Exercises lecture two

1. Starting from the definition

$$\rho_{\text{GW}} = \frac{\langle \dot{h}_{ij}(\mathbf{x}, t)\, \dot{h}_{ij}(\mathbf{x}, t)\rangle}{32\pi G} = \frac{\langle h'_{ij}(\mathbf{x}, \eta)\, h'_{ij}(\mathbf{x}, \eta)\rangle}{32\pi G\, a^2(\eta)} = \int_0^{+\infty} \frac{dk}{k}\, \frac{d\rho_{\text{GW}}}{d\log k}$$

Demonstrate that, for freely-propagating sub-Hubble modes, the power spectrum of the GW energy density can be related to the power spectrum of the GW amplitude as

$$\frac{d\rho_{\text{GW}}}{d\log k} = \frac{k^2\, h_c^2(k, \eta)}{16\pi G\, a^2(\eta)}$$

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2. (OPTIONAL)

Reproduce the figure below, by integrating numerically the homogeneous GW propagation equation (with appropriate initial conditions on super-Hubble modes) through the matter-radiation transition. Choose the mode that crosses the horizon at matter-radiation equality (or any other of your choice).