Cosmology from: gravitational waves

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LECTURE 3
SGWB from the early universe: examples

SGWB from a generic stochastic source in the radiation era

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$

Possible sources of tensor anisotropic stress in the early universe:

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$
- Gauge fields $\Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- ...

First, solution of the GW propagation equation,
keeping the GW source as general as possible

SGWB from a generic stochastic source in the radiation era

For most processes in the early universe, the source must be treated as a stochastic variable

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Anisotropic stress power spectrum at unequal time

Suppose the source operates in a time interval $\eta_{\rm fin}$ - $\eta_{\rm in}$ in the radiation dominated era

$$H_r^{\rm rad}(\mathbf{k},\eta<\eta_{\rm fin}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\,\sin[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

Matching at $\eta_{\rm fin}$ with the homogeneous solution to find the GW signal today

$$H_r^{\rm rad}(\mathbf{k},\eta>\eta_{\rm fin}) = A_r^{\rm rad}(\mathbf{k})\cos(k\eta) + B_r^{\rm rad}(\mathbf{k})\sin(k\eta)$$

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\,\sin(-k\tau)\,\Pi_r(\mathbf{k},\tau),$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{x_{\rm in}}^{x_{\rm fin}} d\tau\, a(\tau)^3\,\cos(k\tau)\,\Pi_r(\mathbf{k},\tau)$$

SGWB from a generic stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle]$$

GW energy density power spectrum today for modes $k\eta_0 \gg 1$

$$\frac{d\rho_{\mathrm{GW}}}{d\log k} = \frac{k^2\, h_c^2(k,\eta_0)}{16\pi G\, a_0^2}$$

(freely propagating sub-Hubble modes)

$$\frac{d\rho_{\mathrm{GW}}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\,\frac{G}{a_0^4}\int_{\eta_{\mathrm{in}}}^{\eta_{\mathrm{fin}}} d\tau\, a^3(\tau)\int_{\eta_{\mathrm{in}}}^{\eta_{\mathrm{fin}}} d\zeta\, a^3(\zeta)\, \cos[k(\eta-\zeta)]\, \Pi(k,\tau,\zeta)$$

SGWB from a generic stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

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a_*^3 $\quad a_*^3 \quad \simeq 1 \quad \Pi(k)$

SUPPOSE:

$\Delta\eta = \eta_{\rm fin} - \eta_{\rm in} \ll \mathcal{H}_*^{-1}$ $\qquad k\eta_{\rm in} \ll 1$ $\qquad \Pi(k,\tau,\eta)$ constant over $\Delta\eta$

SGWB from a generic stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2\tilde{P}_{\rm GW}(k)$$

Exercise: derive the GW energy density power spectrum, and this approximated result

SGWB from a generic stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 \tilde{P}_{\rm GW}(k)$$

$$\Pi(k) = \rho_\Pi \tilde{P}_{\rm GW}(k)$$

From the time integrals

SGWB from a generic stochastic source in the radiation era

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SGWB from a generic stochastic source in the radiation era

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$\mathcal{O}(10^{-11})$ — Value that would guarantee a detection in a not so far future

$\mathcal{O}(10^{-6})$ — Factor depending slightly on the generation epoch through the number of relativistic d.o.f.

$\mathcal{O}(10^{-5})$ — Only slow, very anisotropic processes have the chance to generate detectable SGWB signals!

SGWB from a generic stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

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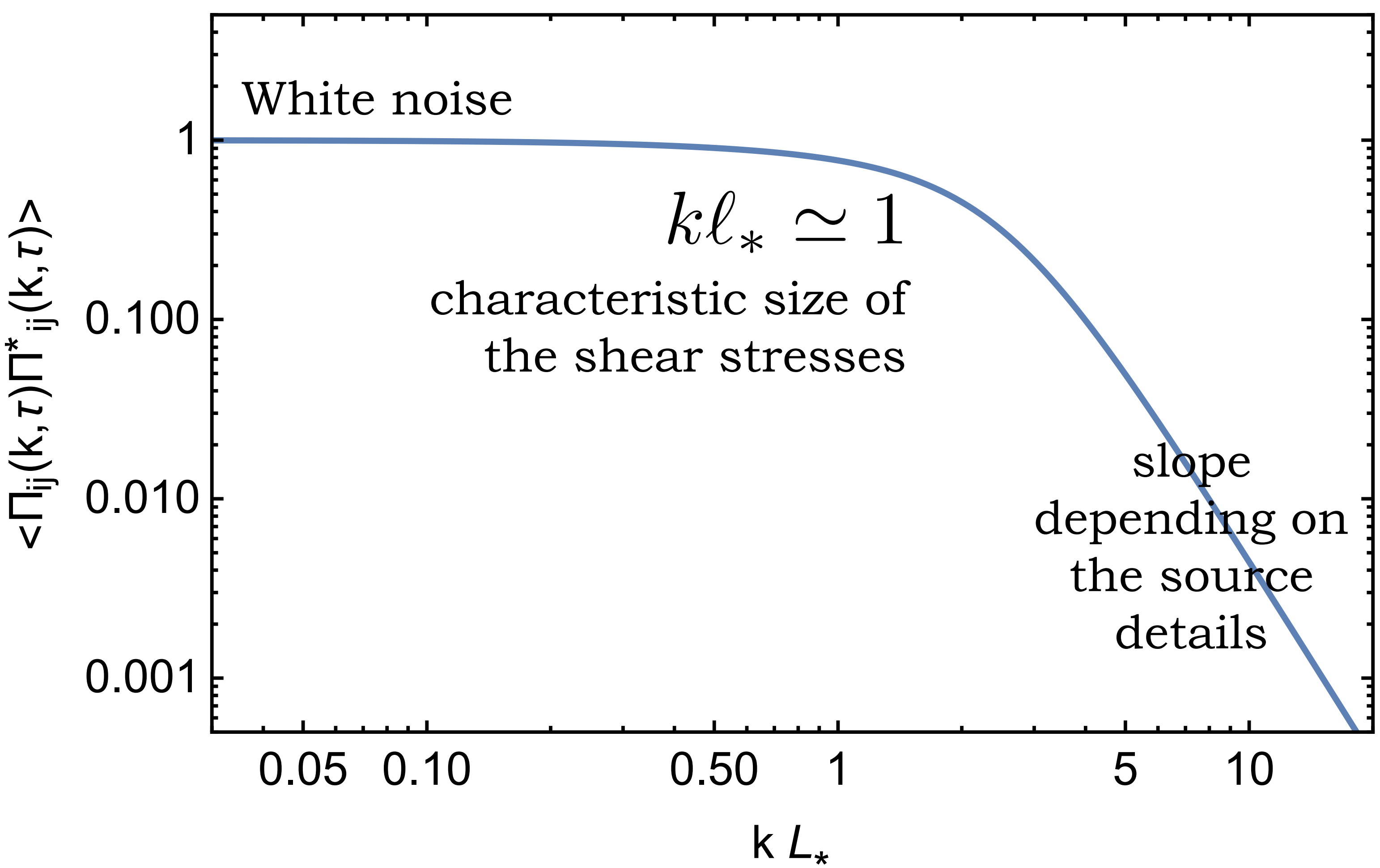
$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi^*_p(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Independent on k for large enough scales (uncorrelated)

$$\ell_* \leq H_*^{-1}$$



SGWB from a generic stochastic source in the radiation era

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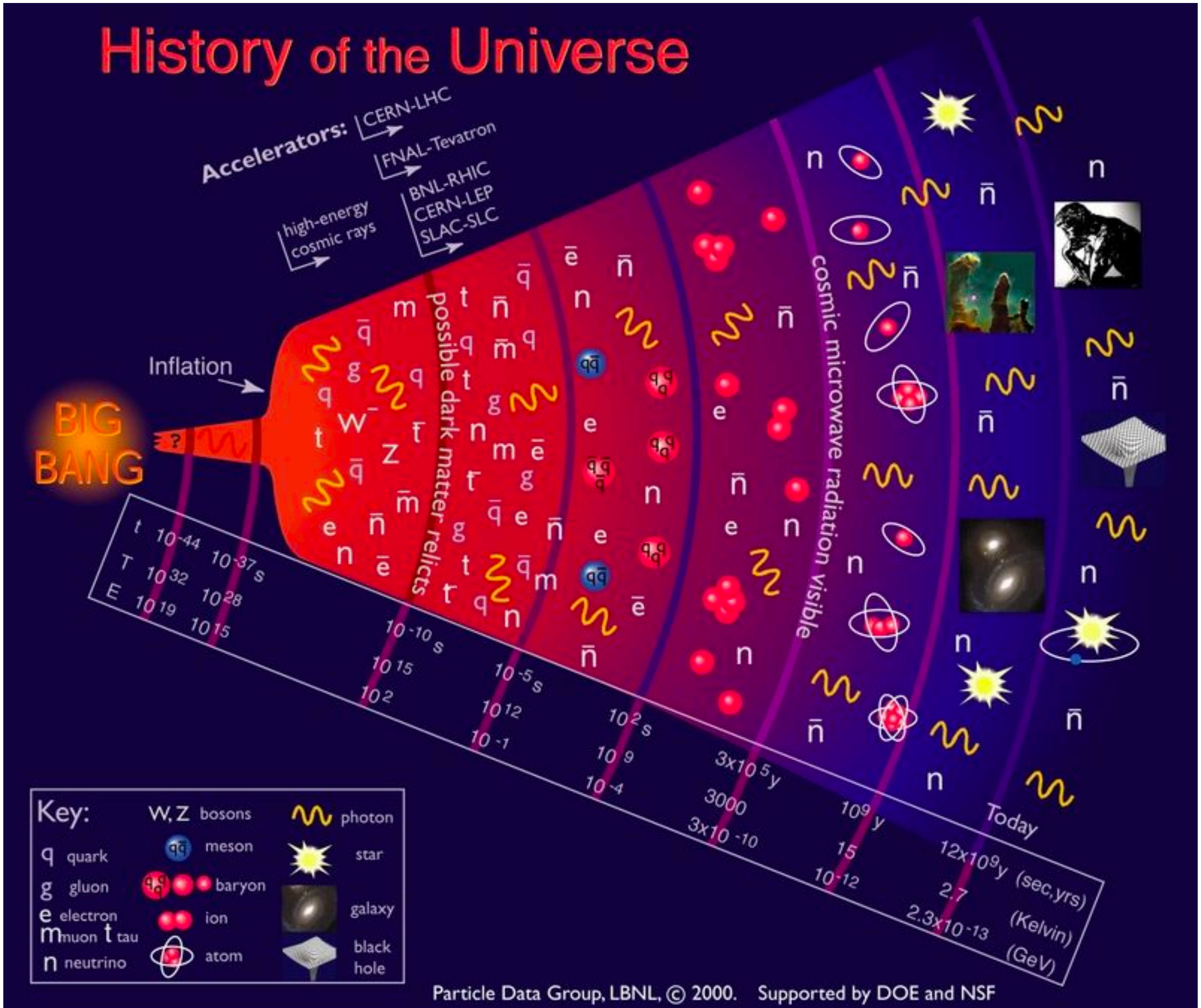
$$\underbrace{1/\eta_0 \ll k \ll}_{\text{Range of validity of the solution}} \mathcal{H}_* \underbrace{\ll 1/(a_*\ell_*)}_{\text{Causality of the sourcing process}}$$

Range of validity of the solution

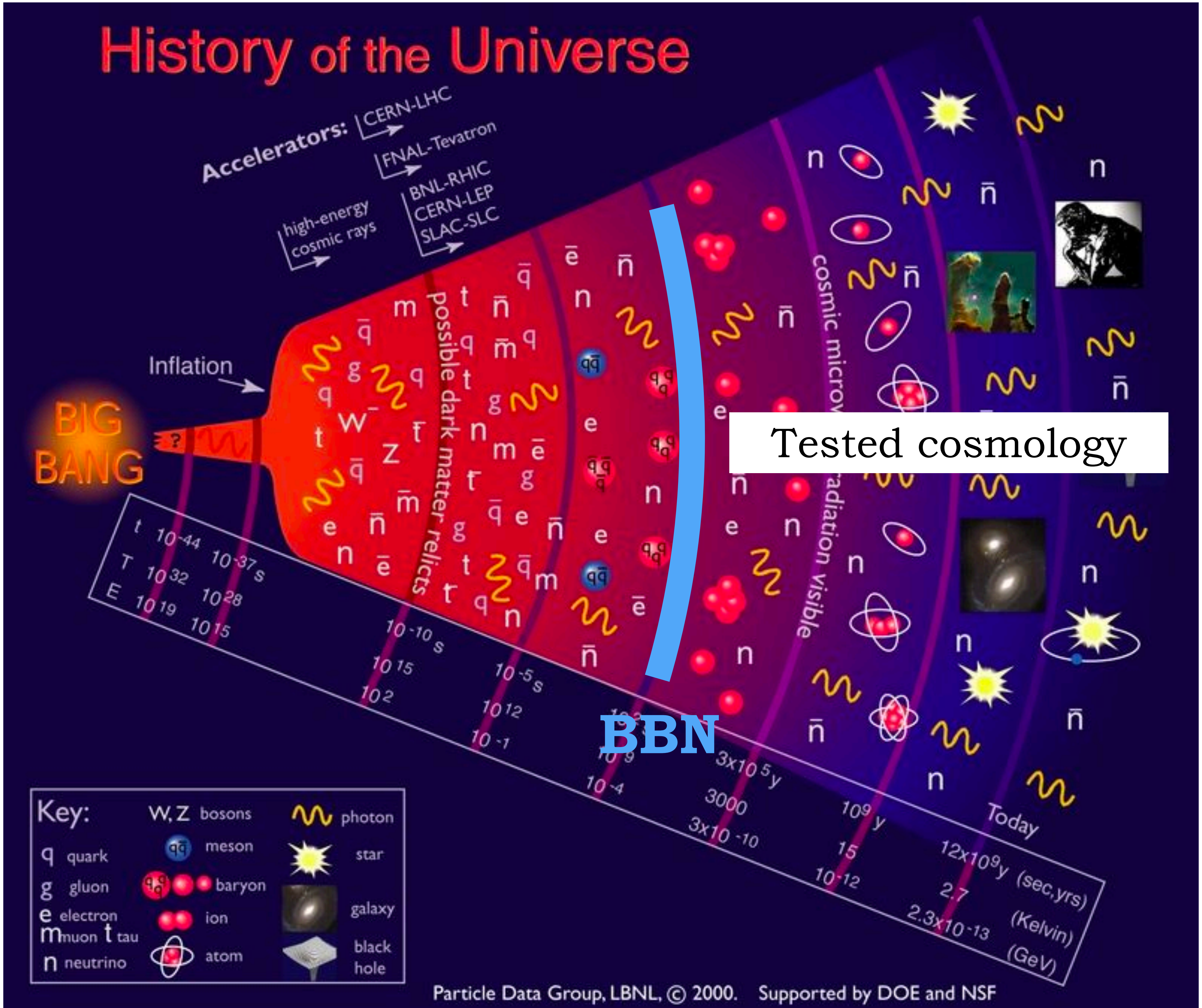
Causality of the sourcing process

$$\Omega_{\rm GW}(k) \propto \tilde{P}_{\rm GW}(k) \propto k^3$$

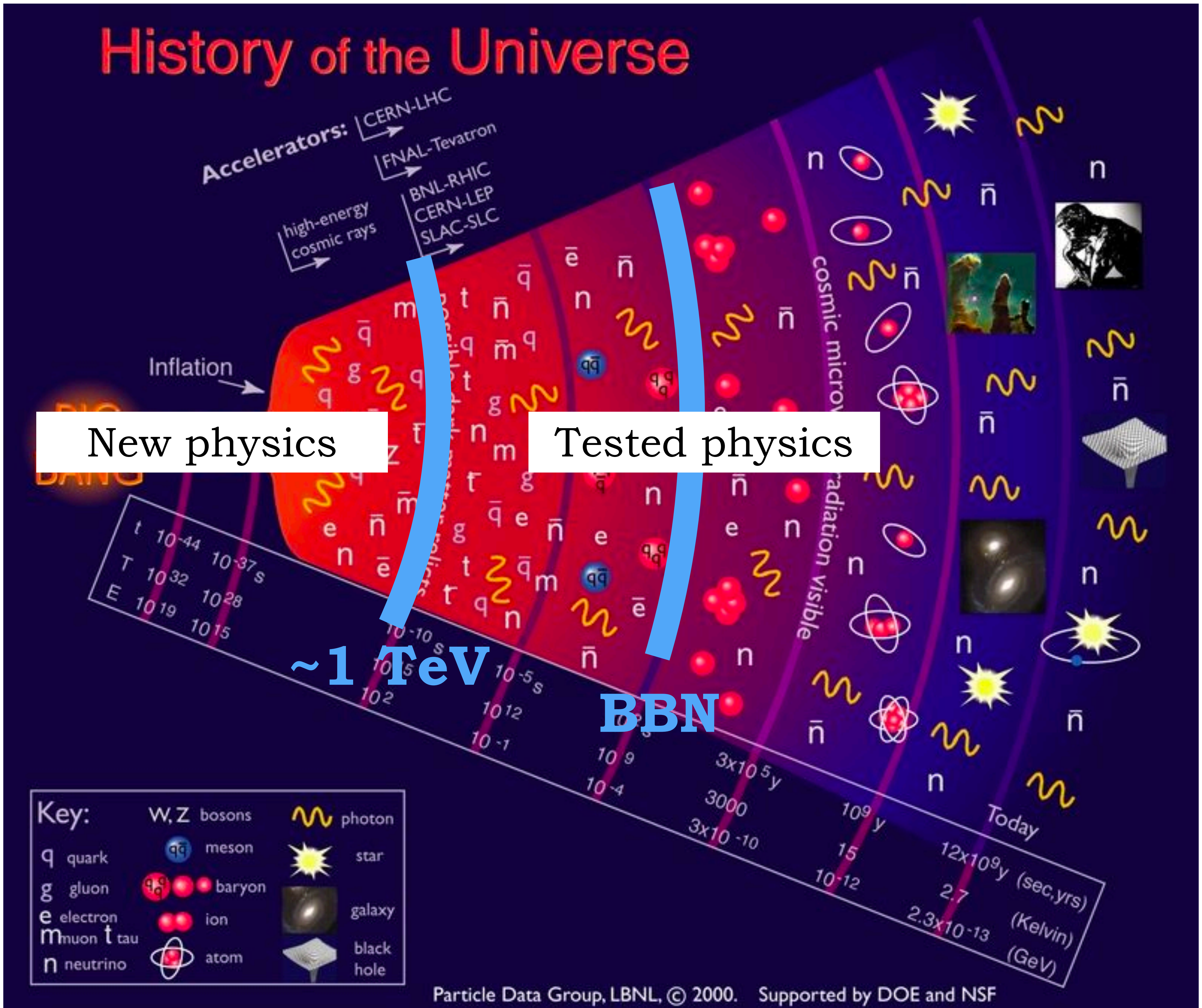
Examples of SGWB sources in the early universe



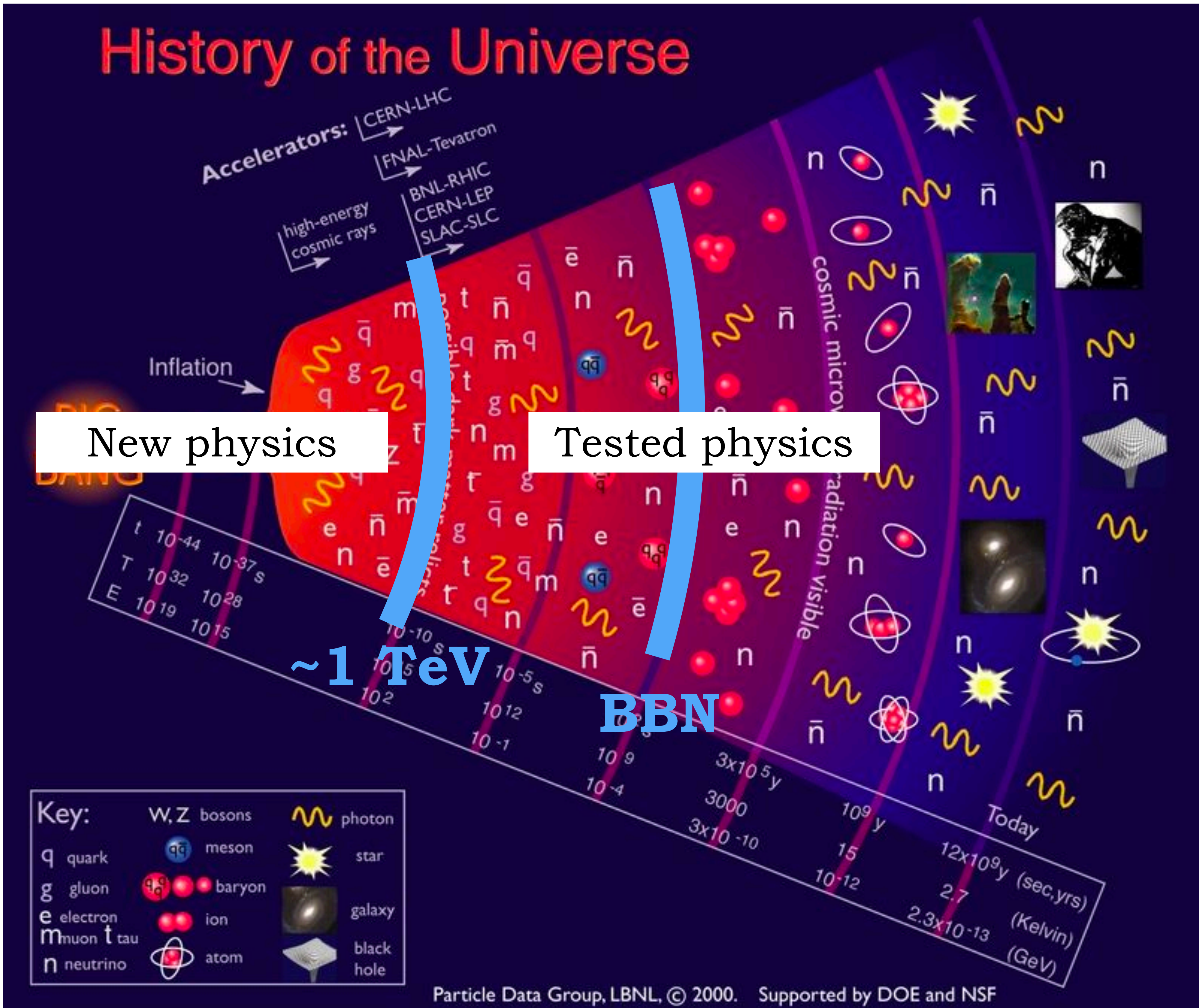
GWs can bring direct information from very early stages of the universe evolution, to which we have no direct access through em radiation —> amazing discovery potential



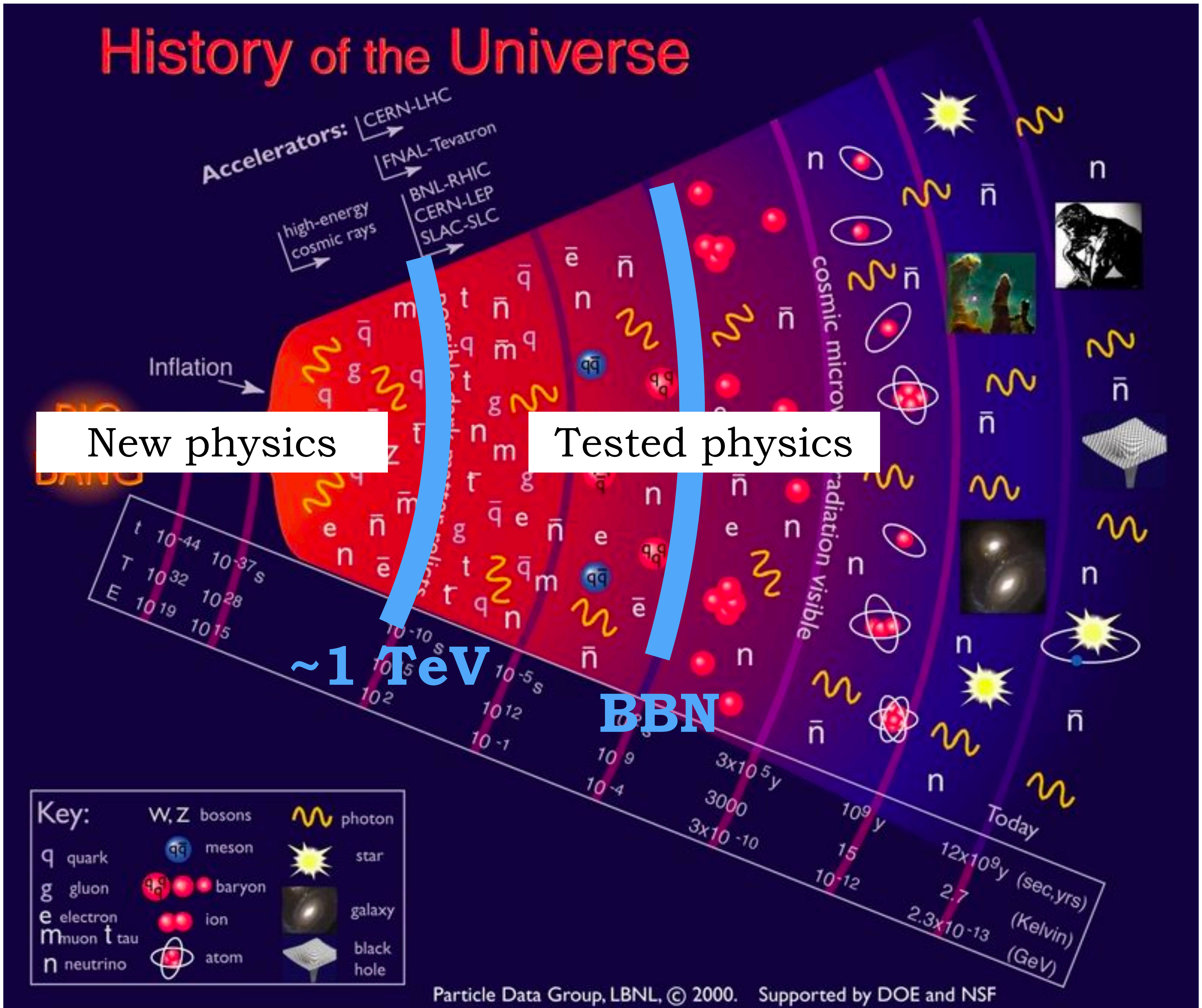
No guaranteed GW signal: predictions rely on untested phenomena, and are often difficult to estimate (non-linear dynamics, strongly coupled theories...)



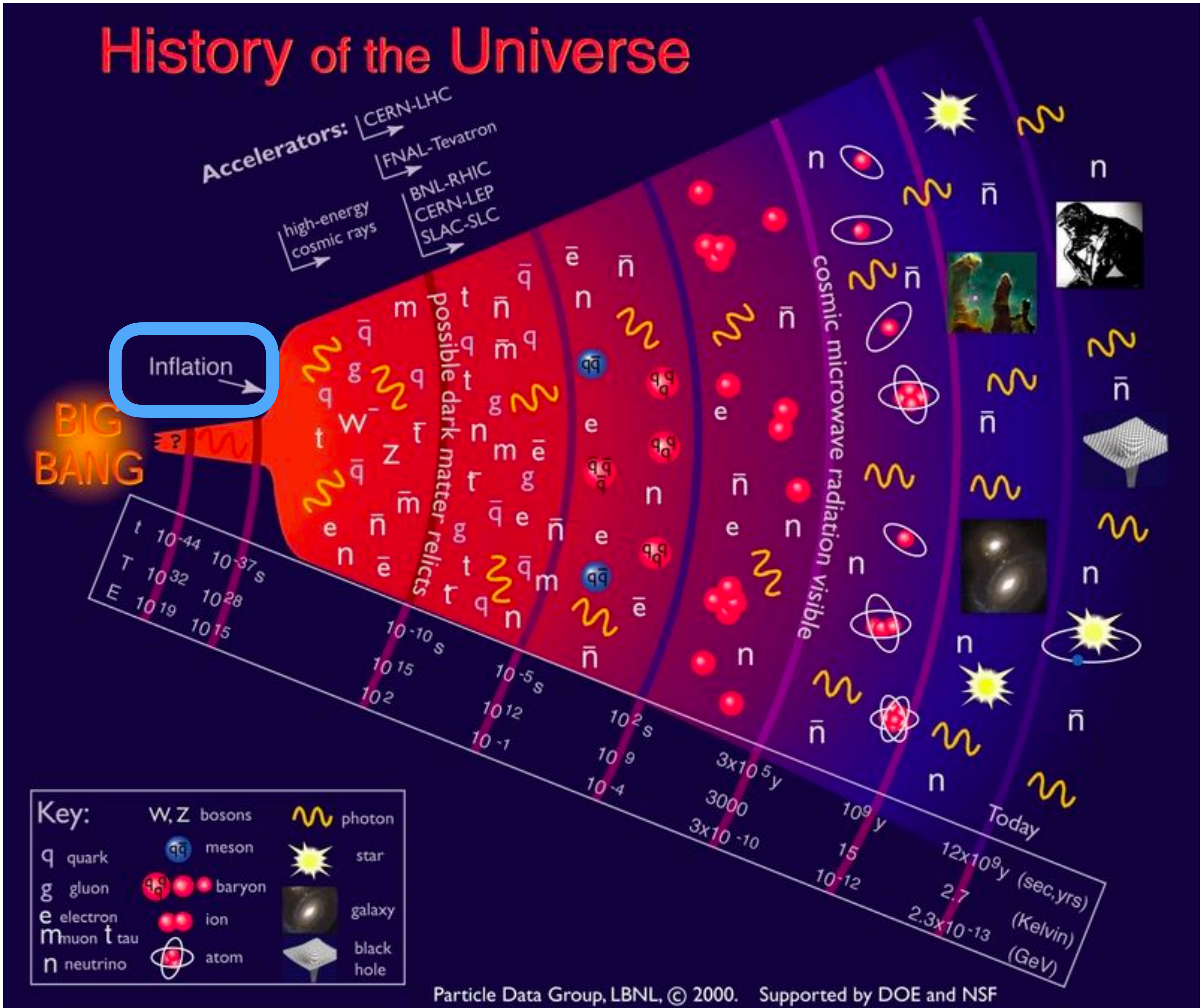
Many GW generation processes are related to PHASE TRANSITIONS



Phase transition: some field in the universe changes from one state to another, which has become more energetically favourable due to a change in external conditions (e.g. a change in temperature)



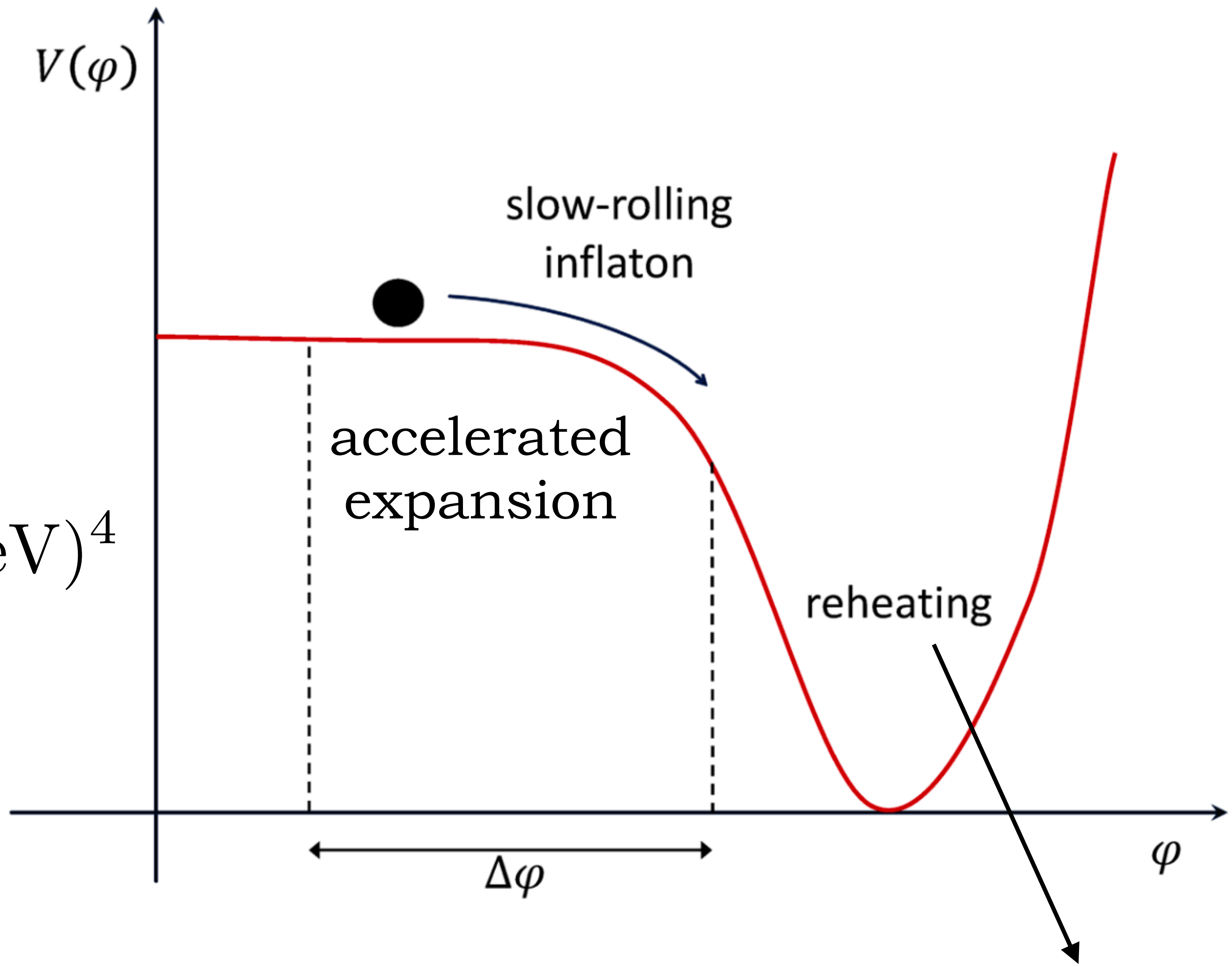
Inflation: phase transition of the Inflaton field



Inflation: phase transition of the Inflaton field

$$\ddot{\varphi} + 3H\dot{\varphi} - \nabla^2\varphi + V'(\varphi) = 0$$

$$V \lesssim (10^{16}\,\mathrm{GeV})^4$$



Generation of a thermodynamical state:
particles in thermal equilibrium,
radiation-dominated phase

Image credit: Guzzetti et al, arXiv:1605.01615

GW signal from inflation

Amplification of tensor metric vacuum fluctuations by the exponential expansion

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\Pi_r(\mathbf{k},\eta)$$

- ✓ canonically normalised free field $v_\pm = a\,M_{Pl}\,h_\pm$
- ✓ quantisation
- ✓ homogeneous wave equation: harmonic oscillator with *time dependent* frequency

$$v_\pm''(t) + (k^2 - a^2 H^2)v_\pm(t) = 0$$

$k \gg a\,H$ sub-Hubble modes

$$\omega^2(t) = k^2$$

free field in vacuum
zero occupation number

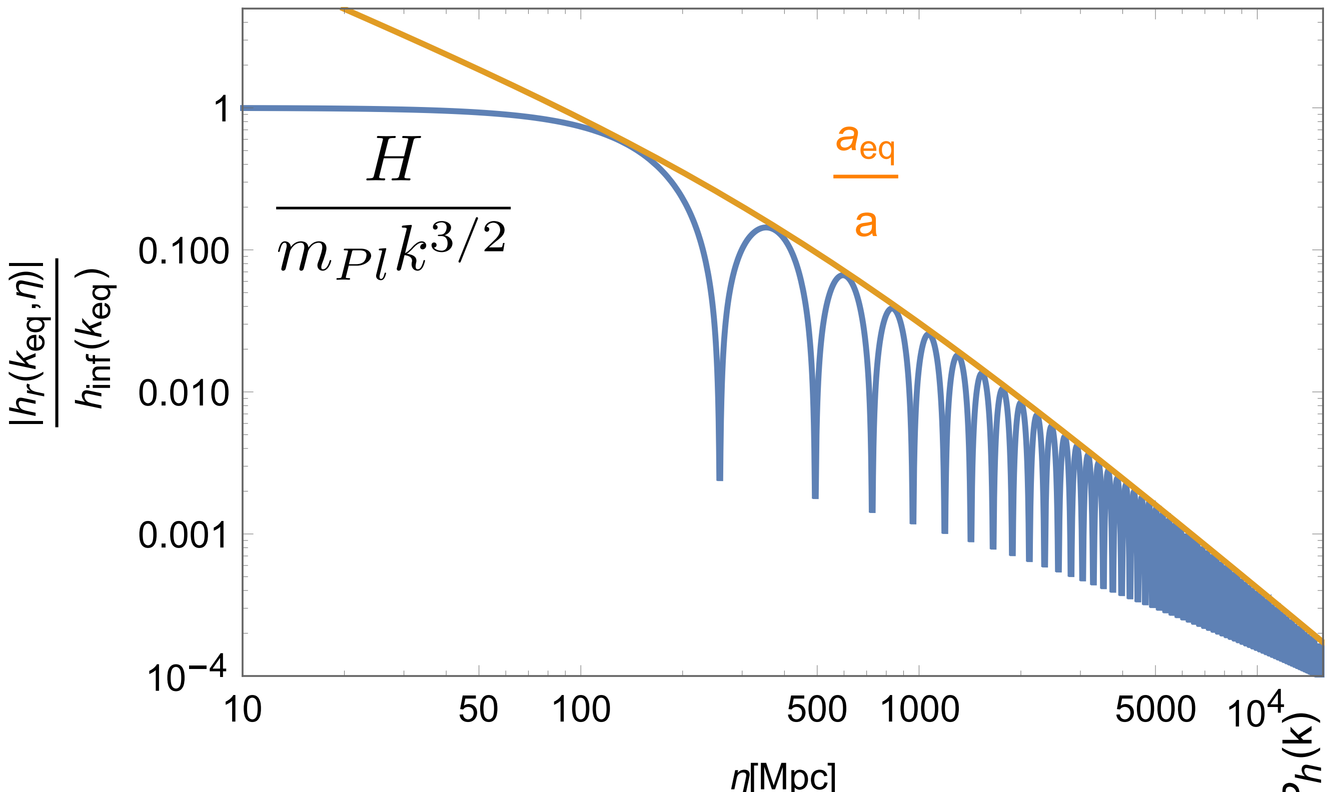
$k \ll a\,H$ super-Hubble modes

$$\omega^2(t) = -a^2 H^2$$

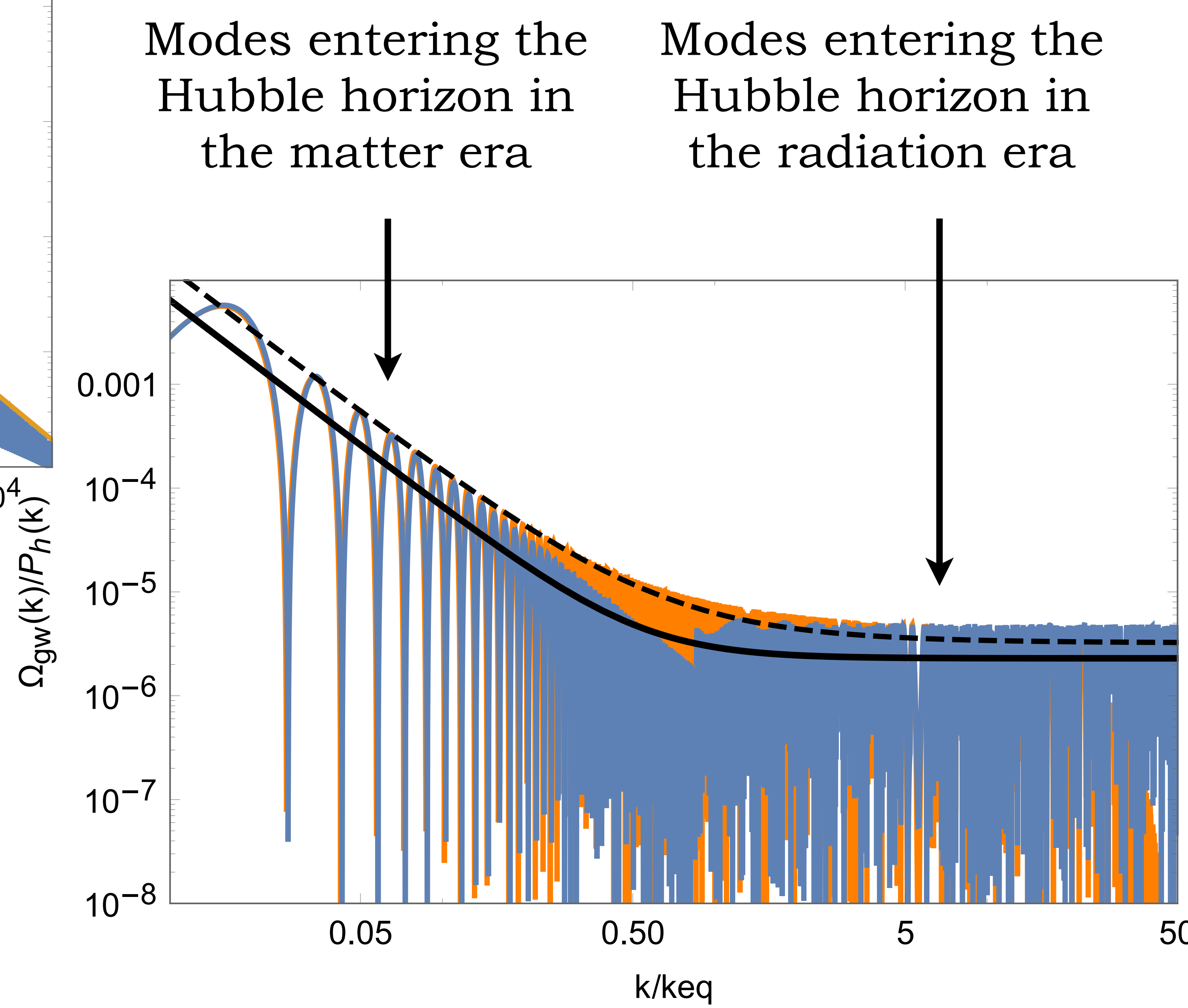
super-Hubble modes have very large occupation number

GW signal from (slow roll) inflation

- tensor spectrum $\mathcal{P}_h = \frac{2}{\pi}\frac{H^2}{m_{Pl}^2}\left(\frac{k}{aH}\right)^{-2\epsilon}$ $\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
- transfer function from inflation to today, as modes re-enter the Hubble horizon



$$\Omega_{\rm GW}(k,\eta_0) = \frac{[T'(k,\eta_0)]^2}{12a_0^2H_0^2}\mathcal{P}_h(k)$$



GW signal from (slow roll) inflation

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$$\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\, r\,\mathcal{P}_{\mathcal{R}}^*\left(\frac{f}{f_*}\right)^{n_T}\left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]$$

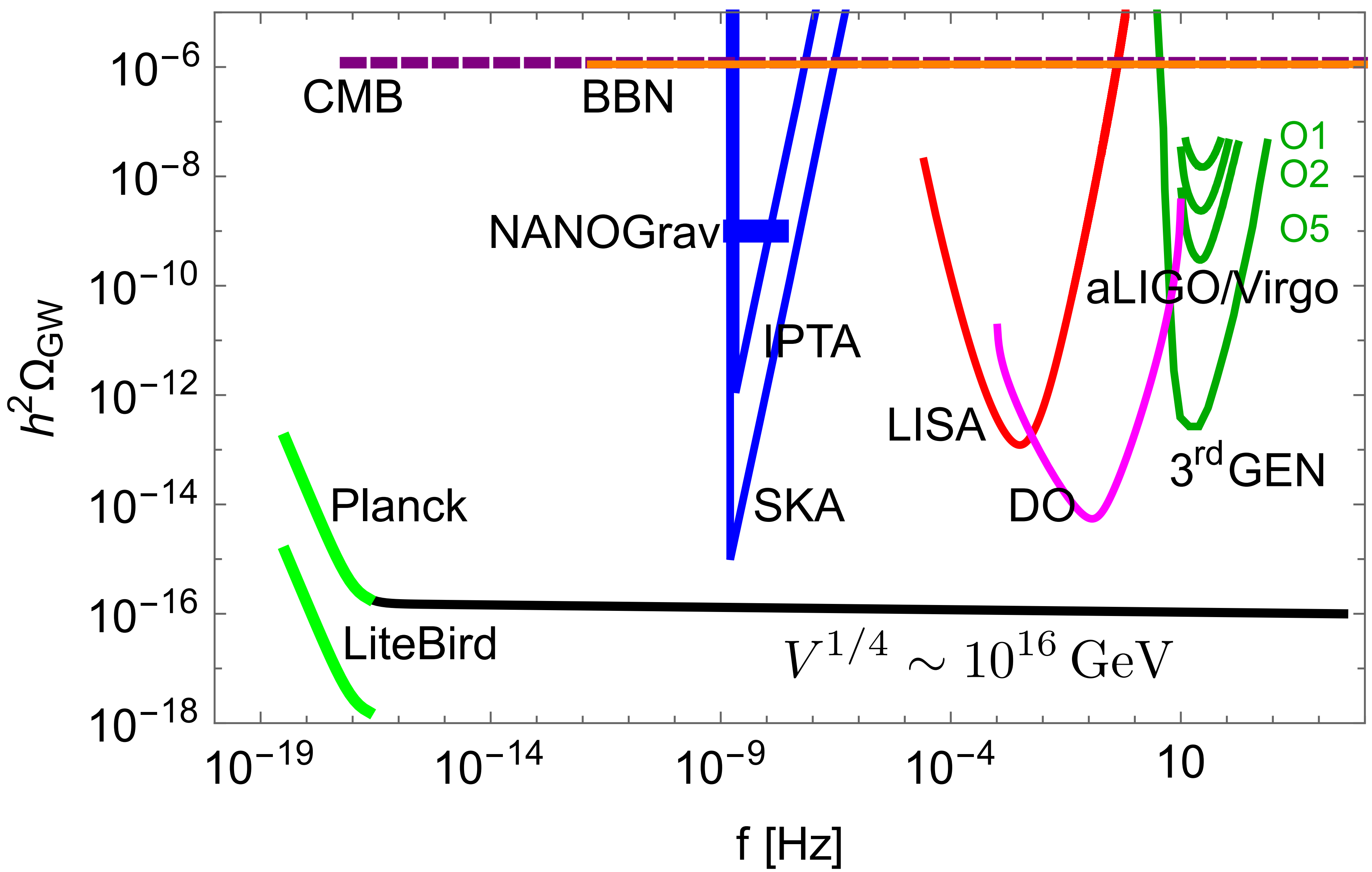
- tensor to scalar ratio $r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}}$ $r_* \leq 0.07$ Planck+BICEP limit
- scalar amplitude at CMB pivot scale $\mathcal{P}_{\mathcal{R}}^* \simeq 2\cdot 10^{-9}$ $k_* = \frac{0.05}{\rm Mpc}$
- GW signal extended in frequency: $H_0 \leq f \leq H_{\rm inf}$

continuous sourcing of GW as modes re-enter the Hubble horizon

GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

BUT! The signal in the standard slow roll scenario is too low



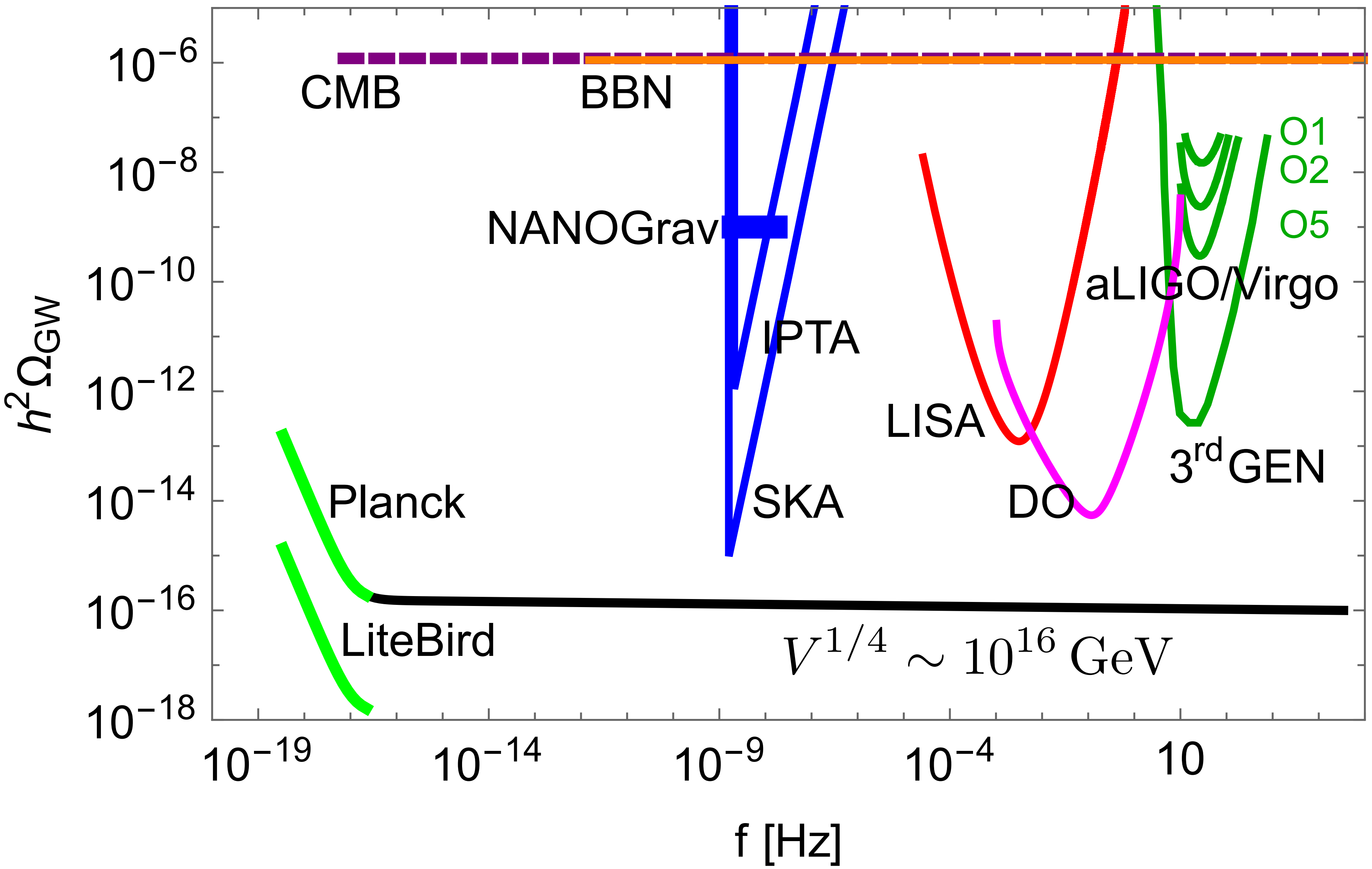
$n_T \simeq -2\epsilon$

Exercise: Why with GW we have potential access to these scales and not with the CMB?

GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

(P)reheating generates a signal with high amplitude, but unfortunately at very high frequencies

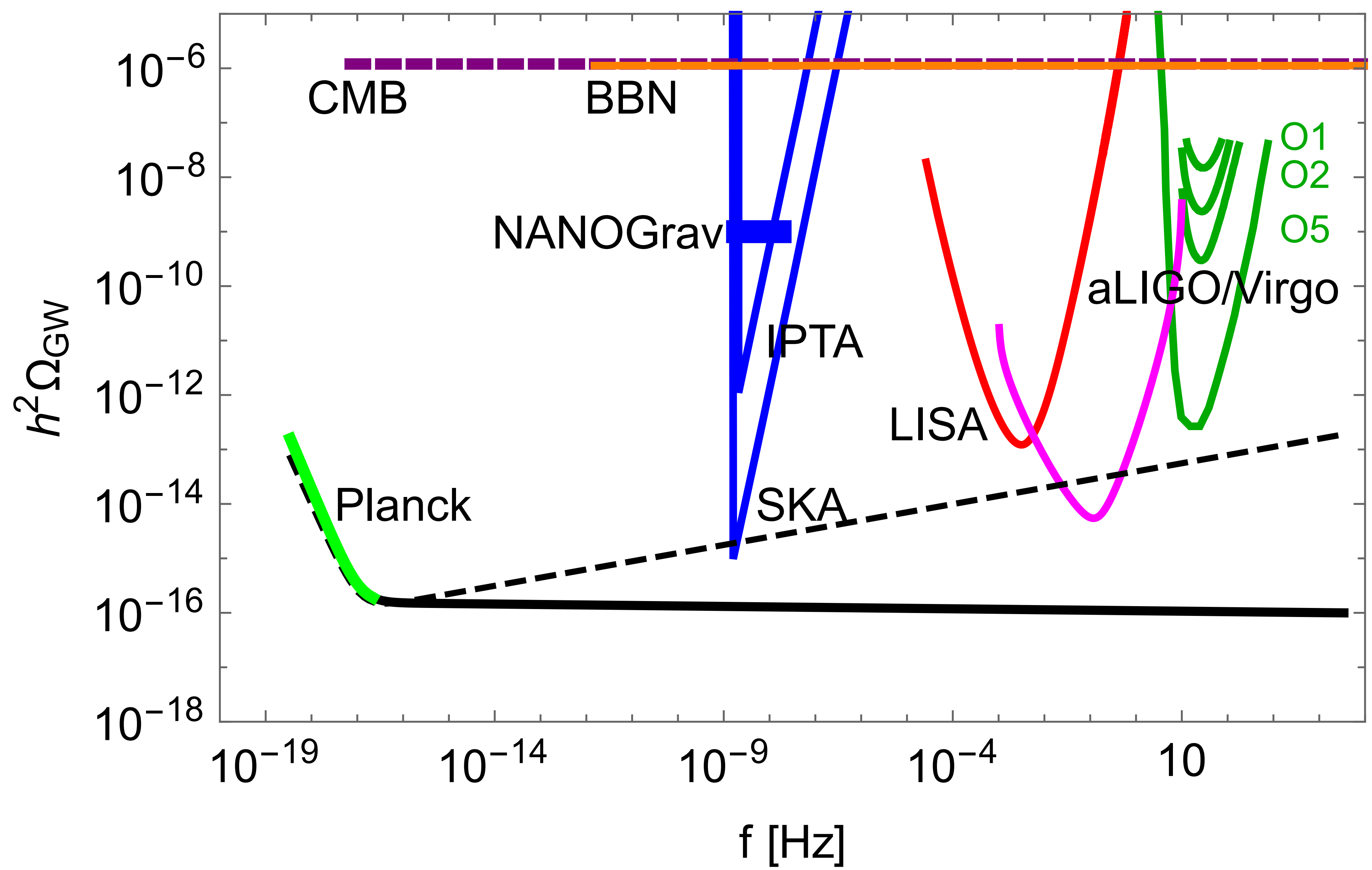


$n_T \simeq -2\epsilon$

GW signal from (non-standard) inflation

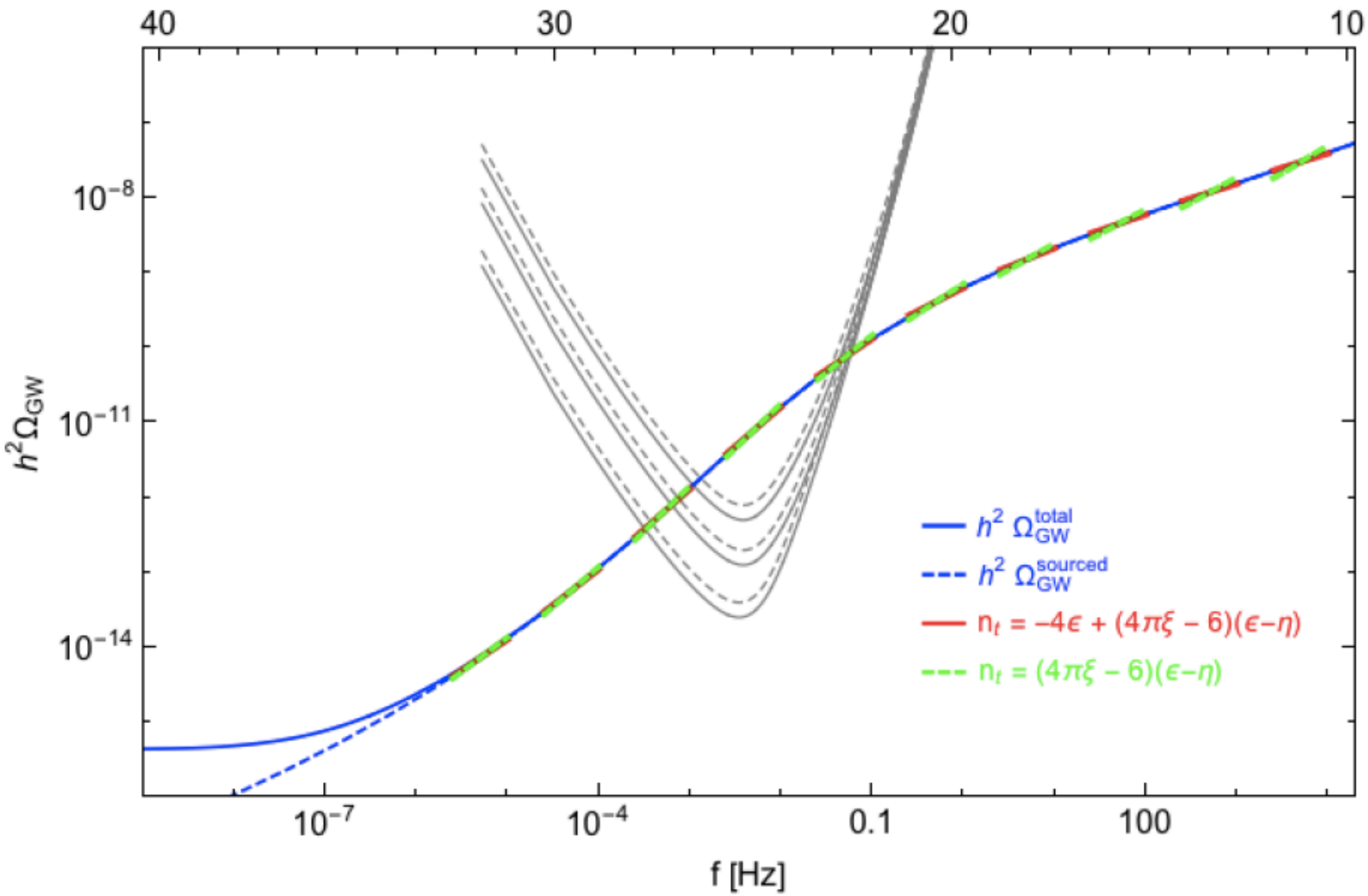
There is the possibility to enhance the signal going beyond the standard inflationary scenario: adding extra fields, modifying the inflaton potential, modifying the gravitational interaction, adding a phase with stiff equation of state…

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$



just one example: inflaton-gauge field coupling

$$\Delta\mathcal{L} = -\frac{1}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$$



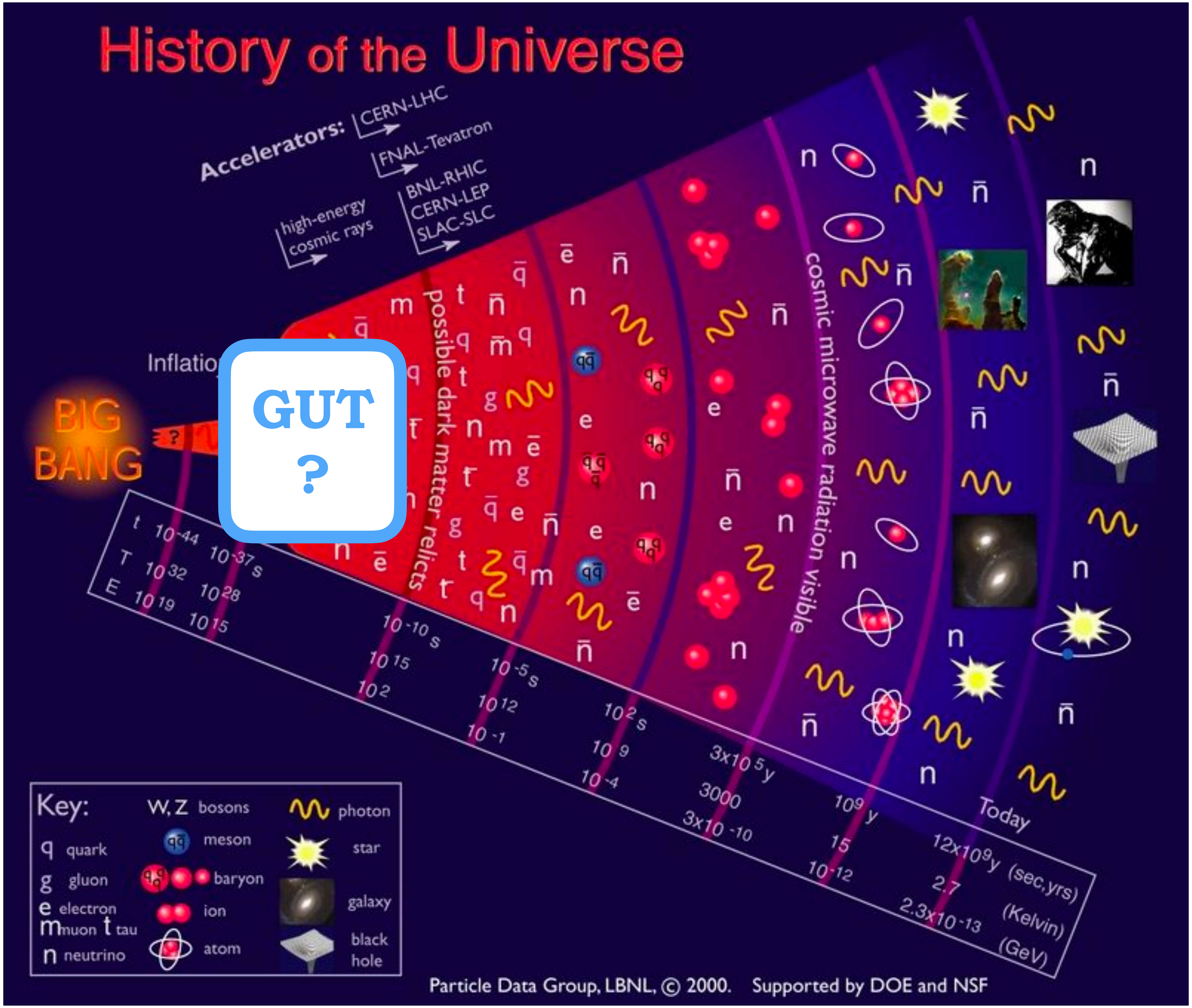
$$\Lambda = \frac{M_{Pl}}{35}$$

quadratic inflaton potential

OTHER SIGNATURES:
non-gaussianity, chirality

N. Bartolo et al, arXiv:1610.06481
N. Bartolo et al, arXiv:1806.02819

GUT phase transition or similar: related to the breaking of the symmetries of the high-energy theory describing the universe

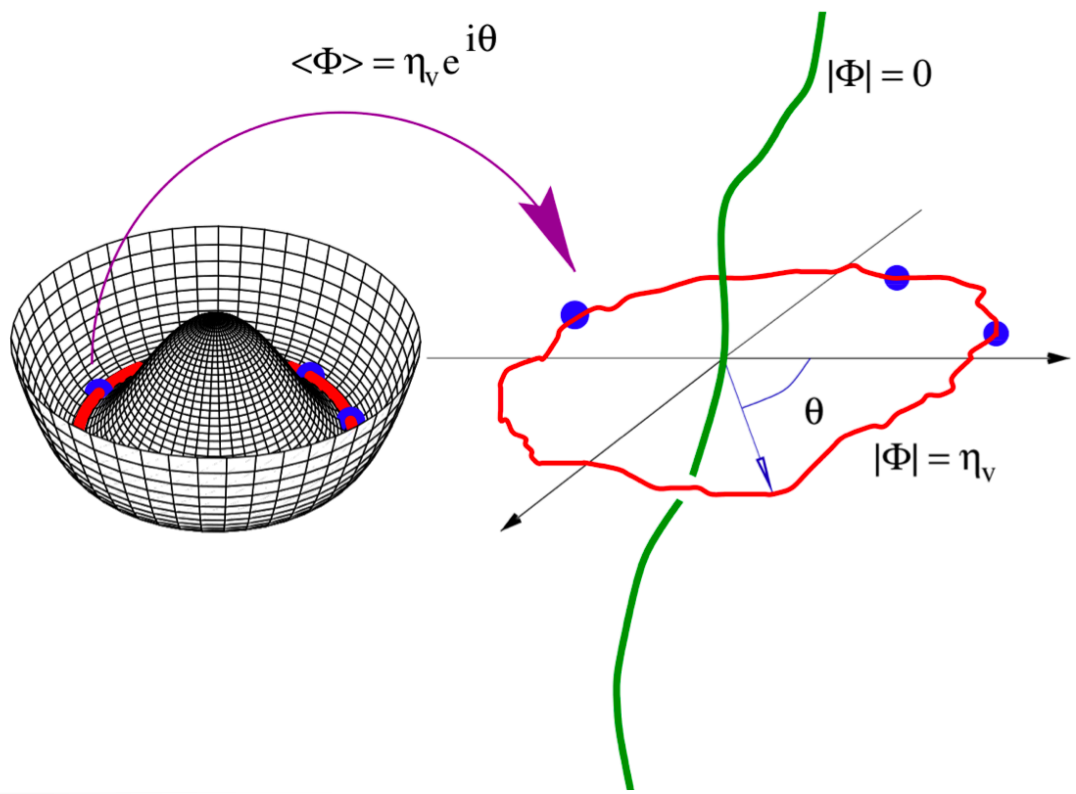


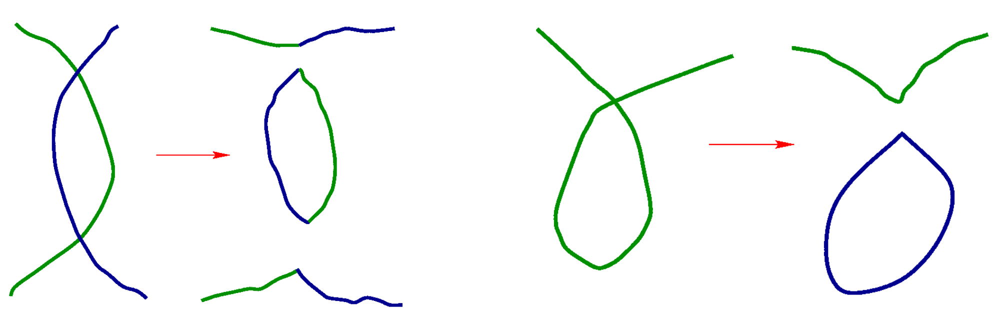
GW signal from cosmic strings

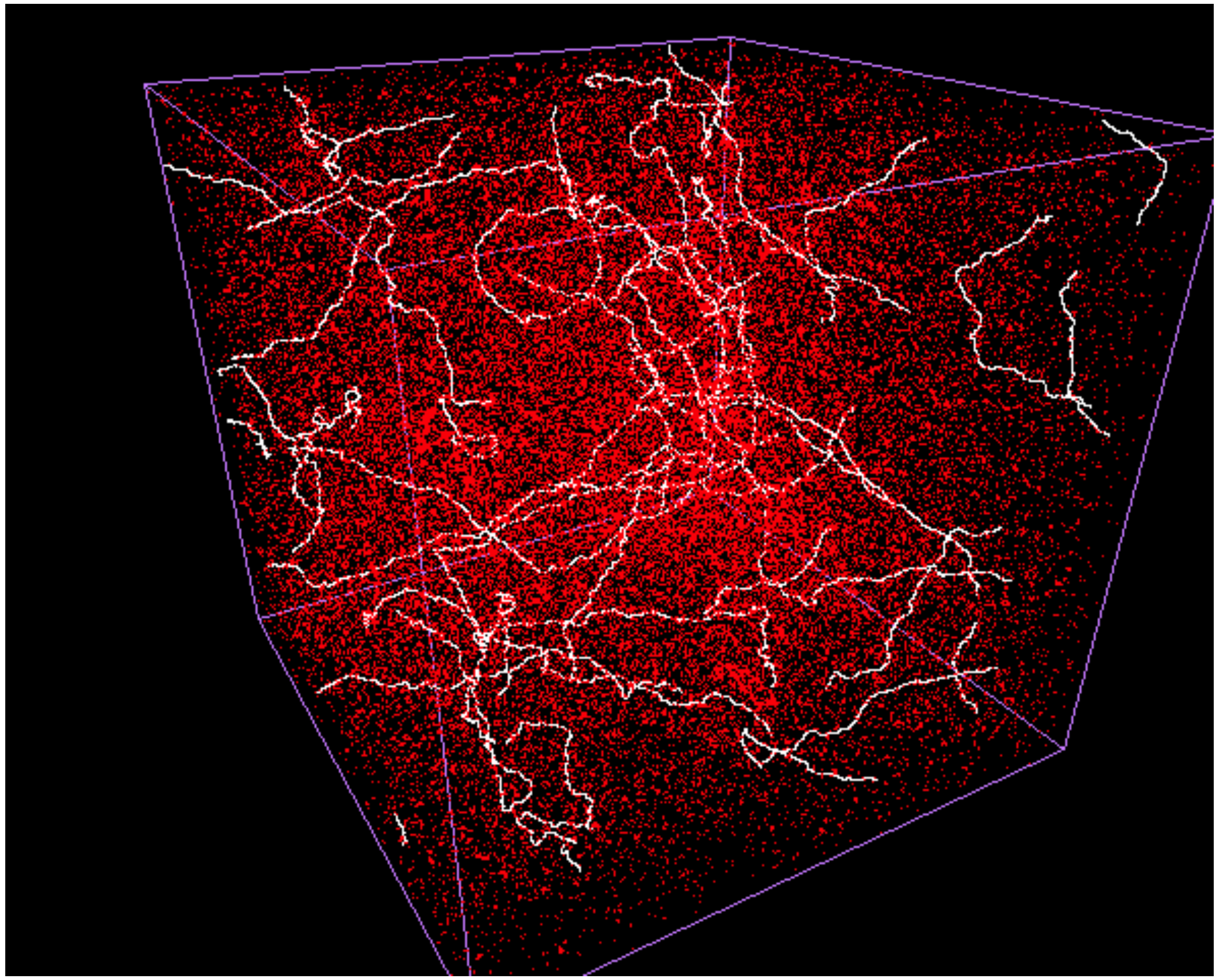
Cosmic strings (or other kind of topological defects) are non-trivial field configurations left-over after the phase transition has completed

A network of cosmic strings emits GWs
(though the results are very model dependent)

$$\Pi_{ij} \sim [\partial\phi_i \partial\phi_j]^{TT}$$

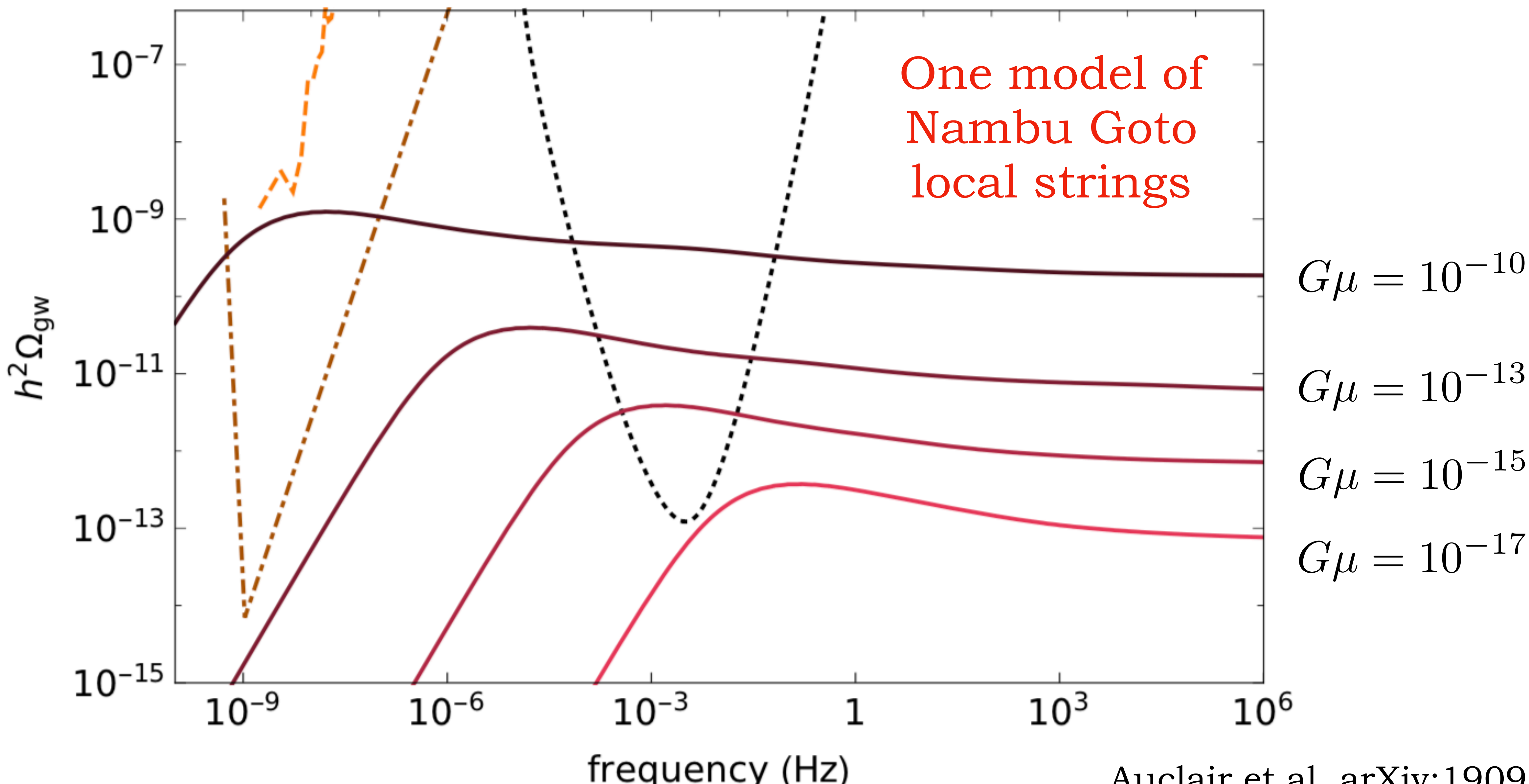






https://curl.irmp.ucl.ac.be/~chris/strings.html

GW signal from cosmic strings

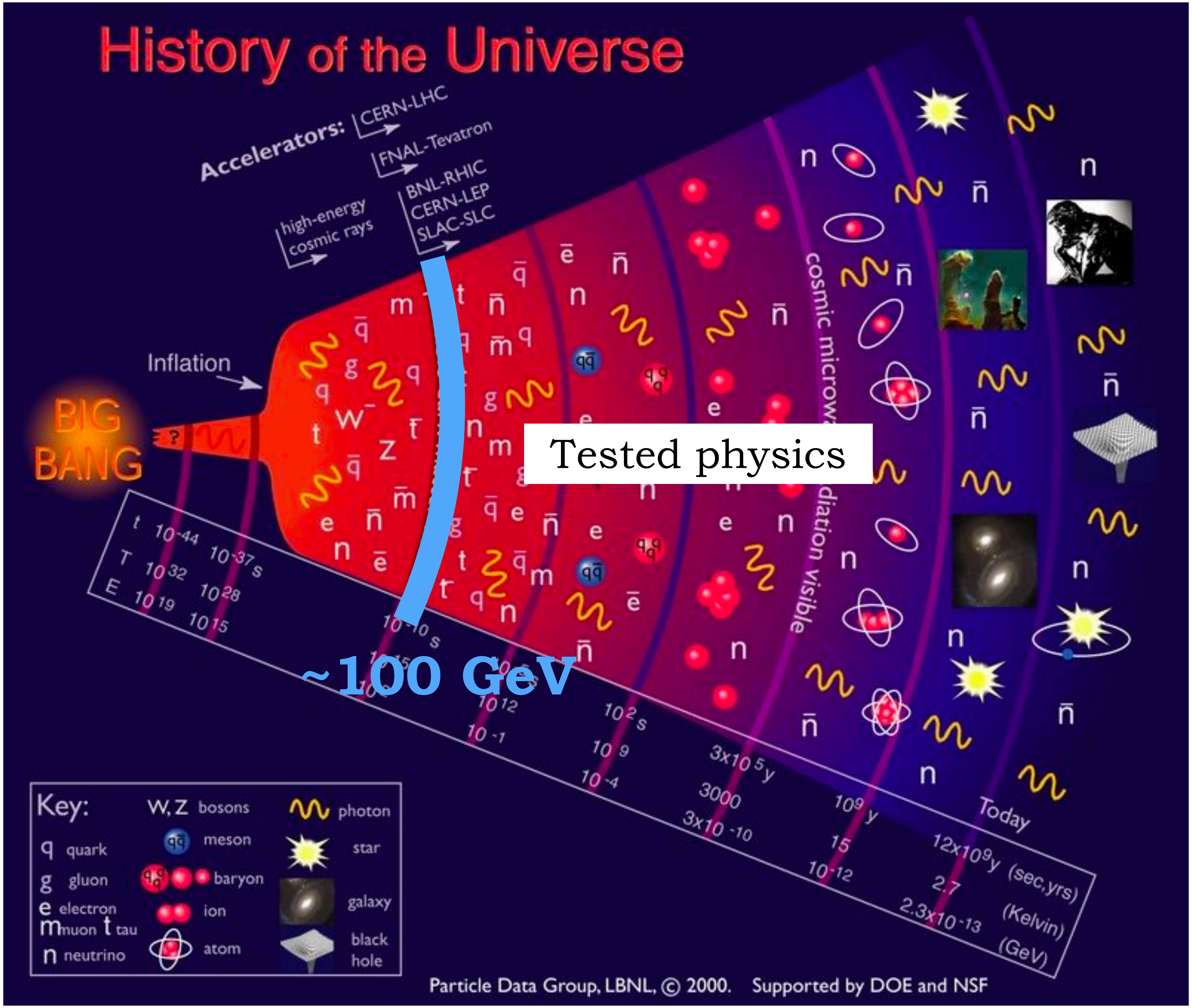


Auclair et al, arXiv:1909.00819

- The signal extends over many frequencies since the GW production is continuous throughout the universe evolution

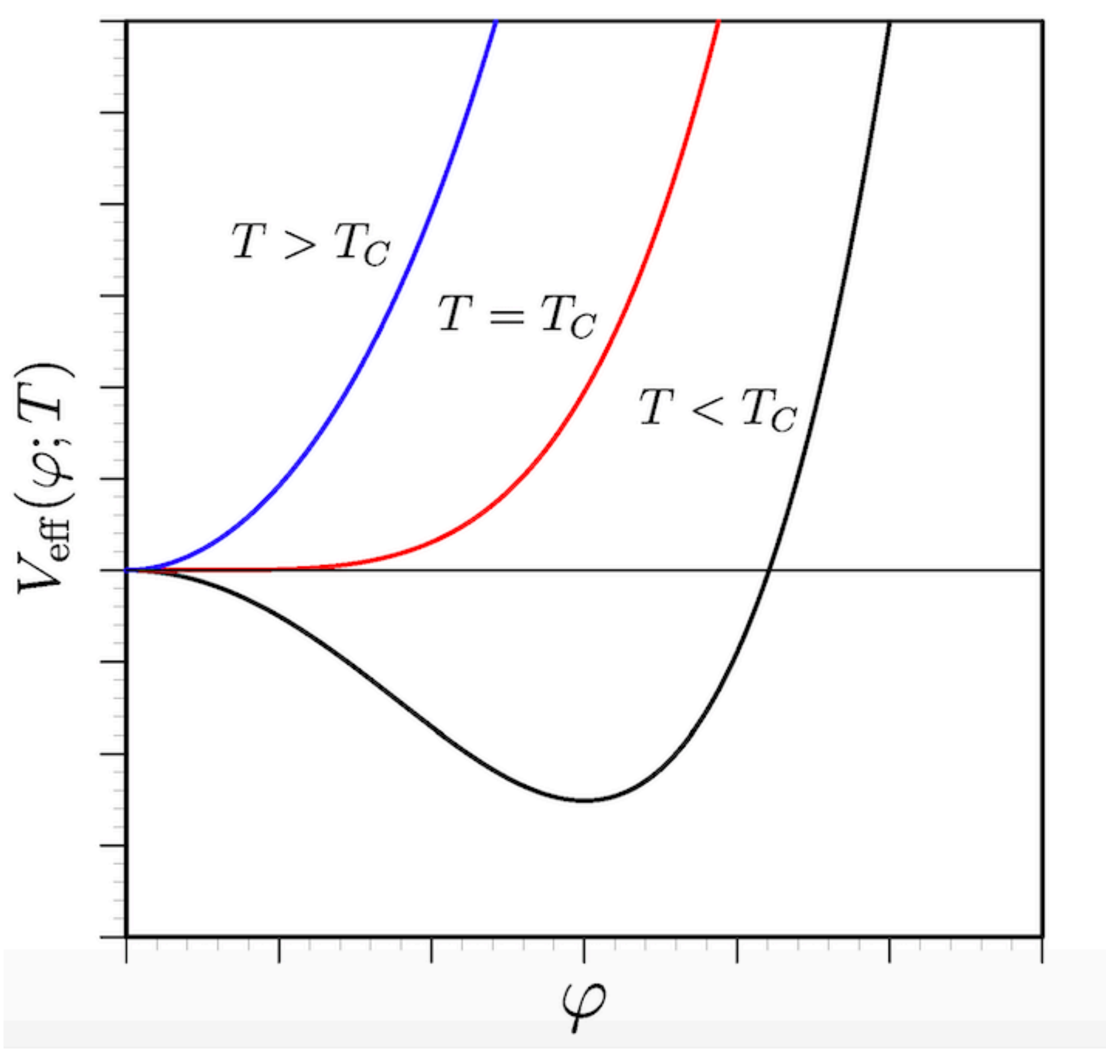
- The energy density of the cosmic string network is a constant fraction of the universe's one

Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands



Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands

Second order phase transition



First order phase transition

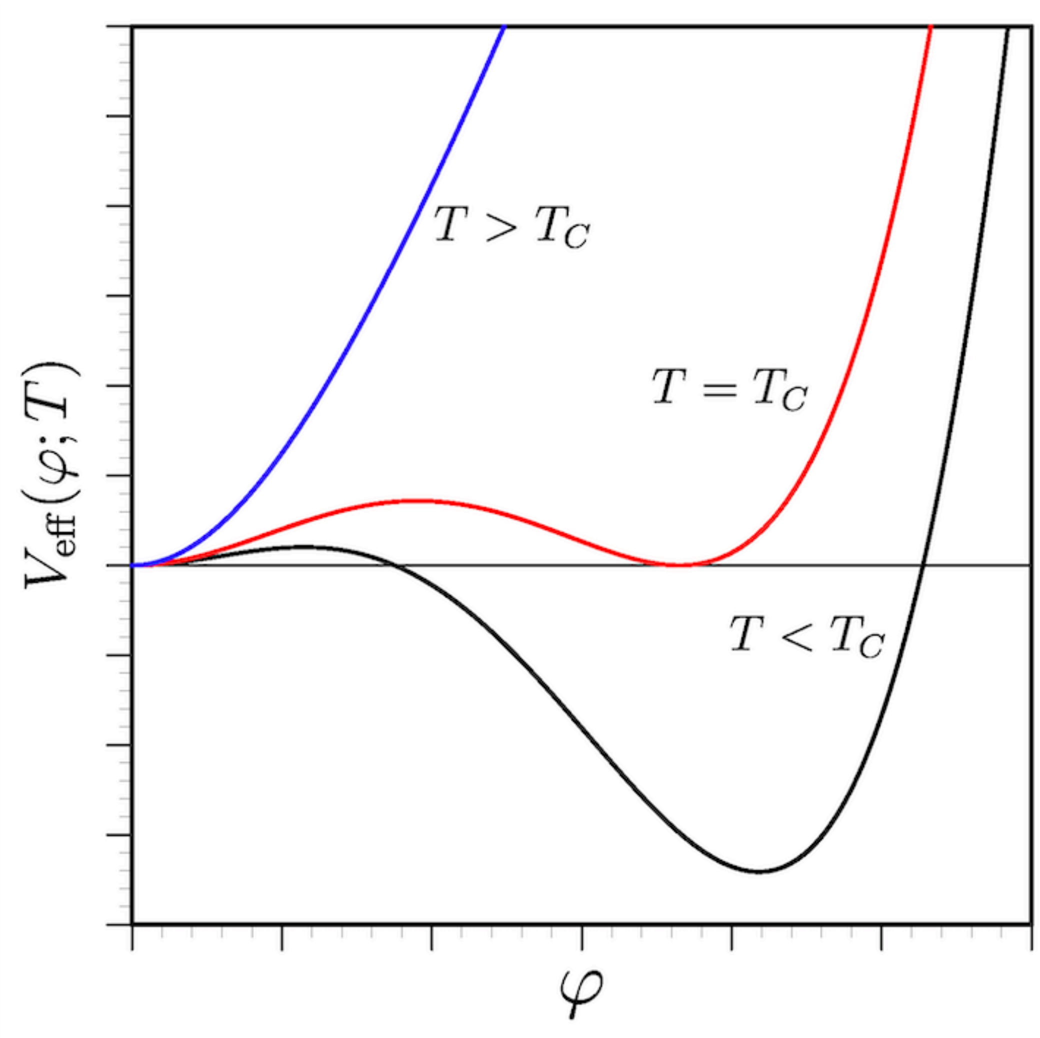
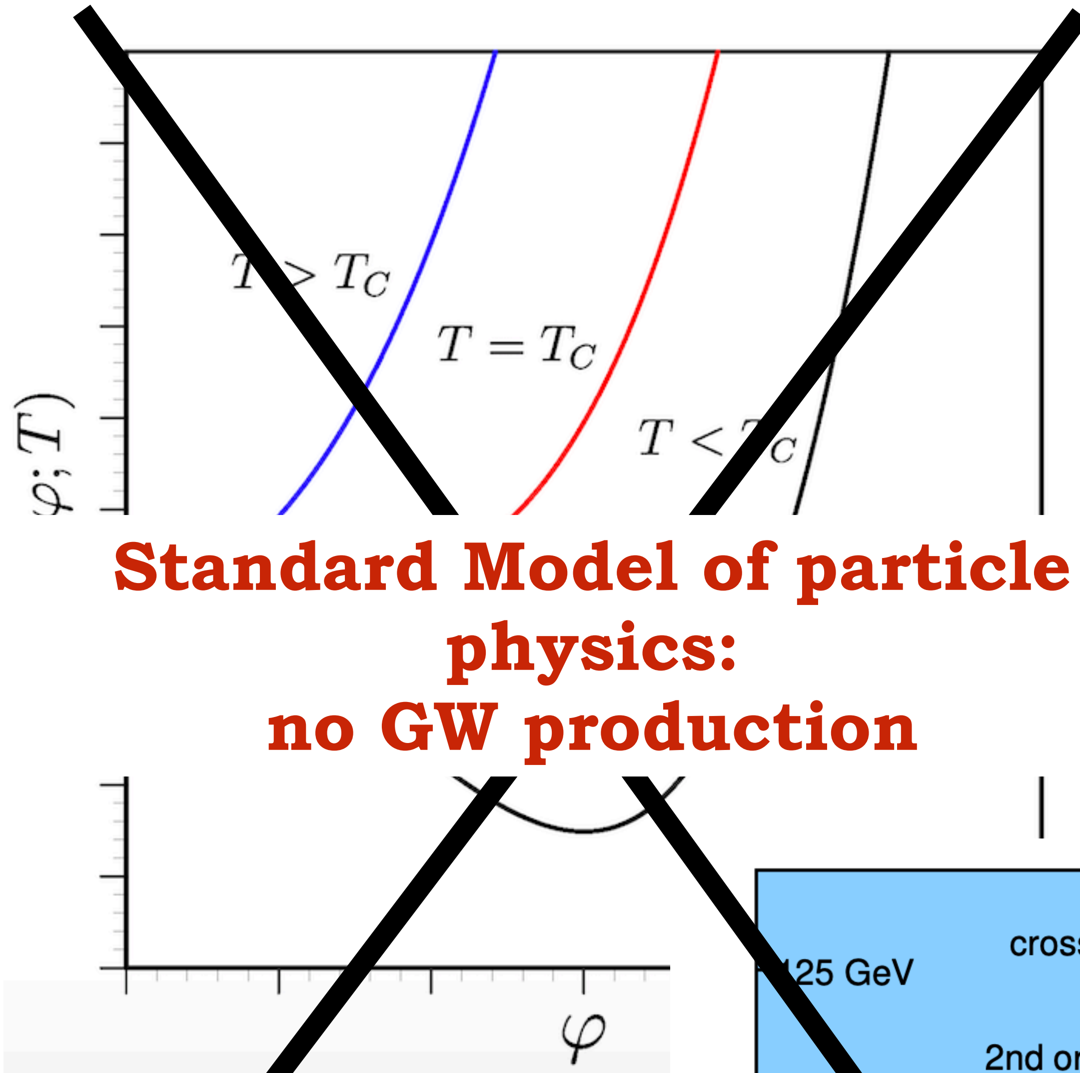


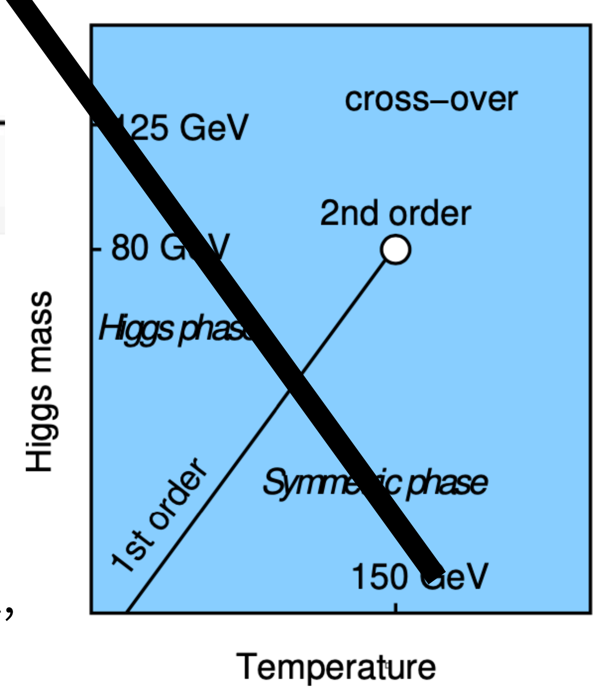
Image credit: E. Senaha, Symmetry 2020

Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands

Second order phase transition

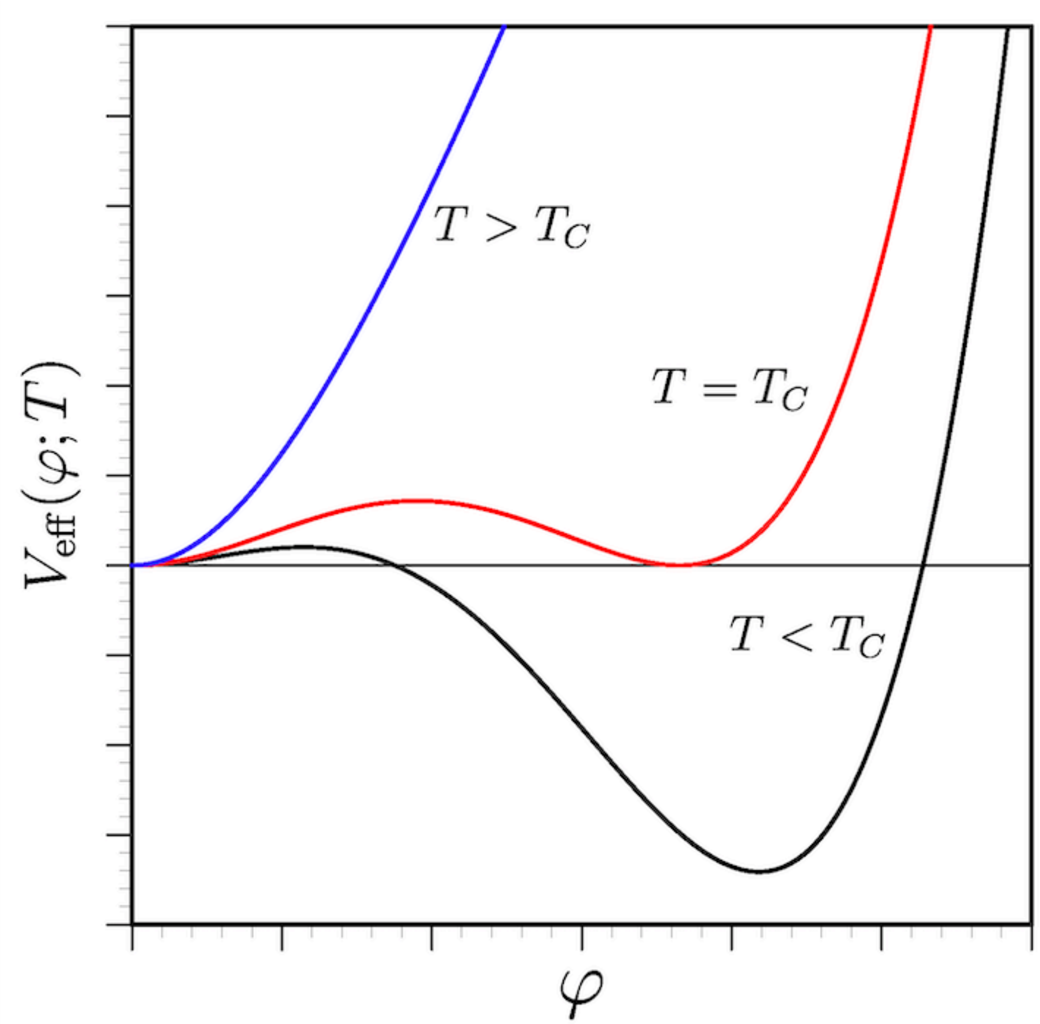


Standard Model of particle physics:
no GW production



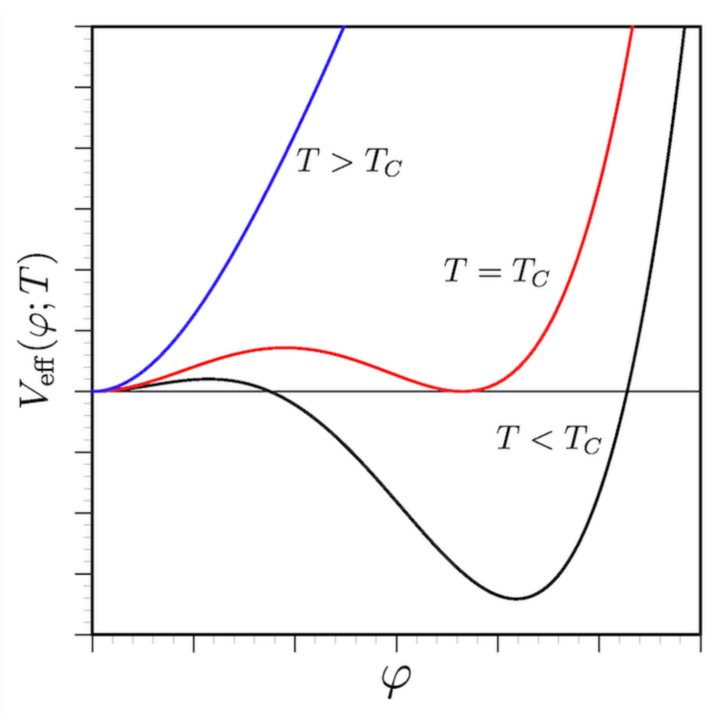
M. Hindmarsh et al,
arXiv:2008.09136

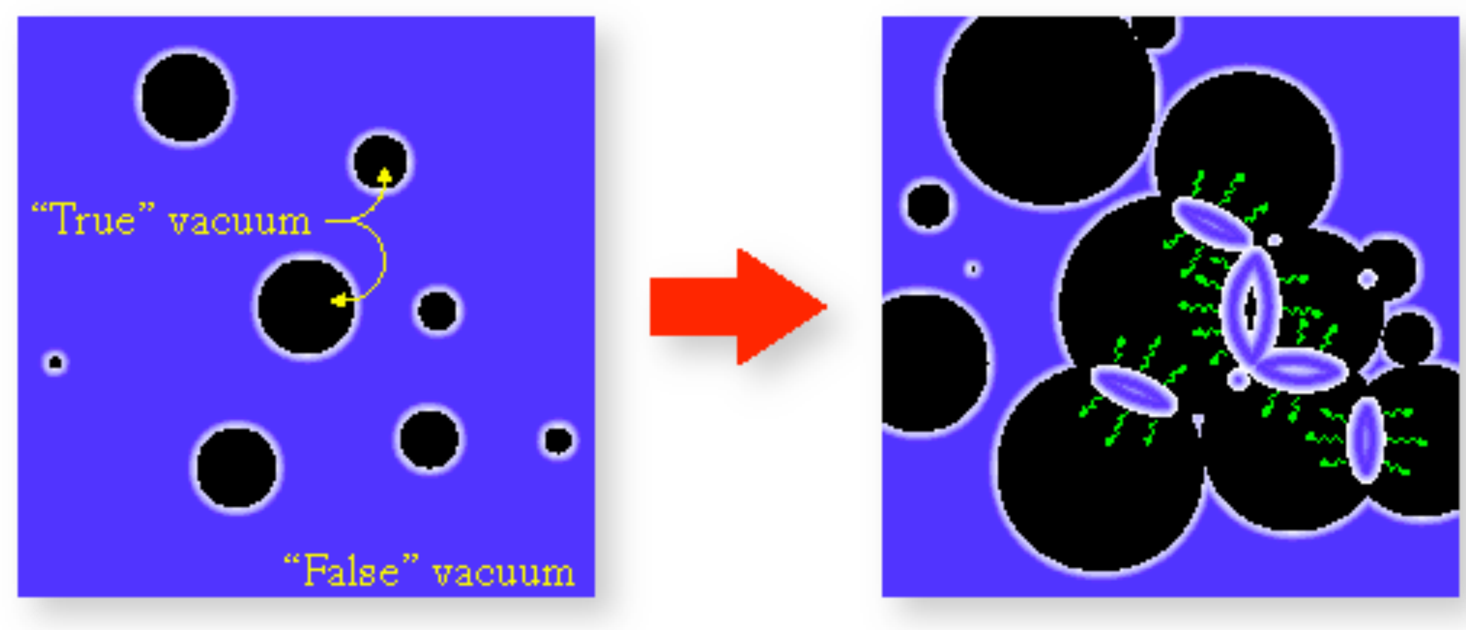
First order phase transition



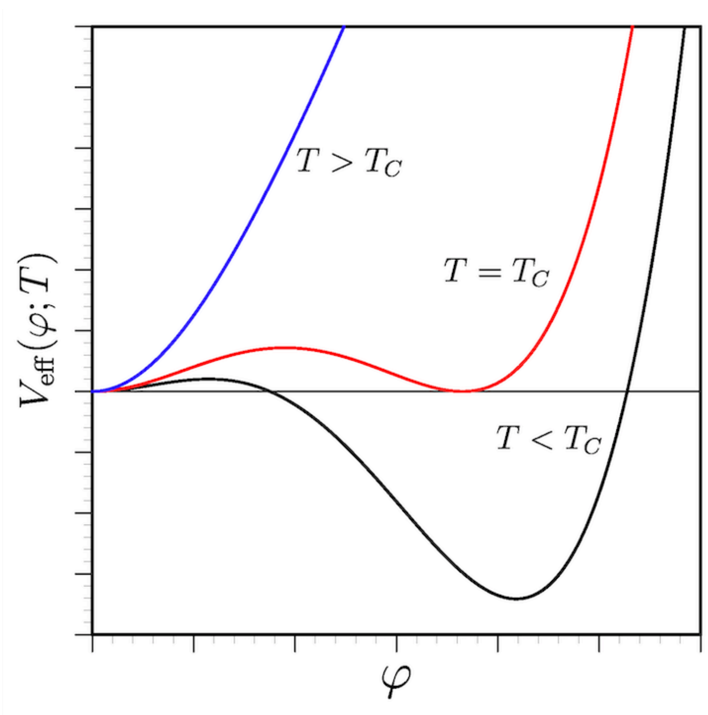
Beyond Standard Model of particle physics:
GW production possible

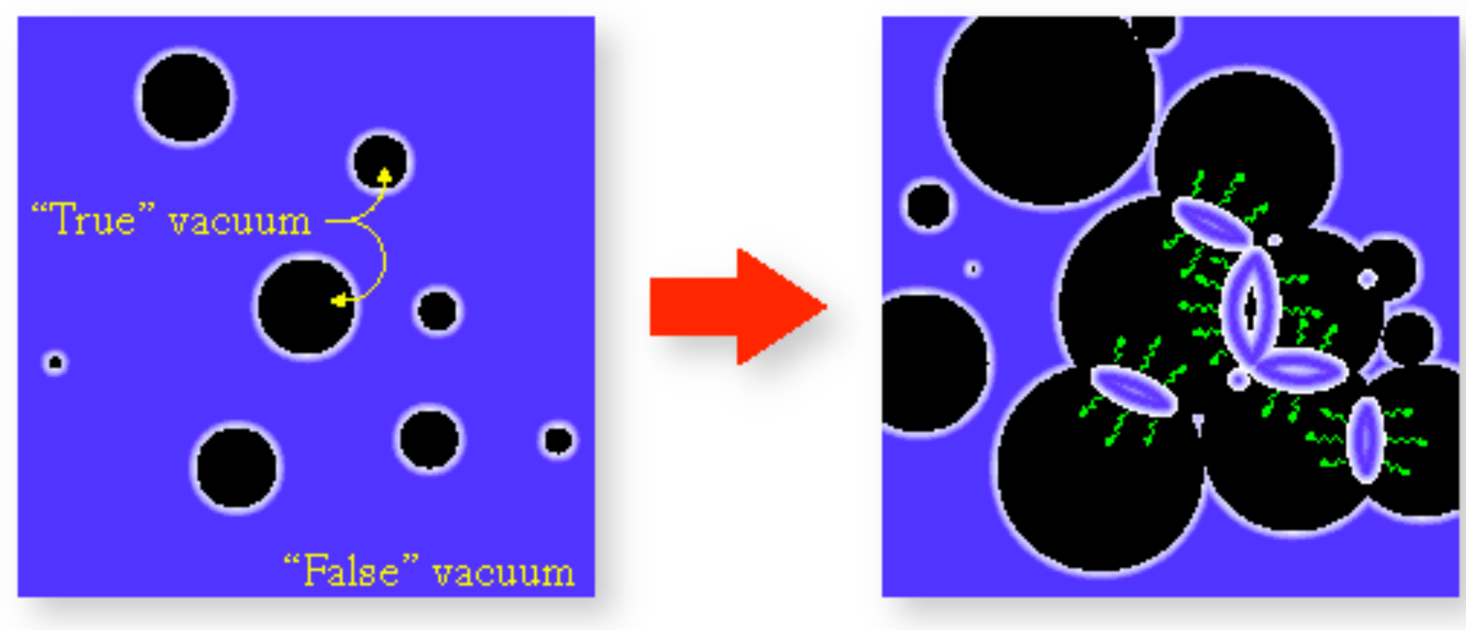
GW signal from the EW phase transition





GW signal from the EW phase transition

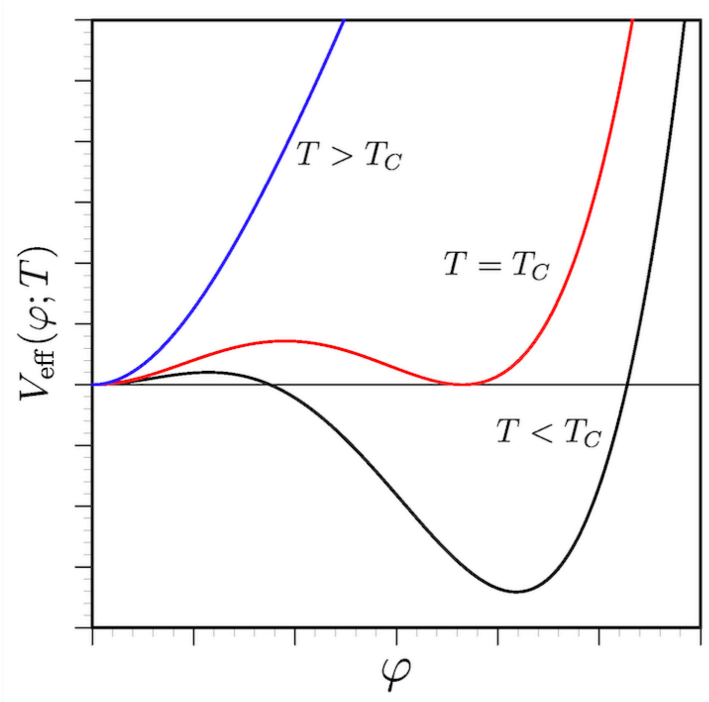


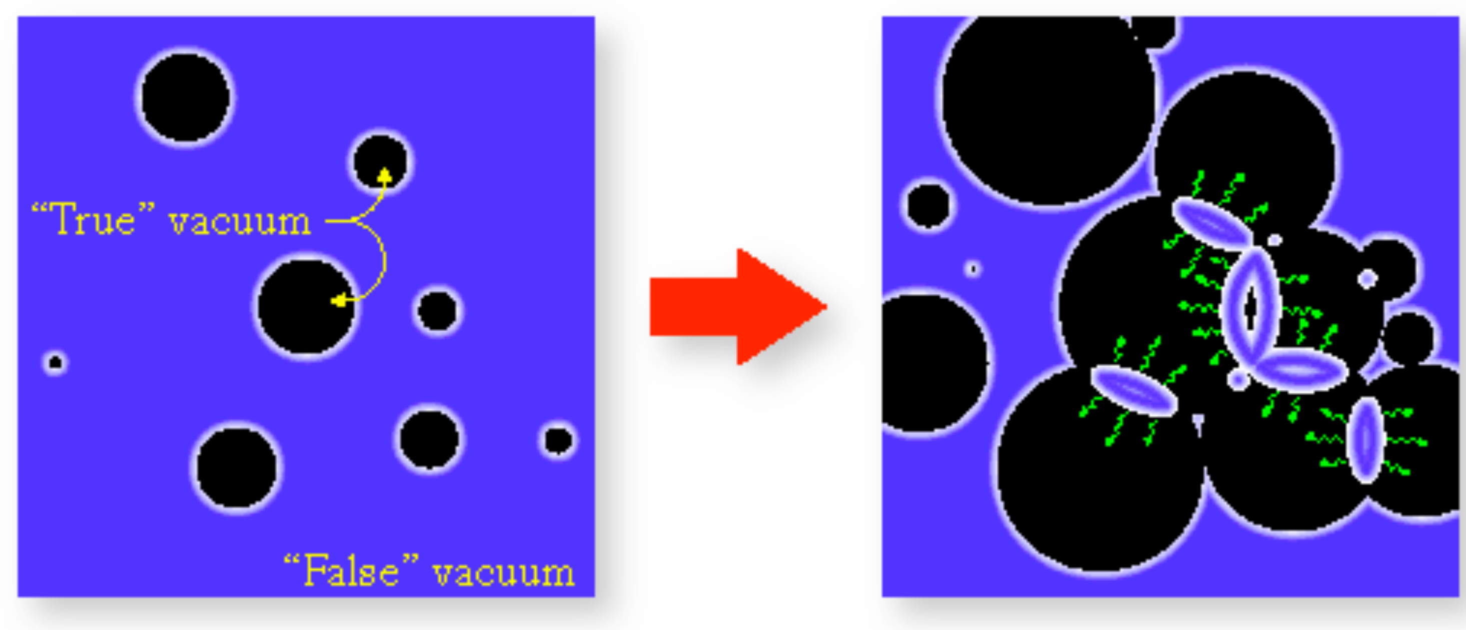


$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$

- collisions of bubble walls $\quad \Pi_{ij} \sim [\partial\phi_i \partial\phi_j]^{TT}$
- sound waves and turbulence in the fluid $\quad \Pi_{ij} \sim [\gamma^2(\rho+p)v_i v_j]^{TT}$
- primordial magnetic fields (MHD turbulence) $\quad \Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$

GW signal from the EW phase transition





The characteristic scale of the tensor stresses determine the GW frequency: connected to the bubble size

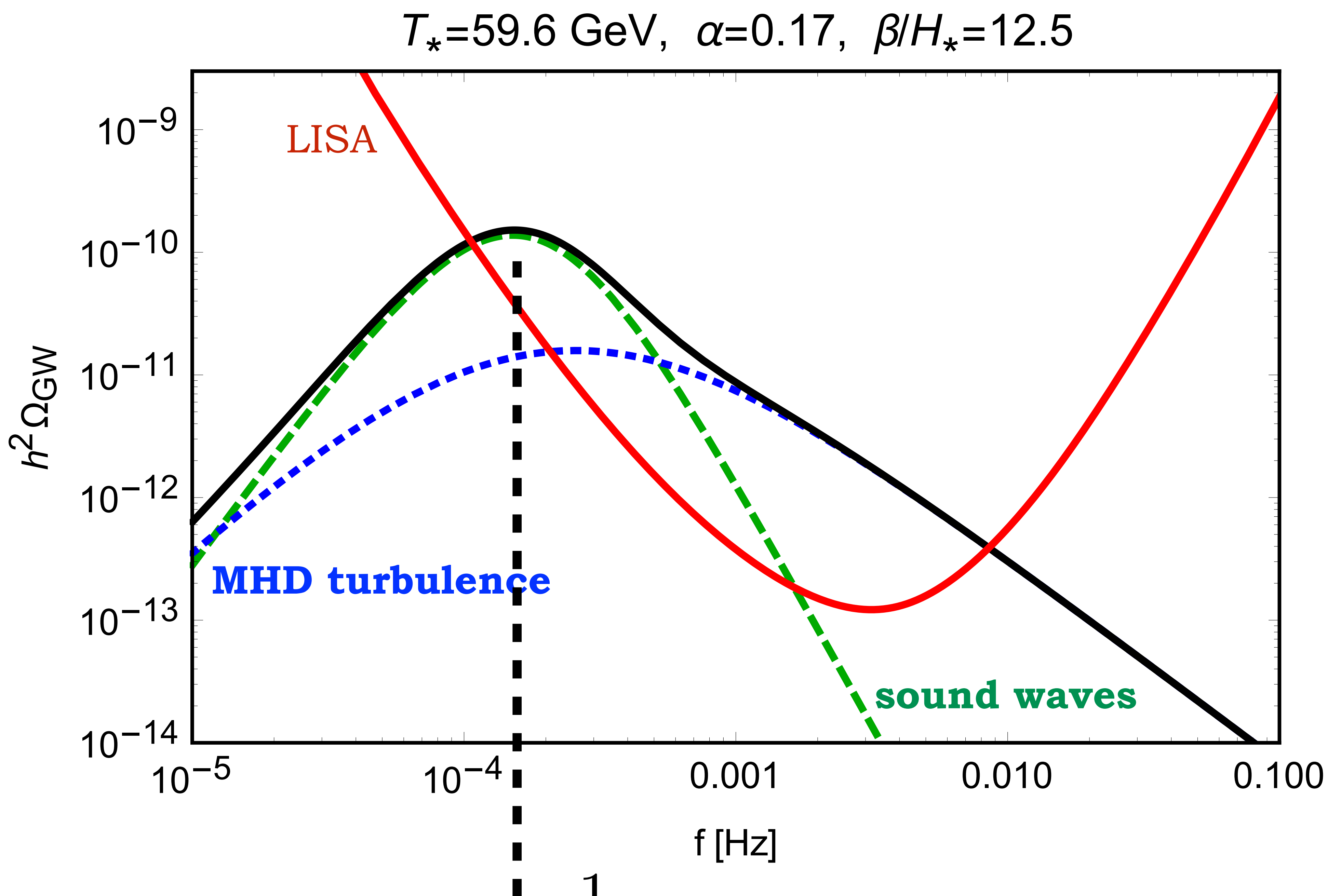
$$\epsilon_* = \ell_* H_*$$

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\rm GeV} \, {\rm Hz}$$

$T_{\rm EW} \sim 100\,{\rm GeV}$ $\quad \ell_* H_* \simeq 0.01$ $\longrightarrow$ $f \sim {\rm mHz}$ LISA

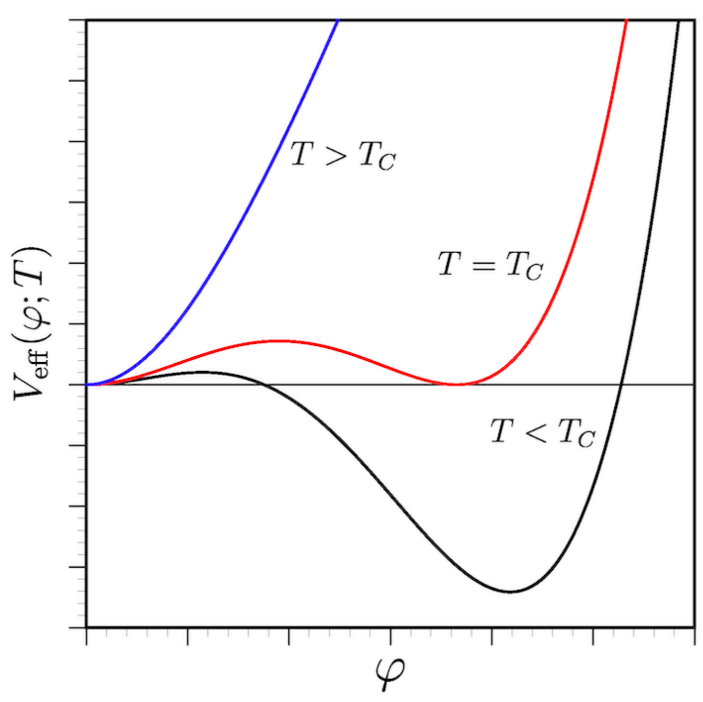
GW signal from the EW phase transition

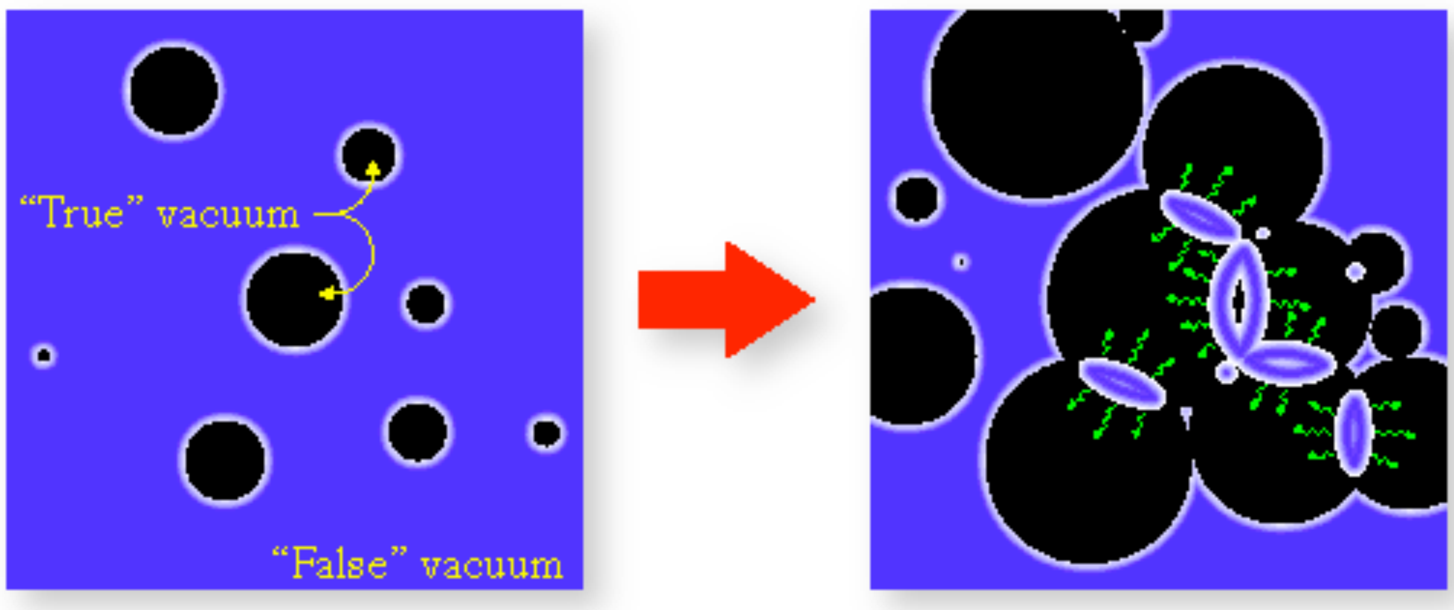
One example of GW signal from the EW phase transition
“Higgs portal” scenario



$$f_* \sim \frac{1}{\ell_*}$$

GW signal from the EW phase transition





The characteristic scale of the tensor stresses determine the GW frequency:
connected to the bubble size

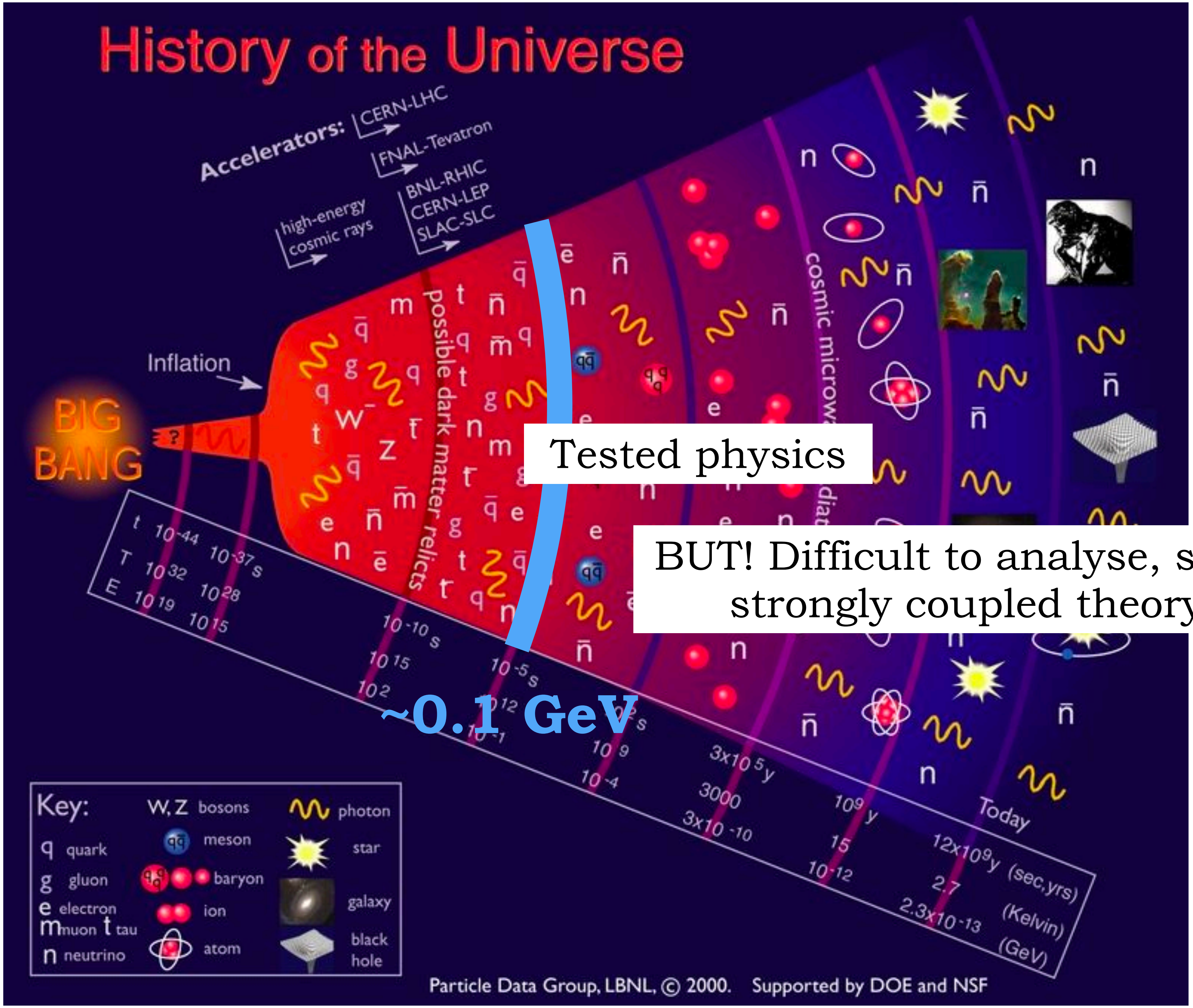
$$\epsilon_* = \ell_* H_*$$

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\rm GeV} \, {\rm Hz}$$

$f \sim 10\,{\rm nHz}$ → $\ell_* H_* \simeq 0.1$ $\quad T_* \sim 0.1\,{\rm GeV}$

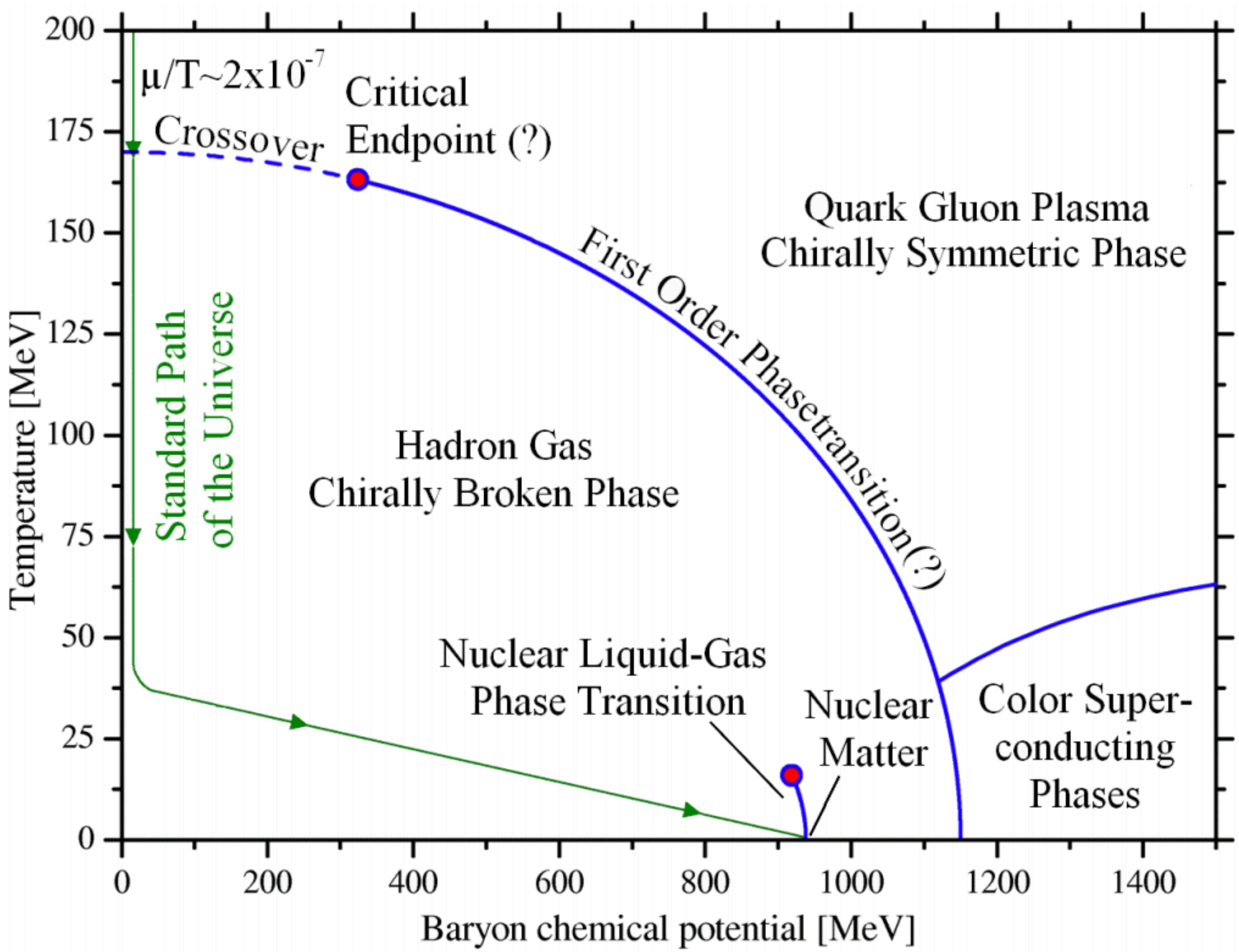
PTA

QCD phase transition: phase transition related to the strong interaction, confinement of quarks into hadrons



QCD phase transition: phase transition related to the strong interaction, confinement of quarks into hadrons

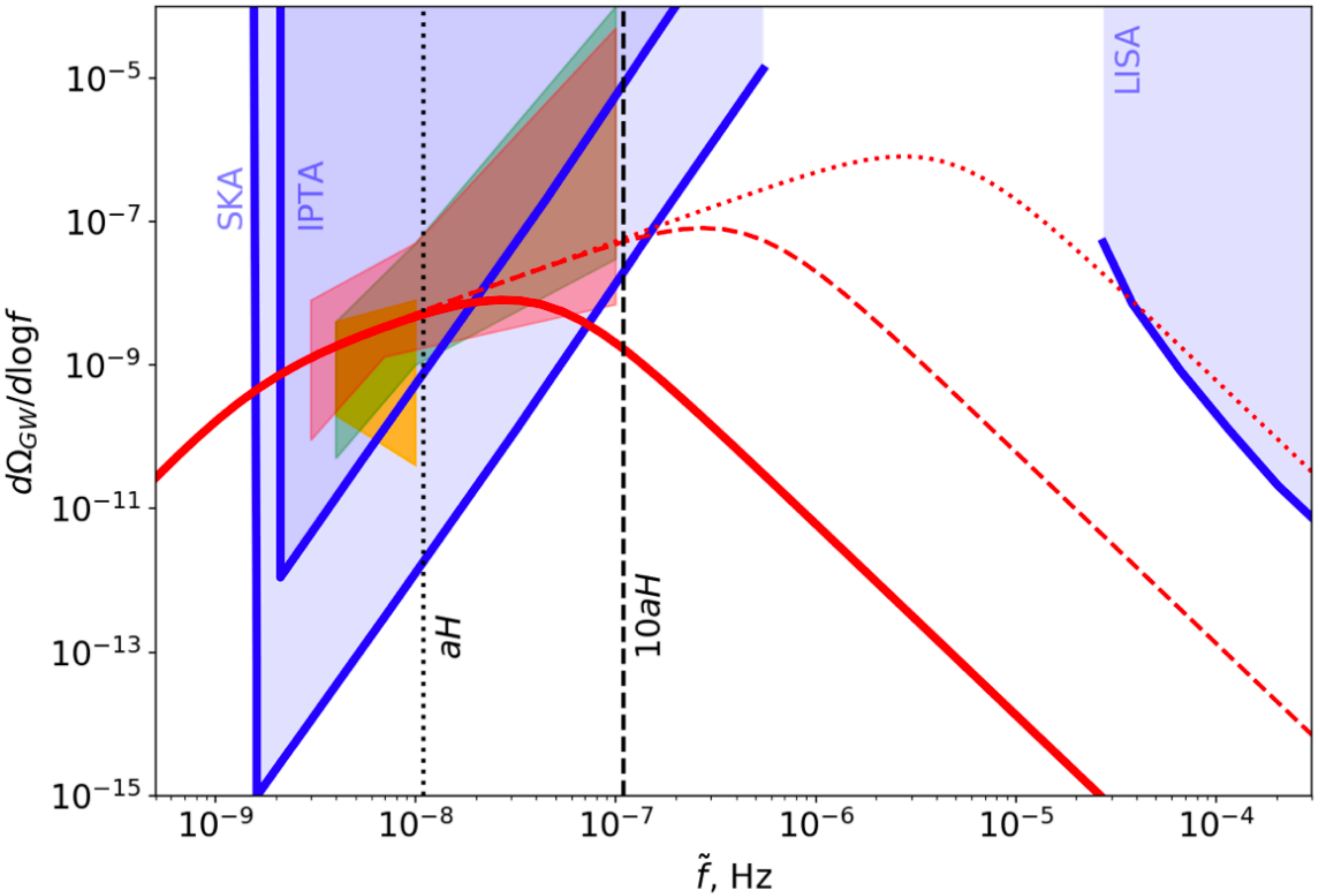
Is it first order?
Probably not, but it depends on the (uncertain) conditions of the early universe



T. Boekel and J. Schaffner-Bielich, arXiv:1105.0832

One example of GW signal from the QCD phase transition: MHD turbulence

A. Neronov et al, arXiv:2009.14174



$$\Pi_{ij} \sim B_i B_j$$

$$R_* \sim \frac{v_A}{H_*}$$

$$v_A \sim \sqrt{\frac{B^2}{\rho^*_{\rm rad}}}$$

- We tune the characteristic size of the stresses to be the one of the “largest processes eddies”, typical of MHD turbulence

- The magnetic field giving rise to the GW signal stays around for the entire universe evolution: at recombination it can modify the CMB spectrum to ease the Hubble tension, and it seeds the magnetic fields observed in matter structures

To summarise:

- Several phase transitions might have occurred in the early universe, leading to GW production

- Inflation: new physics but observationally motivated, extended GW signal in frequency, only accessible by CMB unless one goes beyond the standard slow roll scenario (there are well motivated scenarios!)

- Cosmic strings: new physics but theoretically motivated, GW signal strongly model dependent, extended in frequency, can be accessed/constrained at PTA, LISA, LIGO/Virgo

- Electroweak PT: at the limit of tested physics, GW signal can be accessed/constrained by LISA only for models beyond the standard model of particle physics —> tests of models, complementary to particle colliders

- QCD PT: tested physics but difficult to predict, GW signal can be accessed/constrained by PTA only for models beyond the standard model of particle physics —> test of the early universe

- **SGWBs from the primordial universe might seem speculative but their potential to probe fundamental physics is great and amazing discoveries can be around the corner**