Exercises IV Joint ICTP School on Cosmology

lecture 02

January 19, Valerie Domcke

1 - Slow roll equation of motion. (4pt) Consider the action of a real scalar field,

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right), \qquad (1)$$

with $\sqrt{-g} = (-\det\, g^{\mu\nu})^{1/2}$.

(a) Simplify Eq. (1) assuming in a flat FRW space time for a homogeneous scalar field, i.e. $\phi(t, \vec{x}) = \phi(t)$. (2 pt)

(b) Use the Euler-Lagrange formalism,

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = 0\,,$$

where $S = \int d^4x\mathcal{L}$ to obtain the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \qquad \text{(2 pt)} \qquad (2)$$

2 - Inflation models. (7 pt)

(a) Consider an inflation model determined by $V(\phi) = \lambda\phi^n$ in Eq. (2). Using the slow-roll approximation, compute the value of ϕ where inflation ends. (2 pt)

(b) Solving the horizon problem requires about 60 e-folds of inflation. What was the value of ϕ 60 e-folds before the end of inflation? (2 pt)

(c) Re-write Eq. (2) as a differential equation in e-folds instead of cosmic time, $dN = -Hdt$ (1 pt)

(d) The so-called Starobinsky model, $V(\phi) = V_0\left(1 - \exp\left[-\sqrt{\frac{2}{3}}\phi\right]\right)^2$, gives a particular good fit to the CMB data collected by the PLANCK satellite. Using your favourite computer program, plot $V(\phi)$ and $\phi(N)$.[1] What is the range of ϕ over which the last 60 e-folds occur? (3 pt)

[1] You can hand in a simple sketch of these plots, but please put axis labels and units.