Exercises for CMB physics Lecture 1

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Exercise 1:

Derive the relation between the Helium mass fraction $Y_{\rm He}$ (defined as the fraction of baryon mass in Helium) and the Helium-to-Hydrogen number ratio $f_{\rm He} \equiv N_{\rm He}/N_{\rm H}$, and show that $f_{\rm He} \approx 0.08$ for $Y_{\rm He} \approx 0.24$.

Exercise 2:

• Assuming photons have a perfect blackbody spectrum at temperature $T_0 = 2.73$ K, compute their energy density today, and their density parameter $\omega_\gamma = \Omega_\gamma h^2$.

• Given that $\omega_m \equiv \omega_b + \omega_c \approx 0.14$, at what redshift does the energy density in matter equate that in photons? The redshift of matter-radiation equality is $z_{\rm eq} \approx 3400$. Why is your result different?

• Given that $\omega_b \approx 0.024$, at what redshift does the energy density in baryons equate that in photons?

Exercise 3:

Recall that the Hubble rate is defined as

$$H(a) = \frac{1}{a}\frac{da}{dt} \tag{1}$$

It is related to the density parameters Ω_X by Friedmann's equation,

$$H^2(a) = H_0^2 \left(\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda\right), \tag{2}$$

where here we assumed that neutrinos are massless so the radiation density parameter is $\Omega_r = \Omega_\gamma + \Omega_\nu$.

• Write an integral equation for the age of the Universe (i.e. the coordinate time t) as a function of redshift.

• Find analytic solutions for a radiation-only, matter-only, and cosmological-constant-only Universe.

• Neglecting Ω_r, and assuming a spatially flat universe ($\Omega_\Lambda = 1 - \Omega_m$), compute the age of the Universe today, in years, assuming $H_0 = 70$ km/s/Mpc (you will first have to convert $1/H_0$ to years).