

Worldsheets for field theory amplitudes: Ambitwistor-strings

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With David Skinner [& T Adamo, G Albonico, E Casali, Yvonne Geyer, A Lipstein, R Monteiro, S Nekarov, K Roehrig, R Reid-Edwards, A Sharma, & P Tourkine, . . .]

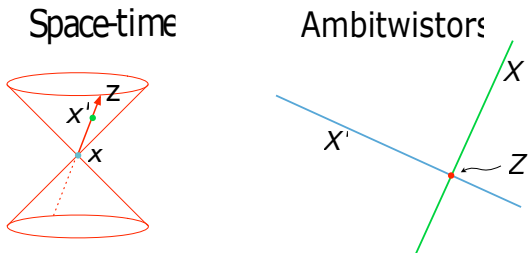
also [N Berkovits, T Azevedo, I Bandos, O Chandia, M Guillem, R Juskinskas, M Lize, W Siegal, . . .]

Builds on twistor-string [Nair, Witten, Berkovits, Roiban, Spradlin, Volovich, Skinner]

& CHY: [Cachazo, He, Yuan, & Feng, Gomez, Guevara, Heydeman, Mizera, Schwarz, Wen, . . .]

Ambitwistor space \mathbb{A} , the space of complex light rays.

- event $x \in M^d \leftrightarrow$ its lightcone $X \subset \mathbb{A}^{2d-2}$.
- Space-time $M =$ space of such $X \subset \mathbb{A}$.



Space-time physics is encoded in complex structure of \mathbb{A} .

Theorem (LeBrun 1983 following Penrose 1976, Witten, Isenberg et. al., 1978)

Complex structure of $\mathbb{A} \leftrightarrow$ space-times $(M, [g])$.

Penrose transform encodes space-time fields in $H^1_{\partial}(\mathbb{A}, \dots)$'s.

What are ambitwistor strings?

Wide-ranging generalization of Twistor-strings

- Chiral strings in *ambitwistor-space* \mathbb{A} .
- vertex ops built from *Penrose transform* of space-time fields on \mathbb{A} , i.e. $H^1_{\bar{\partial}}(\mathbb{A})$.
- Models parallel any worldline theory, i.e., RNS, Green-Schwarz, Pure Spinor, twistorial,
- Robustly gives n -point field theory amplitudes, $\alpha' = 0$, via worldsheet formulae e.g. gauge and gravity

[RSVW, Cachazo-Skinner, CHY, . . .].

Ambitwistors from chiral bosonic strings

Bosonic ambitwistor string action:

$$S_B = \int_{\Sigma} P_{\mu} \bar{\partial} X^{\mu} - e P^2 / 2.$$

- Complexify space-time (M, g) , coords $X \in \mathbb{C}^d$, g hol.

Quantize:

- Gauge fix \rightsquigarrow ghosts & BRST Q
- $Q^2 = 0 \Leftrightarrow D = 26$ or 10 with worldsheet SUSY.
- Amplitudes:

$$\mathcal{M}_n = \langle V_1 \dots V_n \rangle$$

- Vertex op. descent \Rightarrow Penrose transform \rightsquigarrow scattering eqs.
- Add worldsheet susy & matter \rightsquigarrow amplitude formulae.
- Double copy in vertex ops:

$$V = v_l v_r \bar{\delta}(k \cdot P) e^{ik \cdot x}, \quad v_l \in \mathcal{S}_l, \quad v_r \in \mathcal{S}_r.$$

Combining worldsheet matter \rightsquigarrow Double copy

The CHY formulae

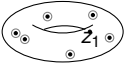
$S' \backslash S^r$	S_Ψ	S_{Ψ_1, Ψ_2}	$S_{\rho, \Psi}^{(\tilde{m})}$	$S_{CS, \Psi}^{(\tilde{N})}$	$S_{CS}^{(\tilde{N})}$
S_Ψ	Einstein				
S_{Ψ_1, Ψ_2}	BI	Galileon			
$S_{\rho, \Psi}^{(m)}$	EM $U(1)^m$	DBI	EMS $U(1)^m \times U(1)^{\tilde{m}}$		
$S_{CS, \Psi}^{(N)}$	EYM	ext. DBI	$EYMS$ $SU(N) \times U(1)^{\tilde{m}}$	$EYMS$ $SU(N) \times SU(\tilde{N})$	
$S_{CS}^{(N)}$	YM	Nonlinear σ	$EYMS$ $SU(N) \times U(1)^{\tilde{m}}$	<i>gen. YMS</i> $SU(N) \times SU(\tilde{N})$	<i>Biadjoint Scalar</i> $SU(N) \times SU(\tilde{N})$

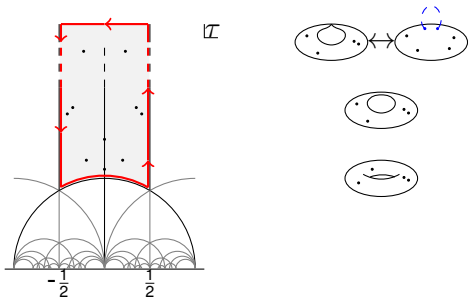
Table: Theories & CHY formulae from choices of matter.

- Plenty of other sectors—Ramond, gravitational etc.
- Twistorial models manifest susy \rightsquigarrow polarized scatt. equs..

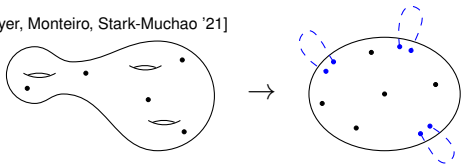
Loops: scattering equations from genus g to sphere

[Adamo, Casali, Skinner 2013, Casali Tourkine 2014 Geyer, M., Monteiro, Tourkine 2015-6

- On torus $\Sigma_\tau =$ , scattering eqs fix moduli τ .
- Integration by parts in $\tau \rightsquigarrow$ formulae on nodal sphere.



- Genus 3 [Geyer, Monteiro, Stark-Muchao '21]



Further developments and questions

Physics:

- **Models in curved space** [Adamo, Casali, Skinner 2014] .
 - **Amplitudes on strong backgrounds** [Adamo, Casali, M., Nekovar, Sharma].
 - **Nonlinear connection to space-time action** [Adamo, M., Sharma '21].
 - **Twistor action now for Einstein gravity** [Sharma].
- **AdS/CFT holography:**
 - **via models in bulk**, [Skinner & Roerhig, Eberhardt Komatsu & Mizera],
 - **or boundary**, [Adamo, M. Sharma] cf. [Gaberdiel & Gopakumar].
 - **Celestial & BMS** [Adamo, Casali, Geyer, Lipstein, M., Sharma, Skinner].
- **Twistorial models in different dims & for 'true string'.**
- $\alpha' \rightarrow 0$ from conventional string?
- **Unreasonable robustness?**

Maths:

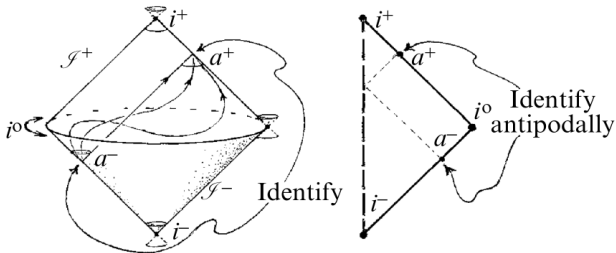
- **Lie polynomials, KLT, colour/kinematics duality** [Frost, M., Mafra].
- **Cluster algebras of surface type** [Frost, Salvatore,...].

Thank You!

Celestial Holography, BMS and soft gravitons

Adamo, Sharma, M. 1905.09224, Geyer, Lipstein & M. 1406.1462, Adamo, Casali & Skinner 1405.5122, 1503.02304.

Asymptotically simple space-time:



Light rays intersect \mathcal{S}^+ and \mathcal{S}^- , $\mathbb{A} = T^* \mathcal{S}^+ = T^* \mathcal{S}^-$.

- **Flat space:** identification is identity.
- **Curved space-time:** identification $\mathbb{A} = T^* \mathcal{S}_C^+ \cup T^* \mathcal{S}_C^-$.
deformed by Hamiltonian h for light ray scattering
- Vertex op $\leftrightarrow \bar{\partial} h \leftrightarrow$ deformation; when soft $h \rightsquigarrow$ BMS.

Twistor models in different dimensions

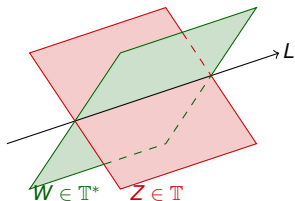


Figure: Ambitwistors in 4d \leftrightarrow twistor+dual twistor \rightsquigarrow Berkovits model.

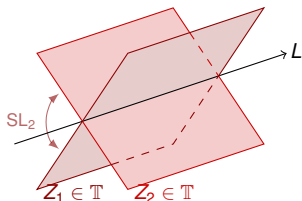


Figure: Ambitwistors in 6d \leftrightarrow pair of twistors Z_a (Albonico, Geyer & M.).

Thank You!