Effective Strings

or:

Bringing Strings back to Strings conferences

Ofer Aharony

Weizmann Institute of Science

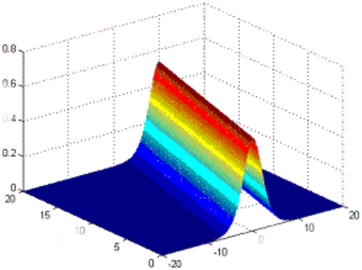
Strings 2021

São Paulo

June 28, 2021

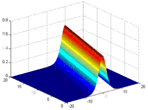
Strings appear in many places

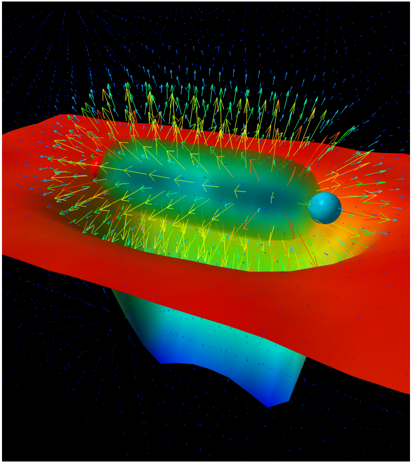
- Solitonic strings (e.g. Abrikosov-Nielsen-Olesen string in $3+1d$ Abelian Higgs model).



Strings appear in many places

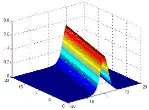
- Solitonic strings (e.g. Abrikosov-Nielsen-Olesen string in $3+1d$ Abelian Higgs model).
- Confining strings in Non-Abelian or (in 2d or 3d) Abelian gauge theories.

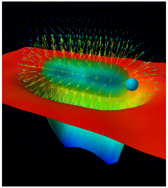


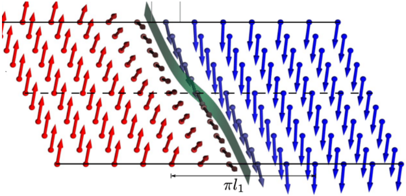


Strings appear in many places

- Solitonic strings (e.g. Abrikosov-Nielsen-Olesen string in $3 + 1d$ Abelian Higgs model).
- Confining strings in Non-Abelian or (in 2d or 3d) Abelian gauge theories.
- Domain walls in 2+1 dimensional theories.

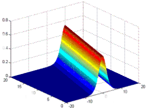


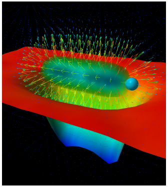


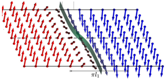


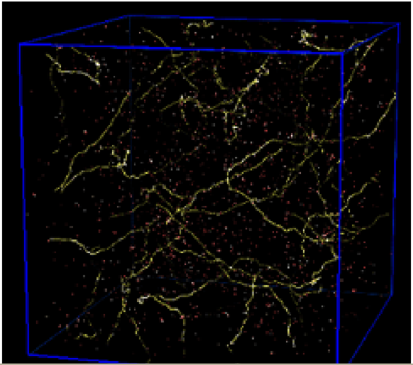
Strings appear in many places

- Solitonic strings (e.g. Abrikosov-Nielsen-Olesen string in $3+1d$ Abelian Higgs model).
- Confining strings in Non-Abelian or (in 2d or 3d) Abelian gauge theories.
- Domain walls in 2+1 dimensional theories.
- Cosmic strings.



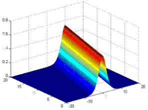


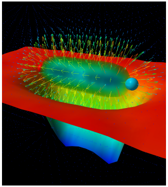


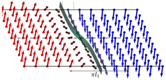


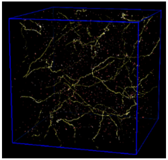
Strings appear in many places

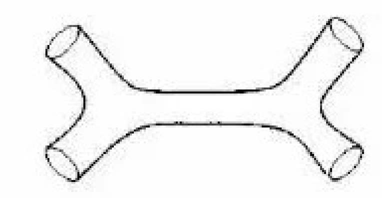
- Solitonic strings (e.g. Abrikosov-Nielsen-Olesen string in $3+1d$ Abelian Higgs model).
- Confining strings in Non-Abelian or (in 2d or 3d) Abelian gauge theories.
- Domain walls in 2+1 dimensional theories.
- Cosmic strings.
- Fundamental strings in string theory (can have overlap).











The long string effective action

Universal aspects of strings I

- In this talk I will discuss **stable** strings (e.g. confining strings when there are no fundamental quarks) with a finite tension T. Generically charged under some 1-form symmetry (continuous or discrete).

Universal aspects of strings I

- In this talk I will discuss **stable** strings (e.g. confining strings when there are no fundamental quarks) with a finite tension T. Generically charged under some 1-form symmetry (continuous or discrete).
- In most of this talk I will describe the universal aspects of these strings when they are *long* and close to being straight, namely the radius of curvature of the worldsheet is much larger than the string scale $l_s = 1/\sqrt{T}$. This is called *Effective String Theory*.

- In this talk I will discuss **stable** strings (e.g. confining strings when there are no fundamental quarks) with a finite tension T. Generically charged under some 1-form symmetry (continuous or discrete).
- In most of this talk I will describe the universal aspects of these strings when they are *long* and close to being straight, namely the radius of curvature of the worldsheet is much larger than the string scale $l_s = 1/\sqrt{T}$. This is called *Effective String Theory*.
- References < 2013 in **[OA-Komargodski]** . **[Akhmedov,Arvis,Athenodorou, Bahns,Baker,Billo,Botelho,Brandt,Bringoltz,Caselle,Chernodub,Dodelson,Drummond,Dubovsky,Field,Flauger, Gliozzi,Gorbenko,Gregory,Jaimungal,Karzbrun,Klinghoffer,Lohmayer,Luscher,Maeda,Majumdar,Makeenko, Mazur,Meineri,Nair,Natsuume,Neuberger,Orland,Pellegrini,Pepe,Polchinski,Polikarpov,Rejzner,Semenoff, Steinke,Strominger,Sundrum,Teper,Turok,Verduci,Vyas,Weisz,Wiese,Zago,Zahn,Zarembo,Zubkov]**

Universal aspects of strings II

- A straight infinite string in a D dimensional QFT breaks the Poincaré group : $ISO(D-1,1) \to ISO(1,1) \times SO(D-2)$.

Universal aspects of strings II

- A straight infinite string in a D dimensional QFT breaks the Poincaré group : $ISO(D-1,1) \to ISO(1,1) \times SO(D-2)$.

- While many space-time symmetry generators are broken, their actions on the worldsheet are not independent, and it turns out there are just $(D-2)$ Nambu-Goldstone bosons, which can be thought of as corresponding to the broken translations in the transverse directions. Namely, they are scalar fields labelling the transverse position of the string, which fluctuate along its worldsheet :

Universal aspects of strings III

- Generically, unless the string breaks extra symmetries, these would be the *only* massless fields on the worldsheet. All other fields are expected to be at scales of order $M_s = \sqrt{T}$; in particular the width of the string is expected to be at this scale (and thus the mass of any internal excitations).

Universal aspects of strings III

- Generically, unless the string breaks extra symmetries, these would be the *only* massless fields on the worldsheet. All other fields are expected to be at scales of order $M_s = \sqrt{T}$; in particular the width of the string is expected to be at this scale (and thus the mass of any internal excitations).
- When the bulk theory has a mass gap M_{gap}, the full low-energy effective action in the presence of a string (below M_{gap}, M_s) is thus given by the $(D-2)$ massless scalars. It has a derivative expansion, with the first few terms universal.

Universal aspects of strings III

- Generically, unless the string breaks extra symmetries, these would be the *only* massless fields on the worldsheet. All other fields are expected to be at scales of order $M_s = \sqrt{T}$; in particular the width of the string is expected to be at this scale (and thus the mass of any internal excitations).
- When the bulk theory has a mass gap M_{gap}, the full low-energy effective action in the presence of a string (below M_{gap}, M_s) is thus given by the $(D-2)$ massless scalars. It has a derivative expansion, with the first few terms universal.
- We can write this effective action using the string embedding

$$S = \int d^2\sigma \mathcal{L}[X^\mu(\sigma)]$$

with diffeomorphism invariance in σ (no preferred choice) and Poincaré invariance in X. Validity limited to long strings.

The long string effective action

- Using the long string effective action we can compute physical observables such as :
  1. Low-energy scattering amplitudes of NGBs. Can be constrained by S-matrix bootstrap.
  2. Energy levels of long strings (e.g. wrapped on a large circle, or stretched between D-branes) : for closed strings

$$E_n(L) = TL + \frac{a_n^{(1)}}{L} + \frac{a_n^{(2)}}{TL^3} + \frac{a_n^{(3)}}{T^2L^5} + \frac{a_n^{(4)}}{T^3L^7} + \cdots$$

All terms up to $O(1/L^5)$ are universal. Open confining string ground state energy = $q - \bar{q}$ potential. Can match to lattice.

The long string effective action

- Using the long string effective action we can compute physical observables such as :
  1. Low-energy scattering amplitudes of NGBs. Can be constrained by S-matrix bootstrap.
  2. Energy levels of long strings (e.g. wrapped on a large circle, or stretched between D-branes) : for closed strings

$$E_n(L) = TL + \frac{a_n^{(1)}}{L} + \frac{a_n^{(2)}}{TL^3} + \frac{a_n^{(3)}}{T^2L^5} + \frac{a_n^{(4)}}{T^3L^7} + \cdots$$

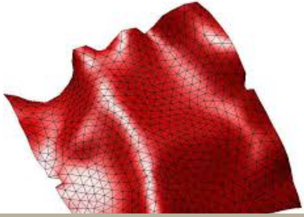
     All terms up to $O(1/L^5)$ are universal. Open confining string ground state energy $= q - \bar{q}$ potential. Can match to lattice.
- We'll discuss 3 different ways to analyze the long string effective action $S = \int d^2\sigma \mathcal{L}[X^\mu(\sigma)]$:
  1. No gauge-fixing
  2. Static gauge
  3. Orthogonal gauge **[Polchinski-Strominger]**

The long string effective action - no gauge fixing I

- At the classical level don't have to gauge-fix, and can look for actions $S = \int d^2\sigma \mathcal{L}[X^\mu(\sigma)]$ with
  - Diffeomorphism invariance in σ^a ($a = 0, 1$)
  - Poincaré invariance $ISO(D-1, 1)$ in X^μ ($\mu = 0, \cdots, D-1$)

The long string effective action - no gauge fixing I

- At the classical level don't have to gauge-fix, and can look for actions $S = \int d^2\sigma \mathcal{L}[X^\mu(\sigma)]$ with
  - Diffeomorphism invariance in σ^a ($a = 0, 1$)
  - Poincaré invariance $ISO(D-1,1)$ in X^μ ($\mu = 0, \cdots, D-1$)
- Such actions can be written using various geometric properties of the string embedding : in the derivative expansion enough to use
  - The induced metric $h_{ab}(\sigma) = \partial_a X^\mu(\sigma)\partial_b X_\mu(\sigma)$, and the curvature $R_{ab}[h]$ constructed from it.
  - The extrinsic curvature $K^\mu_{ab}(\sigma) = \nabla_a \partial_b X^\mu(\sigma)$.



The long string effective action - no gauge fixing II

- Terms in the effective action can be classified by the weight; all X's appear with derivatives so we assign weight $w = 0$ to $\partial_a X^\mu$, and weight $w = 1$ to any derivative acting on it. Thus the metric h_{ab} has $w = 0$ and the curvature K^μ_{ab} has $w = 1$.
- At $w = 0$ there is a single diffeo-invariant term that can be written down, which is just the *Nambu-Goto action*

$$S_{w=0} = -T \int d^2\sigma \sqrt{-\det(h_{ab})}.$$

We assume $T \neq 0$ to have a good derivative expansion around this. The leading order EOM are $\Box X^\mu = 0$.

The long string effective action - no gauge fixing III

- At $w = 2$ there are two independent terms one can write down :

$$S_{w=2} = \int d^2\sigma \sqrt{-\det(h)}(a_1 R[h] + a_2 (K^{\mu}_{ab})^2).$$

- The first term is a topological invariant - no effect on long strings.
- The second term (rigidity term) is proportional to the leading order equations of motion, so it can be eliminated by a field redefinition (shifted to higher orders).
- So, no corrections at expected leading order ! (Unlike most effective actions.)

The long string effective action - no gauge fixing IV

- The leading corrections are thus expected to come from $w = 4$ terms ($O(1/L^7)$):

$$S = \int d^2\sigma \sqrt{-\det(h)}(-T + a_3 R[h]^2 + a_4 K^4 + \text{higher orders}),$$

with two independent coefficients at leading order for $D > 3$ (one for $D = 3$).

The long string effective action - no gauge fixing IV

- The leading corrections are thus expected to come from $w = 4$ terms ($O(1/L^7)$):

$$S = \int d^2\sigma \sqrt{-\det(h)}(-T + a_3 R[h]^2 + a_4 K^4 + \text{higher orders}),$$

  with two independent coefficients at leading order for $D > 3$ (one for $D = 3$).
- Does this remain true quantum mechanically ?

The static gauge effective action I

- For a long string mostly stretched along $X^{0,1}$, natural to gauge-fix $\sigma^0 = X^0$, $\sigma^1 = X^1$ – **static gauge**.
- Remain just with $(D-2)$ scalars $X^i(\sigma)$ ($i = 2, \cdots, D-1$) which are the NGBs; no remaining gauge freedom, no non-trivial Jacobian.
- $S = \int d^2\sigma \mathcal{L}[X^i(\sigma)]$, invariant under $ISO(1,1) \times SO(D-2)$.

The static gauge effective action I

- For a long string mostly stretched along $X^{0,1}$, natural to gauge-fix $\sigma^0 = X^0$, $\sigma^1 = X^1$ – **static gauge**.
- Remain just with $(D-2)$ scalars $X^i(\sigma)$ ($i = 2, \cdots, D-1$) which are the NGBs; no remaining gauge freedom, no non-trivial Jacobian.
- $S = \int d^2\sigma \mathcal{L}[X^i(\sigma)]$, invariant under $ISO(1,1) \times SO(D-2)$.
- Disadvantage : Lorentz invariance is not manifest (non-linearly realized), needs to be carefully checked in quantum theory (as in light-cone gauge).

The static gauge effective action II

- Can again classify terms by their weight. At weight $w = 0$ all terms are polynomials in $y = \partial_+ X^i \partial_- X^i$ and in $z = (\partial_+ X^i \partial_+ X^i)(\partial_- X^j \partial_- X^j)$, and by considering their Lorentz variation, just one invariant combination

$$S_{w=0} = -T \int d^2\sigma \sqrt{-\det(\eta_{ab} + \partial_a X^i \partial_b X^i)}.$$

  Interacting theory ($(\partial X)^4 + \cdots$), like chiral Lagrangian.

The static gauge effective action II

- Can again classify terms by their weight. At weight $w = 0$ all terms are polynomials in $y = \partial_+ X^i \partial_- X^i$ and in $z = (\partial_+ X^i \partial_+ X^i)(\partial_- X^j \partial_- X^j)$, and by considering their Lorentz variation, just one invariant combination

$$S_{w=0} = -T \int d^2\sigma \sqrt{-\det(\eta_{ab} + \partial_a X^i \partial_b X^i)}.$$

  Interacting theory ($(\partial X)^4 + \cdots$), like chiral Lagrangian.

- At weight $w = 2$ there is a unique term whose Lorentz transformation is proportional to EOM, of the form

$$\mathcal{L}_{w=2} = c_4 (\partial_+^2 X^i \partial_-^2 X^i)(\partial_+ X^j \partial_- X^j).$$

  However, adding it to the action modifies the Lorentz algebra, $[M_{+i}, M_{+j}] \neq 0$, so it is not allowed, and allowed terms start at $w = 4$ as in the classical analysis.

The static gauge effective action III

$$\mathcal{L}_{w=2} = c_4(\partial_+^2 X^i \partial_-^2 X^i)(\partial_+ X^j \partial_- X^j).$$

- Computations in static gauge require regularization, which often breaks Lorentz invariance (e.g. cutoff, zeta function). In those cases one generally does find a c_4 term with a specific (regularization-dependent) value which is needed to restore Lorentz invariance in the quantum theory. Not present in dimensional regularization. NOT a Wilson coefficient.
- Non-universal terms start at $w = 4$ as before. Computations in this gauge are conceptually straightforward but complicated.

The orthogonal gauge effective action I

- Another natural gauge-fixing of diffeos is the **orthogonal gauge** $h_{ab}(\sigma) = e^{2\phi}(\sigma)\eta_{ab}$.
- Similar (but not the same as) conformal gauge in fundamental string theory.

The orthogonal gauge effective action I

- Another natural gauge-fixing of diffeos is the **orthogonal gauge** $h_{ab}(\sigma) = e^{2\phi}(\sigma)\eta_{ab}$.
- Similar (but not the same as) conformal gauge in fundamental string theory.
- Advantages of this gauge choice include :
  1. The leading order Nambu-Goto action is free, $S_{w=0} = -2T \int d^2\sigma \partial_+ X^\mu \partial_- X_\mu$.
  2. The full Poincaré symmetry $ISO(D-1,1)$ is manifest.
  3. Remaining diffeo invariance = conformal symmetry, strong constraint on action.

The orthogonal gauge effective action I

- Another natural gauge-fixing of diffeos is the **orthogonal gauge** $h_{ab}(\sigma) = e^{2\phi}(\sigma)\eta_{ab}$.
- Similar (but not the same as) conformal gauge in fundamental string theory.
- Advantages of this gauge choice include :
  1. The leading order Nambu-Goto action is free, $S_{w=0} = -2T \int d^2\sigma \partial_+ X^\mu \partial_- X_\mu$.
  2. The full Poincaré symmetry $ISO(D-1,1)$ is manifest.
  3. Remaining diffeo invariance = conformal symmetry, strong constraint on action.
- Disadvantages : ghosts, complicated constraints on physical states.

The orthogonal gauge effective action II

- [Polchinski-Strominger] conjectured that one can quantize in this gauge by imposing Virasoro constraints on physical states, as for fundamental strings in conformal gauge.
- No direct derivation, but can derive by adding an auxiliary metric as in the Polyakov formalism [Hellerman-Maeda-Maltz-Swanson] (also [Hari Dass-Matlock]).

The orthogonal gauge effective action II

- [Polchinski-Strominger] conjectured that one can quantize in this gauge by imposing Virasoro constraints on physical states, as for fundamental strings in conformal gauge.
- No direct derivation, but can derive by adding an auxiliary metric as in the Polyakov formalism [Hellerman-Maeda-Maltz-Swanson] (also [Hari Dass-Matlock]).
- Naively conformal invariance does not allow any corrections to $S_{w=0} = -2T \int d^2\sigma \partial_+ X^\mu \partial_- X_\mu$.

The orthogonal gauge effective action II

- [Polchinski-Strominger] conjectured that one can quantize in this gauge by imposing Virasoro constraints on physical states, as for fundamental strings in conformal gauge.
- No direct derivation, but can derive by adding an auxiliary metric as in the Polyakov formalism [Hellerman-Maeda-Maltz-Swanson] (also [Hari Dass-Matlock]).
- Naively conformal invariance does not allow any corrections to $S_{w=0} = -2T \int d^2\sigma \partial_+ X^\mu \partial_- X_\mu$.
- However, in the context of the long string effective action where $Z \equiv \partial_+ X^\mu \partial_- X_\mu$ obeys $\langle Z \rangle \neq 0$, can allow negative powers of Z :

$$S = \frac{\beta}{4\pi} \int d^2\sigma \frac{\partial_+ Z \partial_- Z}{Z^2} + \text{higher orders.}$$

Can arise by integrating out other worldsheet fields.

$$S = -2T \int d^2\sigma \partial_+ X^\mu \partial_- X_\mu + \frac{\beta}{4\pi} \int d^2\sigma \frac{\partial_+ Z \partial_- Z}{Z^2} + \text{higher orders.}$$

- Consistency of this approach requires $c = 26$, while $S_{w=0}$ gives $c = D$.

The orthogonal gauge effective action III

$$S = -2T \int d^2\sigma \partial_+ X^\mu \partial_- X_\mu + \frac{\beta}{4\pi} \int d^2\sigma \frac{\partial_+ Z \partial_- Z}{Z^2}$$
$$+ \text{higher orders.}$$

- Consistency of this approach requires $c = 26$, while $S_{w=0}$ gives $c = D$.
- It turns out that the β term also contributes to c (expanding around a long string), while higher order terms do not, and this fixes $\beta = (D - 26)/12$. So the leading interaction is universal and proportional to $(D - 26)$!
- Higher order terms are not fixed, and match with $w \geq 4$ terms in other approaches.

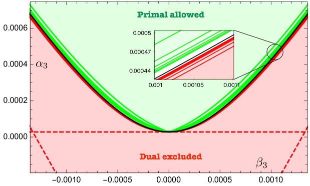
Summary of effective string action

- All approaches indicate leading order terms universal, non-universal terms start at $w = 4$ (corresponding to $O(1/L^7)$ for energy levels of long closed strings, $O(s^4)$ for S-matrix). All computations of energy levels and scattering amplitudes **[Dubovsky-Flauger-Gorbenko]** consistent between different approaches. Leading scattering (for $D > 3$) universal and $\propto (D - 26)$! (Also other approaches.)

Summary of effective string action

- All approaches indicate leading order terms universal, non-universal terms start at $w = 4$ (corresponding to $O(1/L^7)$ for energy levels of long closed strings, $O(s^4)$ for S-matrix). All computations of energy levels and scattering amplitudes [Dubovsky-Flauger-Gorbenko] consistent between different approaches. Leading scattering (for $D > 3$) universal and $\propto (D - 26)$! (Also other approaches.)
- Can compare (successfully) to :
  1. Computation of the effective action when it is under control, e.g. in perturbation theory for solitonic strings, or holographic confining strings in weakly curved backgrounds.
  2. Energy levels of long strings, in particular from lattice simulations in $D = 3$ and $D = 4$ Yang-Mills theories [Athenodorou-Teper,many other lattice groups] .

- Additional constraints on the effective action come from requiring unitarity of the effective action, can be studied using S-matrix bootstrap [Miró-Guerrieri-Hebbar-Penedones-Vieira,Miró-Guerrieri] .
- It turns out that considering just unitarity of $2 \to 2$ scattering can provide lower bounds on the leading non-universal coefficients a_3, a_4. For $D = 3$ where there is just one coefficient, a lower bound on it can be obtained analytically; otherwise lower bounds may be obtained numerically.



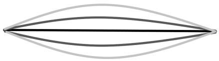
Generalizations I

- The generalization to open strings (Neumann or ending on D-branes) is straightforward, but extra terms appear on the boundary.
- For Neumann boundary conditions the leading boundary term is a mass term $S = -m \int d\tau \sqrt{h_{00}}$.
- Beyond this, the leading non-universal correction for both boundary conditions starts at $O(1/L^4)$.
- Matches with lattice simulations of confining (and other) strings.

Generalizations II

- When there are more broken symmetries, like supersymmetries, have extra light fields, and can generalize the computations.
- In particular the confining string in $D = 4$ $\mathcal{N} = 1$ SYM theory has four Goldstinos, effective action analyzed in [Solberg-Yutushui] . The confining string in $D = 2$ SYM has only a Goldstino.
- In $D = 4, \mathcal{N} = 1, 2$ or $D = 3, \mathcal{N} = 2$ theories can also have BPS strings, where worldsheet effective action is supersymmetric (+extra non-linearly realized supercharges).
- Many generalizations have not yet been studied.

The effective string action also controls



- The width of long strings $\propto \log(L)$.

Additional applications

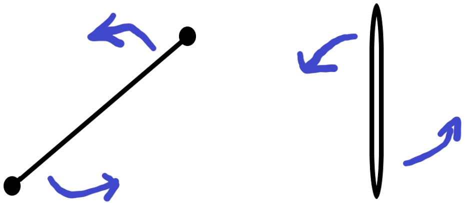
The effective string action also controls

- The width of long strings $\propto \log(L)$.
- Expectation values of large Wilson loops (for confining strings).

Additional applications

The effective string action also controls

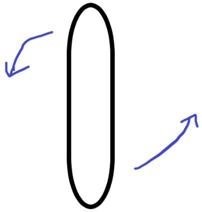
- The width of long strings $\propto \log(L)$.
- Expectation values of large Wilson loops (for confining strings).
- The spectrum of high-spin states (glueballs and mesons) [Hellerman-Swanson] – historical origin of string theory. Subtle for closed strings in $D \leq 4$ due to folds [Sonnenschein-Weissman-Yankielowicz] .



Additional applications

For instance, for rotating closed strings in $D \geq 5$ [Hellerman-Swanson] found

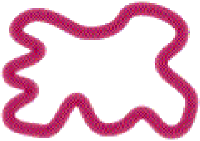
$$M^2 = 2\pi T \left[2(J_1 + J_2) - \frac{D-2}{6} + \frac{26-D}{12} \left(\left(\frac{J_1}{J_2}\right)^{1/4} - \left(\frac{J_2}{J_1}\right)^{1/4} \right)^2 + O(J^{-1}) \right].$$



Additional applications

Cannot use the long-string effective action to compute

- The spectrum of short string states (light mesons and glueballs).



- The Hagedorn temperature, where a string wrapped on a Euclidean circle becomes massless (around $L \simeq 1/\sqrt{T}$).

Beyond the low-energy effective action

Going to higher energies

- Generally that we expect that around the scale $\sqrt{T}$ additional modes, in the bulk and on the worldsheet, will couple to the Nambu-Goldstone bosons, and the effective action will break down.
- However, for some strings there is a limit where they decouple from the bulk fields. In particular this is the case for confining strings in large N gauge theories **['t Hooft]** . In this decoupling limit the $1+1$-dimensional theory on the worldsheet (generally with extra fields) makes sense at all energy scales.

Going to higher energies

- Generally that we expect that around the scale $\sqrt{T}$ additional modes, in the bulk and on the worldsheet, will couple to the Nambu-Goldstone bosons, and the effective action will break down.
- However, for <u>some</u> strings there is a limit where they decouple from the bulk fields. In particular this is the case for confining strings in large N gauge theories **['t Hooft]** . In this decoupling limit the $1+1$-dimensional theory on the worldsheet (generally with extra fields) makes sense at all energy scales.
- We expect that in such cases it should be possible to think of the worldsheet as a fundamental string, such that quantizing it gives rise also to the bulk physics (perturbatively in $1/N$).
- In particular we have examples of this in holographic confining backgrounds **[Witten,Polchinski-Strassler,Klebanov-Strassler,$\cdots$]** .

Going to higher energies II

- In general no reason why this all-scale worldsheet theory should be weakly coupled (though it is in some limits of holographic confining backgrounds which have a supergravity approximation).
- Do not expect worldsheet to be a bosonic string / NSR superstring, probably some formalism that can incorporate RR backgrounds is needed (pure spinor formalism ?).

Going to higher energies II

- In general no reason why this all-scale worldsheet theory should be weakly coupled (though it is in some limits of holographic confining backgrounds which have a supergravity approximation).
- Do not expect worldsheet to be a bosonic string / NSR superstring, probably some formalism that can incorporate RR backgrounds is needed (pure spinor formalism ?).
- So, what can we do ?
    - Bottom-up : approximate integrability
    - Top-down approaches

- Specifically for the confining strings of large N $D = 3$ and $D = 4$ Yang-Mills theories, no reason to expect a weakly coupled worldsheet theory. So the best hope to understand the worldsheet would be if it happens to be an integrable theory.
- Surprisingly, [Dubovsky-Flauger-Gorbenko, Dubovsky-Gorbenko] found that these strings are well-approximated by integrable theories on the worldsheet (whose spectrum can be computed exactly for all L) !
- For $D = 3$ the confining string is well-approximated by the bosonic Nambu-Goto action (integrable for $D = 3, 26$)

$$S = -T \int d^2\sigma \sqrt{h} = -T \int d^2\sigma \sqrt{-\det(\eta_{ab} + \partial_a X \partial_b X)},$$

even down to values of L where the effective action is expected to break down.

Approximate integrability for QCD strings II

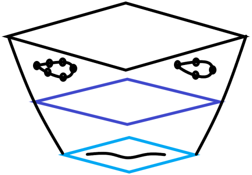
- For $D = 4$ the integrable model includes, in addition to the Nambu-Goldstone bosons, an extra pseudo-scalar field $a(\sigma)$:

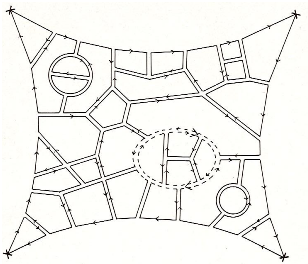
$$S_a = \frac{Q_a}{4} \int d^2\sigma\, a \cdot \sqrt{-h} h^{ab} \epsilon_{\mu\nu\rho\sigma} \partial_a t^{\mu\nu} \partial_b t^{\rho\sigma},$$

  where $t^{\mu\nu} = \frac{\epsilon^{ab}}{\sqrt{-h}} \partial_a X^\mu \partial_b X^\nu$. This theory happens to be integrable for $Q_a = \sqrt{\frac{7}{16\pi}}$.
- For a specific mass m_a this matches the confining string spectrum very well (including pseudo-scalar excitations).
- The reasons for this approximate integrability are not yet clear.
- Can we systematically expand the QCD string action around these integrable theories (additional fields / interactions) ?

Top-down – asymptotic freedom

- Holography : worldsheet theory of QCD string should have some field $\phi(\sigma)$ that can be thought of as the "radial direction". Long strings will sit near the minimum value of ϕ, and there ϕ is massive and can be integrated out to reproduce the effective string action. For large ϕ we should be in the UV where the gauge theory is free – so worldsheets should become "fishnets", and the worldsheet action should become topological ("string bits").





Top-down – asymptotic freedom

- Holography : worldsheet theory of QCD string should have some field $\phi(\sigma)$ that can be thought of as the "radial direction". Long strings will sit near the minimum value of ϕ, and there ϕ is massive and can be integrated out to reproduce the effective string action. For large ϕ we should be in the UV where the gauge theory is free – so worldsheets should become "fishnets", and the worldsheet action should become topological ("string bits").
- From this point of view should start from string dual of a free gauge theory (for $\mathcal{N} = 4$ SYM some versions were suggested by [Berkovits-Vafa,Berkovits,Gaberdiel-Gopakumar] , and these can perhaps be generalized to the non-supersymmetric case) and deform it by the gauge coupling.

Top-down – known duals I

- Alternatively we could start from a theory whose holographic dual is known, and which we can deform to get $D = 3$ or $D = 4$ Yang-Mills. Shows that QCD string is a (limit of a) standard fundamental string (at least if no phase transitions).
- For instance, we could start from $\mathcal{N} = 4$ SYM and add masses to all scalars/fermions. In the limit of $m \to \infty$, $g_{YM}^2 N \to 0$ keeping the low-energy QCD scale $\Lambda_{QCD} \simeq m \exp\left(-\#/g_{YM}^2 N\right)$ fixed, this will give the $D = 4$ QCD string. But this requires understanding the precise string dual of $\mathcal{N} = 4$ mass deformations (complicated backgrounds [Polchinski-Strassler]), and then also taking the free limit $g_{YM}^2 N \to 0$.

Top-down – known duals II

- Or, we could take a similar limit for $\mathcal{N} = 4$ SYM on a circle with anti-periodic boundary conditions for fermions **[Witten]** , and then a limit of $R \to 0$, $g_{YM}^2 N \to 0$ keeping the $D = 3$ gauge coupling fixed would give the $D = 3$ QCD string. This requires understanding the precise string worldsheet for the corresponding background = AdS black brane (also complicated).

Summary

- The effective action of long strings has several universal terms that control many of its low-energy properties.

Summary

- The effective action of long strings has several universal terms that control many of its low-energy properties.
- For large N confining strings there should be a description of the worldsheet at all energy scales, but this is yet to be found. Does this exist for any non-confining strings ?
- Can we learn from well-understood cases like $D = 2$ QCD strings [Gross-Taylor] , or the (topological) closed string duals of Chern-Simons theory [Gopakumar-Vafa,Ooguri-Vafa] ?

Summary

- The effective action of long strings has several universal terms that control many of its low-energy properties.
- For large N confining strings there should be a description of the worldsheet at all energy scales, but this is yet to be found. Does this exist for any non-confining strings ?
- Can we learn from well-understood cases like $D = 2$ QCD strings [Gross-Taylor] , or the (topological) closed string duals of Chern-Simons theory [Gopakumar-Vafa,Ooguri-Vafa] ?
- More discussion in a few hours [Dubovsky-Klebanov] .  
- Thank you for listening !