REAL-TIME HOLOGRAPHY
REPLICAS AND OPEN QUANTUM SYSTEMS

Mukund Rangamani
QMAP, Dept of Physics & Astronomy, UC Davis

Strings 2021
ICTP-SAIFR, São Paulo

June 22, 2021

MOTIVATION

- Broad goal: better understand real-time gravitational path integrals and stationary phase approximation for such functional integrals.

- This has relevance for a wide set of questions:
  - computation of causal response functions as well as real-time correlation functions with various out-of-time-orderings
  - replica observables in dynamical situations.

- Practical and effective strategy in general quantum systems: compute quantities using the Euclidean path integral and thence analytically continue the results.

MOTIVATION

* Successful strategy in gravity: use the Euclidean quantum gravity path integral.

* Lots of new insights:
    * black hole thermodynamics

      Gibbons, Hawking '77

    * correlation functions of gauge invariant observables

      Gubser, Klebanov, Polyakov '98 & Witten '98

    * von Neumann, Rényi entropies, etc., from gravity

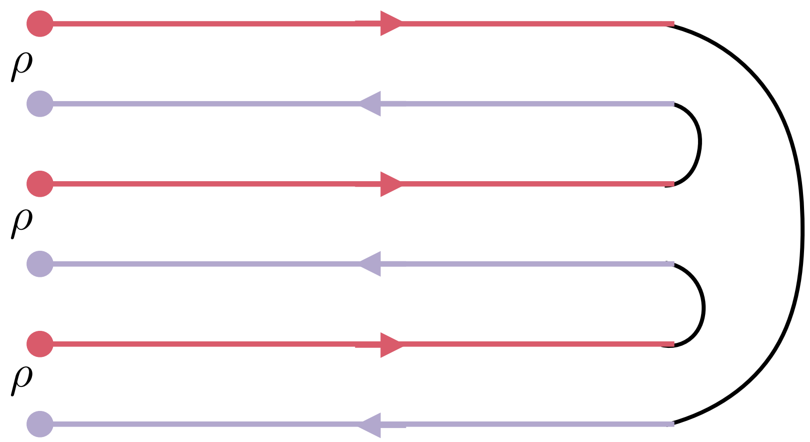
      Lewkowycz, Maldacena '13 Faulkner, Lewkowycz, Maldacena '13

      Dong '16 Dong, Lewkowycz '17

      Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19 Penington, Shenker, Stanford, Yang '19 Marolf, Maxfield '20

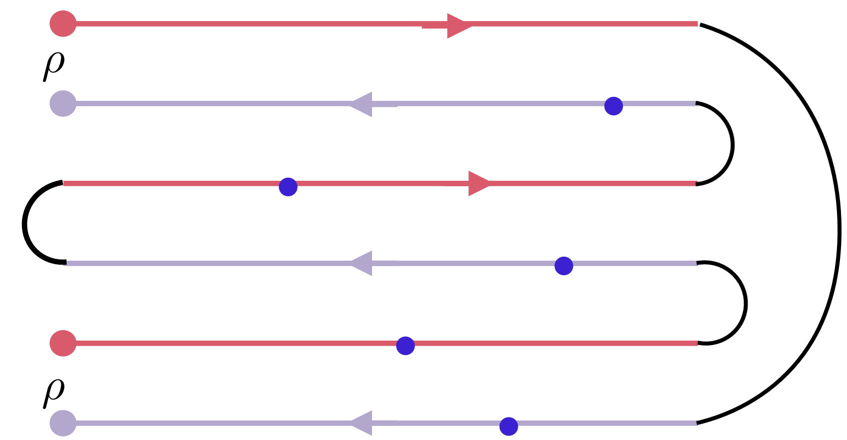
* However, leaves unanswered the question of real-time dynamics.

REPLICA TIMEFOLDS



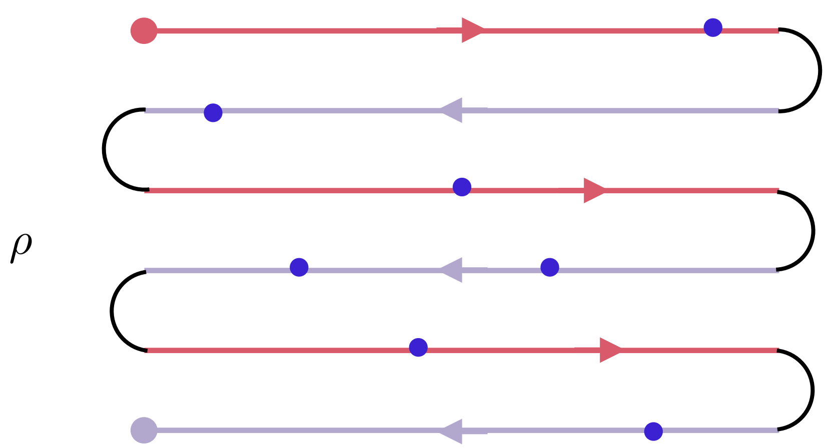
$\mathrm{Tr}(\rho^3)$

Replica contour

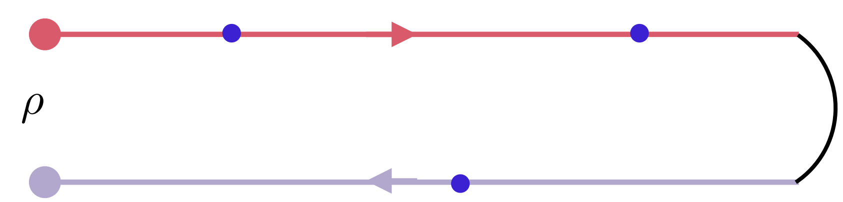


$\mathrm{Tr}(\rho\, \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4\, \rho\, \mathcal{O}_5)$

Replica OTO contour



Out of time order (OTO) contour



Schwinger-Keldysh contour

$\mathrm{Tr}(\rho\, \mathcal{O}_1 \mathcal{O}_2 \cdots \mathcal{O}_n)$

OUTLINE

- ✴ Boundary real-time contours in AdS/CFT give boundary conditions for the the bulk gravity path integral. How do we pick out the class of admissible geometries for a given boundary condition?
- ✴ How do we compute real-time observables and translate them into physical statements in a low energy effective field theory, eg., strongly coupled plasma dynamics?
- ✴ Two main themes of talk:
  - ❖ progress computing thermal observables and obtaining off-shell effective actions for hydrodynamic modes.
  - ❖ computing Renyi entropies in time dependent states.

1. Open Quantum Systems

- Open quantum systems and Schwinger-Keldysh holograms

  arXiv: 2004:02888 w/ C.Jana, R. Loganayagam (P1)

- Effective field theory of stochastic diffusion from gravity

  arXiv: 2012:03999 w/ J. Ghosh, R. Loganayagam, S. Prabhu, A. Sivakumar, V. Vishal (P2)

- To appear….

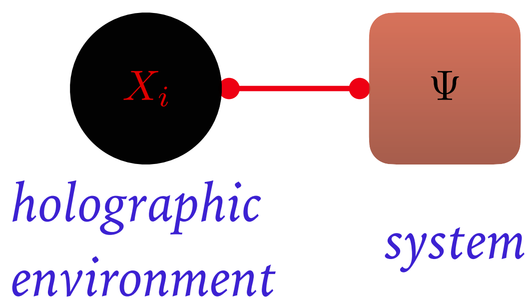
  w/ R. Loganayagam + …. (P3)

  w/ T. He, R. Loganayagam, J. Virrueta (P4)

OPEN QFT PARADIGM

* Consider probing a strongly coupled holographic field theory with an auxiliary system (eg., semi-holographic model).

Polchinski, Faulkner '11



$$S = \int d^d x \, \left(\mathcal{L}[\Psi] + \mathcal{L}[X] + \Psi(x) \, \mathcal{O}_X(x) \right)$$

* Integrating out the holographic environment leads to an open quantum effective field theory: non-trivial information in the influence functionals (IF).

$$\int [D\Psi] \int [DX_i] \, e^{iS} = \int [D\Psi_{\text{L}}] \, [D\Psi_{\text{R}}] \, \exp\left(i \int d^d x \, \mathcal{L}[\Psi_{\text{R}}] - i \int d^d x \, \mathcal{L}[\Psi_{\text{L}}] + i \int d^d x \, \mathcal{L}_{\text{IF}}[\Psi_{\text{R}}, \Psi_{\text{L}}] \right)$$

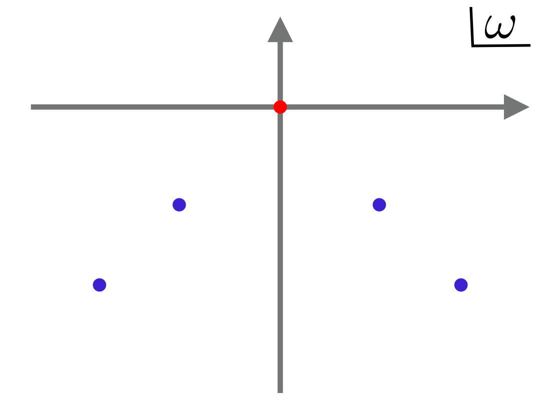
Influence functionals constrained by microscopic unitarity

Feynman, Vernon '63 Caldeira Leggett '83

* Focus on probe system coupled to a thermal holographic plasma.

PLASMA OPERATORS

* Two classes of plasma operators:
  * Markovian/forgetful: rapidly decaying quasinormal modes
  * non-Markovian/memory: long-lived quasinormal modes.



* Stress tensor correlators in a thermal plasma has long-lived hydrodynamic poles: shear momentum diffusion and sound modes.

$$\langle T_{tx}(-\omega, -k\hat{z}), T_{tx}(\omega, k\hat{z})\rangle \propto \frac{1}{i\omega + D\,k^2 + \cdots}$$

$$\omega = -i\,D\,k^2 + \cdots \qquad D = \frac{\eta}{\varepsilon + P}$$

$$\omega = \pm v_s\,k - i\,\Gamma_s\,k^2 + \cdots$$

Policastro, Son, Starinets '01

* Goal: Obtain a low energy effective description of the influence functional.
  * What are the natural variables? Generating function of correlators is non-local for non-Markovian modes…
  * Fluctuations associated with such modes (bulk Hawking quanta)?

Nickel, Son '10

Dubovsky, Hui, Nicolis, Son '11

Grozdanov, Polonyi '13

Kovtun, Moore, Romatschke '14

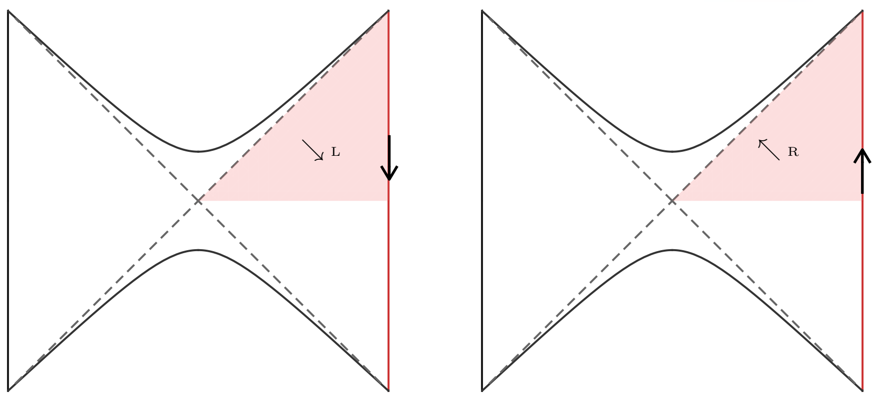
Haehl, Loganayagam, MR '13-'18

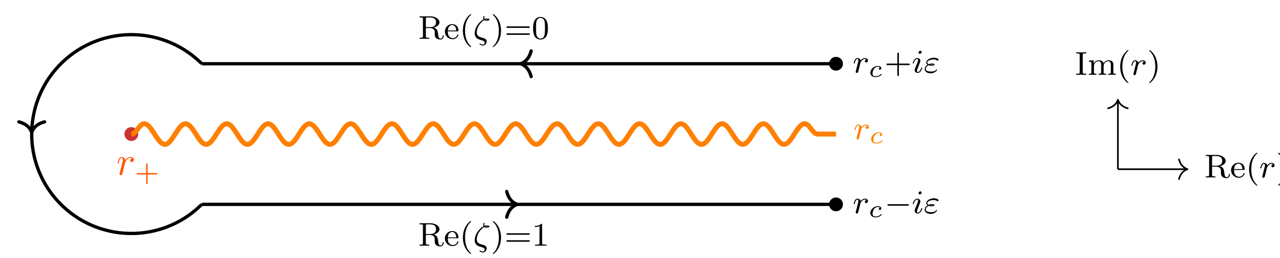
Crossley, Glorioso, Liu '15-'18

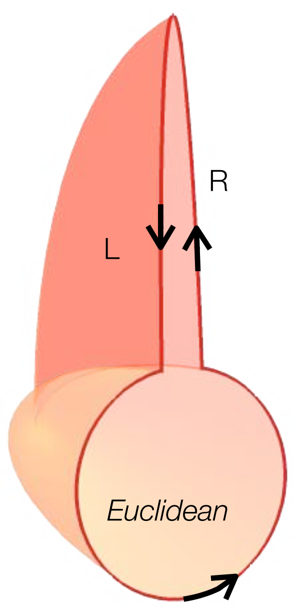
Jensen, Pinzani-Fokeeva, Yarom '17-'18

Chen-Lin, Delacratz, Hartnoll '18

GRAVITY SK GEOMETRY







* grSK geometry: a two-sheeted spacetime, with copies of domains of outer communication of the black hole glued across the future horizon.

Glorioso, Crossley, Liu '18

$$ds^2 = -r^2 f\, dv^2 + i\,\beta f\, dv d\zeta + r^2\, d\mathbf{x}^2\,, \qquad f = 1 - \frac{r_+^d}{r^d} \qquad \beta = \frac{4\pi}{d\, r_+}$$

$$\zeta = \frac{2}{i\beta}\int \frac{dr}{r^2 f}$$

Earlier work: Son, Starinets '02 Herzog, Son '02 Skenderis, van Rees '08 van Rees '08 Arnold, Vaman, Wu, Xiao '11

MARKOVIAN PROBES: GRAVITY

- ✳ Fairly straightforward analysis for Markovian fields: a double Dirichlet problem for bulk fields in terms of boundary data.

- ✳ Determine ingoing bulk-boundary propagator and use grSK time-reversal to get outgoing Green's function.

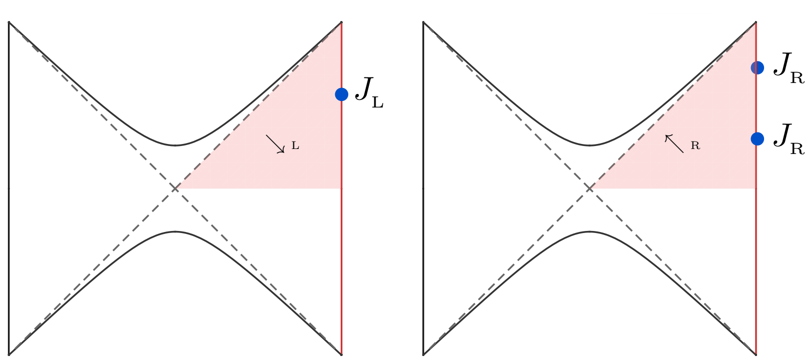
Hawking Green's function

$$\Phi(\zeta,\omega,\mathbf{k}) = G_{\text{in}}(\zeta,\omega,\mathbf{k})\, J_a + \frac{1}{2}\, G^H(\zeta,\omega,\mathbf{k})\, J_d \qquad J_a = \frac{1}{2}(J_{\text{R}} + J_{\text{L}})\,, \qquad J_d = J_{\text{R}} - J_{\text{L}}$$

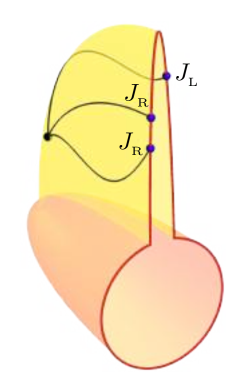
$$G^H(\zeta,\omega,\mathbf{k}) = \coth\left(\frac{\beta\omega}{2}\right) G_{\text{in}}(\zeta,\omega,\mathbf{k}) - e^{\frac{\beta\omega}{2}}\ \text{csch}\left(\frac{\beta\omega}{2}\right) e^{-\beta\omega\zeta} G_{\text{in}}(\zeta,-\omega,\mathbf{k})$$

$e^{-\beta\omega\zeta} G_{\text{in}}(\zeta,-\omega,\mathbf{k}) \rightarrow G_{\text{out}}(\zeta,\omega,\mathbf{k})$

- ✳ Well defined bulk perturbative Witten diagrammatics for extracting influence functionals.



IFs from Witten diagrams



Chakrabarty et al., '19

(P1)

MARKOVIAN OPEN QFTS

* Repackage IFs into stochastic open effective field theory with couplings obeying generalized fluctuation-dissipation relations (nb: probe field acts as a source to leading order)

$$S_\Psi = -\int d^dx\, \Psi_d \left[-K\partial_t^2 + D\,\nabla^2 + \gamma\,\partial_t\right]\Psi_a + i\int d^dx \left[-f\,\frac{(i\Psi_d)^2}{2!} + \sum_{k=1}^{n} \frac{(i\Psi_d)^k}{k!}\left(\theta_k + \bar{\theta}_k\,\partial_t\right)\frac{\Psi_a^{n-k}}{(n-k)!}\right]$$

Fuctuation dissipation: $\gamma = \frac{\beta\, f}{2}\,, \qquad \frac{2}{\beta}\,\bar{\theta}_k + \theta_{k+1} + \frac{1}{4}\theta_{k-1} = 0$

* Salient features:

(P1)

- Explicit formulae for 2d holographic CFTs.
- Gradient expansion results in higher dimensions (interesting structure of iterated Bose-Einstein integrals).
- Insight into renormalization effects in open quantum dynamics.

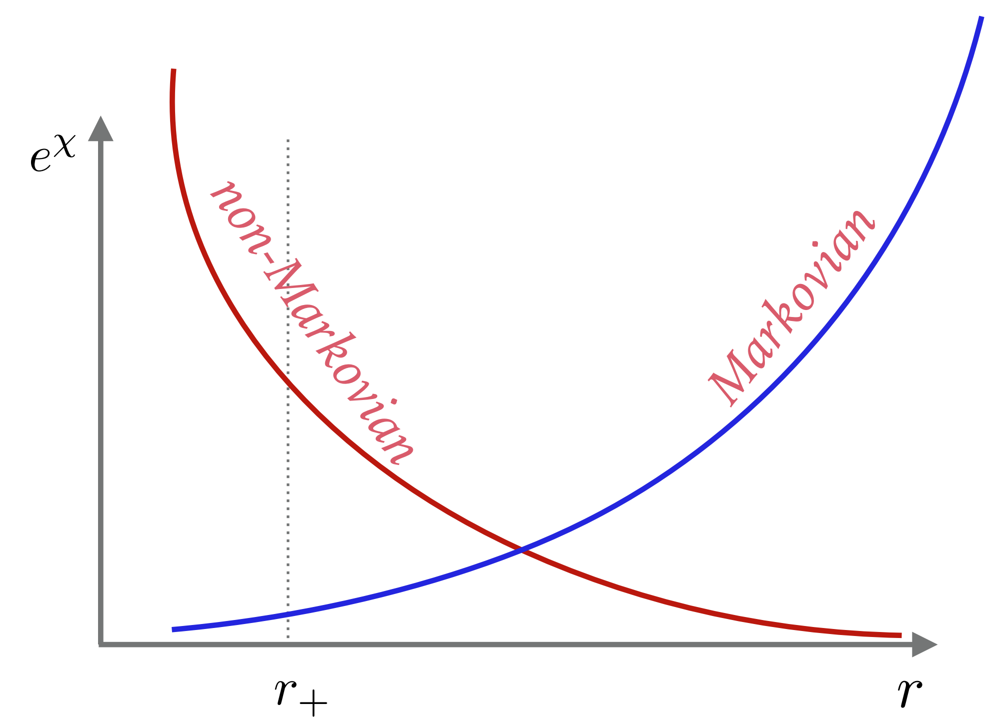
Other developments: Chakrabarty et al., '19 Loganayagam, Ray, Sivakumar '20 + Sharma Chakrabarty, Aswin '20

NON-MARKOVIAN PROBES: GRAVITY

- Bulk dynamics with gauge invariance needs careful analysis:
  - Radial gauge traditionally employed fails to respect SK time-reversal.
  - Radial momentum constraint forces difference currents to be on-shell.
  - Presence of Markovian and non-Markovian degrees of freedom (with mixing).
- Influence functional respecting low energy EFT?
- *Gauge invariant bulk variables repackage dynamics into non-minimally coupled (designer) scalar probes of the thermal plasma.*

$$T^{\mu\nu} = (T^{\mu\nu})_{\mathrm{T}} + (T^{\mu\nu})_{\mathrm{V}} + (T^{\mu\nu})_{\mathrm{S}}$$

$$e^{\chi_{\mathrm{T}}} \qquad e^{\chi_{\mathrm{V}}} \qquad e^{\chi_{\mathrm{S}}}$$



- Universal template for analyzing the dynamics of diffusive modes in thermal systems: a probe scalar with IR modulated coupling characterized by a Markovianity index.

(P2)

(related to hyperscaling violating backgrounds)

REMEMBRANCE OF A BLACK HOLE'S PAST

$$S = -\frac{1}{2}\int d^{d+1}x\,\sqrt{-g}\,e^{\chi}\,\nabla_A\varphi_{\mathcal{M}}\,\nabla^A\varphi_{\mathcal{M}}$$

$$\lim_{r\to\infty} e^{\chi} = r^{\mathcal{M}+1-d}$$

$\mathcal{M} > -1$ Markovian

$\mathcal{M} < -1$ non-Markovian

massless field	Scalar sector	Vector sector	Tensor sector
Scalar	$d-1$	–	–
Gauge field	$-(d-3)$	$d-3$	–
Metric	$-(d-3)$	$-(d-1)$	$d-1$
Nambu-Goto (linearized)	2		

- ✳ Bulk phase space analysis dictates that non-Markovian designer fields satisfy double Neumann boundary conditions.

nb: not an input, dictated by gravitational action+bdy terms

- ✳ Wilsonian influence phase: functional of sources for the Markovian modes and hydrodynamic moduli for non-Markovian modes.

(P2)

$$\mathcal{L}_{\text{WIF}}\left[J_{\text{L}}, J_{\text{R}}, \Phi_{\text{L}}, \Phi_{\text{R}}\right]$$

Markovian sources ← $J_{\text{L}}, J_{\text{R}}$; $\Phi_{\text{L}}, \Phi_{\text{R}}$ → *non-Markovian vevs*

Earlier work: Gubser, Pufu '07 Morgan, Cardoso, Miranda, Zanchin '09

N=4 SYM HYDRODYNAMIC EFFECTIVE ACTION

$$\mathcal{L}_{\rm WIF}[\gamma^\sigma_a, \gamma^\sigma_d, \mathcal{P}^\alpha_a, \mathcal{P}^\alpha_d, \mathcal{S}_a, \mathcal{S}_d] = \mathcal{L}_{\rm ideal} + \mathcal{L}_{\rm T}[\gamma^\sigma_a, \gamma^\sigma_d] + \mathcal{L}_{\rm V}[\mathcal{P}^\alpha_a, \mathcal{P}^\alpha_d] + \mathcal{L}_{\rm S}[\mathcal{S}_a, \mathcal{S}_d] + \text{non-Gaussian}$$

Class L hydro action

$$\mathcal{L}_{\rm ideal} = c\,\pi^4 N^2 \left(T^4_{\rm R} - T^4_{\rm L}\right) - \frac{c\,\pi^2 N^2}{2} \left(T^2_{\rm R} \left[\,{}^W R + \omega^2_{\rm vor} - \log 2\, \sigma^2_{\rm sh}\right]_{\rm R} - ({\rm R} \leftrightarrow {\rm L})\right)$$

Haehl, Loganayagam, MR '15

Gaussian terms

$$\mathcal{L}_{\rm A}[\mathfrak{f}_a, \mathfrak{f}_d] = \mathfrak{f}^\dagger_d\, h_A\, K^A_{\rm in} \left(\mathfrak{f}_a + \frac{1}{2} \coth\left(\frac{\beta\omega}{2}\right) \mathfrak{f}_d\right)$$

$$K^T_{\rm in} = \frac{\pi^2 N^2}{8}\, T^4 \left[-i\,\mathfrak{w} - \frac{\mathfrak{q}^2}{2} + \frac{1-\log 2}{2}\,\mathfrak{w}^2 + \cdots\right], \qquad h_T = 1$$

$$\mathfrak{w} = \frac{\omega}{\pi T} \qquad \mathfrak{q} = \frac{k}{\pi T}$$

(P2)

$$K^V_{\rm in} = \frac{\pi^2 N^2}{8}\, T^4 \left[-i\,\mathfrak{w} + \frac{\mathfrak{q}^2}{4} - \frac{1-\log 2}{2}\,\mathfrak{w}^2 + \cdots\right], \qquad h_V = k^2$$

Results known to quartic order in gradient expansion….

scalar/sound (P3) *charged plasma (mode mixing)* (P4)

Related work: Glorioso, Crossley, Liu '18 de Boer, Heller, Pinzani-Fokeeva '18 Bu, Demircik, Lublinsky '20

II. Real-time gravitational replicas

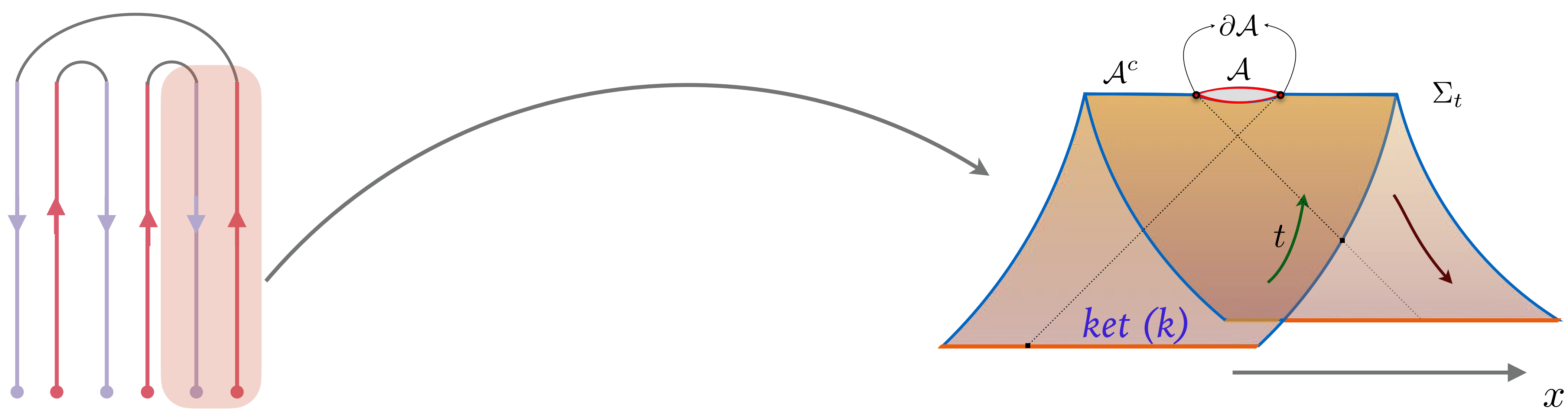
- Deriving covariant holographic entanglement

arXiv: 1607.07506 w/ A. Lewkowycz, X. Dong (PA)

- Real-time gravitational replicas: Formalism and a variational principle
- Real-time gravitational replicas: Low dimensional examples

arXiv: 2012:00828 (PB) arXiv: 2105.07002 (PC) w/ Sean Colin-Ellerin, X. Dong, D. Marolf, Z. Wang

QFT REDUCED DENSITY MATRIX ELEMENTS

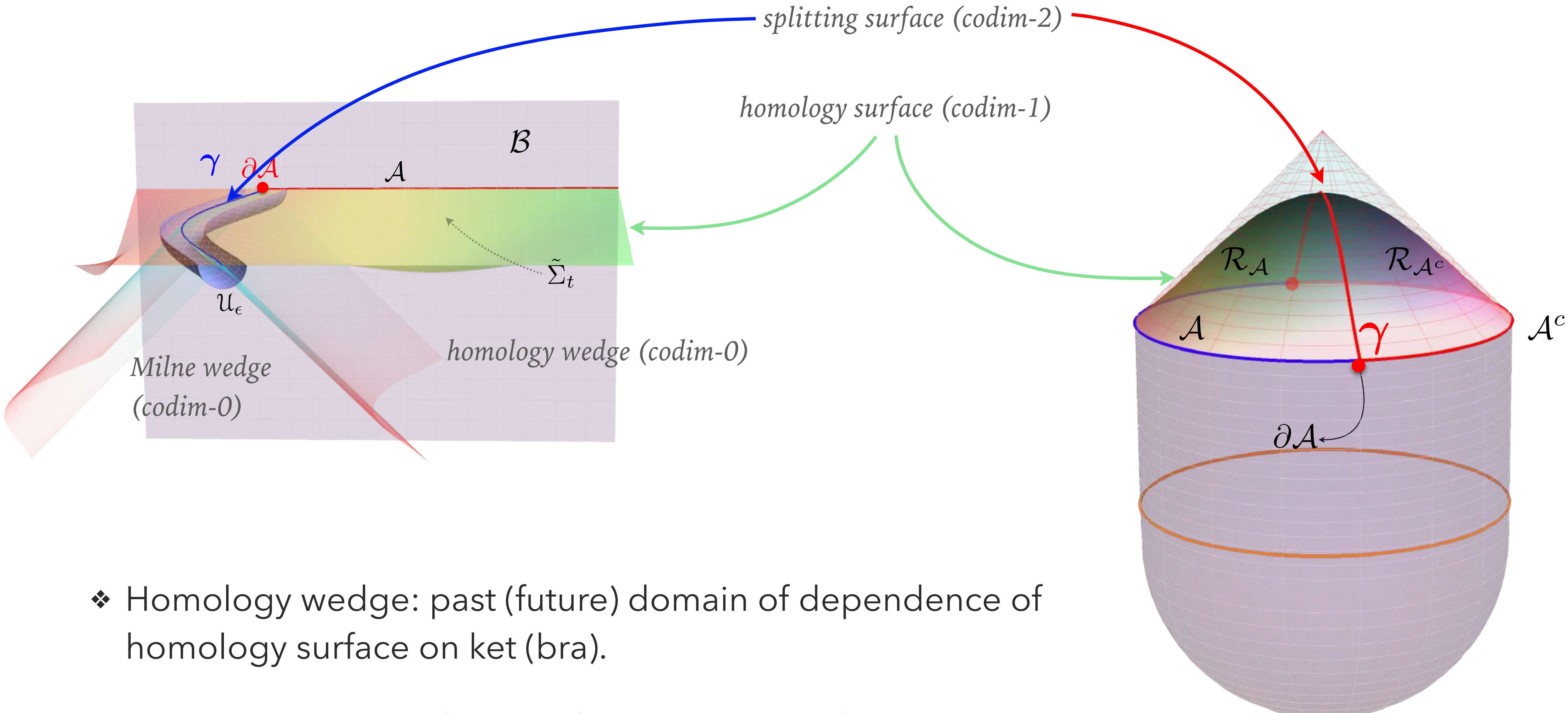


- To obtain matrix elements of the reduced density matrix:
  - cut open the path integral along the region $\mathcal{A}$ on the Cauchy slice Σ_t .
  - Impose suitable boundary conditions in the ket/bra segments.

- Compute the Rényi entropies by the replica trick, cycling gluing the density matrices.

ANSATZ FOR BULK GEOMETRY

* Extend boundary contour into the bulk, picking some bulk Cauchy surface anchored on boundary Cauchy surface: entangling surface extends to codimension-2 splitting surface.



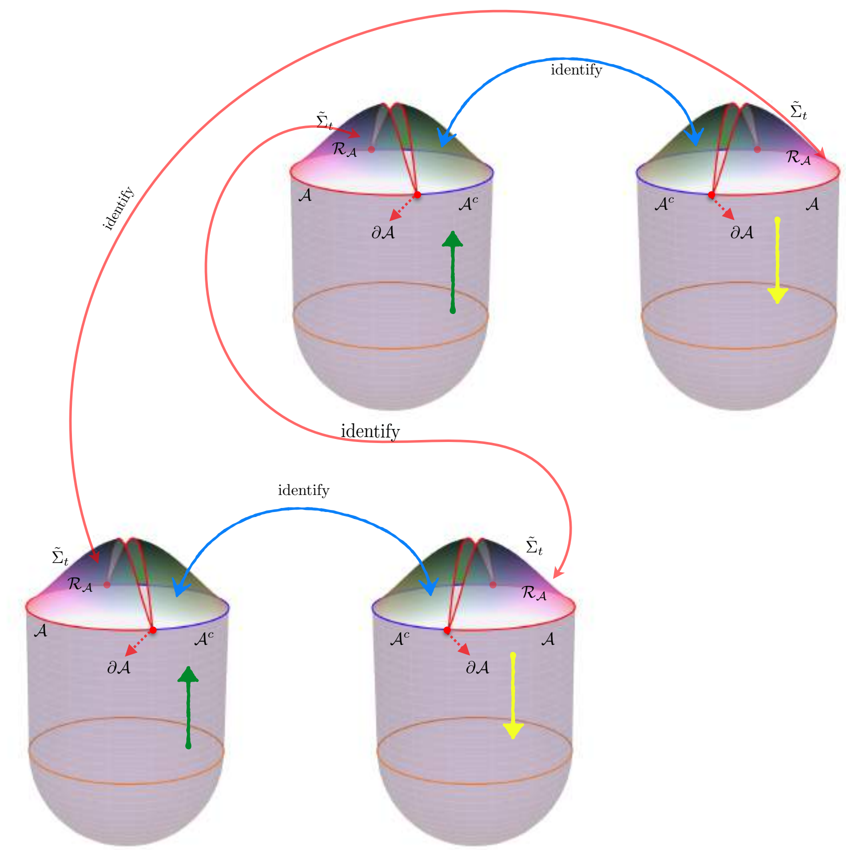
- Homology wedge: past (future) domain of dependence of homology surface on ket (bra).
- Milne wedge: past (future) of the splitting surface on ket (bra).

(PA)

Marolf, Wang '21

BULK REPLICA ANSATZ

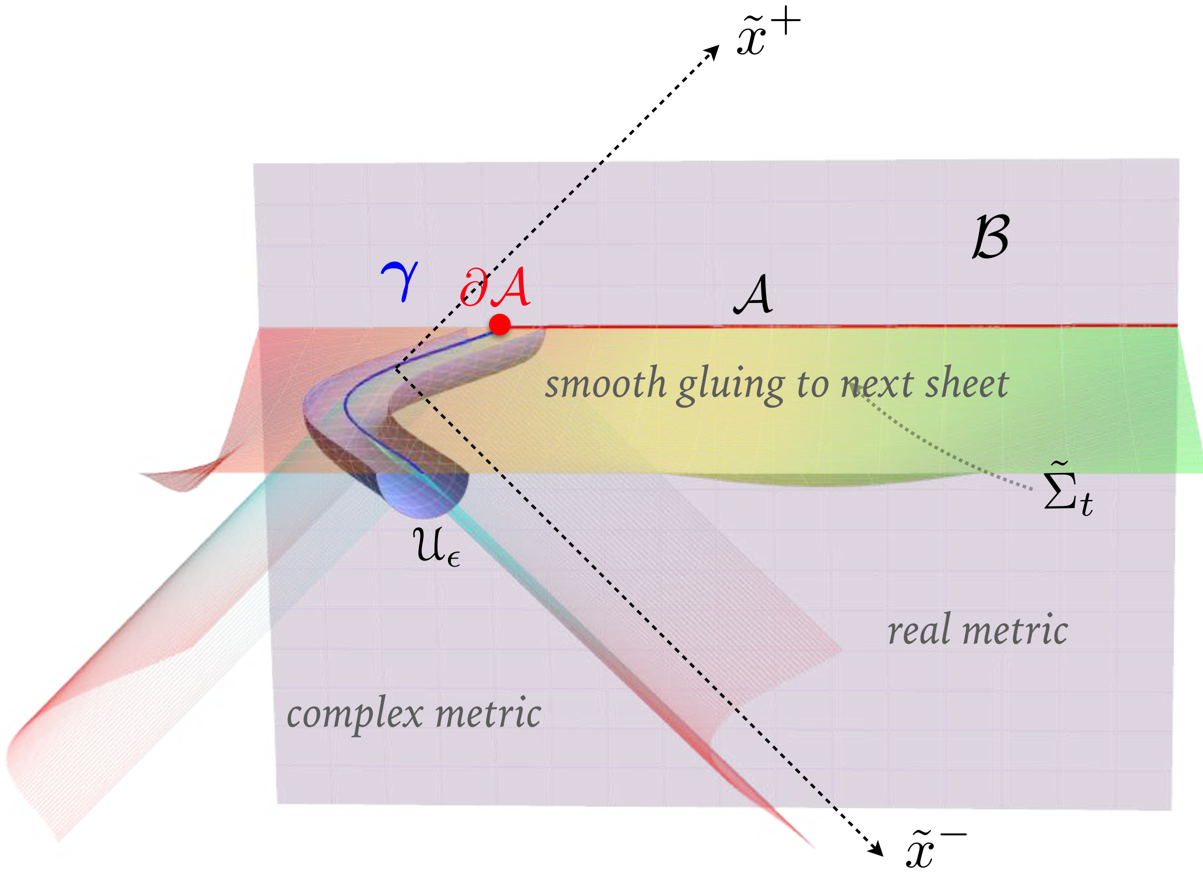
- Bulk replica: copies of bra and ket geometries glued across homology surfaces.
  - Complement region homology surface: glued between i^{th} ket and i^{th} bra.
  - Region homology surface glued between i^{th} ket and $(i+1)^{st}$ bra.



- Demand ansatz give a smooth stationary point of the gravitational action.
- Focus on replica symmetric saddles: complex Lorentz signature geometries owing to replica invariant surface having multiple distinct timelike normals which cannot be deformed to each other.

Euclidean discussion: Lewkowycz, Maldacena '13 Dong '16 Dong, Marolf '19

THE VARIATIONAL PROBLEM



Single fundamental domain: Ket

- real geometry in homology wedge with metric and extrinsic curvature smooth across homology surface
- Local Gaussian normal chart in vicinity of splitting surface (lightcone coordinates) suffices to determine bulk regularity constrains.

(PB)

- local metric depends on combinations $\left((\tilde{x}^+)^{\frac{1}{n}}, (\tilde{x}^-)^{\frac{1}{n}}, \tilde{x}^+\tilde{x}^-\right)$ and is analytic in first two arguments in the vicinity of the origin (splitting surface).
- negative powers of $\tilde{x}^+\tilde{x}^-$ regulated by $\tilde{x}^\pm \to \tilde{x}^\pm \mp i\varepsilon$

Related work: Goto, Hartman, Tajdini '20

RENYI ENTROPIES FROM BULK

$$Z[\mathcal{B}_n] := \int_n [Dg]\, e^{i\,S}$$

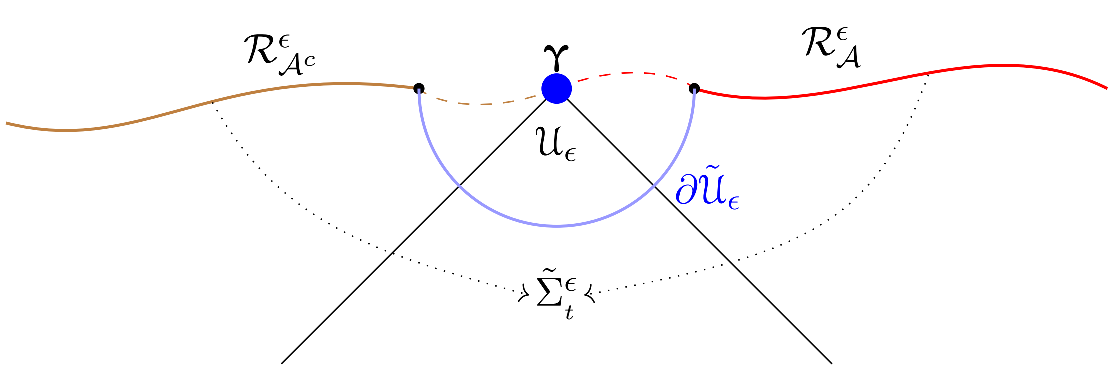
$$S = S^k_{\text{gr}} - S^b_{\text{gr}} + S_{\boldsymbol{\Upsilon}}$$

* Saddle point evaluation of bulk gravitational path integral with boundary replica geometry

$$S^{(n)}_{\mathcal{A}} = \frac{1}{1-n}\,\log\left(\frac{Z[\mathcal{B}_n]}{Z[\mathcal{B}]^n}\right)$$

$$S^{(n)}_{\mathcal{A}} = 2\frac{n}{n-1}\,\text{Im}\left(S^k_{\text{gr},n} - S^k_{\text{gr},1}\right) \qquad \text{(PB)}$$

* On-shell Lorentz signature action is purely imaginary
  * real part of on-shell action cancels between bra and ket.
  * imaginary part from crossing the light-cone



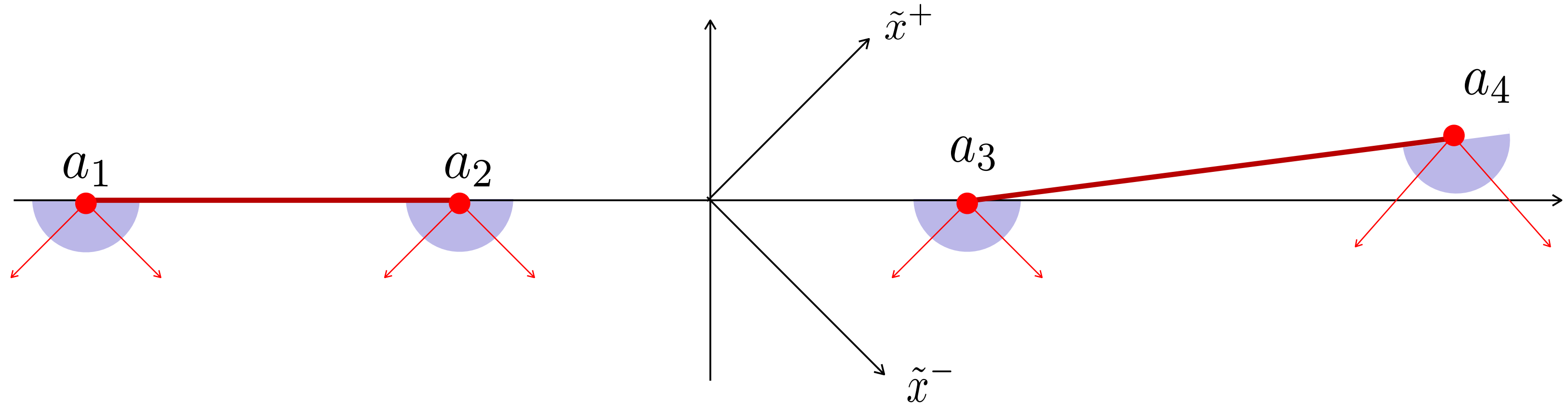
complex Gauss-Bonnet theorem

$$\int_{\mathfrak{U}_\epsilon} d^{d+1}x\,\sqrt{-g}R + \int_{\partial\mathfrak{U}_\epsilon} d^{d+1}x\,\sqrt{|h|}2K = -4\pi i\,\chi(\tilde{\mathfrak{U}}_\epsilon)\,A_{\boldsymbol{\Upsilon}}$$

Louko, Sorkin '95

Related work: Neiman '13

RELATIVELY BOOSTED INTERVALS IN 2D CFT



(PC)

$$T_{\mp\mp} = \sum_{i=1}^{4} \left[\frac{n^2-1}{2\,n^2} \frac{1}{(\tilde{x}^{\mp} - a_i)^2} + \frac{p_i(a_j)}{\tilde{x}^{\mp} - a_i} \right]$$

$p_i(a_j)$ *accessory parameters determine local source of energy-momentum to build the boundary branched cover geometry*

∗ Bulk geometry:

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{d\tilde{x}^+\, d\tilde{x}^-}{\rho} + (A_{\mu\nu} + \rho\, B_{\mu\nu})\, d\tilde{x}^\mu\, d\tilde{x}^\nu$$

functionals of $T_{\mu\nu}$

∗ On shell action:

$$S^k_{\text{gr},n} \equiv \frac{1}{16\pi G_N} \left[\int_{\mathcal{M}_n/\mathbb{Z}_n} d^3x \sqrt{-g}\,(R+2) + 2\int_{\mathcal{B}} d^2x\, \sqrt{-\gamma} K + S_{\text{ct}} \right] = \frac{c}{24\pi} \int_{\mathfrak{R}} d\tilde{x}^+\, d\tilde{x}^- \sqrt{T_{++}\, T_{--}}$$

∗ Rényi entropy:

$$\frac{\partial S^{(n)}}{\partial a_i} = -\frac{n}{6\,(n-1)}\, c\, p_i$$

variation easier to compute because it localizes

local contribution to Euler character set by accessory parameter.

Faulkner ‘13

SUMMARY & OPEN QUESTIONS

- Progress in understanding real-time gravitational path integrals: replica contours for Renyi computations and thermal correlators.
- Holography provides a useful paradigm for analyzing open quantum effective field theories.
- Non-Gaussian features of non-Markovian modes…
- Correlated noise systems…
- Other non-perturbative saddles:
  - long-time behaviour of correlators
  - replicas and islands
- Beyond general relativity….

Thank You!