Achievements, Progress and Open Questions in String Field Theory

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Achievements

We consider here instances where string field theory provided the answer to physical open questions.

- Tachyon condensation, tachyon vacuum, tachyon conjectures

  The tachyon conjectures (Sen, 1999) posited that:

  (a) The tachyon potential has a locally stable minimum, whose energy density measured with respect to that of the unstable critical point, equals minus the tension of the D25-brane

  (b) Lower-dimensional D-branes are solitonic solutions of the string theory on the back- ground of a D25-brane.

  (c) The locally stable vacuum of the system is the closed string vacuum; it has no open string excitations exist.

  Work in SFT established these conjectures by finding the tachyon vacuum, first numerically, and then analytically (Schnabl, 2005). These are non-perturbative results.

- String field theory is the first complete definition of string perturbation theory.

The first-quantized world-sheet formulation of string theory does not define string perturbation theory completely:

– No systematic way of dealing with IR divergences.

– No systematic way of dealing with S-matrix elements for states that undergo mass renormalization.

Work of A. Sen and collaborators demonstrating this:

(a) One loop-mass renormalization of unstable particles in critical string theories.

(b) Fixing ambiguities in two-dimensional string theory: For the one-instanton contribution to N-point scattering amplitudes there are four undetermined constants (Balthazar, Rodriguez, Yin, 2019). Two of them have been fixed with SFT (Sen 2020)

(c) Fixing the normalization of Type IIB D-instanton amplitudes (Sen, 2021). Talk in this conference.

Progress Since 2015

- Analytic solutions for any boundary CFT

  Using Witten's OSFT one can now find solutions describing general open string backgrounds (Erler, Maccaferri, 2014-2020).

  Shows OSFT has non-perturbative information and it is physically background independent.

- Formulations of superstring field theory

  This overcomes a major stumbling block.

  For open superstring field theory we have

  – Kunitomo, Okawa (2015, WZW based)

  – Erler, Okawa, Takezaki (2016, A_∞ based).

  – Konopka, Sachs (2016, A_∞ based)

For Heterotic and type II superstrings we have partial constructions (Berkovits, Okawa, Zwiebach, Erler, Konopka, Sachs, Matsunaga, Goto, Kunitomo, Sugimoto...) and a complete one:

– Sen (arXiv:1508.05387). New insight is the inclusion of an extra copy $\tilde{\Psi}$ of the string field Ψ. The new field represents free decoupled degrees of freedom but solves the longstanding problem of writing a Lorentz-covariant gauge-invariant Lagrangian:

$$S \sim -\tfrac{1}{2}\langle\tilde{\Psi}, Q\mathcal{G}\tilde{\Psi}\rangle + \langle\tilde{\Psi}, Q\Psi\rangle + \sum_{n=3}^{\infty}\frac{1}{n!}\{\Psi^n\}$$

S obeys the Batalin-Vilkovisky master equation. Open-closed theories are also available.

This completes a first construction of all string field theories.

(A very recent WZW formulation by Kunitomo, 2106.07917)

- String vertices and moduli spaces of Riemann surfaces

  The definition of SFT requires string vertices that fix the off-shell amplitudes. Such vertices satisfy the 'geometric' BV master equation.

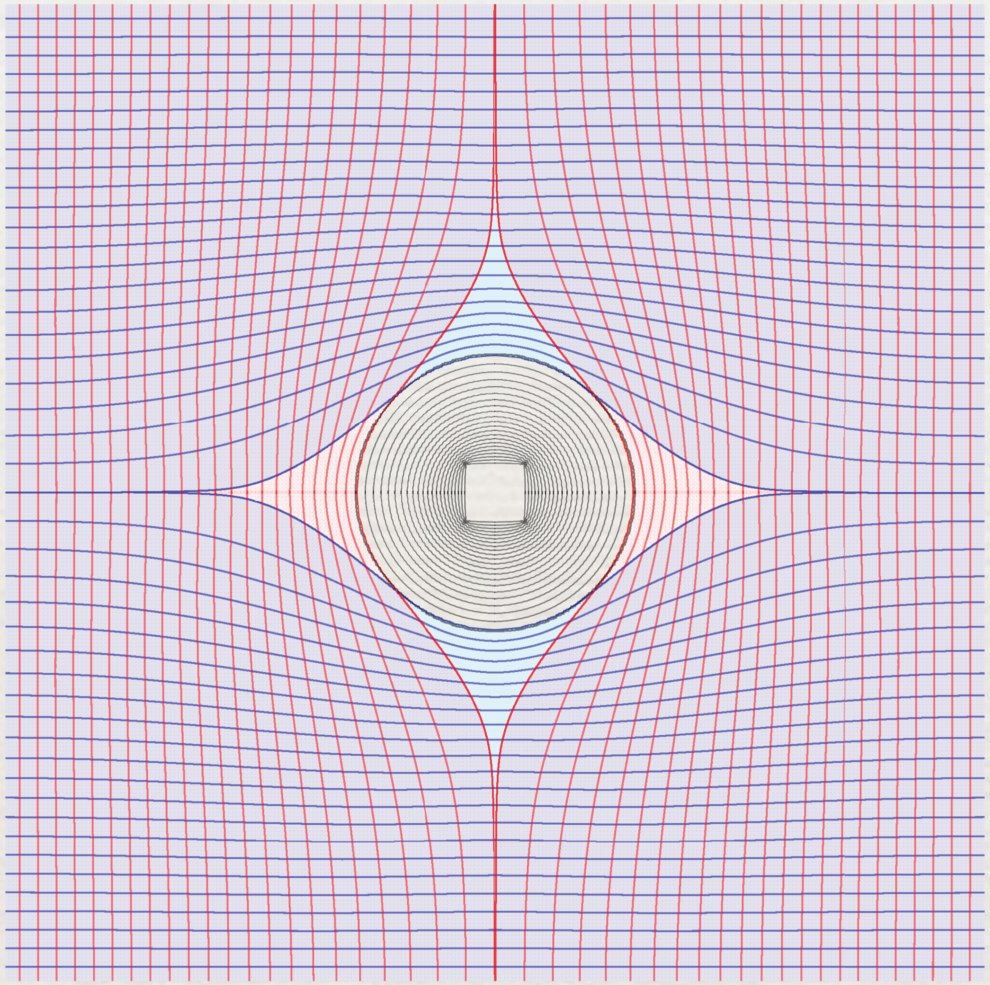
  (a) Minimal-area metrics define Witten's OSFT and classical CSFT.

  Metrics are flat except for negative curvature singularities, and arise from Jenkins-Strebel (JS) quadratic differentials (qds).

  For higher genus the minimal area metrics are not known except when they arise from JS-qds.

  First unknown metrics found (M. Headrick and B. Z, 2018) using the convex-optimization tools that also help in holography (string bits).

  The metrics have intersecting bands of geodesics and have bulk curvature. The results suggest these metrics exists for all genus.



Minimal area metric for a square torus $\tau = i$ with one puncture.

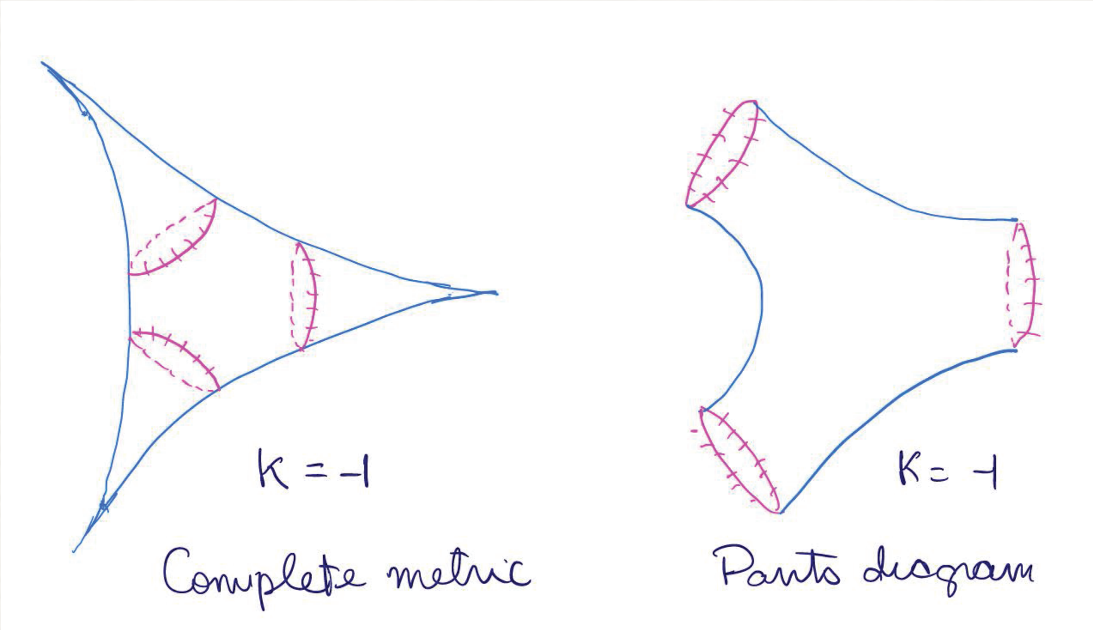
(b) Vertices from hyperbolic geometry

– Approximate description of hyperbolic vertices. Integrals over moduli spaces may be doable (Moosavian and Pius, 2017).

– Exact definition of hyperbolic vertices (K. Costello and B.Z, 2019). Rigorous definition of gauge-invariant string field theory.

– Extension to open-closed string field theory (Cho, 2019).

– Off-shell three-string vertex (Firat, 2021).



It is conceivable that the hyperbolic description of moduli spaces would allow for the calculation of off-shell amplitudes at higher genus.

- Establishing field theory results

A **key realization** (Pius and Sen, 1604.01783) is that because SFT vertices have factors of the form

$$e^{-a^2(p^2+m_i^2)} = e^{a^2(p^0)^2} e^{-a^2\mathbf{p}^2} e^{-a^2 m_i^2}$$

the Feynman diagrams in SFT require integration over loop energies p^0 with a contour that approaches $\pm i\infty$ (the p^0 contour is not rotated back to the real axis).

– Unitarity. Cutkosky rules for SFT (Sen, 2016, Pius and Sen, 2017, 2020). Works with real effective action. In a gauge theory one must also show that only physical states contribute to sums over states.

– Analyticity and crossing symmetry (C. Lacroix, H. Erbin and A. Sen, 2018)

– Wilsonian effective actions (A. Sen, 2016). An action for light fields of the SFT, still satisfying the BV master equation

Recent directions

- Homological perturbation theory of homotopy algebras
  - Traditionally, homotopy algebras (such as A_∞ algebra or L_∞ algebra) were used in construction of string field theory.
    arXiv:hep-th/9206084 by Zwiebach
  - Recently, homotopy algebras are used in different contexts.
    * Low-energy effective theory
      arXiv:1609.00459 by Sen
    * Scattering amplitudes
      arXiv:math/0306332 by Kajiura

The A_∞ algebra describes relations among multi-string products of open string fields.

Consider an action of the form:

$$S = -\frac{1}{2}\langle\, \Psi, Q\Psi\,\rangle - \frac{g}{3}\langle\, \Psi, V_2(\Psi,\Psi)\,\rangle - \frac{g^2}{4}\langle\, \Psi, V_3(\Psi,\Psi,\Psi)\,\rangle + O(g^3)\,.$$

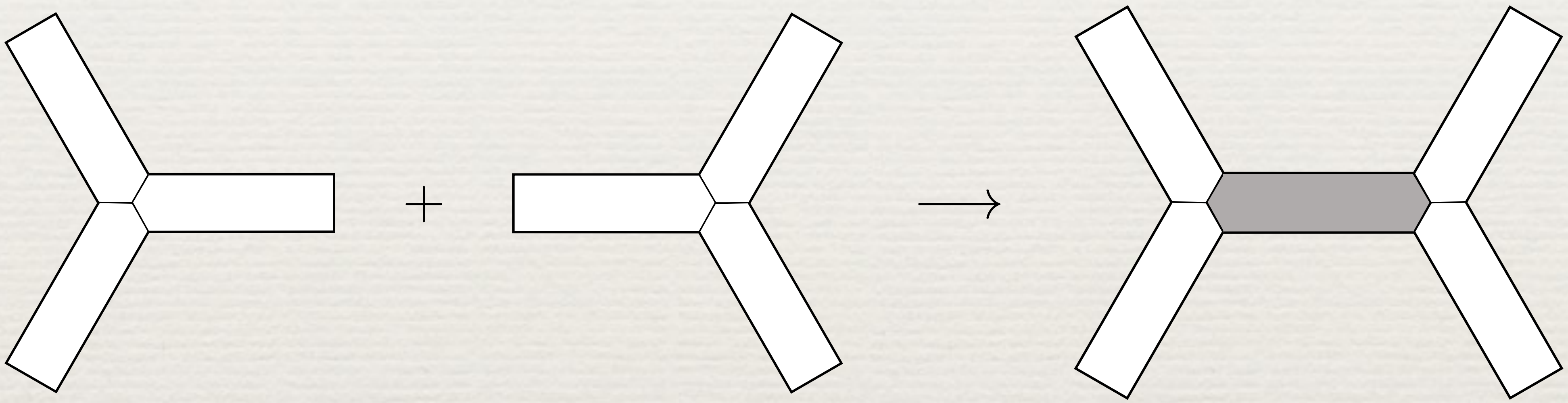
This action is invariant under the gauge transformation given by

$$\delta_\Lambda \Psi = Q\Lambda + g\,(\,V_2(\Psi,\Lambda) - V_2(\Lambda,\Psi)\,)$$
$$+\,g^2\,(\,V_3(\Psi,\Psi,\Lambda) - V_3(\Psi,\Lambda,\Psi) + V_3(\Lambda,\Psi,\Psi)\,) + O(g^3)$$

if multi-string products satisfy a set of relations called A_∞ relations:

$$Q^2 A_1 = 0\,,$$
$$Q\,(V_2(A_1,A_2)) - V_2(QA_1,A_2) - (-1)^{A_1} V_2(A_1,QA_2) = 0\,,$$
$$Q\,(V_3(A_1,A_2,A_3)) + V_3(QA_1,A_2,A_3) + (-1)^{A_1} V_3(A_1,QA_2,A_3)$$
$$+\,(-1)^{A_1+A_2} V_3(A_1,A_2,QA_3) - V_2(V_2(A_1,A_2),A_3) + V_2(A_1,V_2(A_2,A_3)) = 0\,,$$
$$\vdots$$

The path integral over massive fields to obtain the effective action for massless fields generates new multi-string products, but the A_∞ structure is preserved.



A_∞ structure in the original theory

$\downarrow$ Homological perturbation theory

A_∞ structure in the low-energy effective theory
or A_∞ structure in the scattering amplitudes

Multi-string products satisfying the A_∞ relations can be efficiently described by linear operators acting on the vector space $T\mathcal{H}$ defined by

$$T\mathcal{H} = \mathcal{H}^{\otimes 0} \oplus \mathcal{H} \oplus \mathcal{H}^{\otimes 2} \oplus \mathcal{H}^{\otimes 3} \oplus \ldots\,,$$

where we denoted the tensor product of n copies of the Hilbert space $\mathcal{H}$ by $\mathcal{H}^{\otimes n}$.

The A_∞ relations can be compactly expressed in terms of a linear operator $\mathbf{M}$ on $T\mathcal{H}$ which squares to zero:

$$\mathbf{M}^2 = 0\,.$$

For the action of open bosonic string field theory, the A_∞ structure can be described in terms of $\mathbf{M}$ given by

$$\mathbf{M} = \mathbf{Q} + \boldsymbol{m}_2 \,,$$

where $\mathbf{Q}$ is associated with the BRST operator and $\boldsymbol{m}_2$ is associated with the two-string product (the star product).

The homological perturbation theory provides $\mathbf{M}$ for the A_∞ structure in the effective action for massless fields as

$$\mathbf{M} = \mathbf{P}\,\mathbf{Q}\,\mathbf{P} + \mathbf{P}\,\boldsymbol{m}_2\,\frac{1}{\mathbf{I} + \boldsymbol{h}\,\boldsymbol{m}_2}\,\mathbf{P} \,,$$

where $\mathbf{P}$ is associated with the projection onto the massless sector, $\boldsymbol{h}$ is associated with the propagator, and $\mathbf{I}$ is the identity operator.

Examples from recent string field theory papers

- Classical algebraic structures in string theory effective actions
  arXiv:2006.16270 by Erbin, Maccaferri, Schnabl and Vošmera

- Closed string deformations in open string field theory
  arXiv:2103.04919, 2103.04920, 2103.04921
  by Maccaferri and Vošmera

- Mapping between Witten and Lightcone String Field Theories
  arXiv:2012.09521 by Erler and Matsunaga

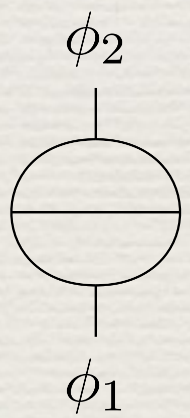
- Gauge-invariant operators of open bosonic string field theory in the low-energy limit
  arXiv:2006.16710 by Koyama, Okawa and Suzuki

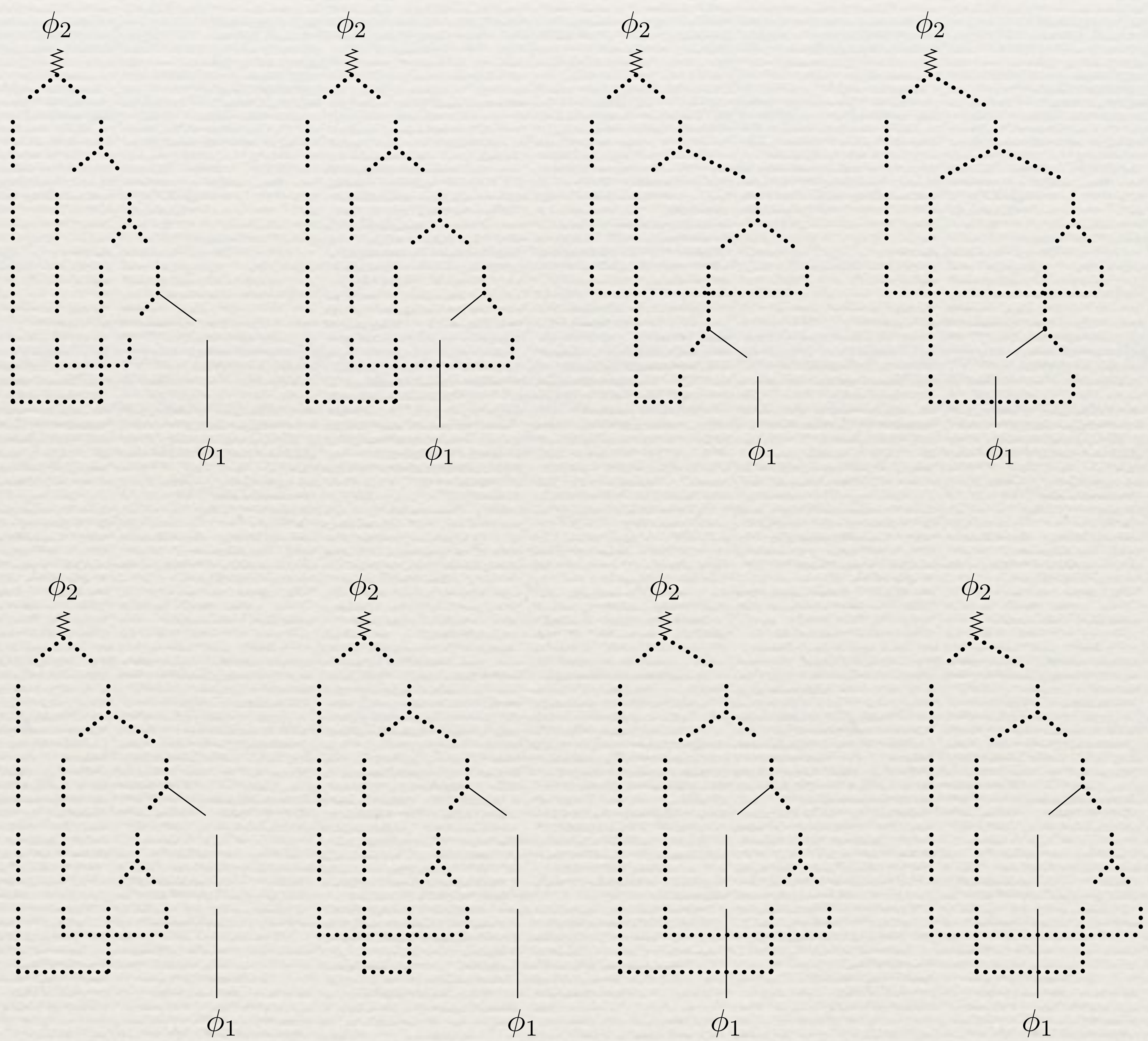
These papers are based on totally different motivations, but essentially the same tool from homological perturbation theory is used.

Homotopy algebras can be also useful in ordinary quantum field theory.

- We can handle Feynman diagrams (tree and loop) algebraically.
- We expect fruitful interactions among various research areas.

Figures from arXiv:2009.12616 by Saemann and Sfinarolakis





Open questions

- Initial value formulation and related matters

  – Initial value formulation (Eliezer and Woodard 1989, Erler and Gross, 2004)

  – Micro-causality, and causality (Bogoliubov causality condition?)

  – Non-locality in string theory (black hole physics).

- Classical solutions of closed string field theory (CSFT)

  – Suspect that CSFT, just as OSFT, contains nonperturbative physics.

  –We should be able to obtain classical solutions (possibly in the minimal-area version of the theory).

  – Alternatively, a cubic (or simpler) formulation, could help in this task.

- Proof of the AdS/CFT correspondence!
  - The AdS/CFT correspondence is typically realized in the low-energy limit of the theory on D-branes.
  - The theory before taking the low-energy limit is considered to be open-closed string field theory (which is difficult to be defined nonperturbatively), but I claim that open string field theory can do the job.
  - Instead of on-shell scattering amplitudes of open strings we are interested in correlation functions of gauge-invariant operators in this context.
  - The $1/N$ expansion of such correlation functions should be a perturbation theory containing gravity.
  - Is open superstring field theory a consistent quantum theory? If yes, use it to prove the AdS/CFT correspondence!

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Let's see what Barton said ten years ago...

String Field Theory: Achievements, Challenges, and Future Directions

Field Theory and String Theory

– Yukawa Institute, 24 July 2011 –

Barton Zwiebach

Future directions, Challenges

The missing string field theories!

- The R sector of Heterotic strings.
- NS- NS sector of type II strings
- NS- R sector of type II strings
- RR sector of type II strings.

We had to learn much to formulate each of the presently known string field theories. Much will be learned to find relatively natural versions of the above (even if we do not presently have the ability to calculate with them!).

The present lack of type II closed superstring field theory is probably just a reflection of our incomplete understanding of superstring perturbation theory.

Even BRST cohomology has not yet been defined properly in the RR- sector.

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The missing string analytic solutions!

- Multibrane solutions: finding the solution that represents multiple branes starting on a background with just one brane. Work in progress (Murata and Schnabl). High possible impact
- Tachyon vacuum in superstring field theory.
- Lump solutions (see work by Bonora, et.al.)

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For open string theory, the correct physics of tachyons was anticipated and OSFT benefitted from it, providing a clean cut proof of the tachyon conjectures.

The physics of bulk closed string tachyons remains mysterious. Their condensation presumably destroys spacetime or may produce non-critical string theories. Closed string field theory could help find the correct physics!

No analytic tools presently exist that apply to closed strings!

A challenge for SFT practitioners!

Many other aspects of string theory (AdS/CFT, holography, F-theory, Matrix theory...) could help develop SFT

But the question most likely to determine the future of SFT is the question of ......

Background independence

This issue arises because all string field theories can only be written after choosing what amounts to a *classical solutions*, namely CFT's with $c = 0$. We then use the Q and the state space of the CFT to formulate the SFT.

Even at the early stage of SFT, one could prove that SFT's formulated on nearby CFT's related by infinitesimal marginal deformation are the same theory.

If the theory on a given background contains solutions that describe all other possible backgrounds the theory is physically background independent (if not manifestly so). Thus the existence of the tachyon vacuum solution was crucial, and so is the existence of multibrane solutions.

We need manifest background independence.

- Manifest background independence

Three approaches that seemed promising but did not pan out yet!

– Boundary string field theory (Witten hep-th/9208027, 9210065). Used BV in the space of 'all open string worldsheet theories' . Difficulties with non-renormalizable interactions.

– String field theory from L_∞ with 'zeroth-product' , a product with no input (Zwiebach, hep-th/9606153) $[\Psi, \Psi] = \Psi \star \Psi$, $[\Psi] = Q\Psi$, $[\cdot] = F$ (with $QF = 0$, and $Q^2 = F$). Realize these structures in some suitable space of 2D field theories.

– Vacuum string field theory: Formulates OSFT around the tachyon vacuum, by attempting to use a pure 'ghost' BRST operator (Rastelli, Sen, Zwiebach, 2001, Okawa 2002, $\cdots$).

Annual workshops on string field theory:
Brazil (2020, online), Italy (2019), India (2018), Israel (2017), Brazil (2016), China (2015), Italy (2014), Israel (2012), Czech (2011), Japan (2010), Russia (2009) and Germany (2008)

If you are interested, please join us at

SFT@Cloud 2021

September 20-24, 2021 (online)

https://indico.cern.ch/e/SFT-2021

Organizers: Chakrabarti, Erbin, Lipinski, Vošmera