Towards a String Dual of SYK

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SYK Model

- Model of N interacting Majorana fermions with Gaussian random couplings - **[Sachdev, Ye; Kitaev; Maldacena, Stanford; ...]**

$$S_{SYK} = \int_0^{\beta=2\pi} d\tau \left(\sum_i \psi_i \partial_\tau \psi_i - H_{SYK} \right),$$

$$H_{SYK} = i^{p/2} \sum_{i_1 \dots i_p} J_{i_1 \dots i_p} \psi_{i_1} \psi_{i_2} \dots \psi_{i_p}, \qquad \langle J^2_{i_1 \dots i_p} \rangle = \mathcal{J}^2 \frac{p!}{2p^2 N^{p-1}}.$$

- Admits a bilocal effective action -

$$\frac{S}{N} = \log \mathrm{Pf} \left(\partial_\tau - \Sigma \right) - \int d\tau_1 d\tau_2 \left[G\Sigma - \frac{\mathcal{J}^2}{2p^2} G^p \right].$$

- Has an effective Lorentzian Liouville description in the double scaling limit - $p \to \infty$, $N \to \infty$ with $\frac{2p^2}{N} = \lambda$ fixed.
- Is a useful toy model for quantum gravity - would like to embed within string theory.

(p, q) Minimal String Theory

- Minimal CFT + Liouville gravity, $c_{MM} + c_L = 26$, with $c_{MM} = 1 - 6\left(b - b^{-1}\right)^2$, $b^2 = p/q$.
- Operator Spectrum
  - Tachyons, $\hat{\mathcal{T}}_{r,s} = c\bar{c}\mathcal{O}_{r,s}e^{2\alpha_{r,s}\phi}$.
  - Ground ring - $\hat{\mathcal{O}}_{r,s} = \mathcal{L}_{r,s} \cdot \mathcal{O}_{r,s}e^{2\beta_{r,s}\phi}$ generated by $\hat{\mathcal{O}}_{1,2} = \hat{x}$, $\hat{\mathcal{O}}_{2,1} = \hat{y}$.
- Branes **[Seiberg, Shih '04]**
  - FZZT_σ branes with boundary cosmological constant, $x = \mu_B = \cosh(\pi b\sigma)$, $y = \partial_{\mu_B} Z$.
  - $\text{ZZ}_{r,s}$ branes.
  - Live on semiclassical target space - $T_q(x) = T_p(y)$.
- Two-matrix model dual **[Daul, Kazakov, Kostov '93]**

$$Z_{p,q} = \int dAdB\, e^{-\text{Tr}(V_p(A)+V_q(B)-AB)}, \qquad V(A) = \sum_{n=1}^{p} g_n A^n,\ W(B) = \sum_{k=1}^{q} t_k B^k.$$

- (p, q) minimal string - tune to (p, q) multicritical point in double scaling limit.

Setup : Brane Construction

- FZZT brane insertion at location x,

$$\Psi(x) = \det(x - B) = \exp(\mathrm{Tr}\log(x - B)).$$

- Insertion of Q FZZT branes,

$$\Psi(X_Q) = \det(X_Q \otimes \mathbb{1}_N - \mathbb{1}_Q \otimes B) = \int d\psi^\dagger d\psi \, e^{\psi^\dagger (X_Q \otimes \mathbb{1}_N - \mathbb{1}_Q \otimes B)\psi}.$$

- ψ_{ia} ZZ-FZZT open string with $i = 1, \ldots, N$, $a = 1, \ldots, Q$.
- Consider $(p, 1)$ MST with Q FZZT branes

$$\int dA dB \, e^{-\mathrm{Tr}(V_p(A) - AB)} \int d\psi^\dagger d\psi \, e^{\psi^\dagger (X_Q \otimes \mathbb{1}_N - \mathbb{1}_Q \otimes B)\psi}$$

- Colour-flavour map : $\mathrm{Tr}_N \left(\psi\psi^\dagger\right)^k = \mathrm{tr}_Q \left(\psi^\dagger\psi\right)^k$.
- FZZT worldvolume description in double scaling limit equivalent to Kontsevich matrix integral $\mathrm{Ai}_p(X_Q)$.

[Maldacena, Moore, Seiberg, Shih '04; Hashimoto, Huang, Klemm, Shih '05;. . .]

Mapping to Non-Commutative Torus

- Matrix SYK model: $\int d\psi d\psi^\dagger \, e^{\sum_i \psi_i^\dagger X_Q \psi_i - \tilde{\mathcal{J}}^2 \mathrm{tr}_Q G^p}$, $G_{ab} = \frac{1}{N} \sum_{i=1}^{N} \psi_{ia}^\dagger \psi_{ib}$.
- Map $G_{Q \times Q}$ to bilocal functions,
  - Introduce clock and shift matrices.

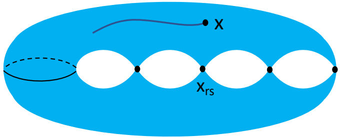
$$UV = VU\xi, \quad U^Q = V^Q = 1 \quad , \qquad \xi = e^{i\hbar}, \quad \hbar = 2\pi/Q,$$

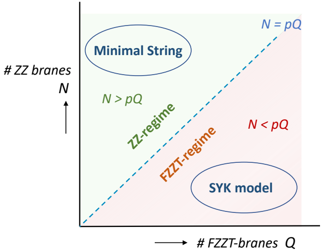
$$U = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \xi^{Q-1} \end{bmatrix} \quad , \qquad V = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

  - $G = \sum_{n,m=1}^{Q} G_{nm} \, U^n V^m \to \mathcal{G}(u,v) = \sum_{n,m=1}^{Q} G_{nm} \, e^{inu} e^{imv}$.
  - $G \cdot G \to \mathcal{G} * \mathcal{G} = e^{i\hbar(\partial_u \partial_{\tilde{v}} - \partial_v \partial_{\tilde{u}})} \mathcal{G}(u,v) \mathcal{G}(\tilde{u},\tilde{v}) \Big|_{\substack{u=\tilde{u} \\ v=\tilde{v}}}$; $\mathrm{tr}_Q \to \int \frac{du\, dv}{2\pi\hbar}$.
  - $\psi_a \to \sum_u \psi(u) |u\rangle$, $U|u\rangle = e^{iu}|u\rangle$, $V|u\rangle = |u - \hbar\rangle$.
- We choose $iX_Q = V^{-1/2} - V^{1/2}$ such that, $iX_Q \psi(u) = \psi\left(u + \frac{\hbar}{2}\right) - \psi\left(u - \frac{\hbar}{2}\right) \equiv \hbar \hat{\partial}_u \psi(u)$.

Conclusions and Comments

- $S = \int \frac{dudv}{2\pi\hbar} \left[\sum_{i=1}^{N} \psi_i^\dagger \left(\hat{\partial}_u - \Sigma \right) \psi_i + N \left(\Sigma * \mathcal{G} - \frac{\mathcal{J}^2}{2p^2} \mathcal{G}^{*p} \right) \right],$
  $\mathcal{G}(u, v) = \frac{1}{N} \sum_i \psi_i^\dagger(u) \psi_i(v)$ with $\mathcal{G}(u, v)^\dagger = \mathcal{G}(v, u)$.
- X_Q - singularities of spectral curve of (p, Q) minimal string.
- Phase structure -





- Double-scaled SYK limit $\rightarrow S[g_+, g_-]$,
  with $c_\pm = 13 \pm i\gamma$ satisfying $c_+ + c_- = 26$.