Cosmology from: gravitational waves

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LECTURE 5
GW emission from binaries: probe of the universe expansion

Using GWs to measure the background expansion of the universe

$$H(z) = H_0\sqrt{\Omega_M\,(z+1)^3 + (1-\Omega_\Lambda-\Omega_M)\,(z+1)^2 + \Omega_\Lambda\,\exp\left[-\frac{3w_a z}{z+1}\right](z+1)^{3(1+w_0+w_a)}}$$

- Hubble factor, written for one specific model of dark energy
- No contribution from radiation, negligible in the late universe

$$d_L(z) = (1+z)\,\mathcal{G}\left(\int_0^z \frac{dz'}{H(z')}\right)$$

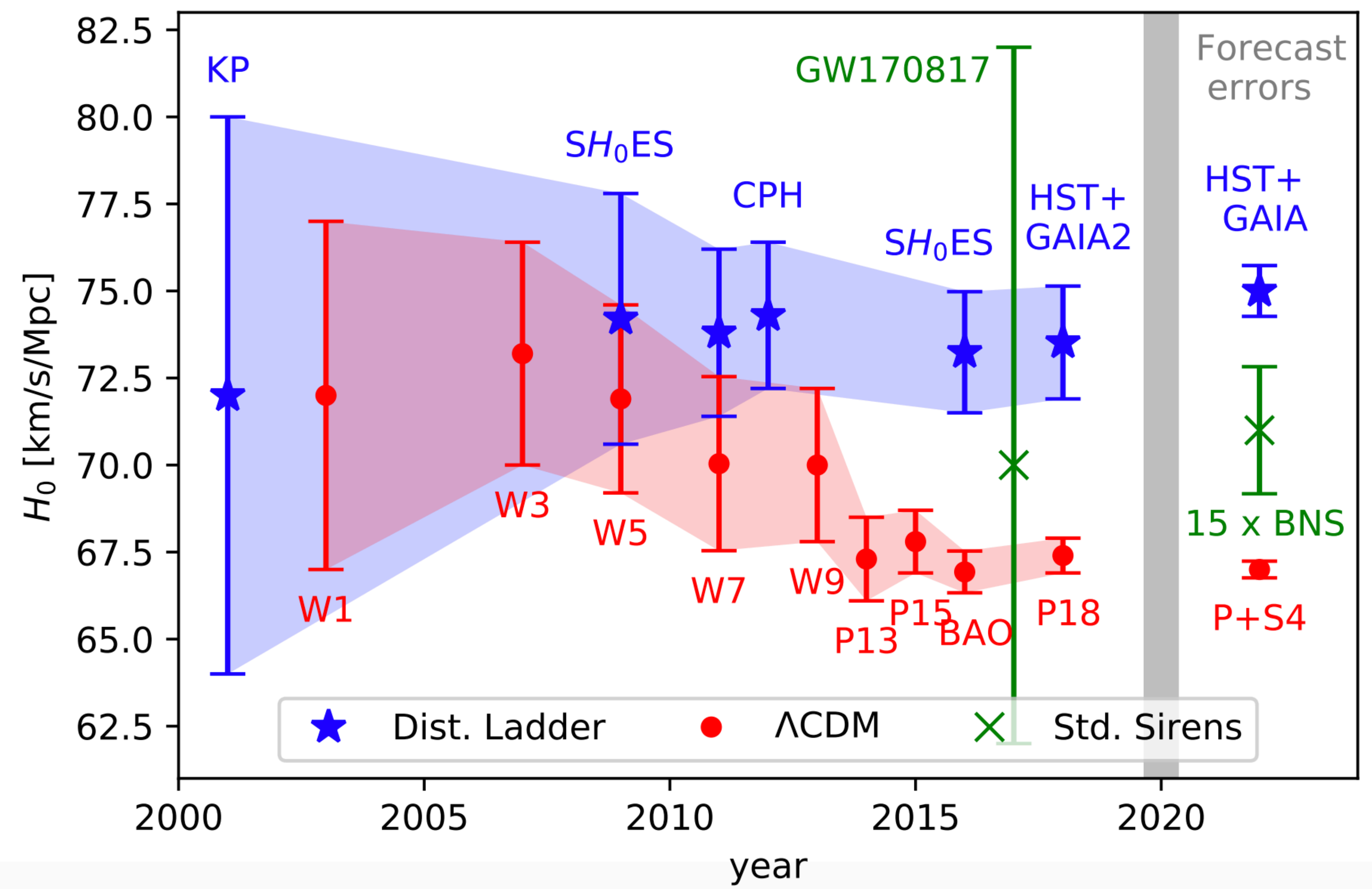
$\mathcal{G} = 1$

Flat space hyper surfaces

Measuring the luminosity distance as a function of redshift provides access to the cosmological parameters, in particular H_0 at low redshift

$z \ll 1$ $\qquad$ $c\,z = H_0\,d_L(z)$ $\qquad$ Hubble law

Using GWs to measure the background expansion of the universe

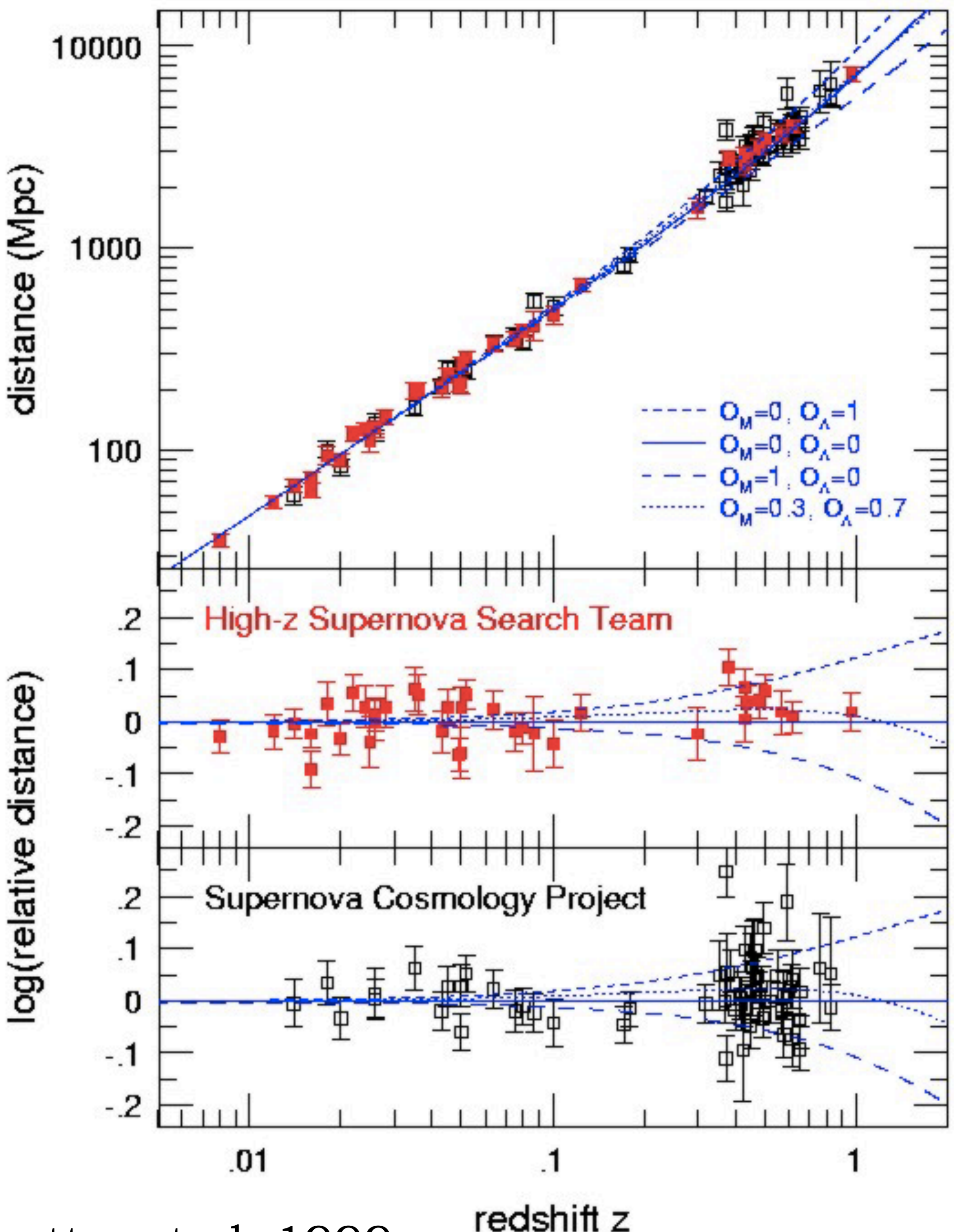


- Remarkable agreement if ones thinks that the two methods measure physical phenomena that are 13 billion years apart
- Not enough in the era of precision cosmology
- Does this require new physics?

J.M. Ezquiaga and M. Zumacalaregui, arXiv:1807.09241 Nissanke et al, arXiv:1307.2638

Measurement of $d_L(z)$: standard candles

Nobel prize in physics 2011:
discovery of the late-time acceleration of the universe



Riess et al. 1998 Perlmutter et al. 1999

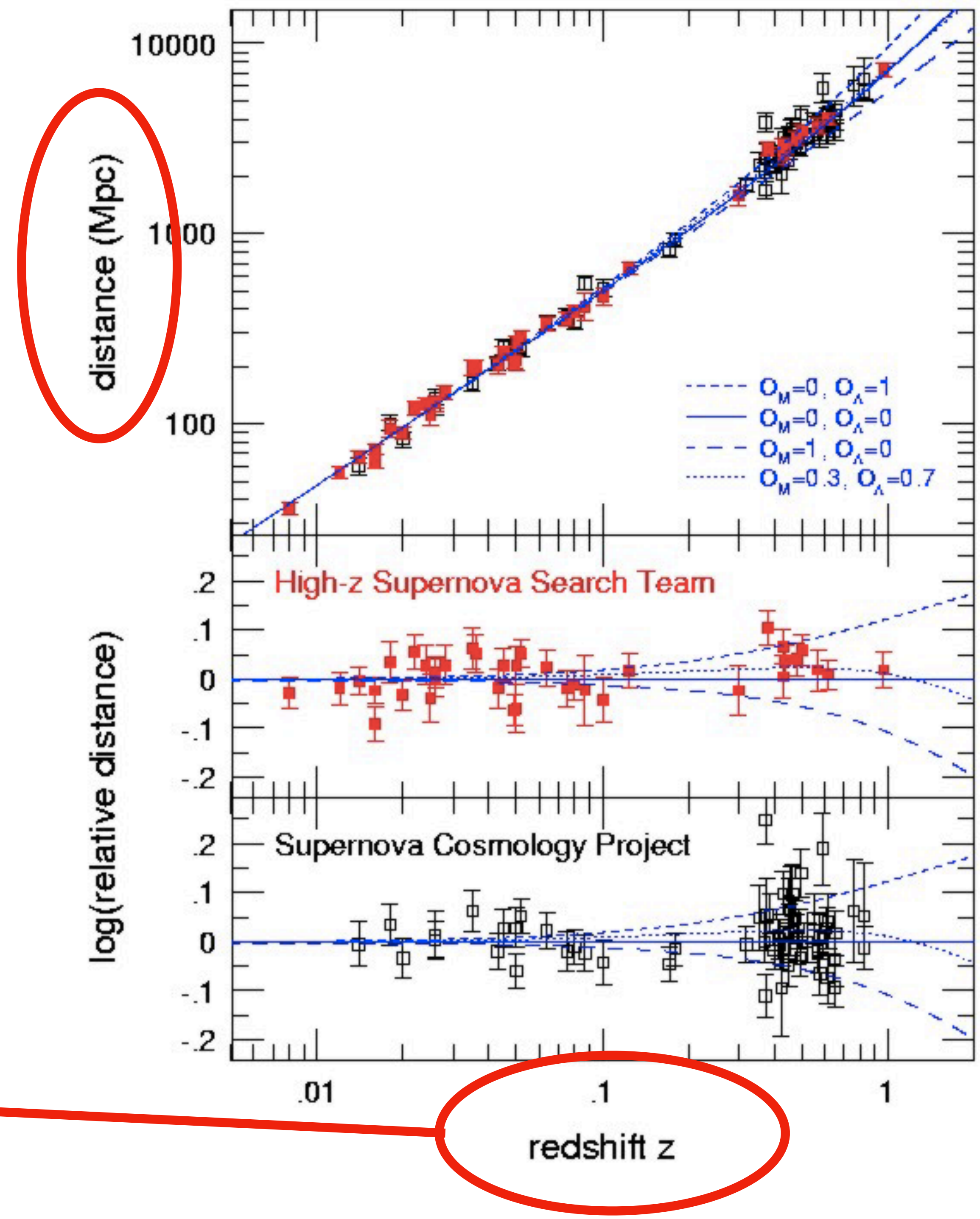
Measurement of $d_L(z)$: standard candles

Nobel prize in physics 2011:
discovery of the late-time acceleration of the universe

$$d_L(z) = \sqrt{\frac{L}{4\pi\mathcal{F}}}$$

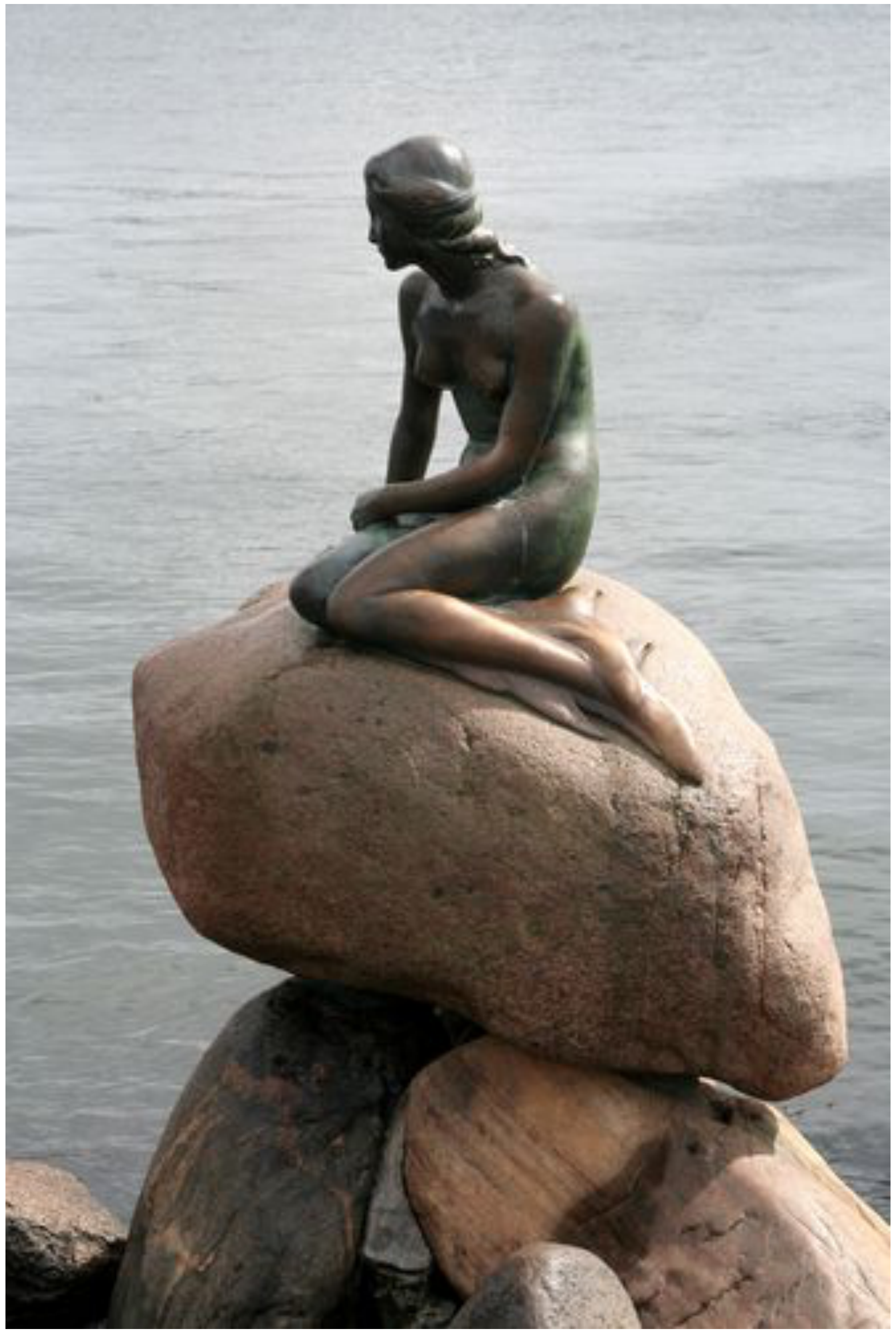
- Flux measured directly
- Intrinsic luminosity known from calibration
- Measurement of the luminosity distance is NOT SO EASY

Redshift measured directly
from the optical emission
EASY!



Measurement of $d_L(z)$: standard sirens

GW emission by compact binaries
can also be used to test the expansion of the universe

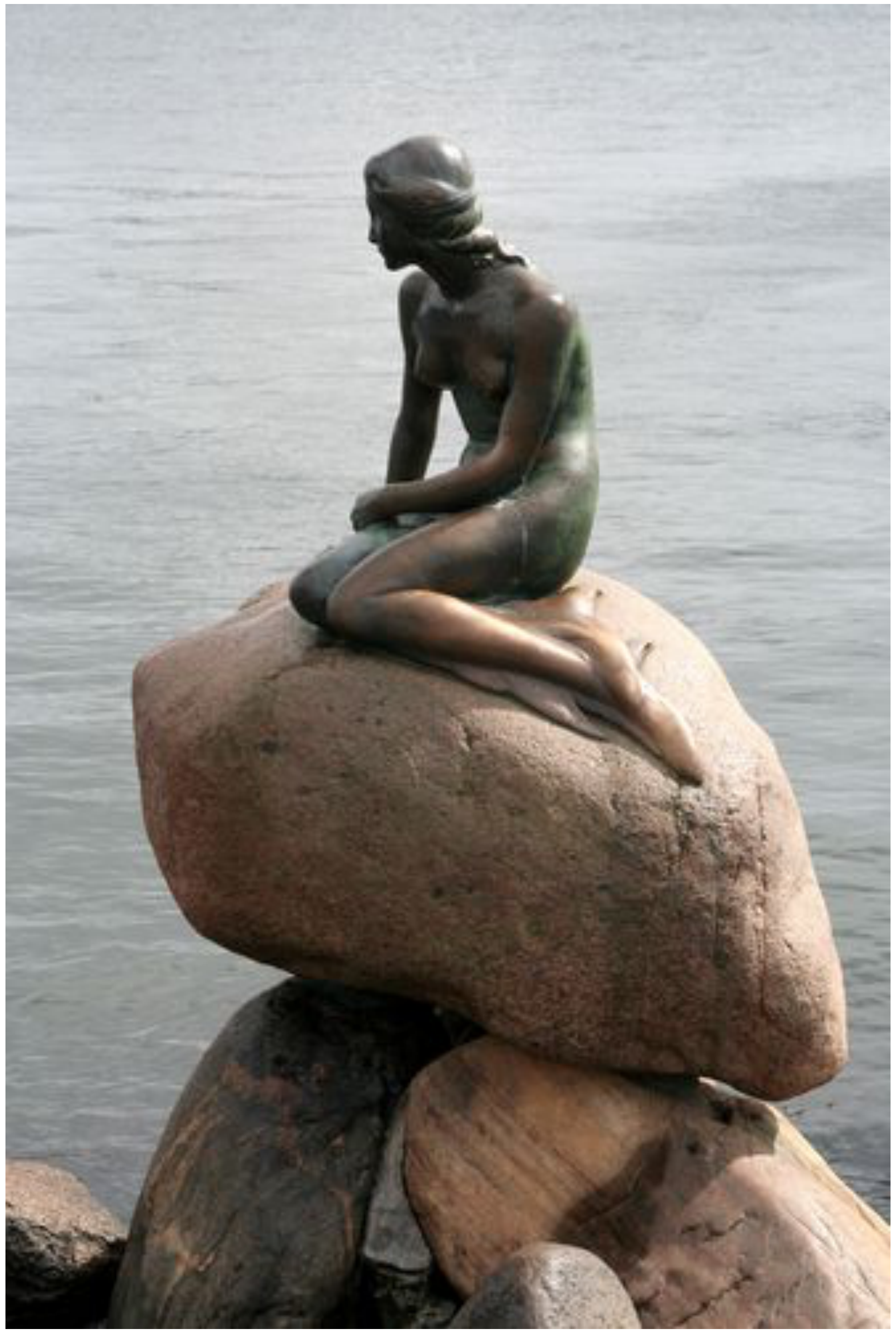


B. Schutz, Nature 323, 310 (1986)

Measurement of $d_L(z)$: standard sirens

GW emission by compact binaries
can also be used to test the expansion of the universe

- Measurement of the luminosity distance: no calibration needed, EASY AND DIRECT
- Measurement of the redshift: IMPOSSIBLE!



B. Schutz, Nature 323, 310 (1986)

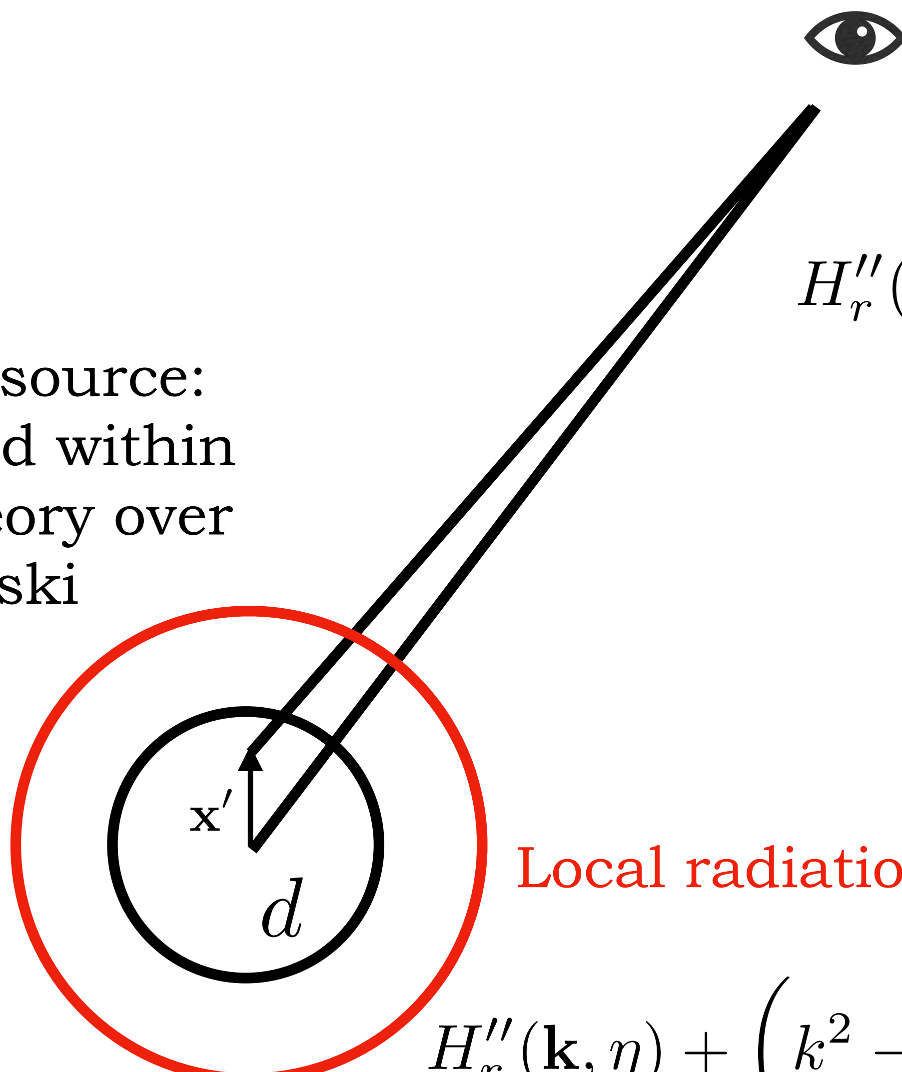
Inspiral of compact binaries at cosmological distance

To reach the observer:
free propagation in
FLRW space-time

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \cancel{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$

$$H_r(\mathbf{k},\eta) = a\, h_r(\mathbf{k},\eta)$$

Close to the source:
we have solved within
linearised theory over
Minkowski



Local radiation zone

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \cancel{\frac{a''}{a}}\right) H_r(\mathbf{k},\eta) = 16\pi G\, \cancel{a^3}\, \Pi_r(\mathbf{k},\eta)$$

$$h_+(t,\theta,\varphi) = \frac{4}{r}(G\,M_c)^{5/3}[\pi f(t_{\rm ret})]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(t_{\rm ret}))$$

$$h_\times(t,\theta,\varphi) = \frac{4}{r}(G\,M_c)^{5/3}[\pi f(t_{\rm ret})]^{2/3}\,\cos\theta\,\sin(2\Phi(t_{\rm ret}))$$

Inspiral of compact binaries at cosmological distance

$$h_+(t_S,\theta,\varphi) = \frac{4}{a(t_O)r}(G\,M_c)^{5/3}[\pi f(t_S^{\rm ret})]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(t_S^{\rm ret}))$$

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Propagation effect at
the observer

"Gravitational Waves", M. Maggiore, Oxford University Press 2008

Inspiral of compact binaries at cosmological distance

$$h_+(t_S,\theta,\varphi) = \frac{4}{a(t_O)r}(G\,M_c)^{5/3}[\pi f(t_S^{\rm ret})]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(t_S^{\rm ret}))$$

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Still measured by the source clock,
we want it in the observer's clock

$$f_S = f_O(1+z)$$

$$\Phi_S = \Phi_O$$

The phase is constant along null geodesics

$$\Phi_S(t_S) = 2\pi\int_{t_{c,S}}^{t_S} dt'_S f_S(t'_S) = 2\pi\int_{t_{c,O}}^{t_O} dt'_O f_O(t'_O) = \Phi_O(t_O)$$

$$dt_O = (1+z)dt_S$$

"Gravitational Waves", M. Maggiore, Oxford University Press 2008

Inspiral of compact binaries at cosmological distance

$$h_+(t_O,\theta,\varphi) = \frac{4}{a(t_O)r}(G\,M_c)^{5/3}[\pi f(t_O^{\rm ret})(1+z)]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(t_O^{\rm ret}))$$

$$h_\times(t_O,\theta,\varphi) = \frac{4}{a(t_O)r}(G\,M_c)^{5/3}[\pi f(t_O^{\rm ret})(1+z)]^{2/3}\cos\theta\,\sin(2\Phi(t_O^{\rm ret}))$$

$$\frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f_O]^{2/3}$$

$$\mathcal{M}_c = (1+z)M_c$$

Redshifted chirp mass

Inspiral of compact binaries at cosmological distance

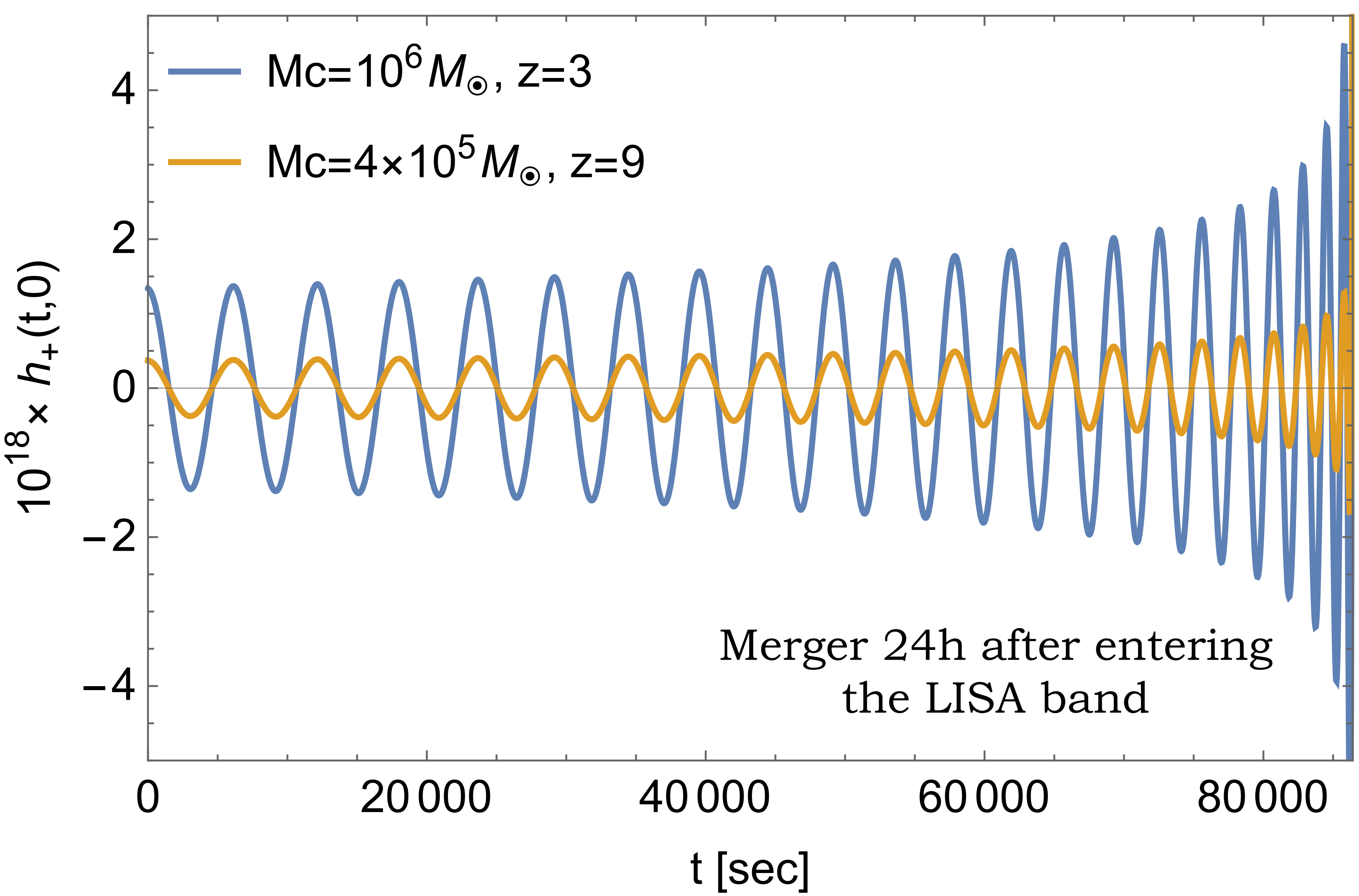
$$h_+(\tau,\theta,\varphi) = \frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f(\tau)]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(\tau))$$

$$h_\times(\tau,\theta,\varphi) = \frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f(\tau)]^{2/3}\cos\theta\,\sin(2\Phi(\tau))$$

time to coalescence at the observer

The signal does depend on redshift, but there is a degeneracy among the redshift and the true chirp mass

$$\mathcal{M}_c = (1+z)M_c$$



Inspiral of compact binaries at cosmological distance

$$h_+(\tau,\theta,\varphi) = \frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f(\tau)]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(\tau))$$

$$h_\times(\tau,\theta,\varphi) = \frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f(\tau)]^{2/3}\,\cos\theta\,\sin(2\Phi(\tau))$$

What happens to the relation expressing the time variation of the frequency?

$$\dot{f}_S = \frac{96\pi^{8/3}}{5}(G\,M_c)^{5/3} f_S^{11/3}$$

$$(1+z)\frac{d}{dt_O}[(1+z)f_O(t_O)] = \frac{96\pi^{8/3}}{5}(G\,M_c)^{5/3}[f_O(1+z)]^{11/3}$$

If the redshift is constant $\qquad \dot{f}_O = \frac{96\pi^{8/3}}{5}(G\,\mathcal{M}_c)^{5/3} f_O{}^{11/3}$

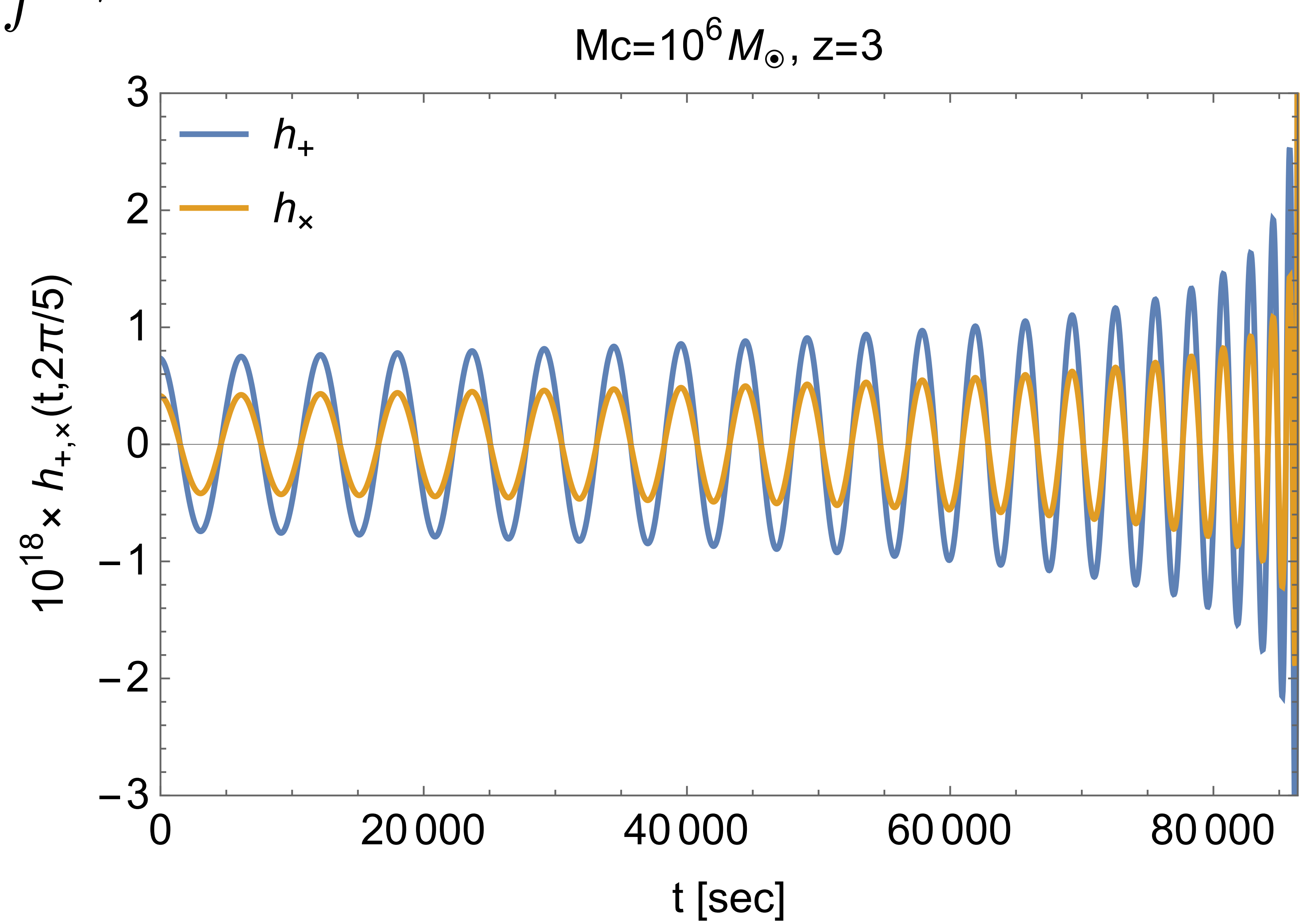
Measurement of $d_L(z)$

$$h_+(\tau,\theta,\varphi) = \frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f(\tau)]^{2/3}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\Phi(\tau))$$

$$h_\times(\tau,\theta,\varphi) = \frac{4}{d_L(z)}(G\,\mathcal{M}_c)^{5/3}[\pi f(\tau)]^{2/3}\,\cos\theta\,\sin(2\Phi(\tau))$$

$$\dot{f} = \frac{96\pi^{8/3}}{5}(G\,\mathcal{M}_c)^{5/3} f^{11/3}$$

- From f and $\dot{f}$ measure the redshifted chirp mass
- From the ratio among h_+ and $h_\times$ measure the inclination of the orbit
- Get a direct measurement of d_L



Inspiral of compact binaries at cosmological distance

Note that in general the redshift is not constant, and in certain cases GW observation can be precise enough to be sensitive to redshift variation

$$1 + z = \frac{a_O}{a_S}\left[1 + \mathbf{v}_S \cdot \mathbf{n} - \mathbf{v}_O \cdot \mathbf{n} + \Psi_O - \Psi_S - \int_{t_S}^{t_O} dt(\dot{\Phi} + \dot{\Psi})\right]$$

1. The background expansion of the universe varies during the time of observation of the binary
2. The redshift perturbations vary in time during the time of observation of the binary

These terms lead to a -4PN effect in the phase of the signal, and a variation in the amplitude, which are proportional to

$$Y(z_c) = \frac{1}{2}\left[H_0 - \frac{H_S}{1 + z_c} + \frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1 + z_c} - \dot{\mathbf{v}}_O \cdot \mathbf{n}\right]$$

A. Nishizawa et al, arXiv:1110.2865

C. Bonvin et al, arXiv:1609.08093

Inspiral of compact binaries at cosmological distance

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$$Y(z_c) = \frac{1}{2}\left[H_0 - \frac{H_S}{1 + z_c} - \frac{\dot{\mathbf{v}}_S \cdot \mathbf{n}}{1 + z_c} - \dot{\mathbf{v}}_O \cdot \mathbf{n}\right]$$

The peculiar acceleration dominates the effect, hiding the dependence on the cosmological parameters, so it doesn't work

But at least one can measure the peculiar acceleration of the binary…

K. Inayoshi et al, arXiv:1702.06529 N. Tamanini et al, arXiv:1907.02018 A. Toubiana et al, arXiv:2010.06056

Measurement of $d_L(z)$

$$\mathcal{M}_c = (1+z)M_c$$

How can we break this degeneracy and use GW emission to build the Hubble diagram $d_L(z)$?

There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- Direct method: directly identify the galaxy hosting the event, via the measurement of a (transient) electromagnetic counterpart
- Statistical method: cross-correlate the sky position given by the GW measurement with galaxy catalogues
- Assume that one knows the intrinsic mass of the object

Measurement of $d_L(z)$

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There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- Direct method: directly identify the galaxy hosting the event, via the measurement of a (transient) electromagnetic counterpart

    - Earth-based interferometers: sources with counterparts are NS-NS binaries and perhaps NS-BH binaries

    - LISA: sources with expected counterparts are massive BH-BH binaries at the centre of galaxies

Measurement of $d_L(z)$

$$\mathcal{M}_c = (1+z)M_c$$

How can we break this degeneracy and use GW emission to build the Hubble diagram $d_L(z)$?

There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- Statistical method: in the absence of a counterpart, one can cross-correlate with a galaxy catalogue to associate to the GW event a group of galaxies with redshift compatible with the redshift range inferred from the GW measurement with priors on the cosmological parameters. One then searches for the unique set of cosmological parameters aligning each event on the same dL(z) relation

  - Earth-based interferometers: this method can be used with stellar mass BH-BH binaries (the most numerous sources)

  - LISA: this method can be used with stellar mass BH-BH binaries and Extreme Mass Ratio Inspirals (EMRIs)

McLeod and Hogan, PDR77 043512 (2008)

Measurement of $d_L(z)$

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How can we break this degeneracy and use GW emission to build the Hubble diagram $d_L(z)$?

There are a few methods to obtain the redshift information, depending on the nature of the source and on the detector

- Assume that one knows the intrinsic mass of the object: one can then infer the redshift of the event

  - this method can only be used with NS-NS mergers
  - one can assume that the mass function for the NS is known and uses it as a priori information, or that its equation of state is known
  - this method works only with the next generation earth-based detectors (ET, CE)

Messenger and Read, arXiv:1107.5725 Taylor and Gair, arXiv:1204.6739

Direct method with Earth based detectors

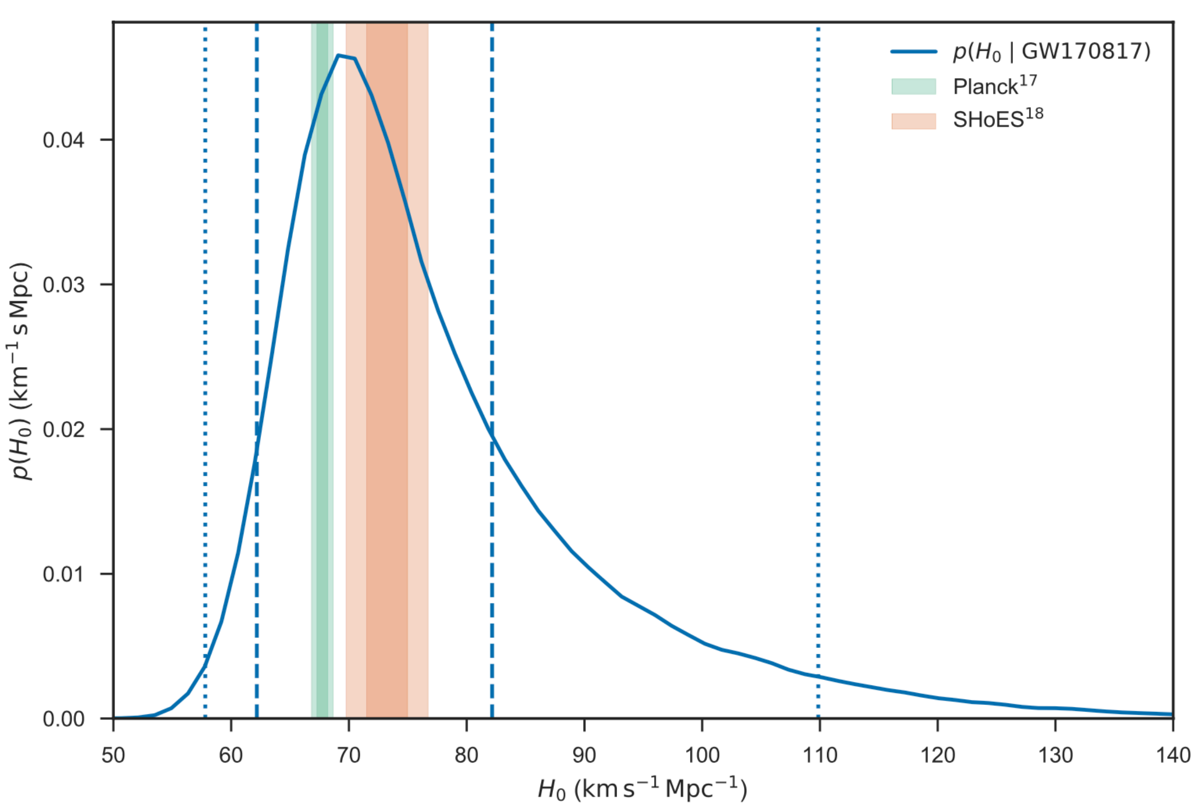
GW170817, the first ever standard siren

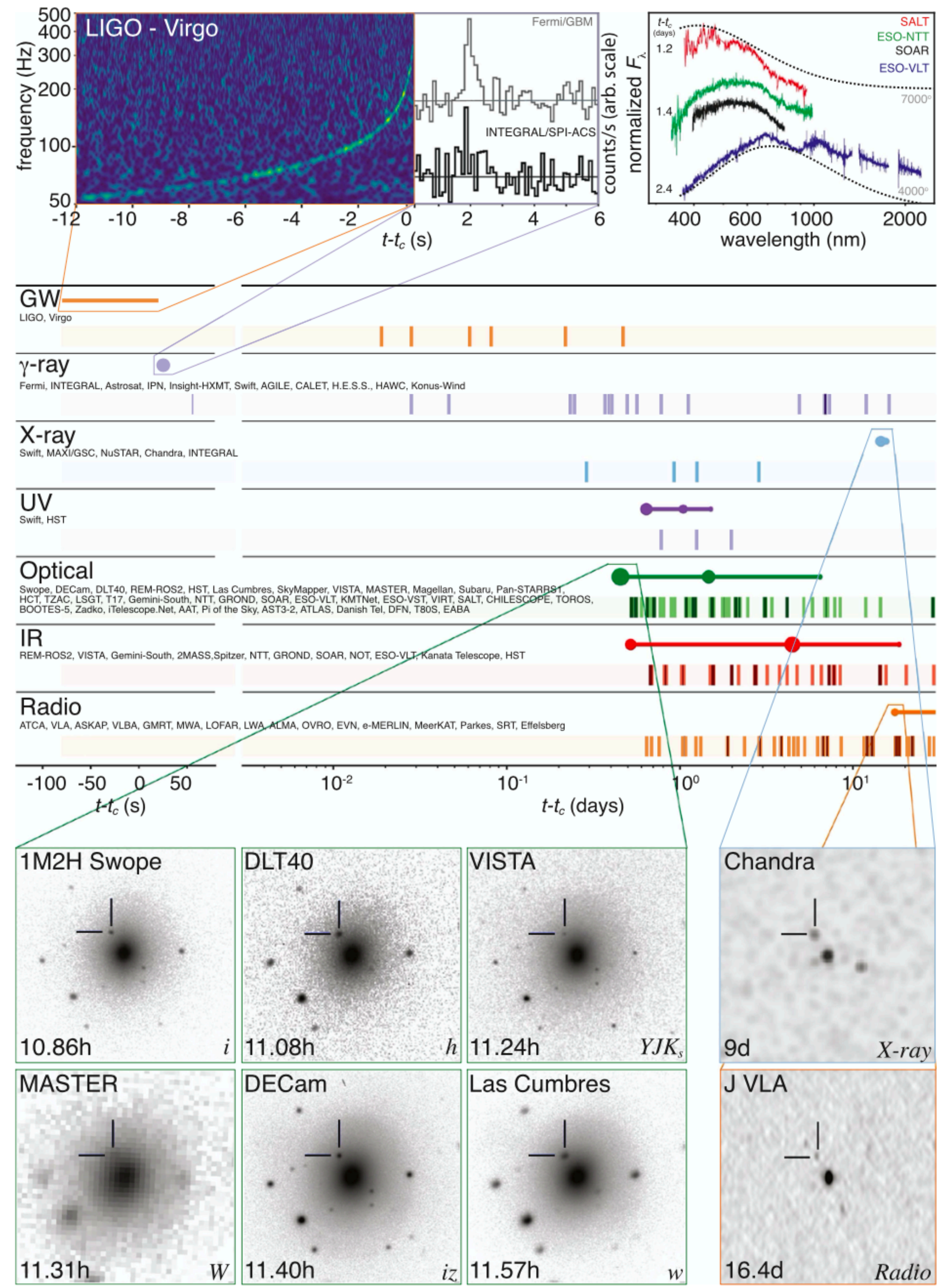
LIGO/Virgo et al, arXiv:1710.05883

Neutron star binary merger at 40 Mpc seen by LIGO/Virgo, and independently as a gamma-ray burst by Fermi.
An optical transient followed allowing identification of the host galaxy

$$H_0 = 70^{+12}_{-8}\,\mathrm{km\,s^{-1}Mpc^{-1}}$$

LIGO/Virgo, arXiv:1710.05935





Direct method with Earth based detectors

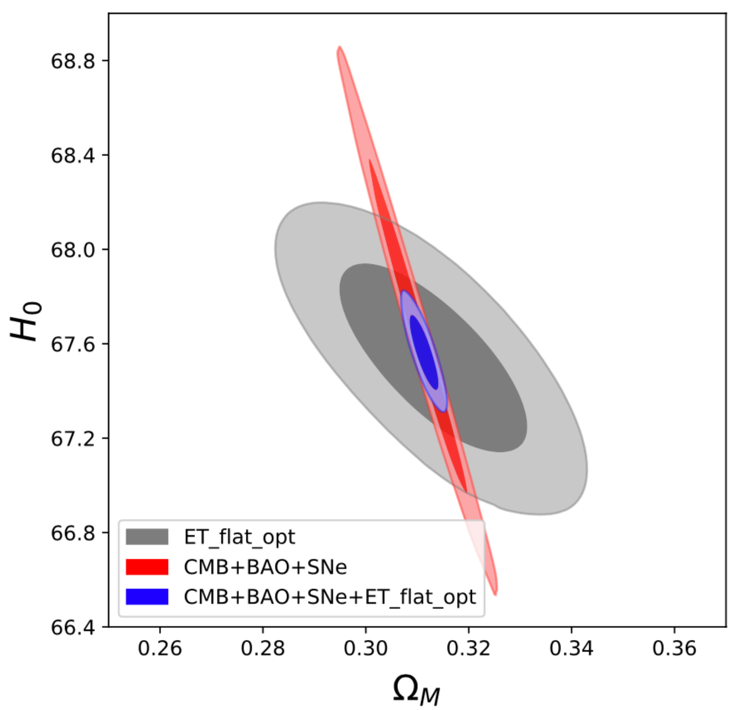
GW170817, the first ever standard siren

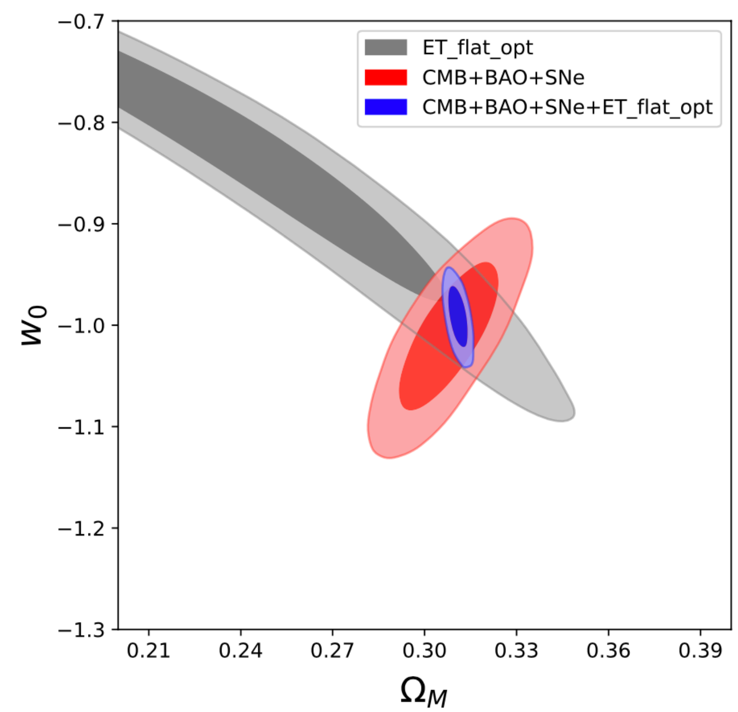
- The measurement of H_0 is not yet competitive with CMB and SNIa, but it is fully independent from them
- GW170817 allowed constraints on fundamental physics: speed of light, Lorentz invariance violation, equivalence principle. These have in turn be used to constrain modified gravity theories
- The measurement of H_0 is expected to get below the 10 percent level by 2030 with the current earth-based detectors network (forecasts depend on the NS binary rates)
- ET and CE will in principle turn into cosmological probes of other cosmological parameters, since they can detect many more events and at higher redshift

Chen et al, Nature (2018)

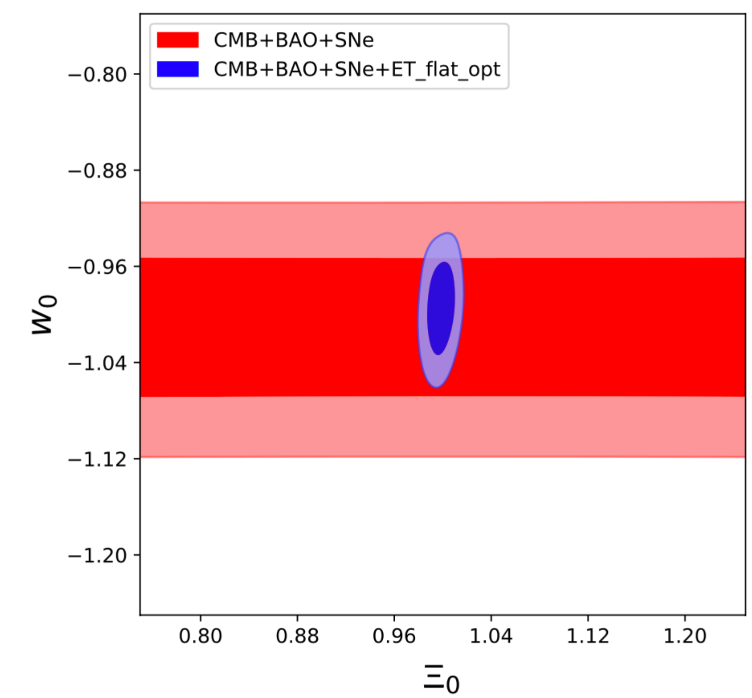
LIGO/Virgo, arXiv:1710.05834

Belgacem et al, arXiv:1907.01487





Ezquiaga and Zumacalaregui, arXiv:1807.09241

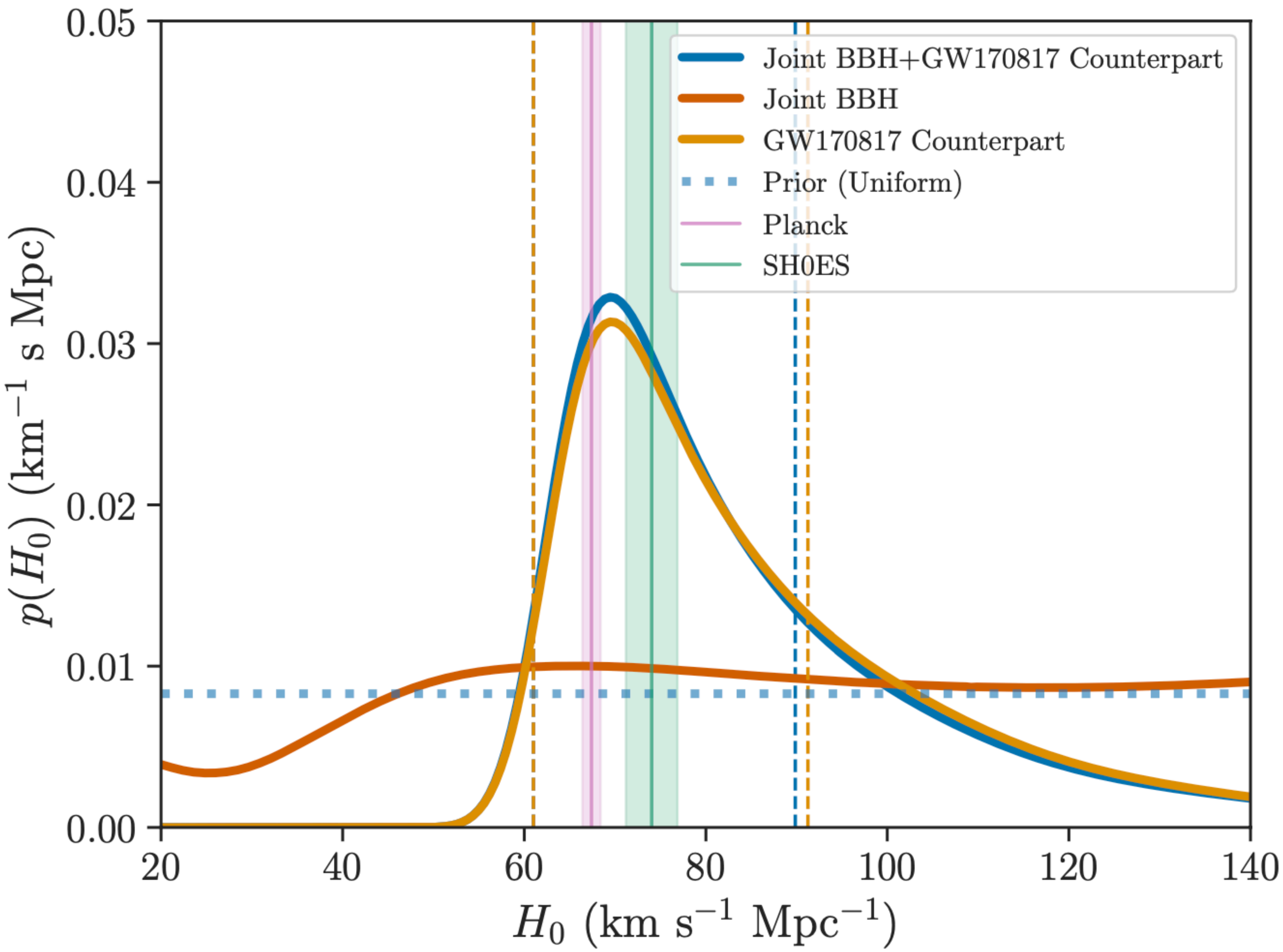


Creminelli and Vernizzi, arXiv:1710.05877

Statistical method with Earth based detectors

- Not competitive with counterpart method as the statistic is low (LV runs O1+O2)
- The completeness of the galaxy catalogue is the main limitation concerning the statistical method

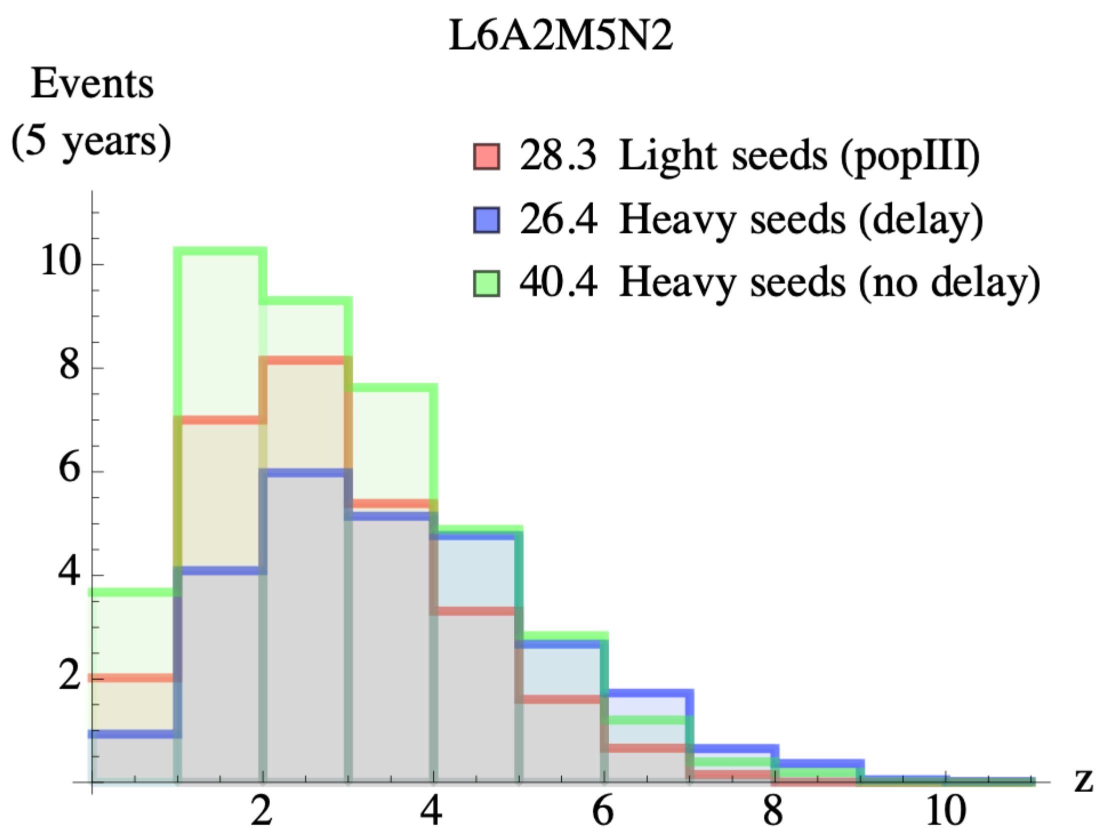
$$H_0 = 69^{+16}_{-8}\,\mathrm{km\,s^{-1}Mpc^{-1}}$$

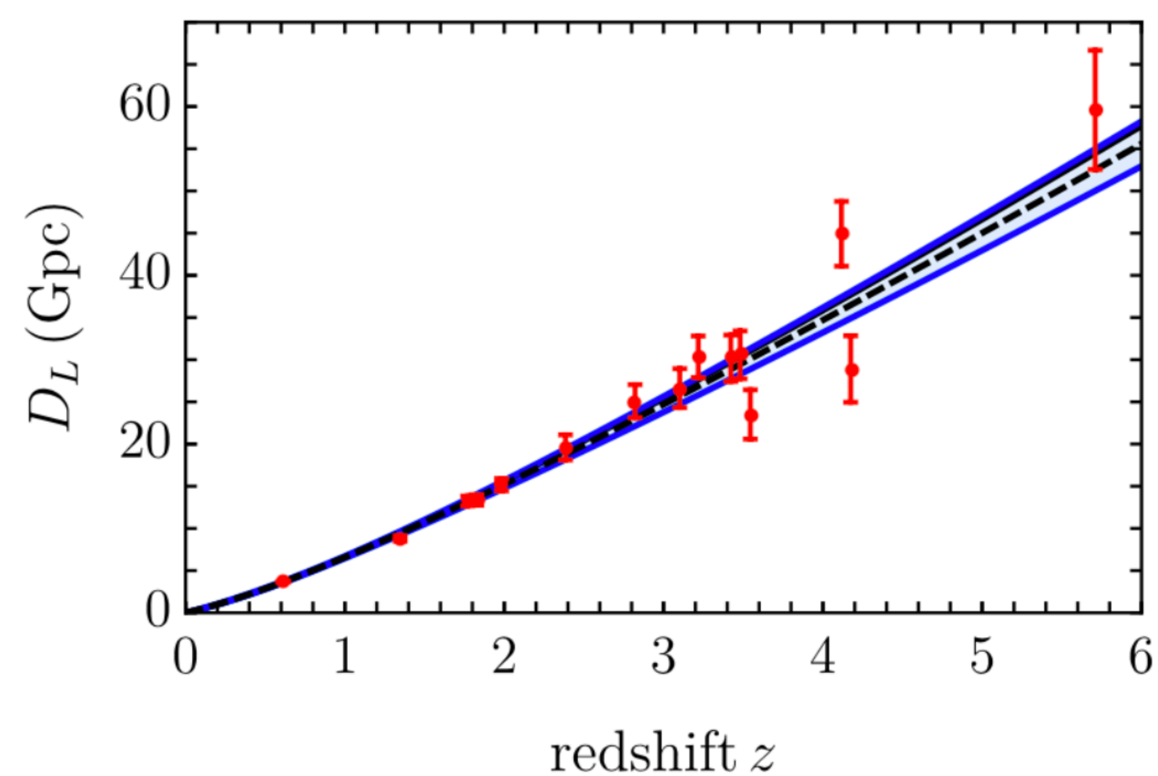


LIGO/Virgo, arXiv:1908.06060

Direct method with LISA

- Massive BH binaries mergers are expected to have counterparts if they occur in gaseous disks at the centre of galaxies (the rate of these events is uncertain though)
- The counterpart is expected in the radio emission, followed by optical identification of the host galaxy
- One must select events with high SNR and good sky localisation (few!)
- Weak lensing (and peculiar velocity errors) affect the measurement of d_L
- The precision on the determination of H0 is expected to be of a few percent
- LISA offers the opportunity to test the cosmic expansion at high redshift





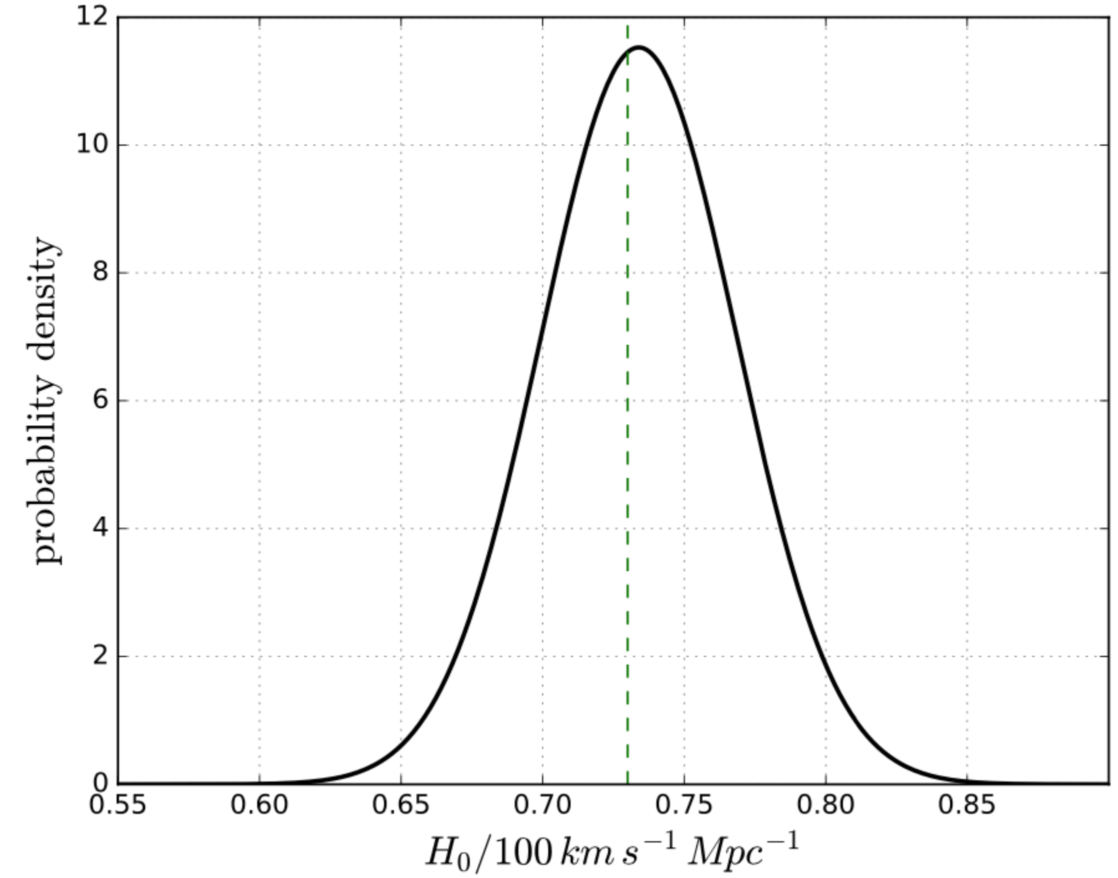
Tamanini et al, arXiv:1601.07112 Speri et al, arXiv:2010.09049

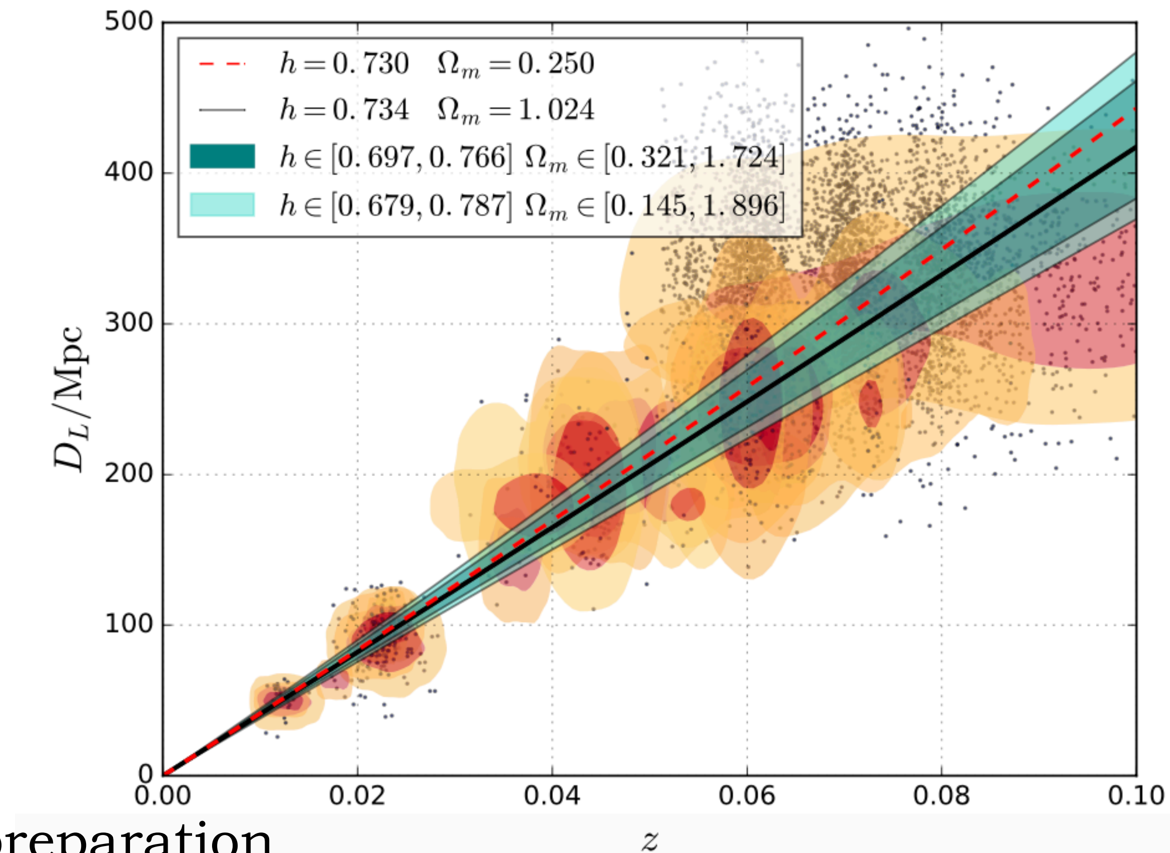
Statistical method with LISA

It can be applied to sources for which no counterpart is expected: stellar origin BHB and EMRIs

- SOBHB sources are at low redshift (which helps concerning the completeness of the galaxy catalogues)
  - The error on the luminosity distance must be smaller than 2%, and the sky localisation better than 1 deg^2
  - LISA could detect around 5 such events per year, allowing a percent measurement of H_0

- EMRIs sources extend up to redshift of about one
  - They should provide a measurement of other cosmological parameters as well
  - The rates are extremely uncertain



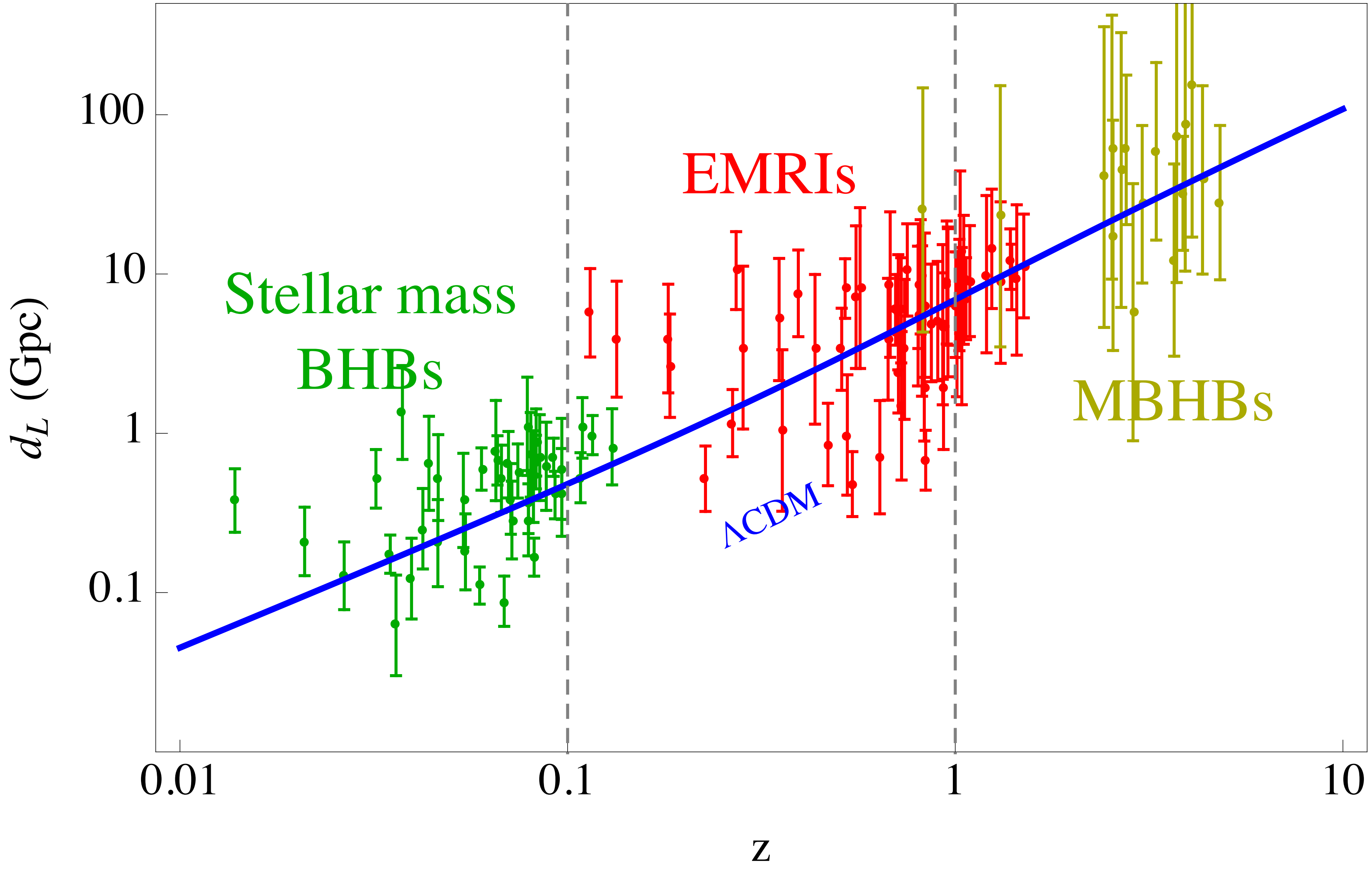


Del Pozzo et al, arXiv:1703.01300 Tamanini et al, in preparation

The advantage of LISA

LISA could provide sub-percent measurement of H_0 as well as constraints on the expansion of the universe at high redshift by combining the results of the three categories of standard sirens

Example of possible LISA cosmological data



N. Tamanini, arXiv:1612.02634

To summarise:

- GWs emission from compact binaries provide clean measurements of the luminosity distance which does not require calibration
- The construction of the Hubble diagram $d_L(z)$ can be performed with and without electromagnetic counterpart
- One can then determine H_0 in a way which is fully independent of both CMB and SNIa
- The network of Earth-based interferometers has already started to provide results, especially thanks to GW170817
- LISA will offer the opportunity to test the expansion at high redshift
- **The future is bright in what concerns GWs as a cosmological probe of the late expansion of the universe, and the construction of the Hubble diagram with GWs will help pinning down the present tension on the measurement of the Hubble constant**