

Strings 2021

June 30, 2021

Discussion on Integrability

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“It does not matter if the solution is exact,
it matters if it is correct”

Alexey Zamolodchikov

Disclaimer: no citations! No complaints accepted



$\mathcal{N}=4$ SYM and ABJM model are integrable in planar limit

- What does it mean?
- Origins of integrability?
- What is achieved due integrability?
- What else might be achieved?

The origins of this integrability still mysterious

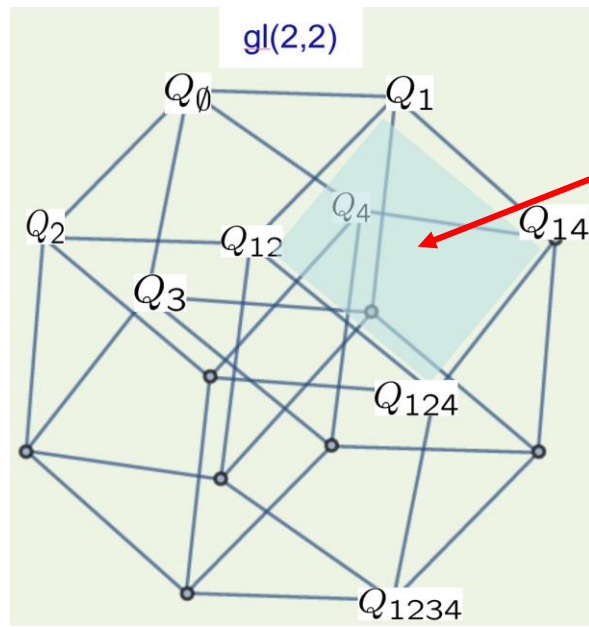
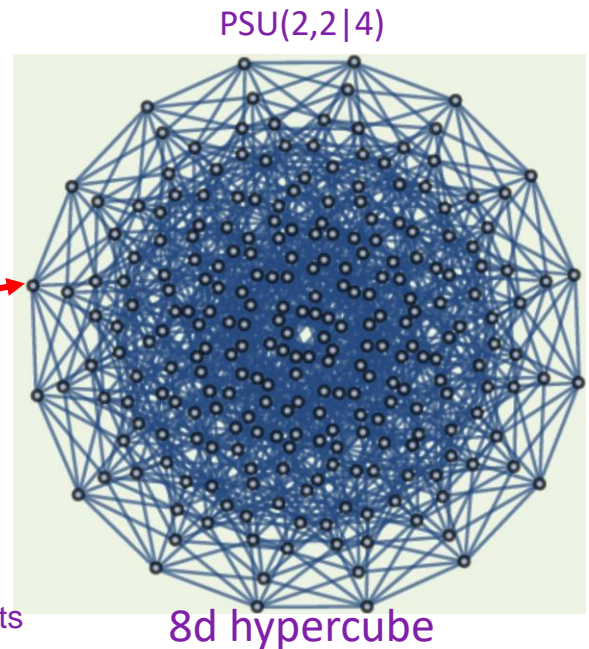
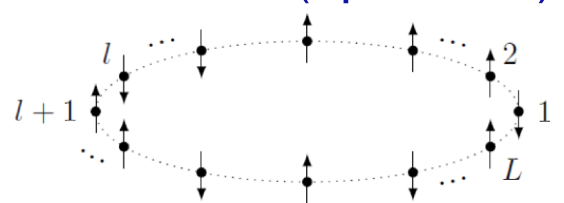
- Some observations :
 - Explicit integrability in lowest orders of PT
 - (Quasi)-classical integrability of the worldsheet sigma-model
 - Mysterious cancellations in planar PT (highest transcendentality)
 - Fishnet CFT limit of χ -twisted $\mathcal{N}=4$ SYM - a chance for understanding its integrability

- Problem of spectrum of anomalous dimensions is solved (any local, and some non-local operators, any coupling...)

- Ultimate formalism: **Quantum Spectral Curve (QSC)** : a Riemann-Hilbert problem on a few Baxter functions $Q_{ijk\dots}(u)$ with the known algebraic (grassmanian) and analytic structure.

- It is a generalization of Baxter functions (operators) in Heisenberg spin chain

$$Q(u) = \prod_{j=1}^{\text{Length}} (u - u_j)$$



Hasse diagram (8d hypercube)

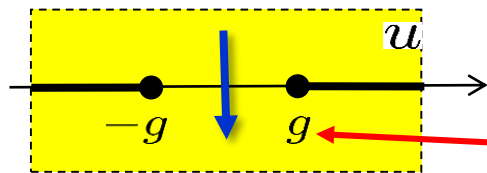
Plücker relations

$$Q_1(u)Q_{124}(u) = \begin{vmatrix} Q_{12}(u + \frac{i}{2}) & Q_{14}(u + \frac{i}{2}) \\ Q_{12}(u - \frac{i}{2}) & Q_{14}(u - \frac{i}{2}) \end{vmatrix}$$

large u asymptotics

$$Q_{123} \sim u^{(\pm\Delta \pm S_1 \pm S_2)/2}$$

cut on physical sheet

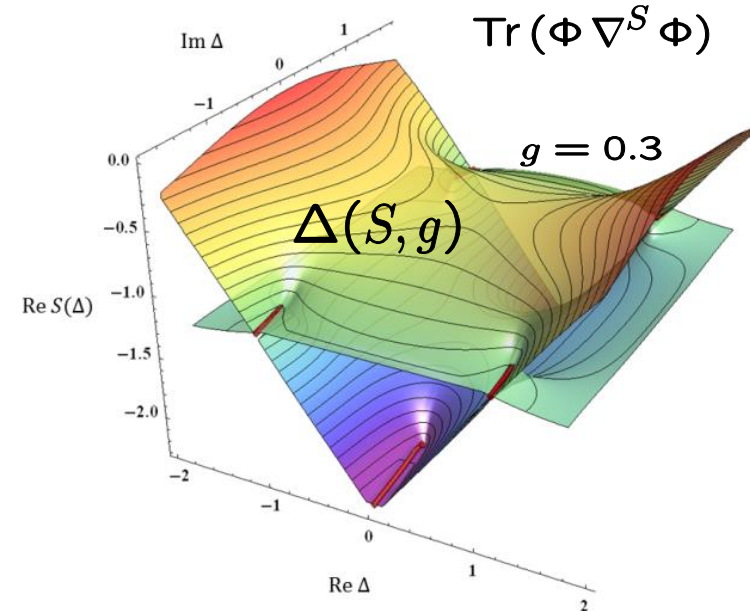
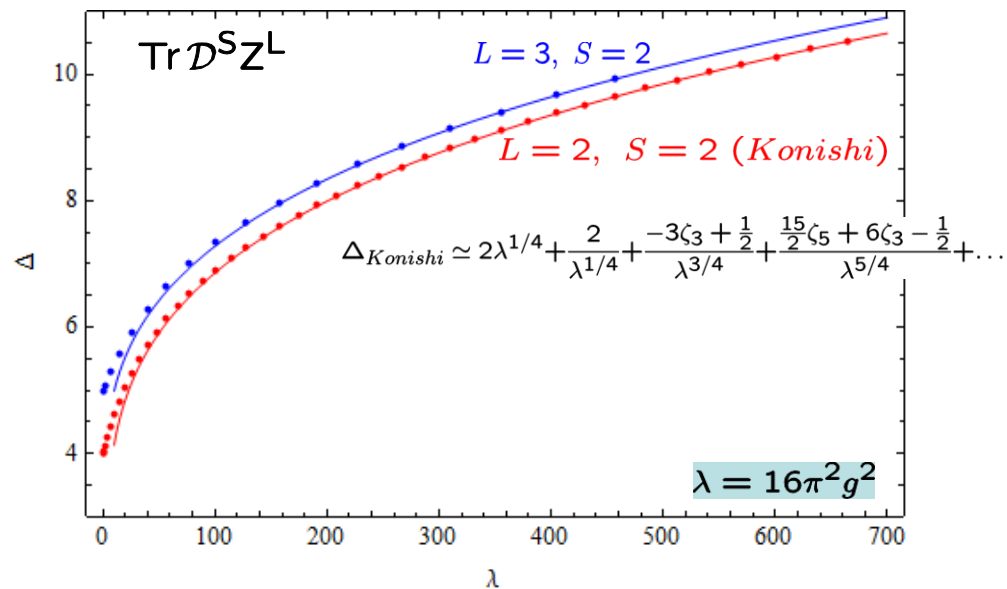


gluing rules (monodromy)

$$Q_{123} \propto \overline{Q_{25678}}, \dots$$

- QSC provides for anomalous dimensions and qq-potentials:

- regular weak coupling expansion (~20 orders...)
- strong coupling (a few orders, no regular expansion yet)
- other approximations for QSC: large spins, next-to-next-to-LO for BFKL spectrum ...
- quasi-exact numerics (easily >50 digits)



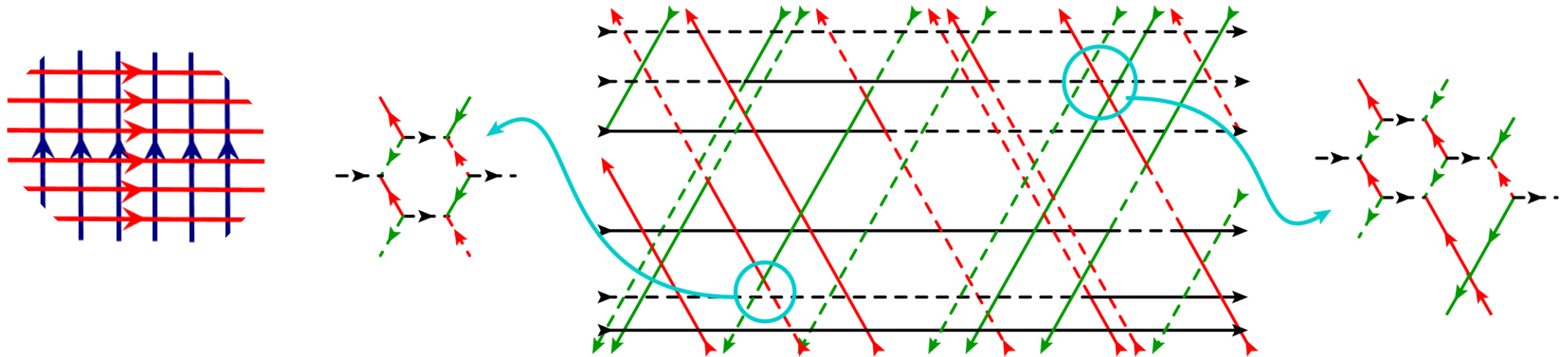
- Perfect fit with other data: perturbation theory, quasiclassical computations in string sigma model,... AdS/CFT seen in full glory!

- **Fishnet CFT limit of \mathfrak{r} -twisted $\mathcal{N}=4$ SYM:**

large complex twist & weak coupling

$$\mathcal{L} = \text{tr} \left[\partial \bar{X} \partial X + \partial \bar{Z} \partial Z + \xi^2 \bar{X} \bar{Z} X Z \right]$$

- **explicitly integrable in all loops** (non-compact SU(2,2) spin chain). QSC in form of TBA...
- Feynman graphs have a regular lattice, “fishnet” structure
- In most general case (with 3 couplings) one gets more complicated “dynamical fishnet”, still regular planar structure



- Hidden regular dynamical lattice structure of planar graphs of full $\mathcal{N}=4$ SYM ?
- Explicit AdS/CFT picture : “Fish-chain” (chain of particles on AdS)
- Integrable O(1,5) sigma model for dense fishnet graphs
- Integrable Fishnet CFT exists in any dimension

