Exercises lecture three

1. Derive the GW energy density power spectrum today, for a generic stochastic source:

$$\frac{d\rho_{\rm GW}}{d{\rm log}k}(k,\eta_0) = \frac{4}{\pi}\,\frac{G}{a_0^4}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a^3(\tau)\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, a^3(\zeta)\,\cos[k(\eta-\zeta)]\,\Pi(k,\tau,\zeta)$$

From the above expression, derive the GW energy density parameter today, for modes inside the Hubble scale today but larger than the Hubble scale at the generation time, when the GW source lasts less than one Hubble time (at the generation time), and the anisotropic stresses are otherwise constant. The result is

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\,h^2\Omega_{\rm rad}^0\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2\tilde{P}_{\rm GW}(k)$$

Exercises lecture two

2. (OPTIONAL)

Think about the reason why GW detectors provide the opportunity to test the inflationary power spectrum at very small scales, while these scales remain inaccessible with the CMB or other cosmological probes