Distinguishing Random States and Black Holes

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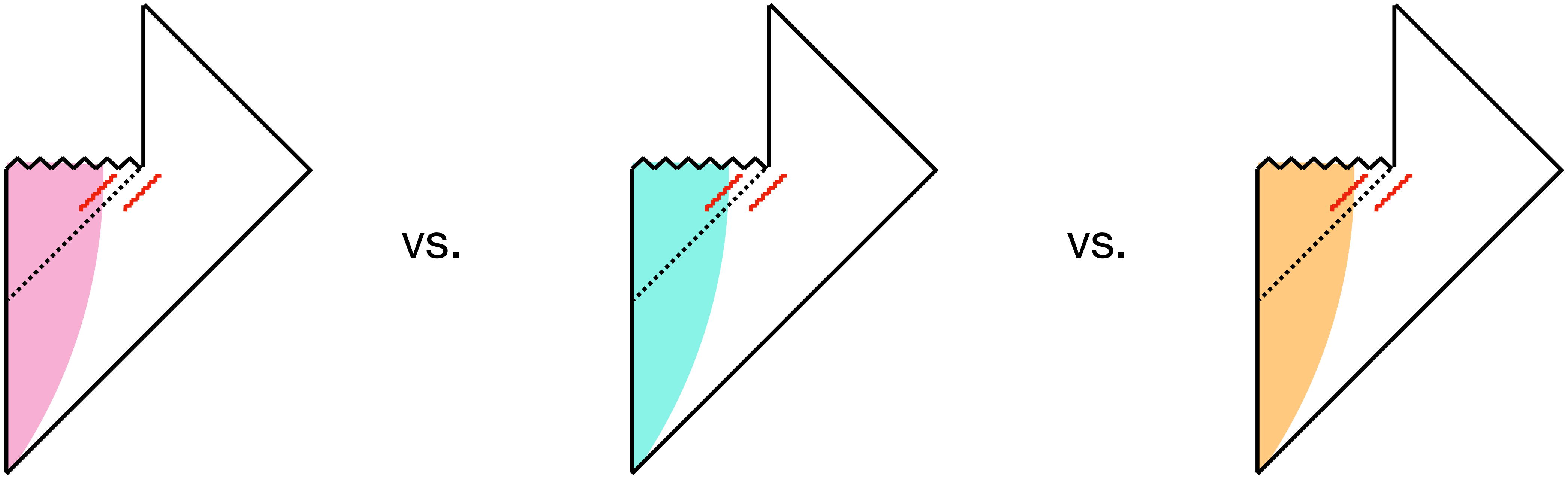
June 23, 2021
Strings 2021

Based on:

1. arXiv:2102.05053 [hep-th] Phys. Rev. Lett. 126, 171603 (2021)
2. arXiv:210X.xxxxx (w/ V. Narovlansky & S. Ryu)

A Black Hole Information Problem

- Hawking told us that black holes will radiate thermal radiation
- This is a breakdown of quantum mechanics (unitarity) — different black hole microstates (with same M, Q, J) evolve to the same final state
- How can we distinguish different black hole microstates from the radiation?
- Must probe physics beyond the Page curve for von Neumann entropy



Quantum Hypothesis Testing

- In quantum information theory, this problem is referred to as quantum hypothesis testing

- What is the best we can do at distinguishing states ρ and σ using a quantum measurement (POVM)?

- For POVM $\{\hat{A}, 1-\hat{A}\}$, we conclude we have ρ if we get measurement outcome corresponding to $\hat{A}$ and σ otherwise

- Type I error: probability of concluding we have σ when we really have ρ, $\alpha(\hat{A}) := Tr[(1-\hat{A})\rho]$. Type II error: probability of concluding we have ρ when we really have σ, $\beta(\hat{A}) := Tr[\hat{A}\sigma]$.

- We optimize our POVM to minimize the error probability:

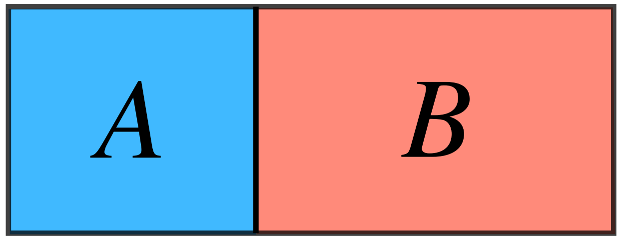
$$\min_{\hat{A}} \left[\alpha(\hat{A}) + \beta(\hat{A})\right] := 1 - T(\rho||\sigma)$$

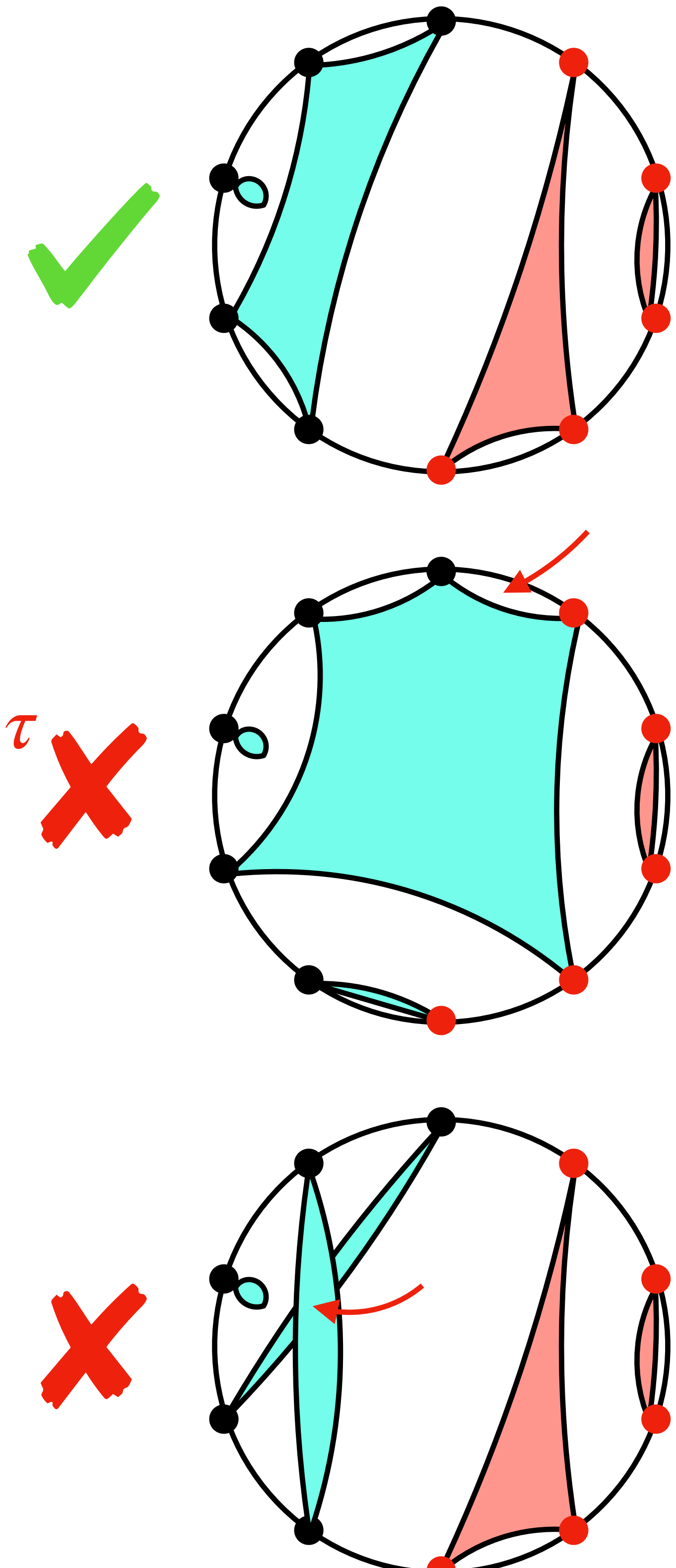
Trace distance

[Helstrom (1969)]

Replica tricks for distinguishability measures

$$\mathscr{H} = \mathscr{H}_A \otimes \mathscr{H}_B$$





- Petz Renyi Relative Entropy measures the rate at which states can be distinguished as one is given more copies

$$D_\alpha(\rho_A||\sigma_A) = \lim_{m\to 1-\alpha} \frac{1}{\alpha - 1} \log\left[Tr\left[\rho_A^\alpha \sigma_A^m\right]\right]$$

- ρ_A and σ_A are independently Haar random

$$\overline{Tr\left[\rho_A^\alpha \sigma_A^m\right]} = \frac{1}{(d_A d_B)^{\alpha+m}} \sum_{\tau \in S_\alpha \times S_m} d_A^{C(\eta^{-1}\circ\tau)} d_B^{C(\tau)}$$

# of cycles in τ

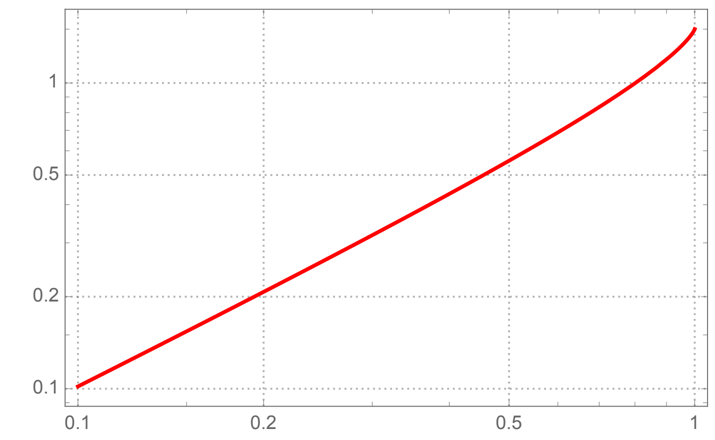
- Can be exactly evaluated using the combinatorics of non-crossing partitions

$$D_\alpha(\rho_A||\sigma_A) = \frac{1}{\alpha-1}\begin{cases} \log\left[{}_2F_1\left(1-\alpha,-\alpha;2;\frac{d_A}{d_B}\right)\ {}_2F_1\left(\alpha-1,\alpha;2;\frac{d_A}{d_B}\right)\right], & d_A < d_B \\ \log\left[\frac{d_B\,{}_2F_1\left(1-\alpha,-\alpha;2;\frac{d_B}{d_A}\right)\,{}_2F_1\left(\alpha-1,\alpha;2;\frac{d_B}{d_A}\right)}{d_A}\right], & d_A > d_B \end{cases}$$

Page Curves for Relative Entropy

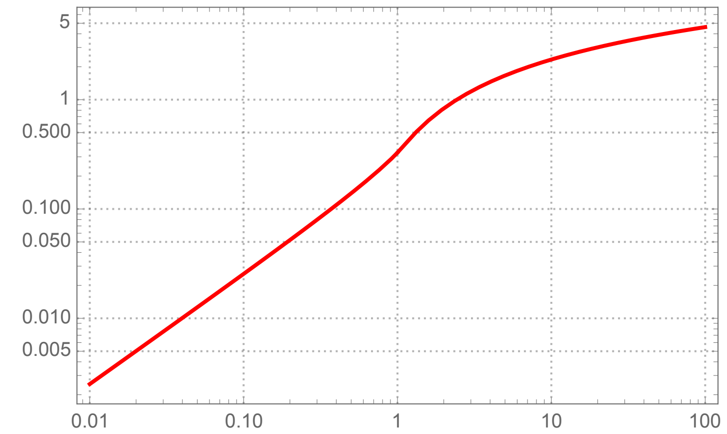
Relative Entropy:

$$D(\rho_A || \sigma_A) = \begin{cases} 1 + \frac{d_A}{2d_B} + \left(\frac{d_B}{d_A} - 1\right) \log\left(1 - \frac{d_A}{d_B}\right), & d_A < d_B \\ \infty, & d_A > d_B \end{cases}$$



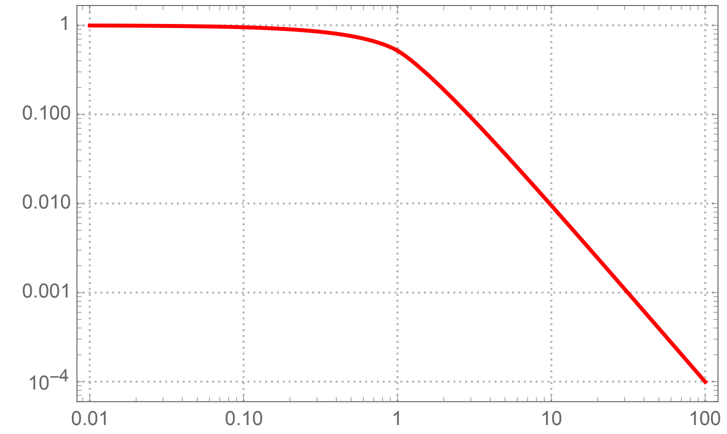
Chernoff Distance:

$$\xi(\rho_A || \sigma_A) = \begin{cases} -2 \log\left[{}_2F_1\left(\frac{1}{2}, -\frac{1}{2}; 2; \frac{d_A}{d_B}\right)\right], & d_A < d_B \\ -\log\left[\frac{d_B}{d_A}\, {}_2F_1\left(\frac{1}{2}, -\frac{1}{2}; 2; \frac{d_B}{d_A}\right)^2\right], & d_A > d_B \end{cases}$$



Fidelity:

$$F_H(\rho_A || \sigma_A) = \begin{cases} {}_2F_1\left(\frac{1}{2}, -\frac{1}{2}; 2; \frac{d_A}{d_B}\right)^4, & d_A < d_B \\ \frac{d_B^2}{d_A^2}\, {}_2F_1\left(\frac{1}{2}, -\frac{1}{2}; 2; \frac{d_B}{d_A}\right)^4, & d_A > d_B \end{cases}$$

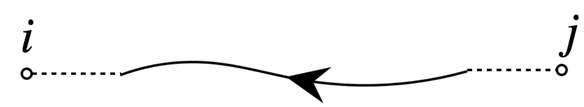


West Coast Model

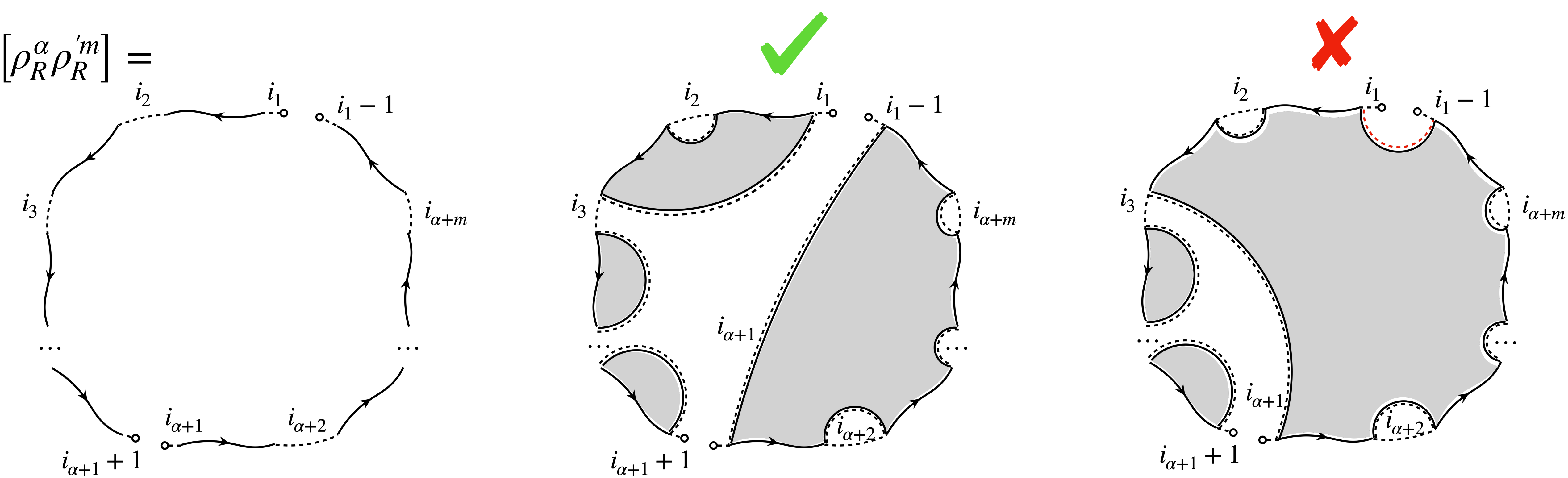
- We use the PSSY (west coast) model as a toy model of black hole evaporation — JT gravity decorated with non dynamical end-of-world branes with k flavors

$$|\Psi\rangle = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} |\psi_i\rangle_B |i\rangle_R \qquad |\Psi'\rangle = \frac{1}{\sqrt{k}} \sum_{i=1}^{k} |\psi_i\rangle_B |i+1\rangle_R$$

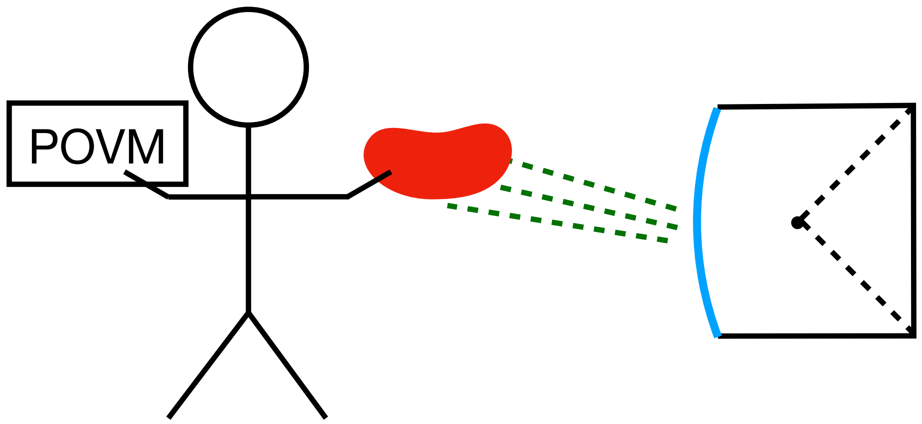
$$\langle \psi_i | \psi_j \rangle =$$



$$Tr\left[\rho_R^{\alpha} \rho_R^{'m}\right] =$$



Implications



- Before the Page time, we need $O\left(e^{S_{BH}-S_R}\log\left[\epsilon^{-1}\right]\right)$ copies of the radiation to distinguish the microstates with error ϵ
- After the Page time, we only need a single copy of the radiation to distinguish the microstates with exponentially small error. However, this will be a "complex" measurement
- If we did not include the replica wormholes in the gravitational path integral, all relative entropies would be zero at all times i.e. information loss

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- "Page curves" for relative entropy, Petz Renyi Relative entropy, Sandwiched Renyi Relative entropy, trace distance, fidelity — agreement with small matrices

- Characterization of distinguishability of black hole microstates in AdS/CFT from subsets of boundary data (with and without fixed areas) — correction to JLMS

- Solution to distinguishing generic random tensor network states using flow network techniques

- Derivation of subsystem ETH (up until $f = 1/2$) for generic chaotic systems and holographic CFTs in all dimensions

Not in this talk but in:

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