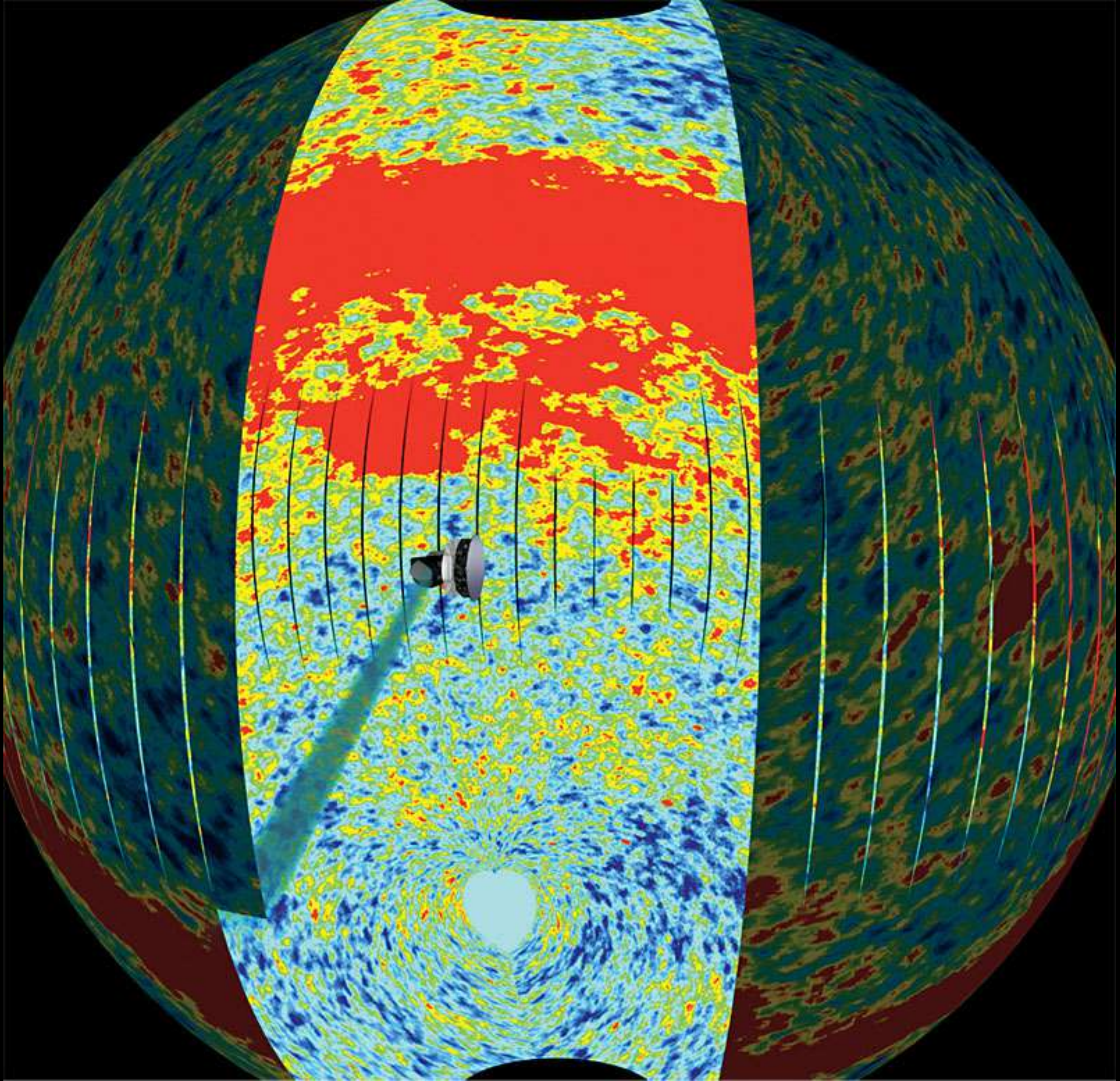
last scattering epoch



last scattering epoch
time
diffuse hot gas
first stars
proto-
galaxies
modern
galaxies

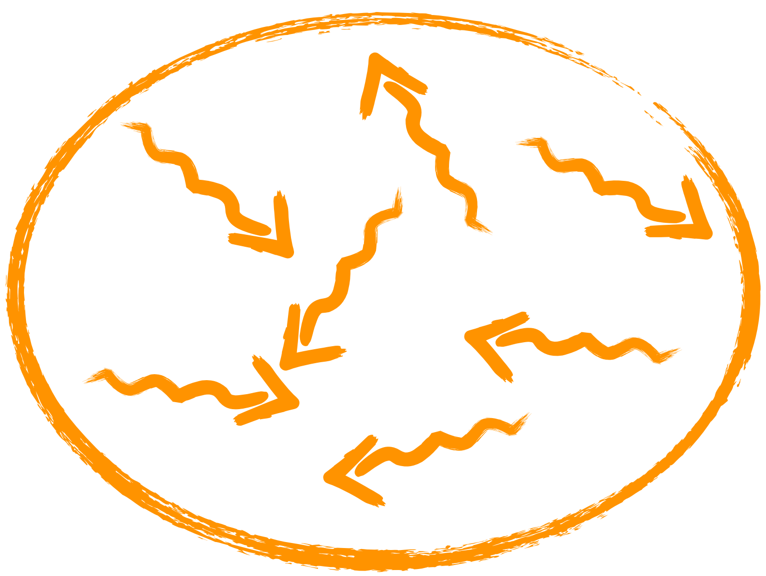


last scattering
"surface"
Big Bang
Big Bang
Big Bang
time
time



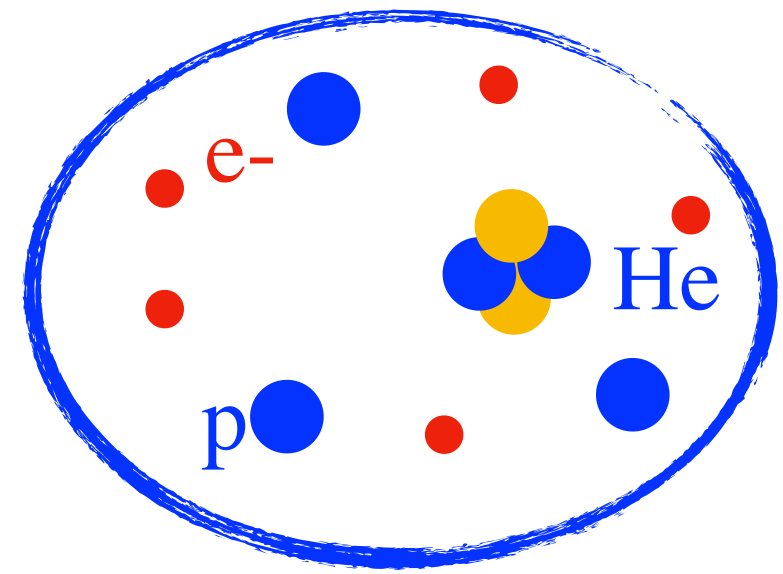
What are we seeing exactly?

before last scattering:



photons

provide pressure

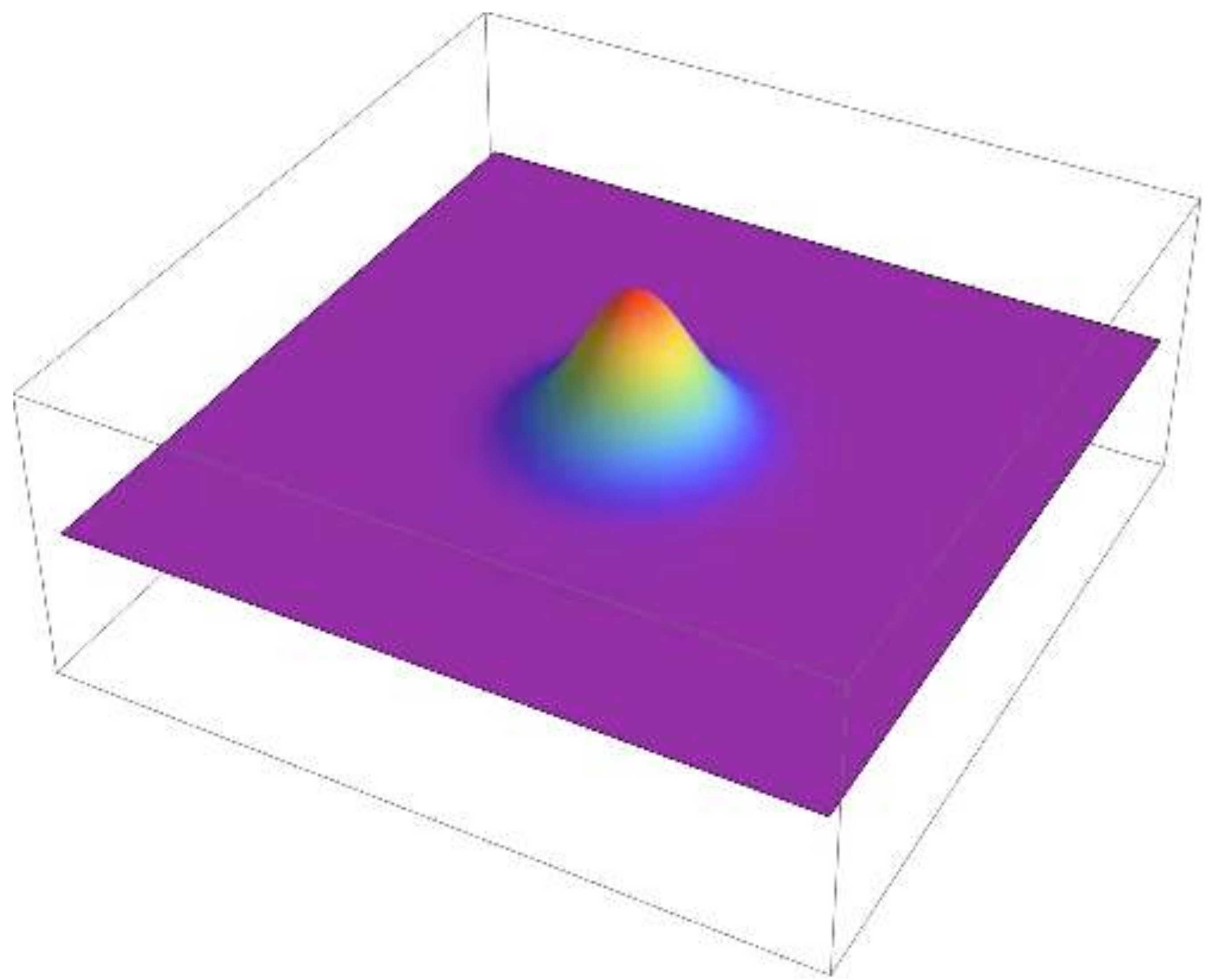


"baryons"

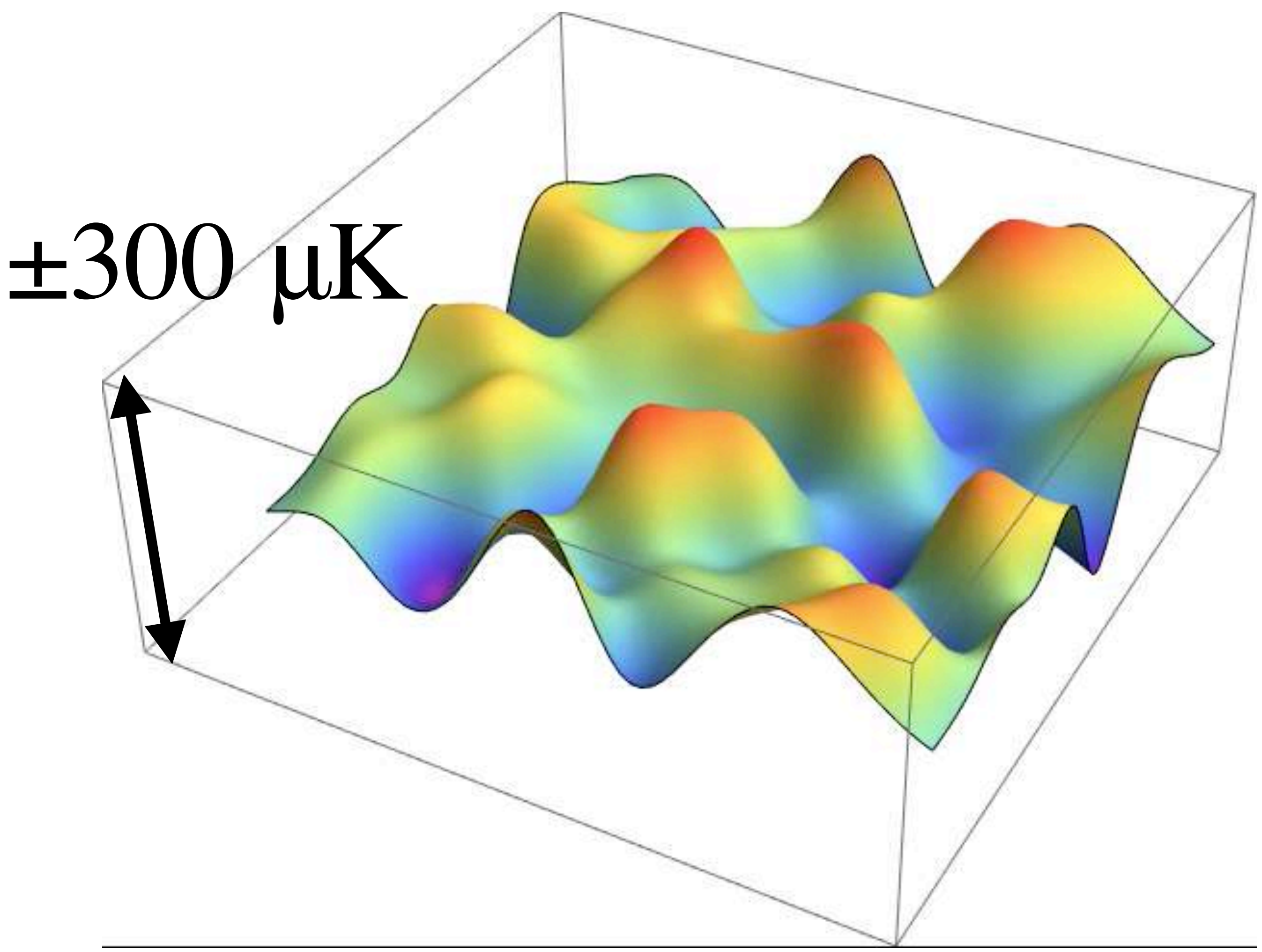
provide containment
(impede free streaming)

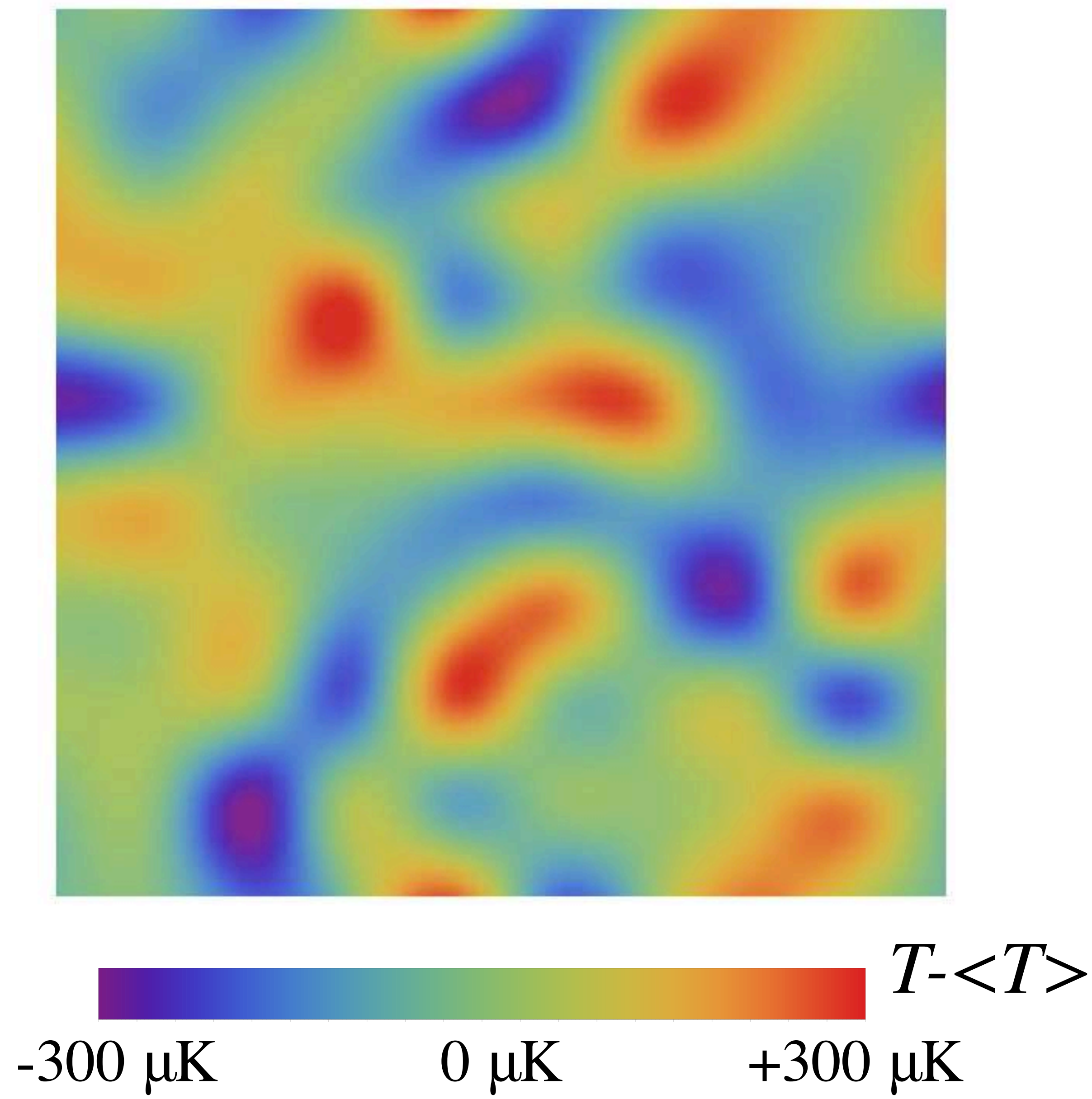
together: ideal fluid with large pressure

Small initial overdensities generate sound waves in photon-baryon fluid, propagating for ~400,000 years

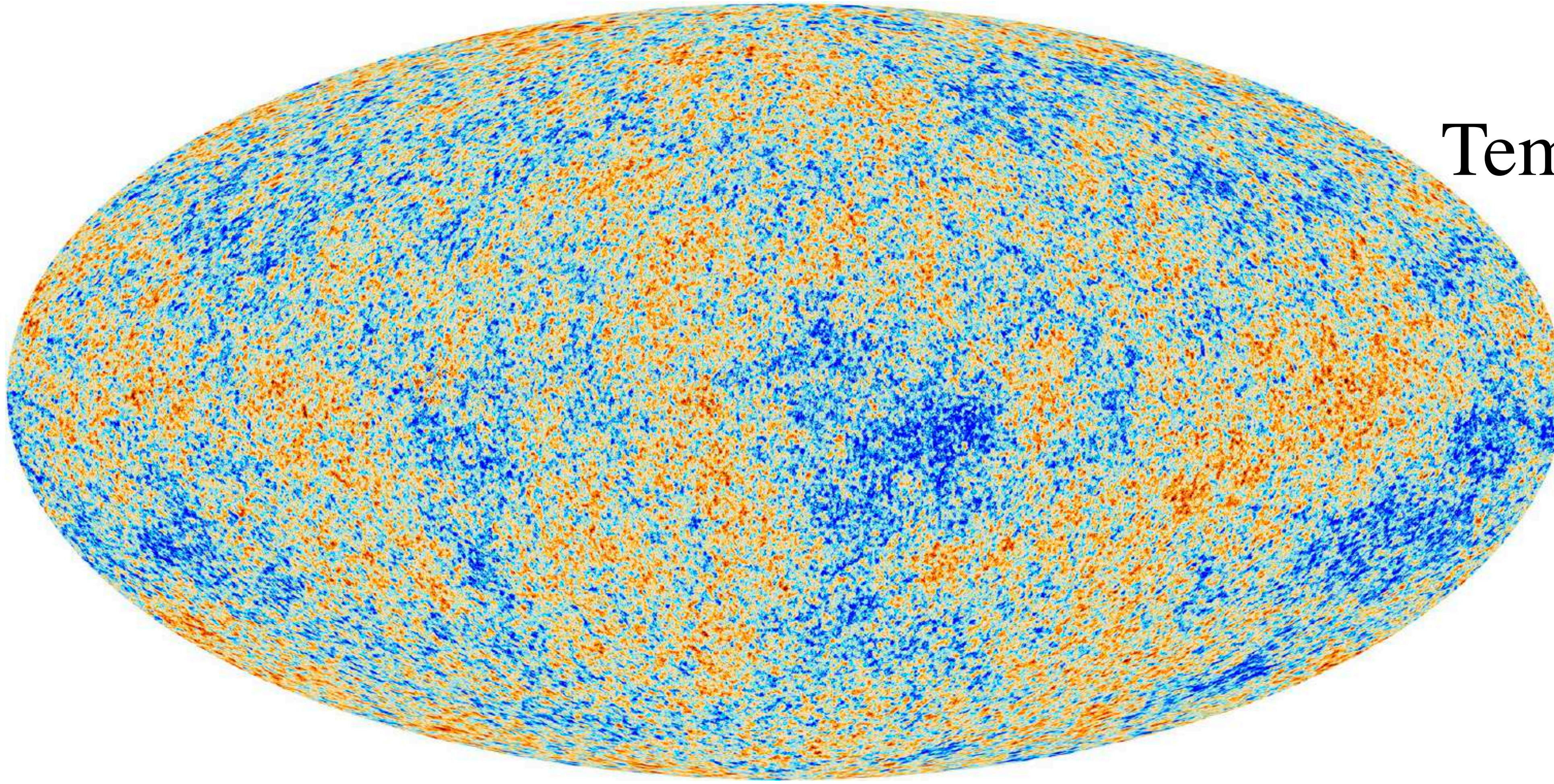


Superposition of many incoherent sound waves, oscillating for ~400,000 years

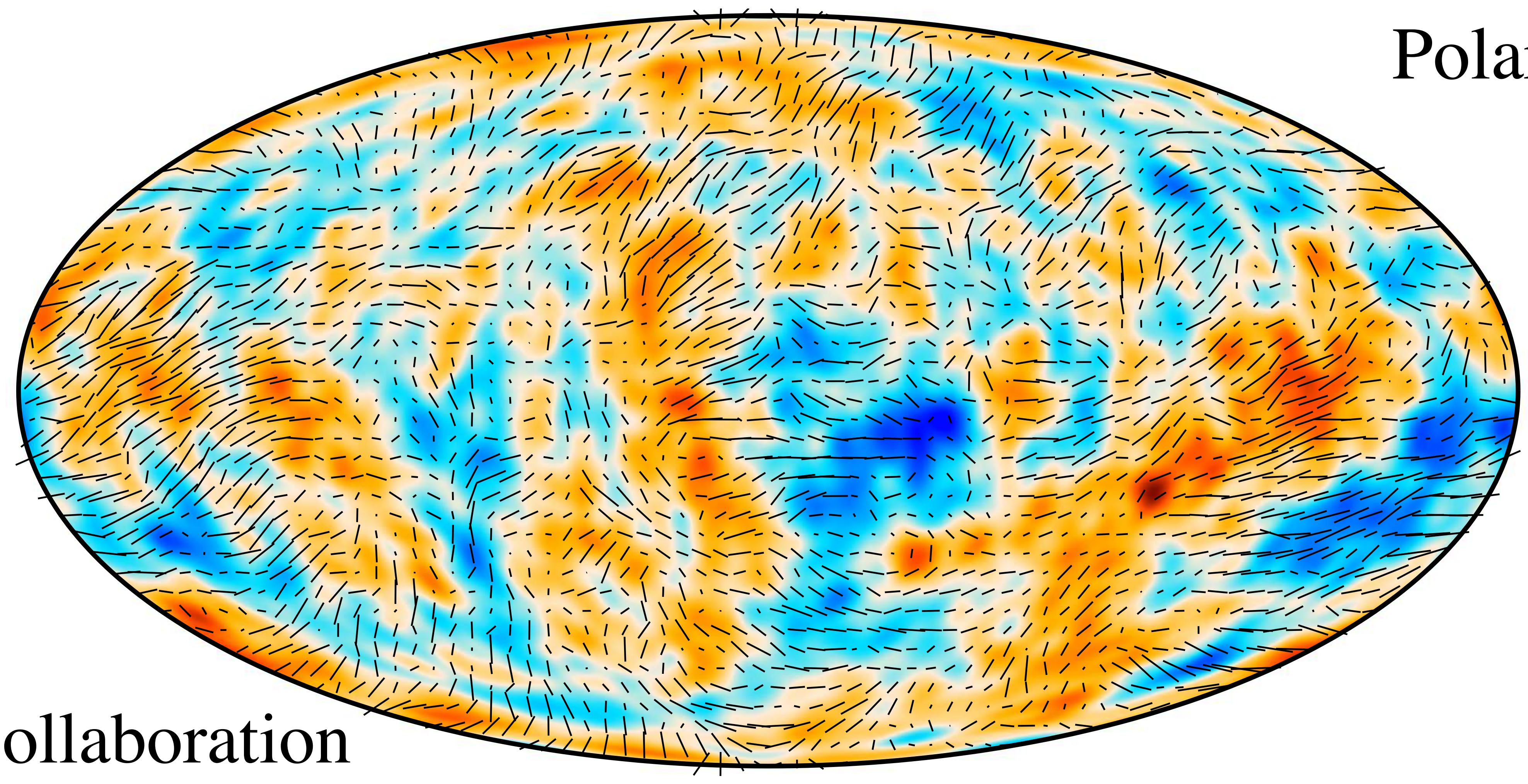




Last ``snapshot” is imprinted on the last scattering surface



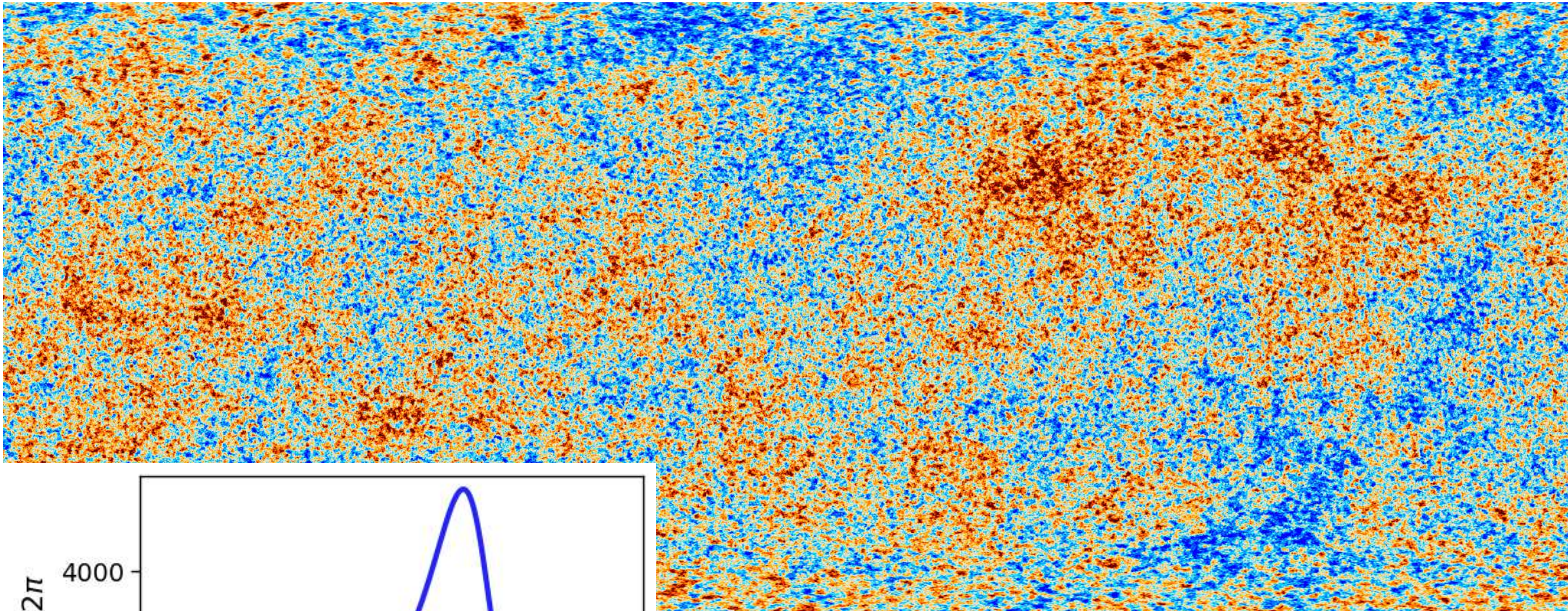
Temperature

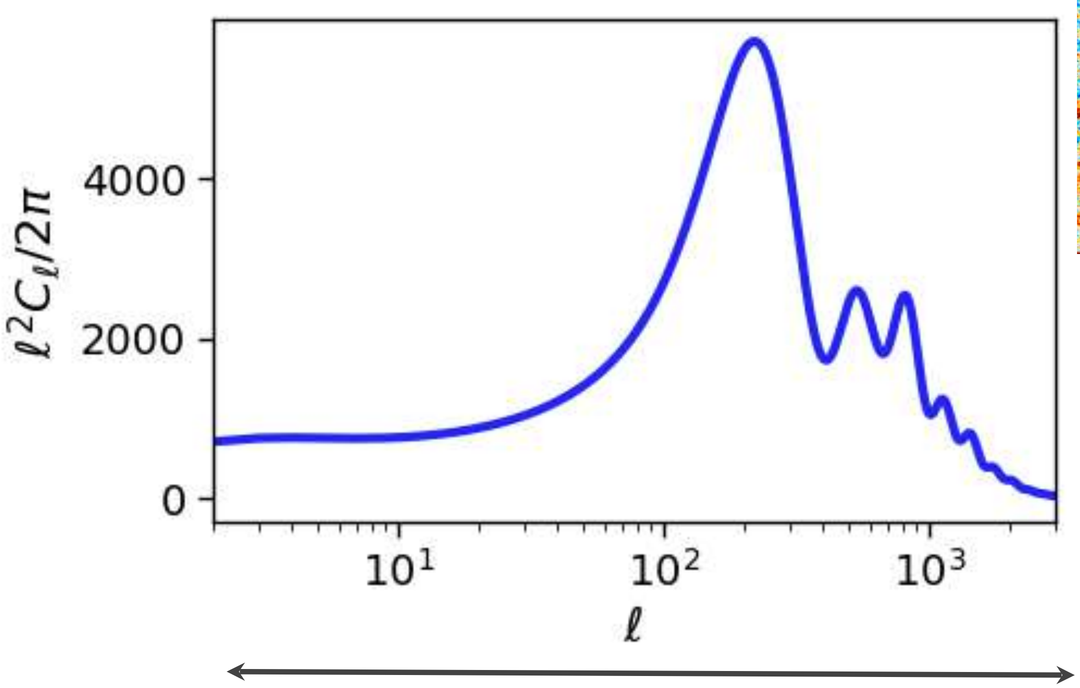


Polarization

Planck collaboration

CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**





Courtesy of Mathew Madhavacheril

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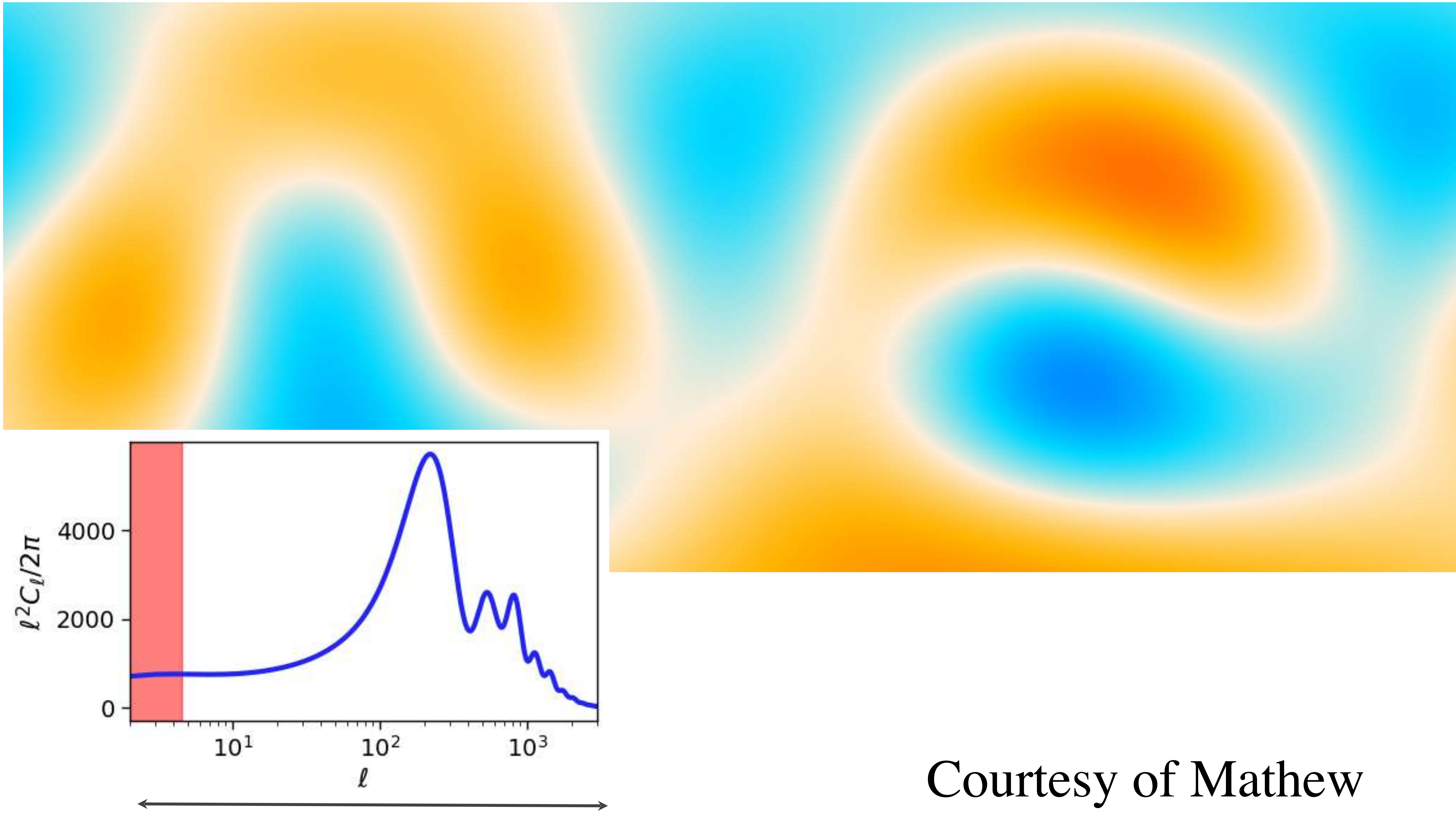

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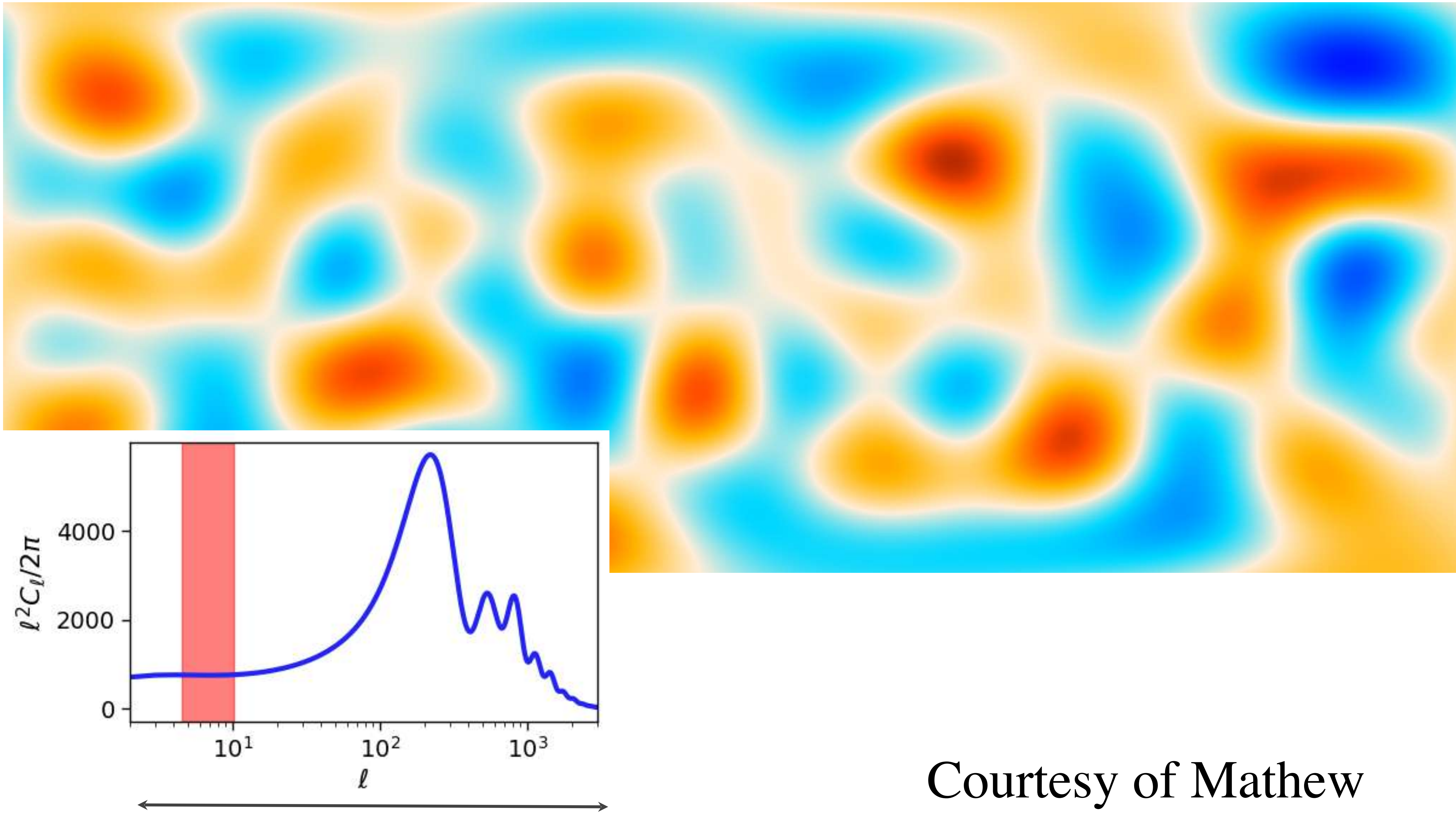

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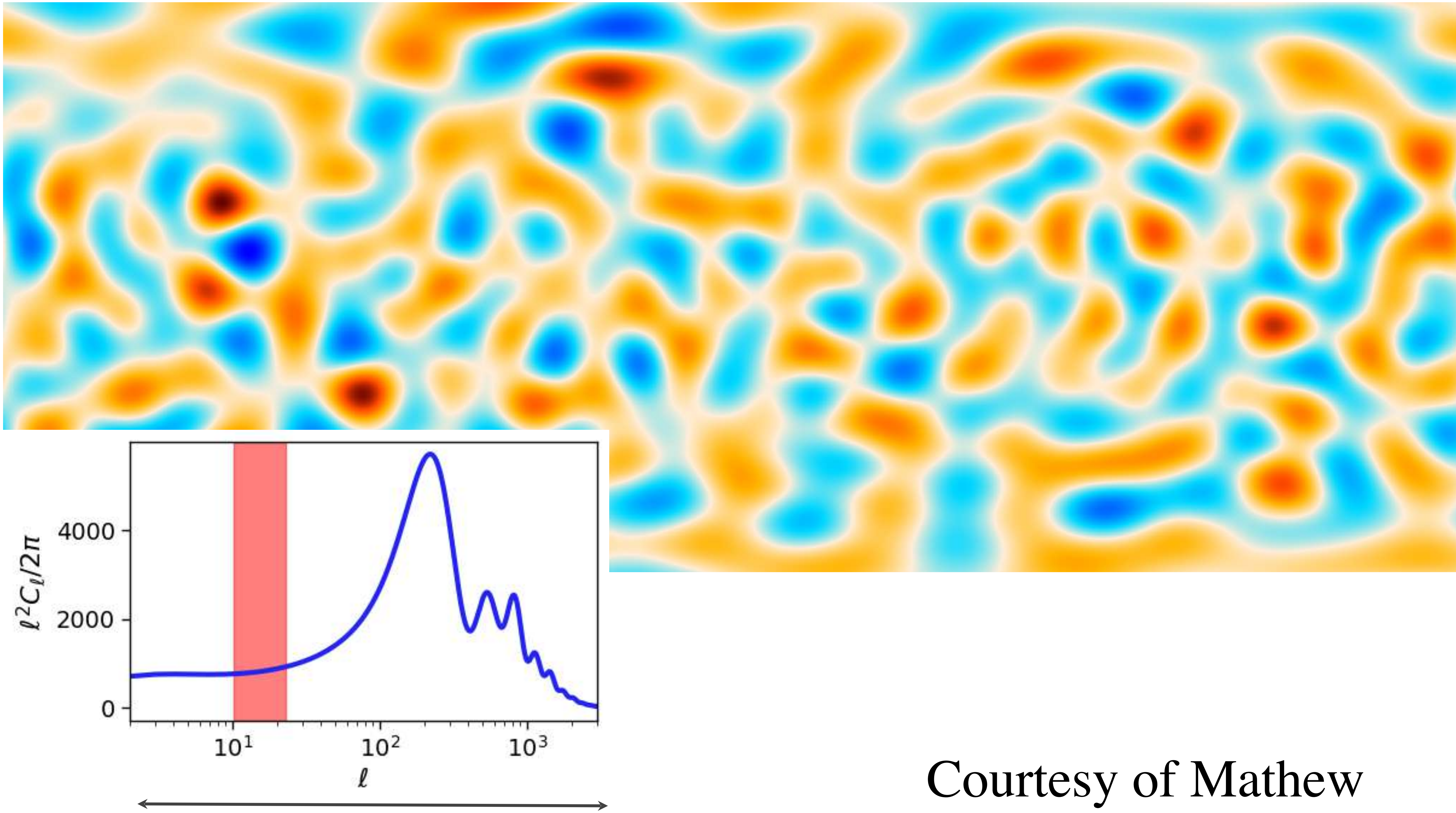

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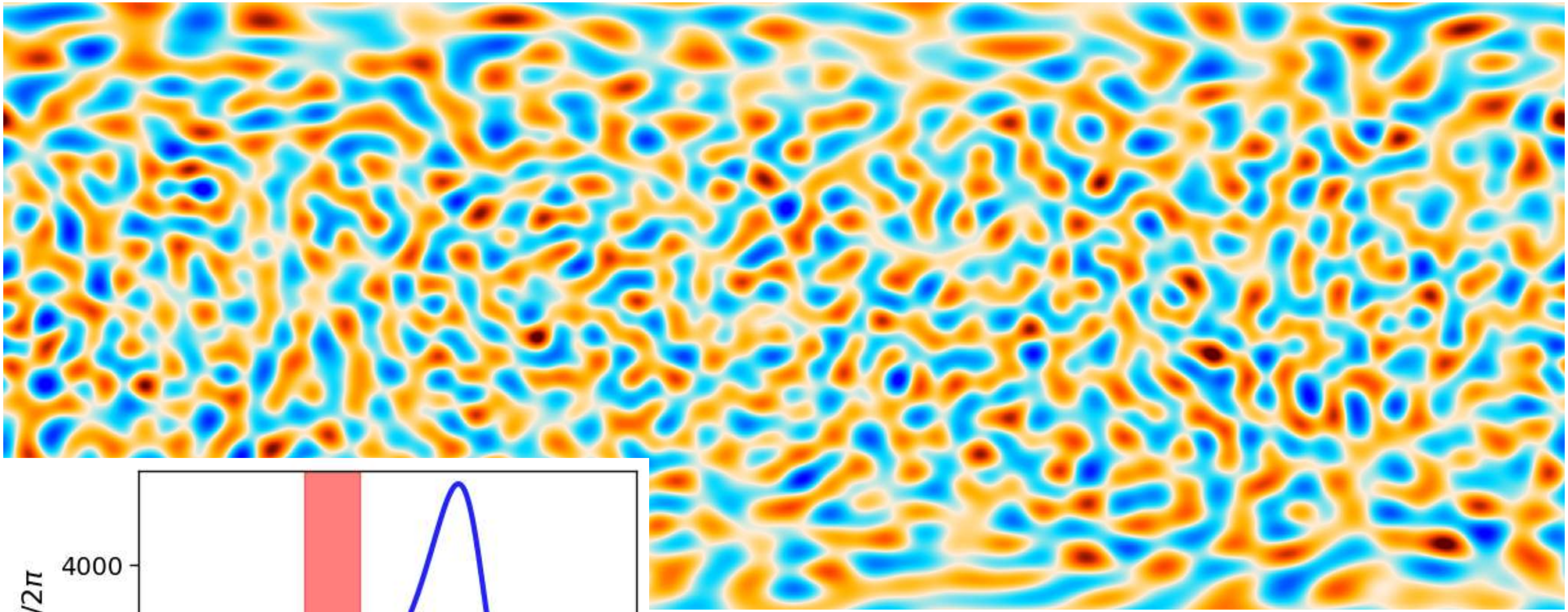

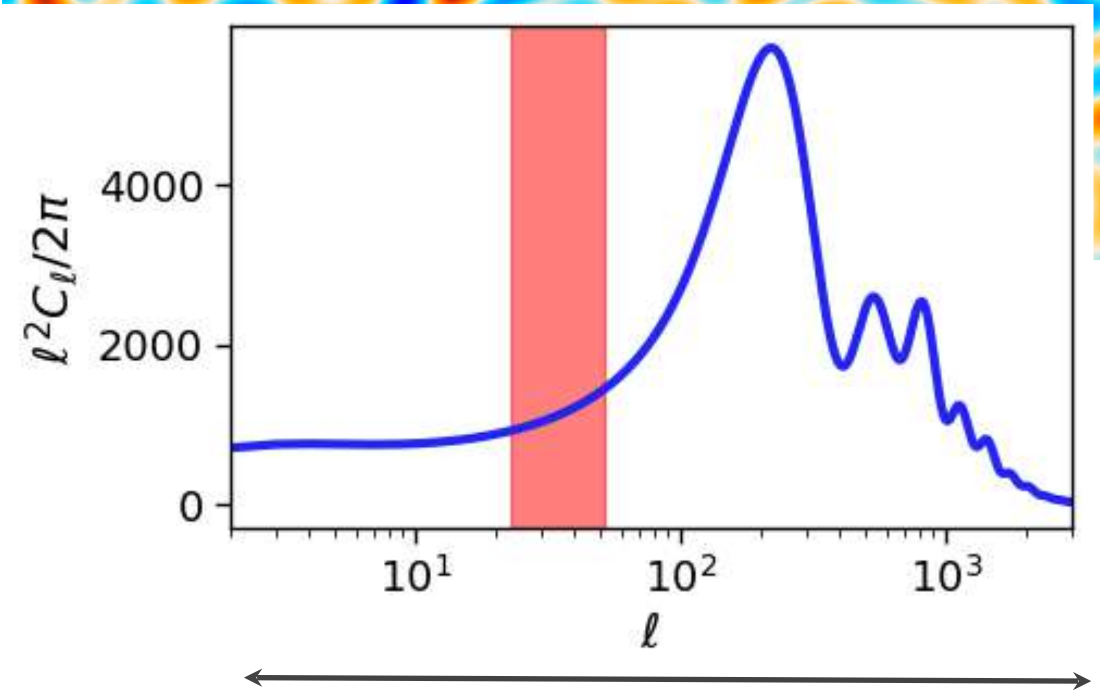

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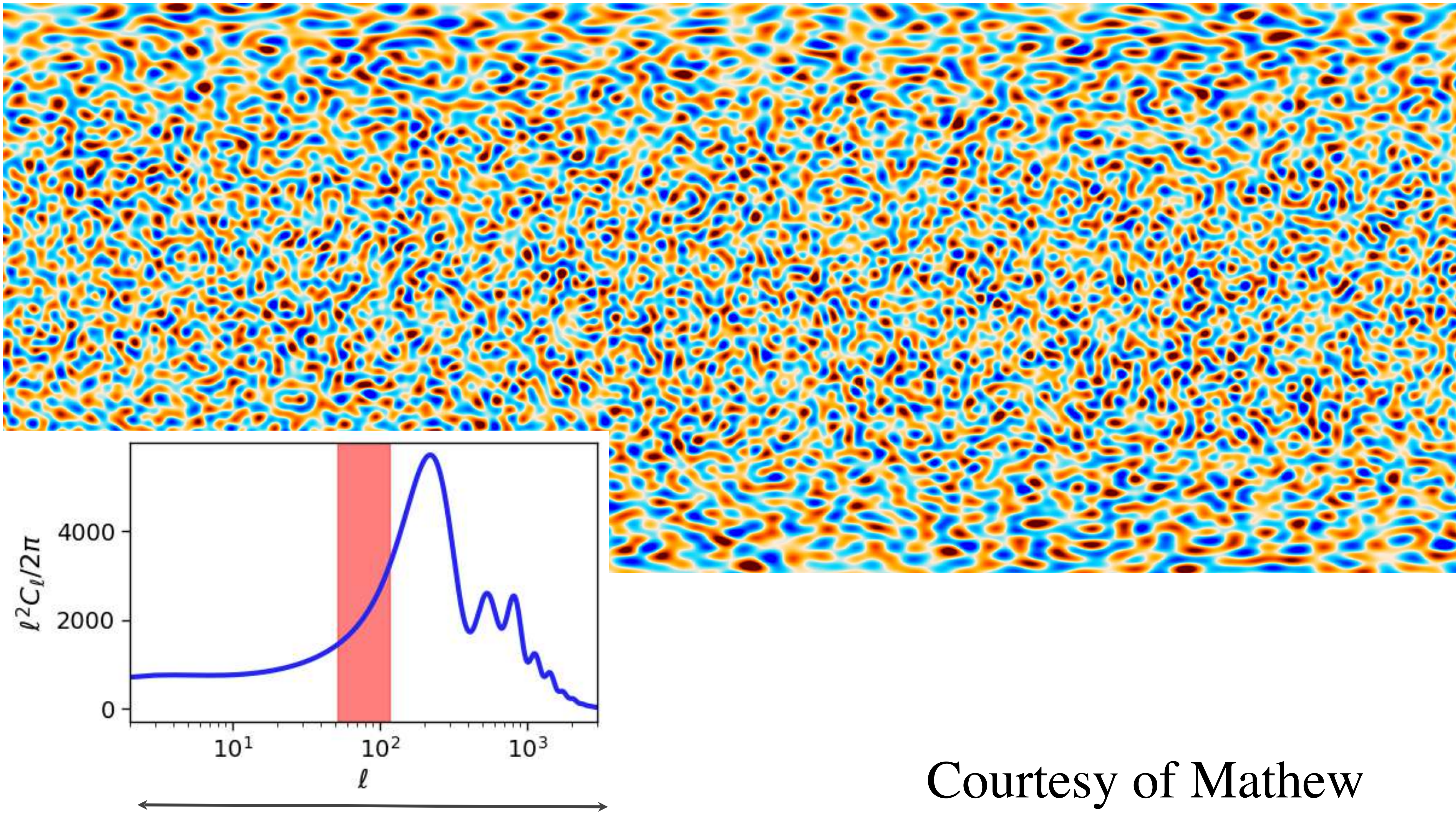

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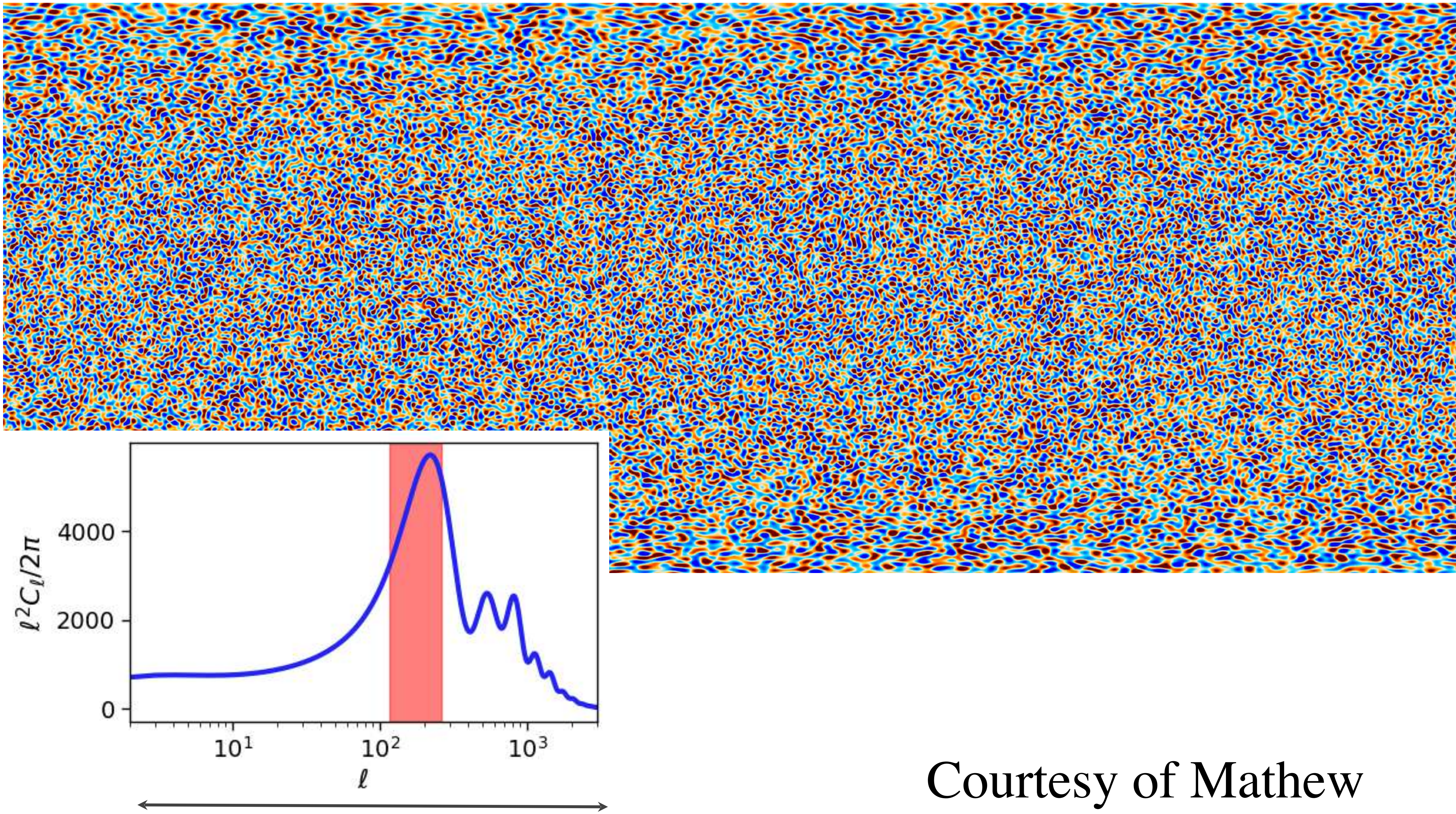

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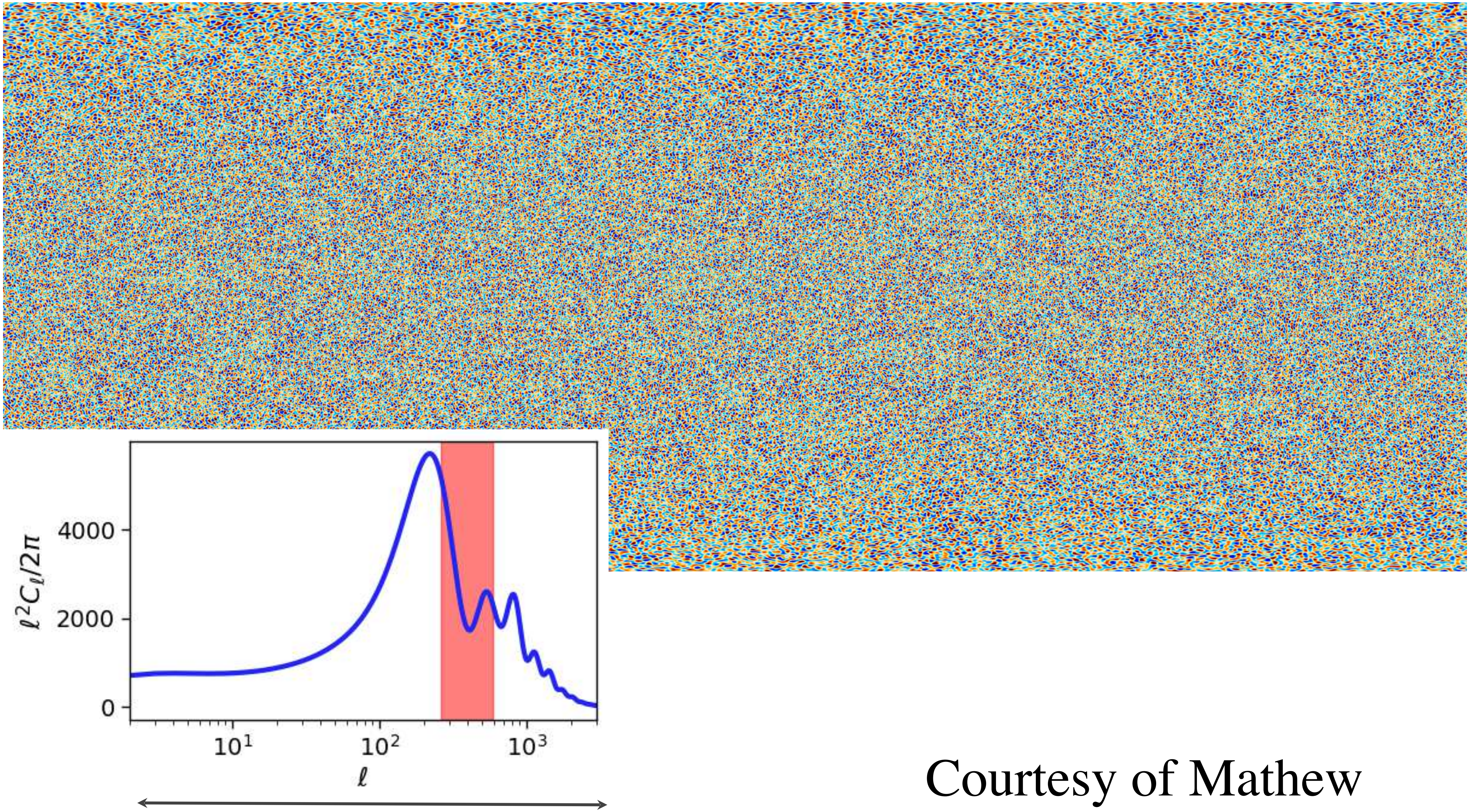
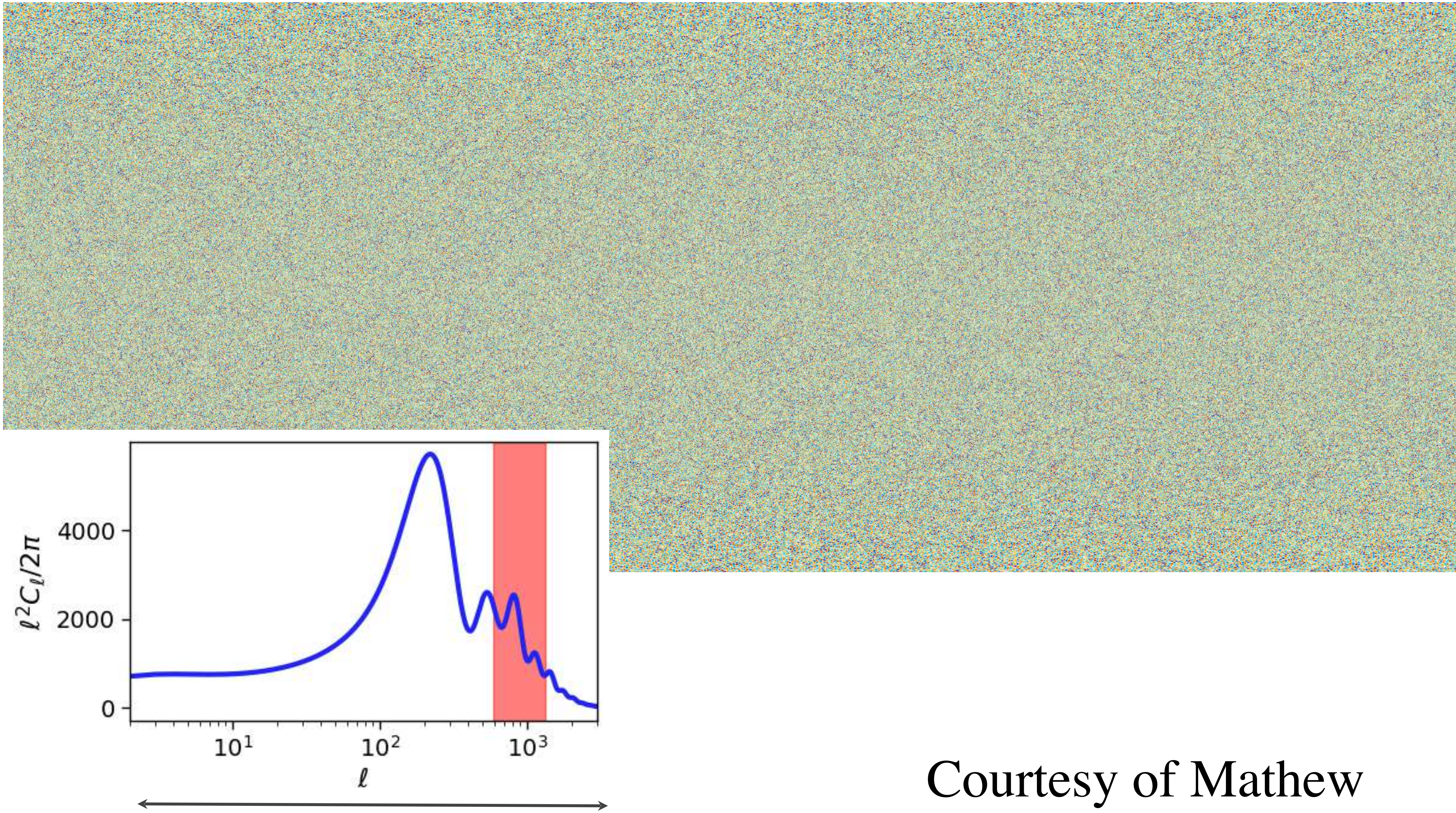


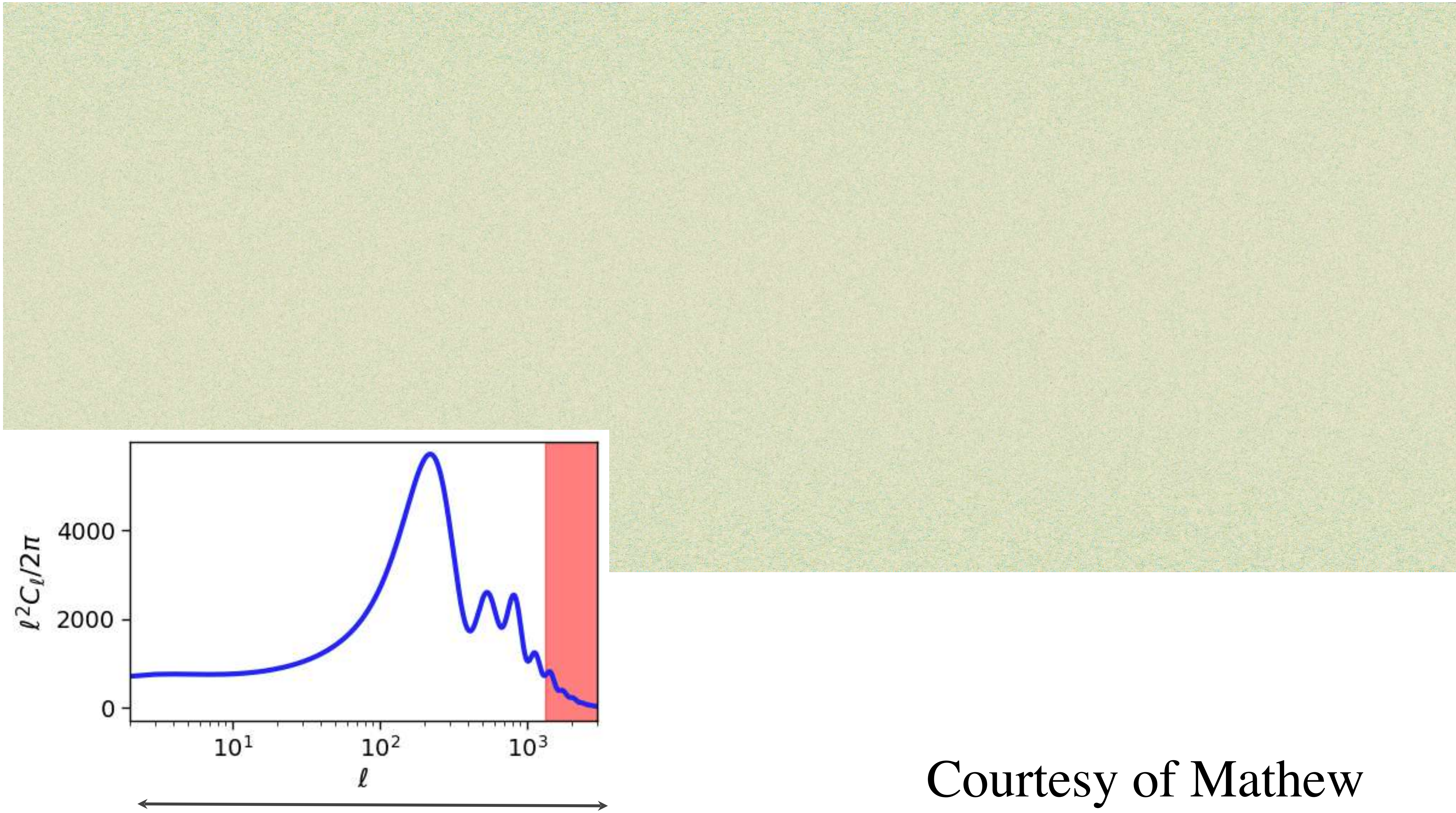
Courtesy of Mathew Madhavacheril

CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**



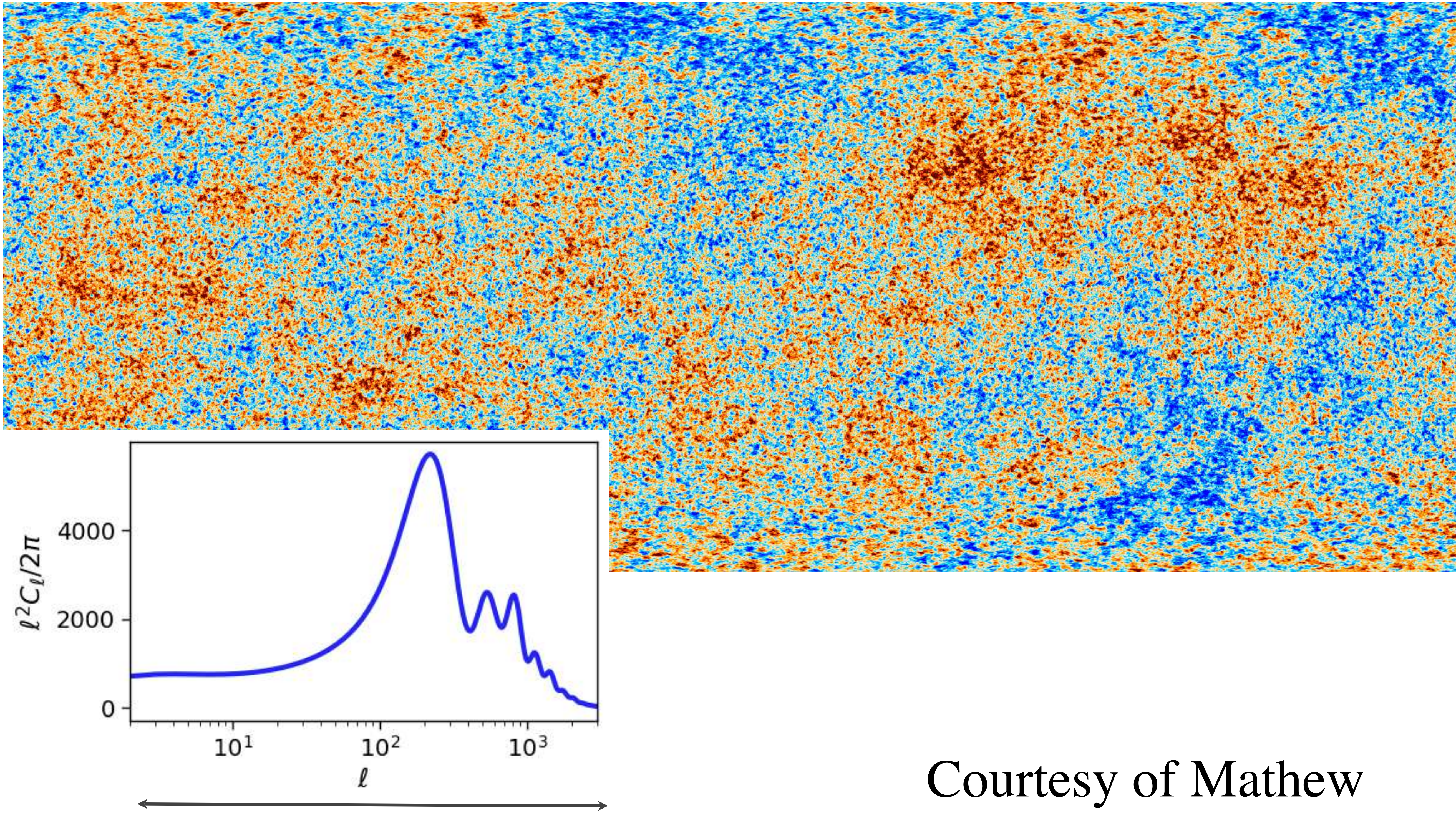
Courtesy of Mathew Madhavacheril

CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**

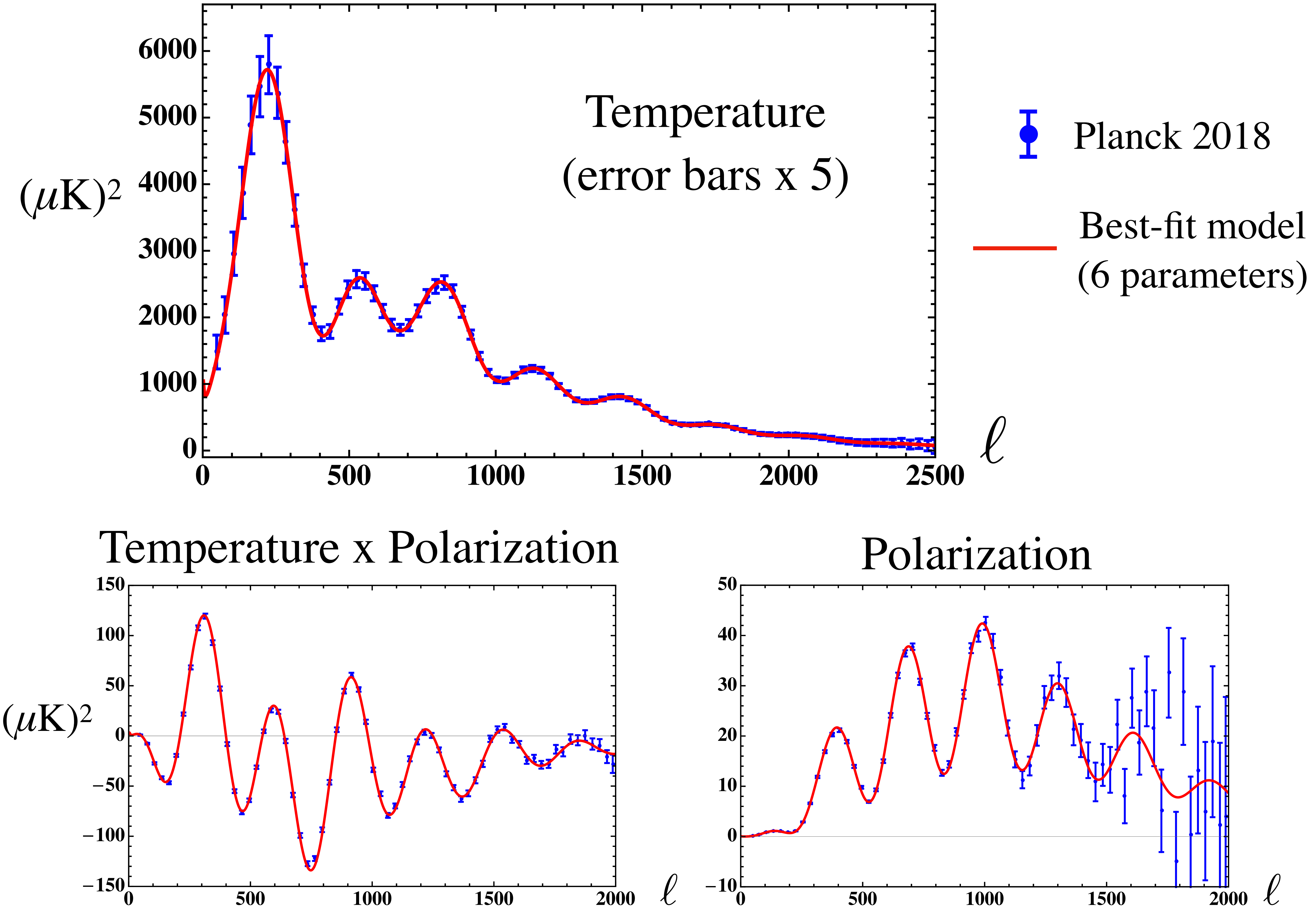


Courtesy of Mathew Madhavacheril

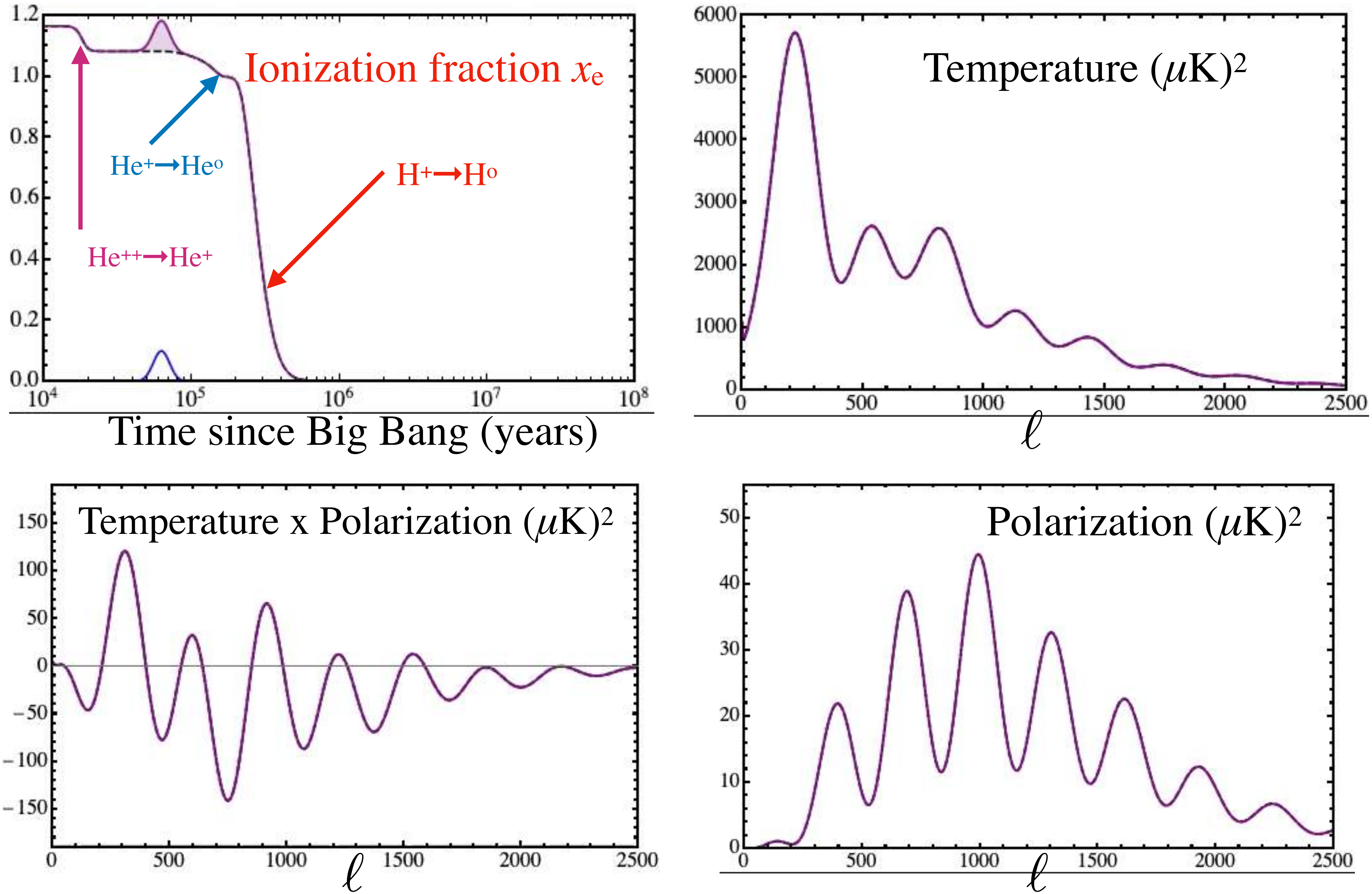
CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**

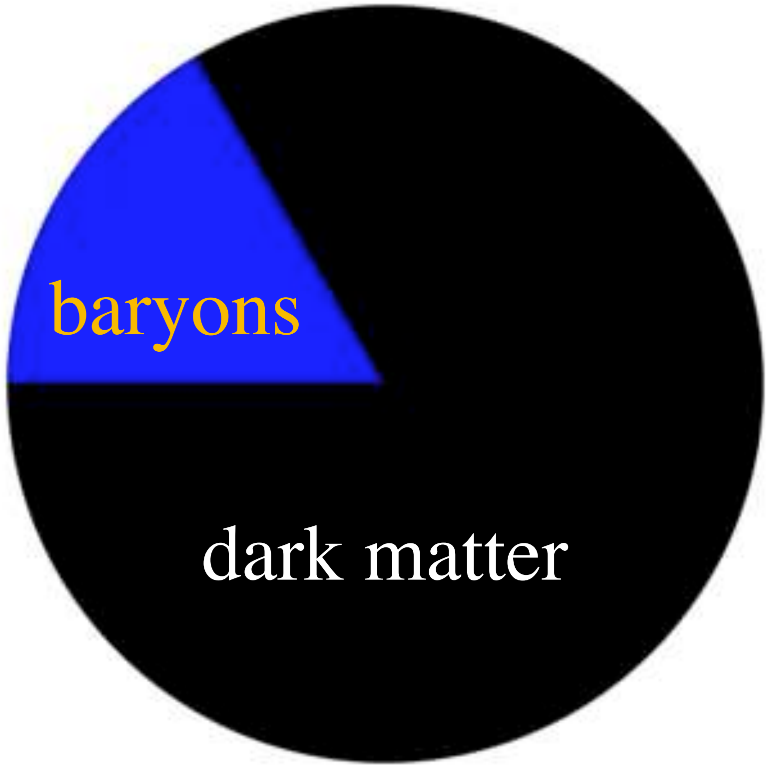


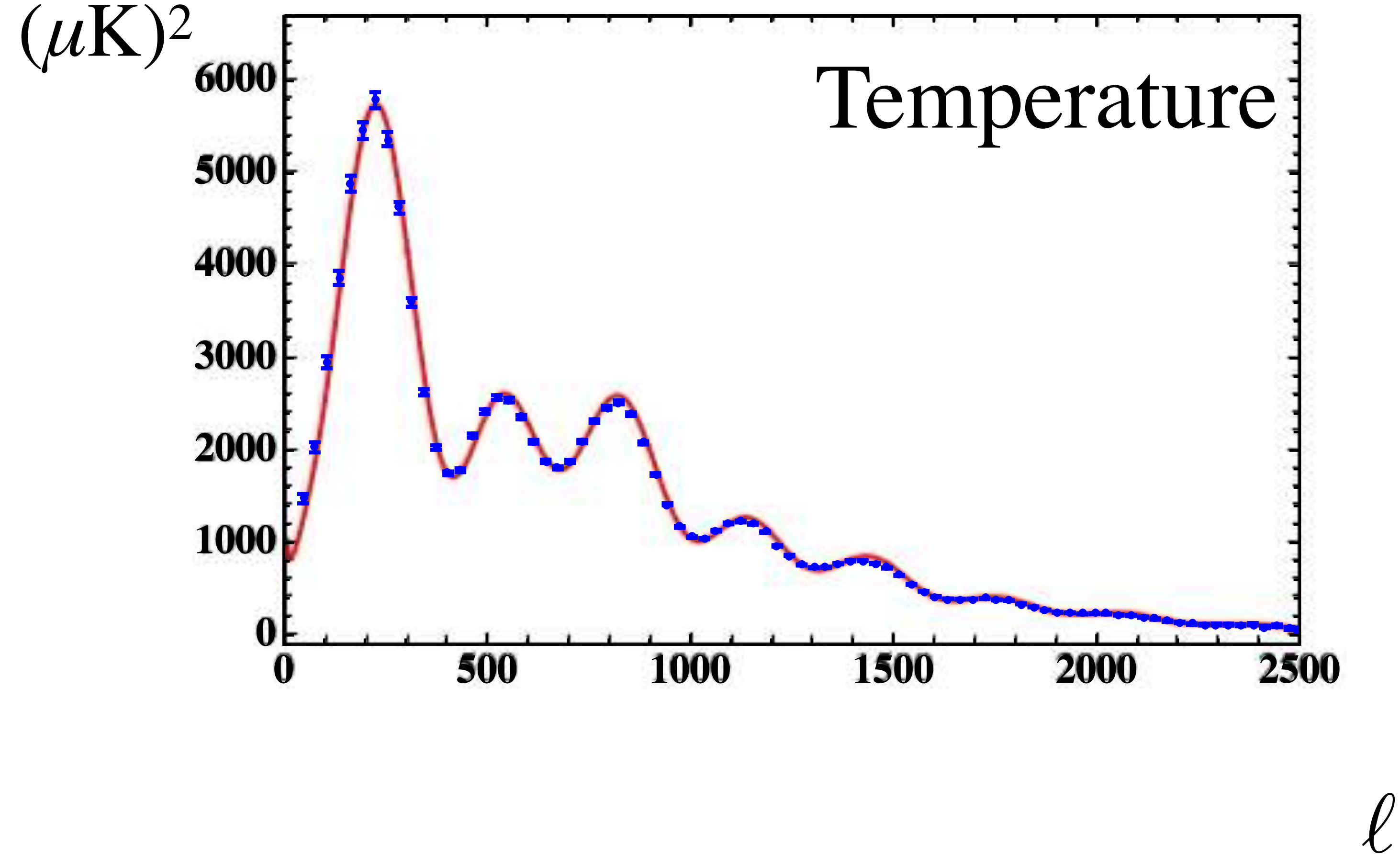
Courtesy of Mathew Madhavacheril


Temperature
(error bars x 5)
Planck 2018
Best-fit model
(6 parameters)
(μK)²
6000
5000
4000
3000
2000
1000
0
500
1000
1500
2000
2500
ℓ
Temperature x Polarization
150
100
50
0
−50
−100
−150
(μK)²
0
500
1000
1500
2000
ℓ
Polarization
50
40
30
20
10
0
−10
0
500
1000
1500
2000
ℓ

Effect of non-standard x_e on CMB power spectra

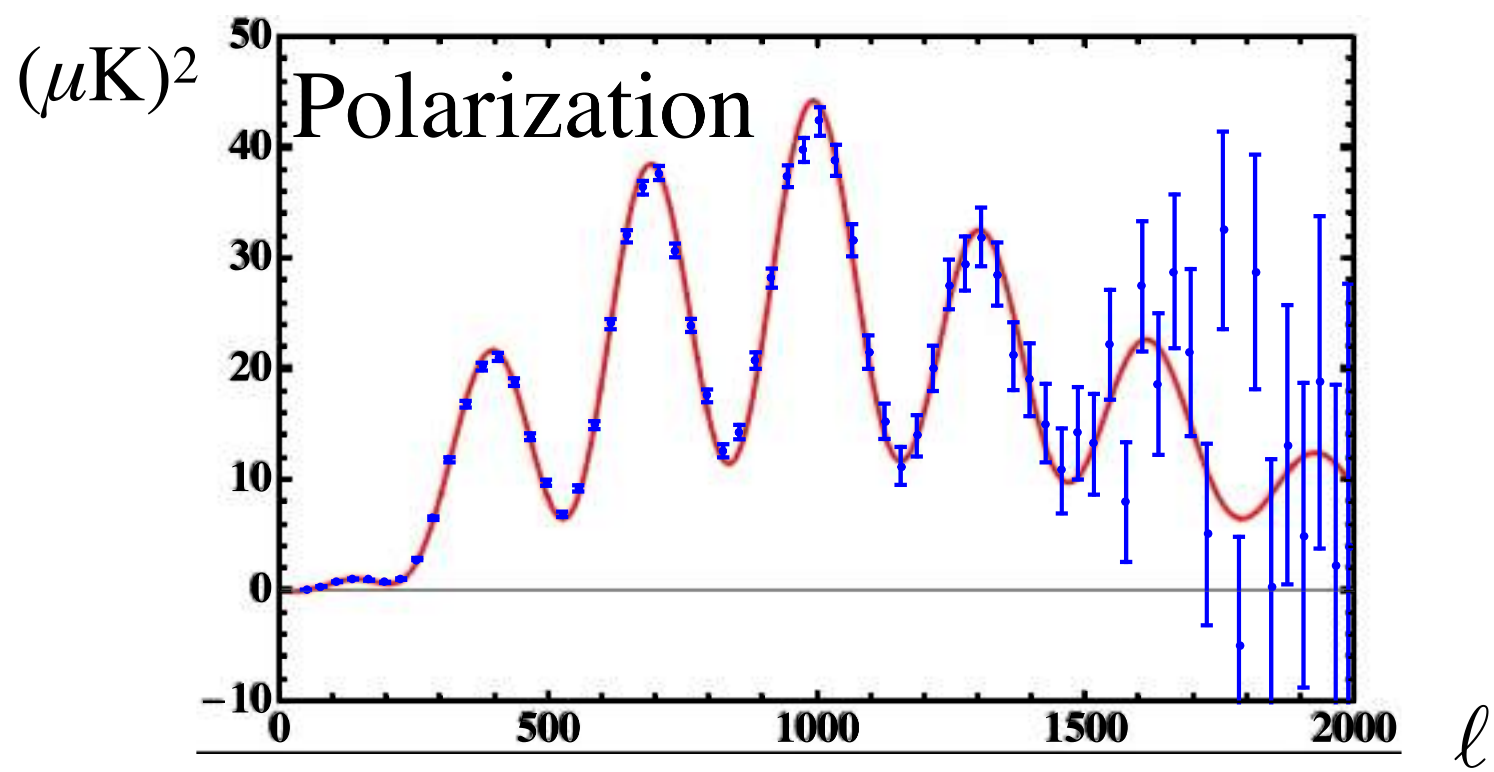






Parameter	`Plik` [1]
$\Omega_b h^2$	0.02237 ± 0.00015
$\Omega_c h^2$	0.1200 ± 0.0012
$100\theta_{MC}$	1.04092 ± 0.00031
τ	0.0544 ± 0.0073
$\ln(10^{10} A_s)$	3.044 ± 0.014
n_s	0.9649 ± 0.0042

Planck collaboration 2018



Plan for the next lectures

- **Lecture 2**: cosmological recombination
- **Lecture 3**: derivation of the photon Boltzmann equation
- **Lecture 4**: solutions of the photon Boltzmann equation
- **Lecture 5**: introduction to CMB polarization and lensing, and/or cosmological parameter estimation