STATE-OPERATOR CORRESPONDENCE IN CELESTIAL CONFORMAL FIELD THEORY

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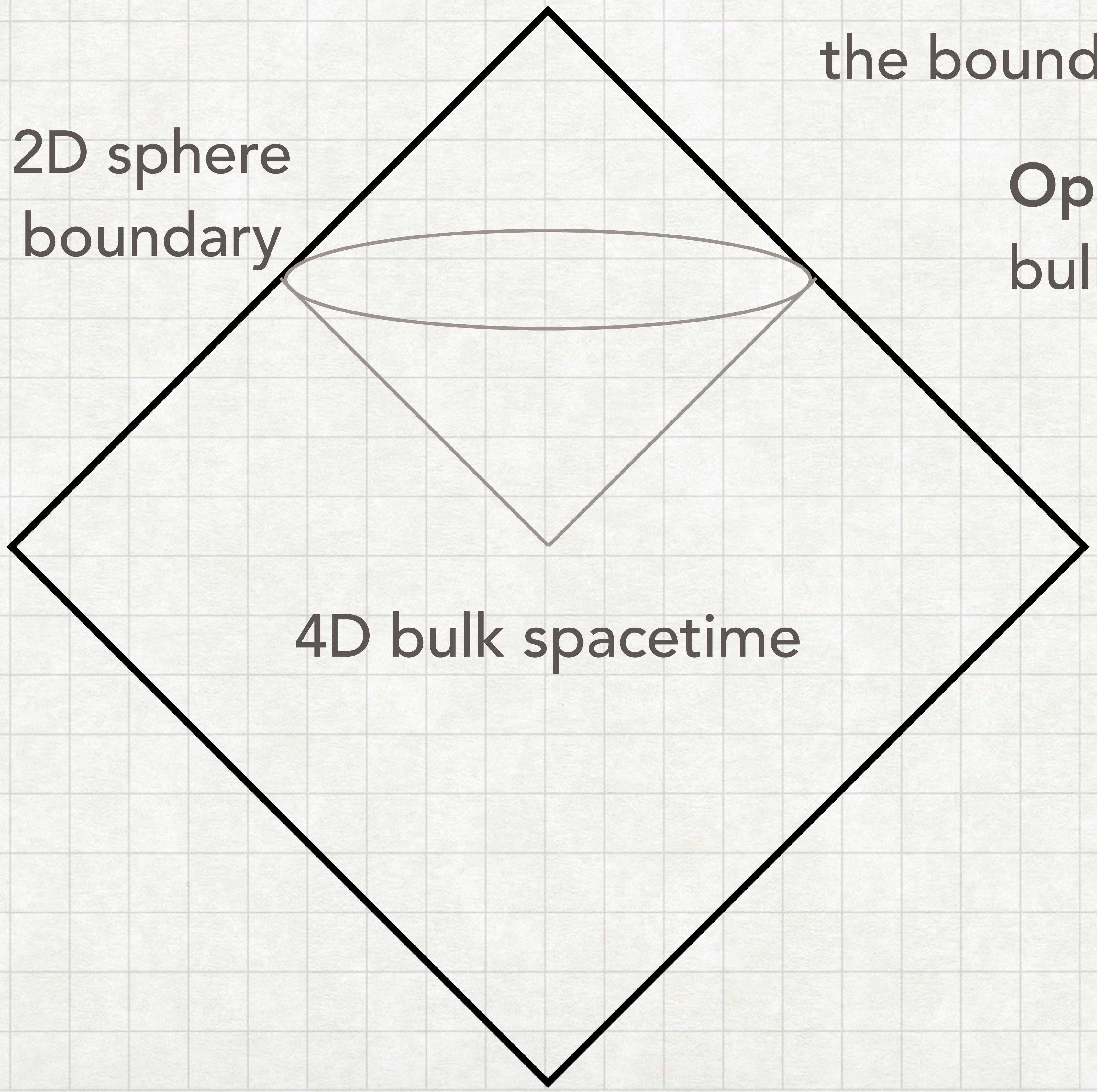
A BRIEF INTRODUCTION TO CELESTIAL CONFORMAL FIELD THEORY

Four-dimensional (4D) Lorentz symmetries in the bulk are manifest as two-dimensional (2D) conformal symmetries on the boundary.

This allows 4D S-matrix elements to be **holographically** recast as 2D conformal correlators in the "celestial conformal field theory" that lives on the boundary celestial sphere.



Operators in this CCFT correspond to both incoming and outgoing bulk fields that puncture the celestial sphere at a point.

OUTLINE:

1. Find a bulk product that make sense as a CFT two point function
2. Use the state-operator correspondence to define operators in CCFT
3. Apply the BPZ construction to the two point function to get an inner product
4. Discuss 2D vs. 4D scattering

INNER PRODUCTS AND CONFORMAL PRIMARY WAVEFUNCTIONS

Solutions to the 4D scalar, **massless** wave equation in (-,+,+,+) admit a conserved current J_μ which can be used to define the symplectic product between two solutions

$$(\Phi_1, \Phi_2)_{\text{sym}} = \int_{\Sigma_3} d^3\Sigma^\mu J_\mu(\Phi_1, \Phi_2) = \int_{\Sigma_3} d^3\Sigma^\mu(\Phi_1 \partial_\mu \Phi_2 - \partial_\mu \Phi_1 \Phi_2)$$

This is true for modified solutions as well e.g $(\Phi_1, \Phi_2)_{\text{KG}} = -i(\Phi_1, \Phi_2^*)_{\text{sym}}$ gives the usual Klein-Gordon inner product used in the normalization of conformal primary wavefunctions.

Conformal primary wavefunctions allow us to transform explicitly from a momentum basis to a conformal primary basis

$$\varphi_{\pm,h,\bar{h}}(X^\mu; \vec{w}) = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q(\vec{w}) \cdot X \mp i\epsilon)^\Delta} = \boxed{\int_0^\infty d\omega \omega^{\Delta-1} e^{\pm i\omega \hat{q}\cdot X - \omega\epsilon}}$$

Analogous currents and products can be constructed for arbitrary spin. What follows is also true for massive fields which require an integral over a hyperbolic slice rather than a Mellin transform.

SHADOW PRODUCT

BUT: $\left(\varphi_{\pm,h_1,\bar{h}_1}(X^\mu;\overrightarrow{w}_1),\varphi_{\mp,h_2,\bar{h}_2}(X^\mu;\overrightarrow{w}_2)\right)_{\text{KG}} \propto \delta^{(2)}(w_1-w_2)$ $\qquad L_n^\dagger = -\bar{L}_n$

A bulk inner product between CPWs can be thought of as a transformed S-matrix element i.e a CFT two point function

A 2D CFT two point function is a power law in $w,\bar{w}$ rather than a delta function.

To try and fix this we can introduce the shadow transform:

$$\tilde{\phi}^{\mu_1\cdots\mu_s}_{\pm,1-h,1-\bar{h}}(X^\mu;\overrightarrow{w}) = \frac{\Gamma(2-2\bar{h})}{\pi\Gamma(2h-1)}\int d^2z\frac{1}{(w-z)^{2-2h}(\bar{w}-\bar{z})^{2-2\bar{h}}}\phi^{\mu_1\cdots\mu_s}_{\pm,h,\bar{h}}(X^\mu;\vec{z})$$

Which serves as a modification to the inner product giving a "shadow product"

$$\left(\varphi_{\pm,h_1,\bar{h}_1}(X^\mu;\overrightarrow{w}_1),\varphi_{\mp,h_2,\bar{h}_2}(X^\mu;\overrightarrow{w}_2)\right)_S = \left(\tilde{\varphi}_{\pm,h_1,\bar{h}_1}(X^\mu;\overrightarrow{w}_1),\varphi_{\mp,h_2,\bar{h}_2}(X^\mu;\overrightarrow{w}_2)\right)_{\text{sym}} \propto \boxed{\frac{1}{(w_1-w_2)^{2h_2}(\bar{w}_1-\bar{w}_2)^{2\bar{h}_2}}}$$

STATE-OPERATOR CORRESPONDENCE

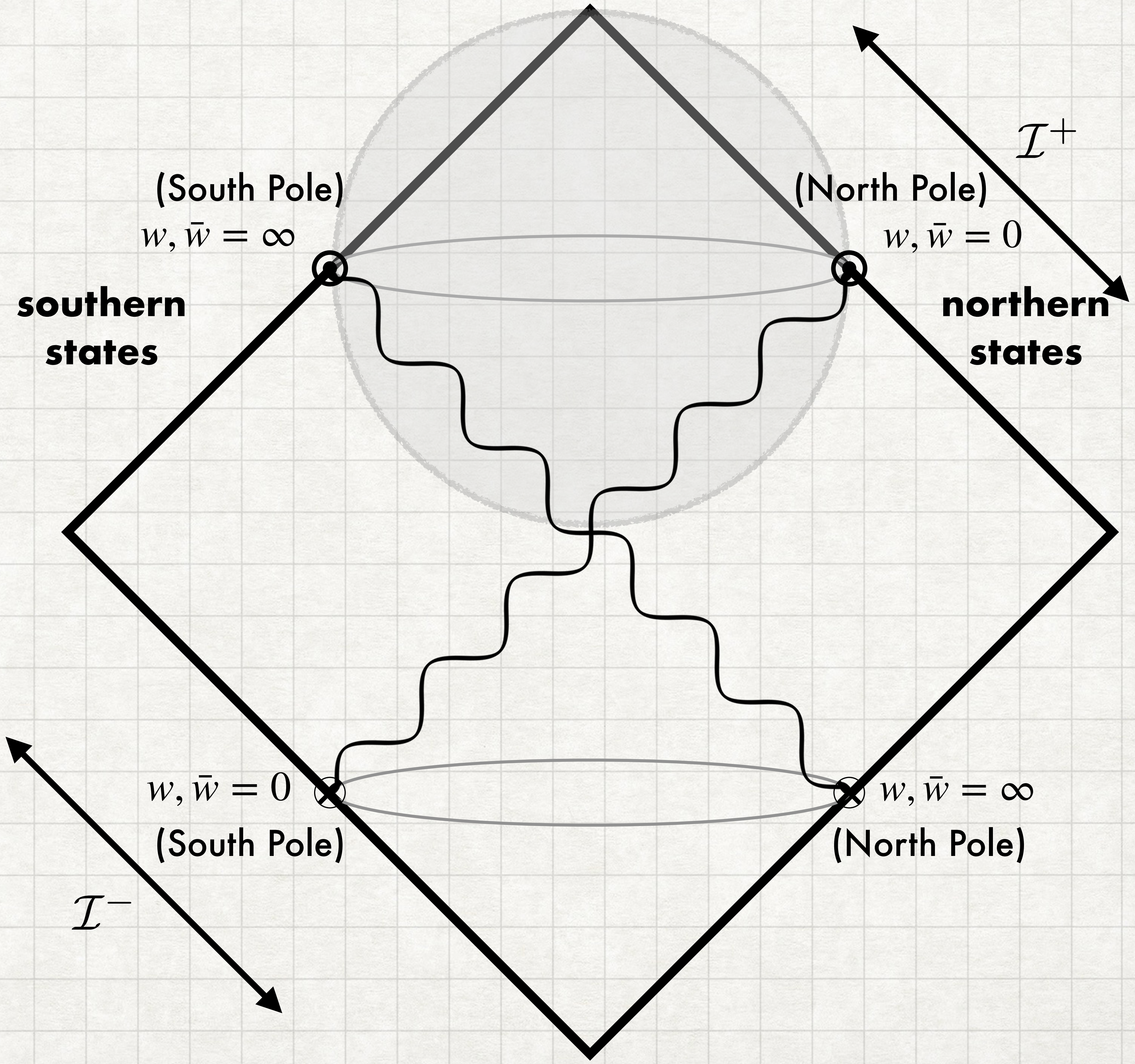
An operator in the CCFT of conformal weights $(h, \bar{h})$ is given by $O^{\pm,h,\bar{h}}(w, \bar{w})$.

A **northern state** is a state that enters at the south pole of the past celestial sphere and exits at the north pole of the future celestial sphere

$$O^{\pm,h,\bar{h}}(w,\bar{w})\,|0_2\rangle = \sum_{m,\bar{m}} \frac{O^{\pm,h,\bar{h}}_{m,\bar{m}}}{w^{h+m}\bar{w}^{\bar{h}+\bar{m}}}\,|0_2\rangle, \quad (m,\bar{m}) \in \mathbb{Z} - (h,\bar{h})$$

A **southern state** is a state that enters at the north pole of the past celestial sphere and exits at the south pole of the future celestial sphere

$$\langle 0_2|\,B'^{\pm,h,\bar{h}}(z,\bar{z}) = \langle 0_2|\sum_{n,\bar{n}} \frac{B'^{\pm,h,\bar{h}}_{n,\bar{n}}}{z^{n-h}\bar{z}^{\bar{n}-\bar{h}}}, \quad (n,\bar{n}) \in \mathbb{Z} + (h,\bar{h})$$



BPZ INNER PRODUCT

The form of a two point function in 2D CFT is dictated by conformal invariance to be a power law. We thereby identify the two point function with the bulk shadow product

$$\langle 0_2 | \tilde{O}^{\pm,1-h_1,1-\bar{h}_1}(\vec{w}_1) O^{\mp,h_2,\bar{h}_2}(\vec{w}_2) | 0_2 \rangle = \left(\phi^{\mu_1 \ldots \mu_s}_{\pm,h_1,\bar{h}_1}(X^\mu; \vec{w}_1), \phi^{\nu_1 \ldots \nu_s}_{\mp,h_2,\bar{h}_2}(X^\mu; \vec{w}_2) \right)_S \propto \frac{1}{(w_1 - w_2)^{2h_2}(\bar{w}_1 - \bar{w}_2)^{2\bar{h}_2}}$$

Contour integrating this expression one obtains the BPZ inner product between operators in the CCFT

$$\langle 0_2 | \tilde{O}'^{\pm,1-h_1,1-\bar{h}_1}_{n,\bar{n}} O^{\mp,h_2,\bar{h}_2}_{m,\bar{m}} | 0_2 \rangle \propto \delta_{h_1-\bar{h}_1+h_2-\bar{h}_2} \delta(\lambda_1 + \lambda_2) \Theta(n - h_2) \Theta(\bar{n} - \bar{h}_2) \delta_{m+n} \delta_{\bar{m}+\bar{n}}$$

This identifies the adjoint relation $\langle 0_2 | \tilde{O}'^{\pm,h,\bar{h}}_{-m,-\bar{m}} = (O^{\mp,h,\bar{h}}_{m,\bar{m}} | 0_2 \rangle)^\dagger$, so the adjoints of operators are given by shadowed operators of the same conformal dimension.

$$[L_n, O^{\pm,h,\bar{h}}_{m,\bar{m}}] | 0_2 \rangle = ((h-1)n - m) O^{\pm,h,\bar{h}}_{m+n,\bar{m}} | 0_2 \rangle$$

FINALLY: → $L_n^\dagger = L_{-n}$

$$\langle 0_2 | [L_{-n}, \tilde{O}'^{\mp,h,\bar{h}}_{-m,-\bar{m}}] = -((h-1)n - m) \langle 0_2 | \tilde{O}'^{\mp,h,\bar{h}}_{-m-n,-\bar{m}}$$

Desired 2D CFT adjoint relation!

2D VERSUS 4D SCATTERING PROBLEMS