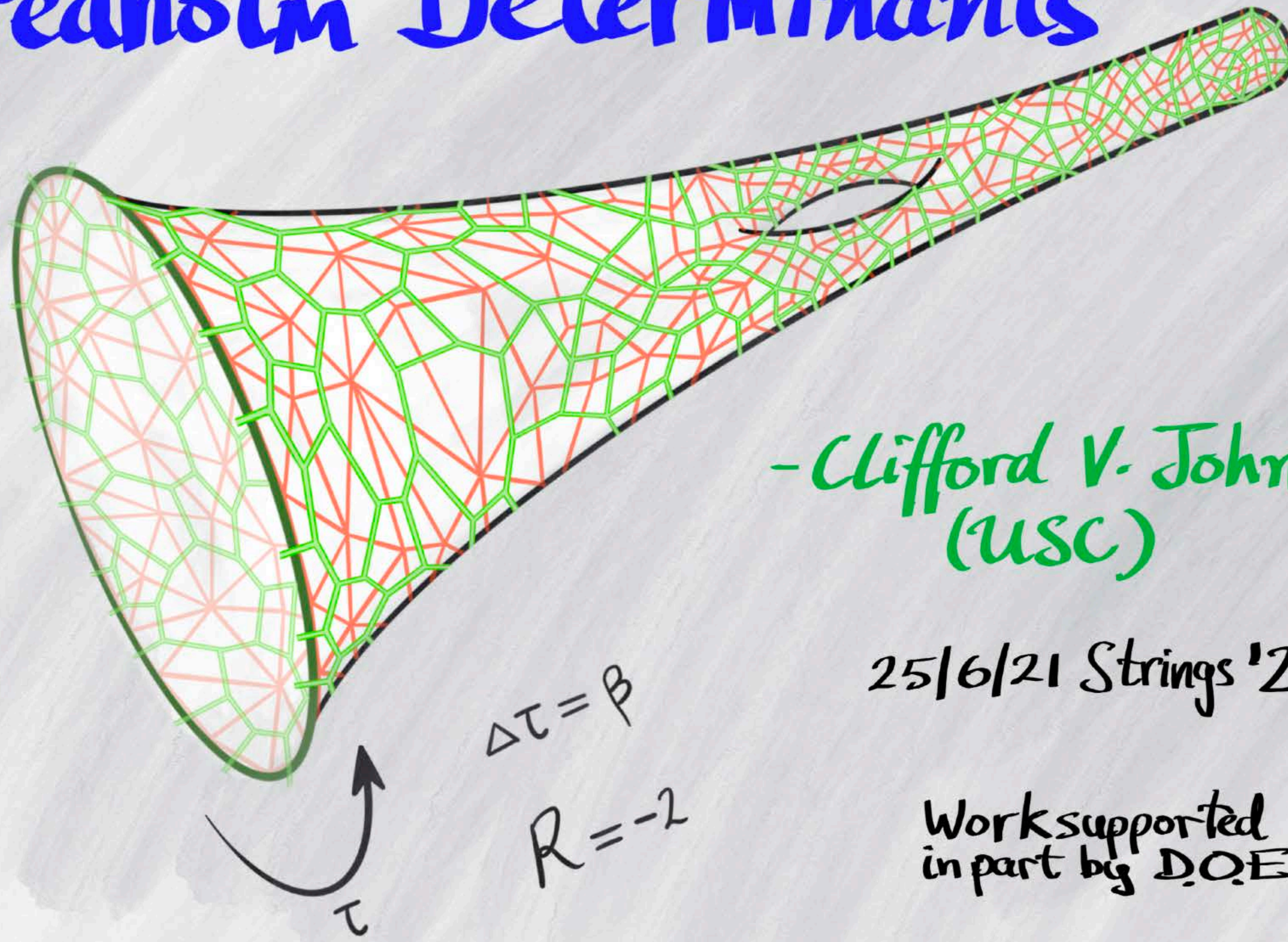
Quantum Gravity Microstates from Fredholm Determinants



- Clifford V. Johnson
(USC)

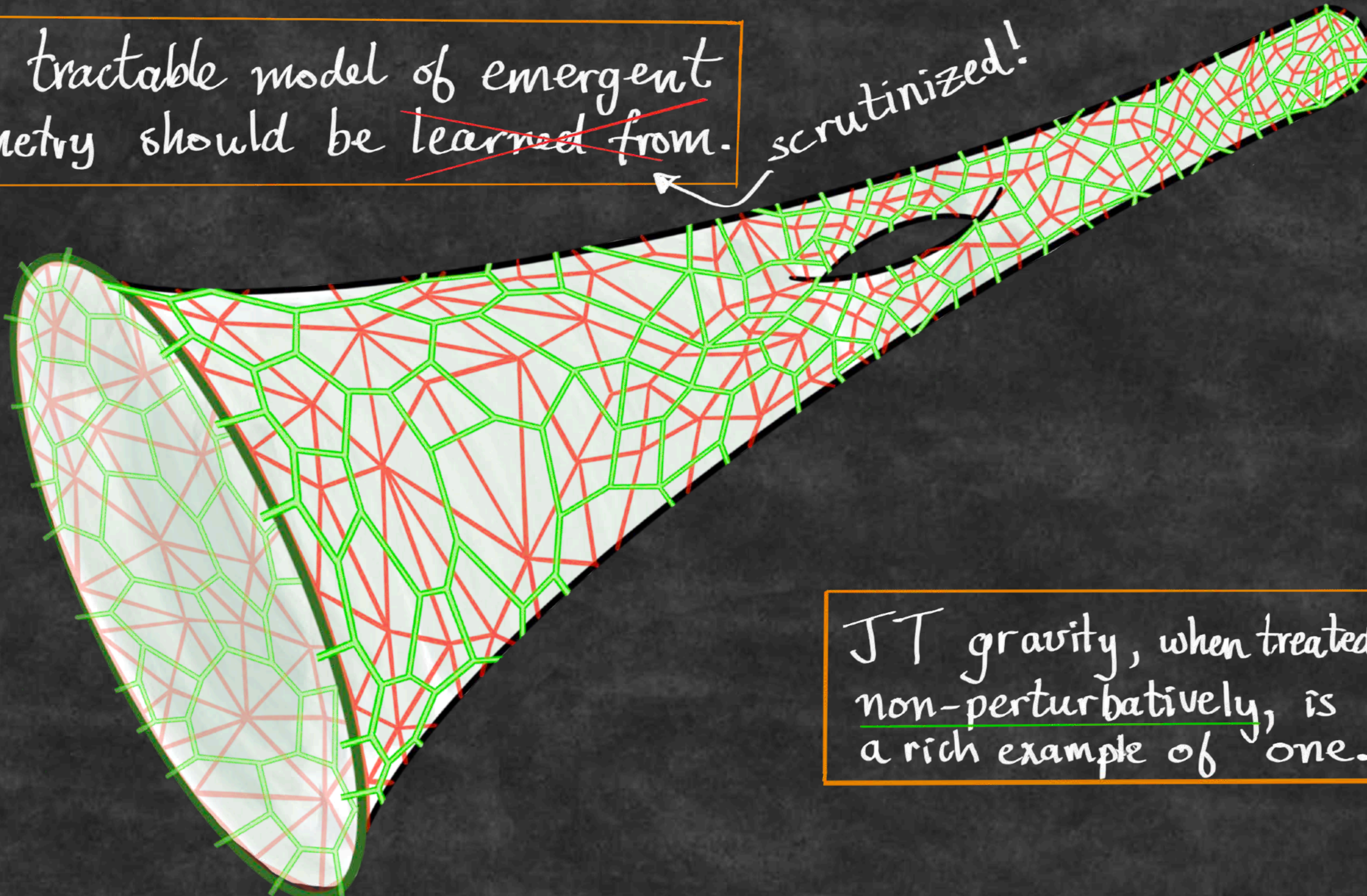
25/6/21 Strings '21

Work supported in part by D.O.E.

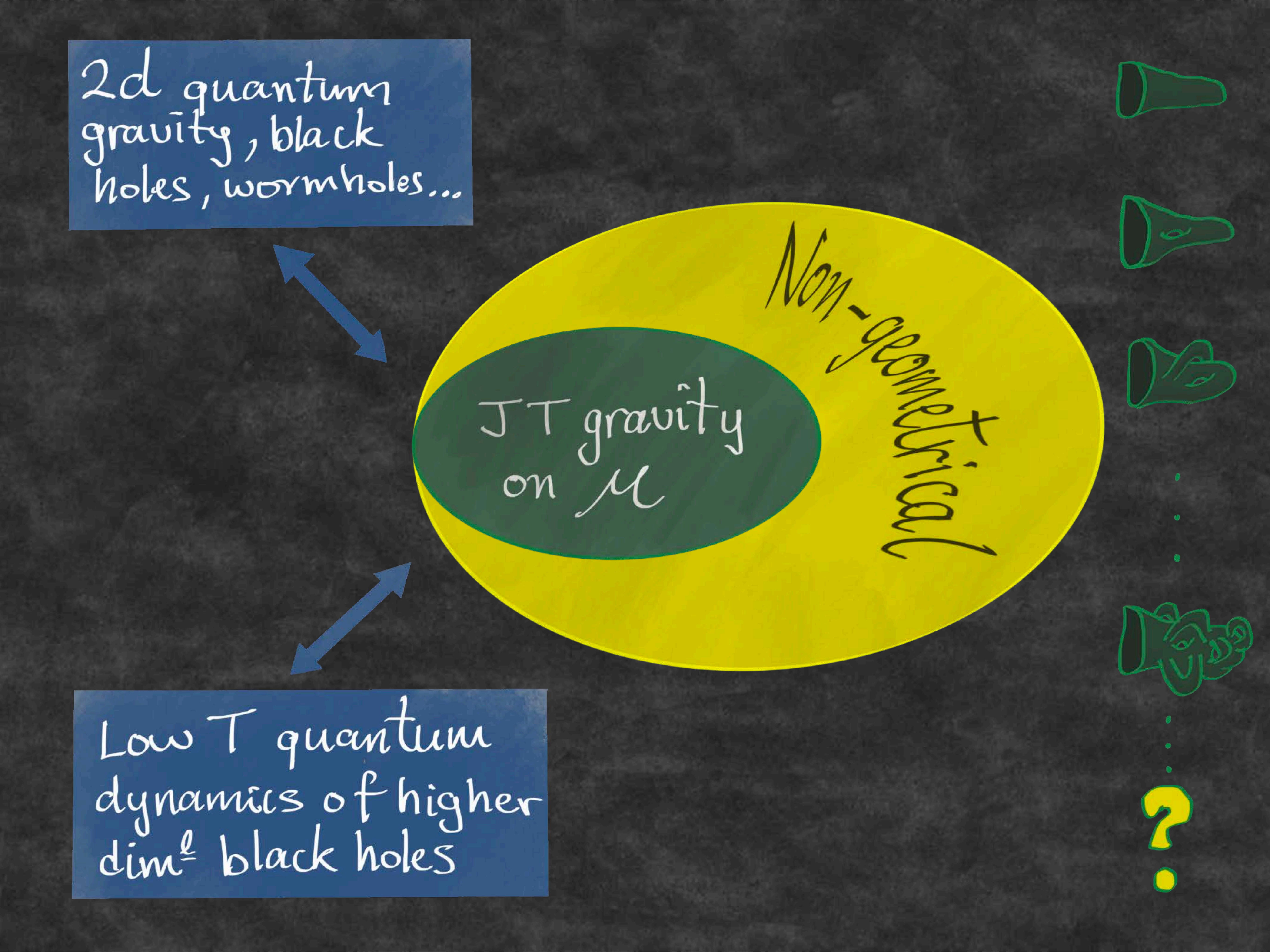
- We're unlikely to fully understand black holes (+…) to everyone's satisfaction using approaches too linked to spacetime geometry.
- There have been many hints that geometry is likely to be emergent in a full quantum theory of gravity.

Any tractable model of emergent geometry should be ~~learned from~~.

scrutinized!



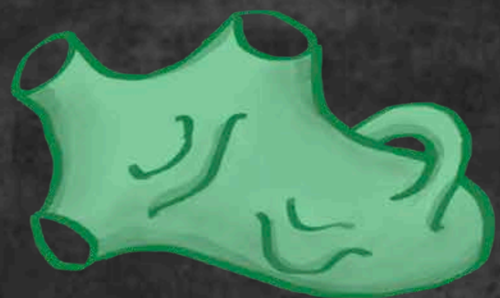
JT gravity, when treated non-perturbatively, is a rich example of one.


2d quantum gravity, black holes, wormholes...
JT gravity on $\mathcal{M}$
Non-geometrical
Low T quantum dynamics of higher dim^{l} black holes
?

JT gravity (briefly):

Jackiw '83
Teitelboim '85

$$I = -\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\,\phi(R+2) - \int_{\partial\mathcal{M}}\sqrt{h}\,\phi(K-1) - \frac{S_0}{2\pi}\left(\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}R + \int_{\partial\mathcal{M}}\sqrt{h}K\right)$$

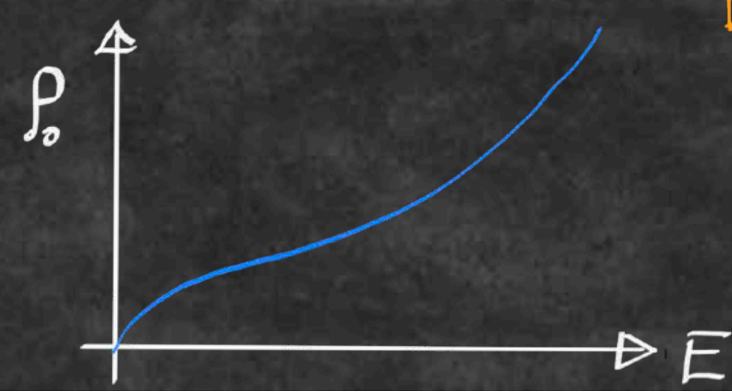
$\mathcal{M} =$ 

$\chi(\mathcal{M}) = 2-2g-b$

- $e^{\chi S_0}$ factor

Partition function: $(b=1)$

- $Z(\beta) = \sum_g Z_g(\beta) + \dots$
- $\int \mathcal{D}\phi \rightarrow R=-2$; Schwarzian boundary dynamics
- $Z_0(\beta) = \dfrac{e^{S_0} e^{\pi^2/\beta}}{4\sqrt{\pi}\,\beta^{3/2}} \Rightarrow$ density of states $\boxed{\rho_0(E) = e^{S_0}\dfrac{\sinh(2\pi\sqrt{E})}{4\pi^2}}$



Maldacena, Stanford, Yang
Jensen
Engelsöy-Mertens-Verlinde
} '16

Landmark result:

Saad, Shenker, Stanford '19

JT path integral on any $\mathcal{M}$

$e^{\chi S_0}$

$\Longleftrightarrow$

double-scaled large N random Hermitian matrix model $\left(\frac{1}{N}\right)^{-\chi}$

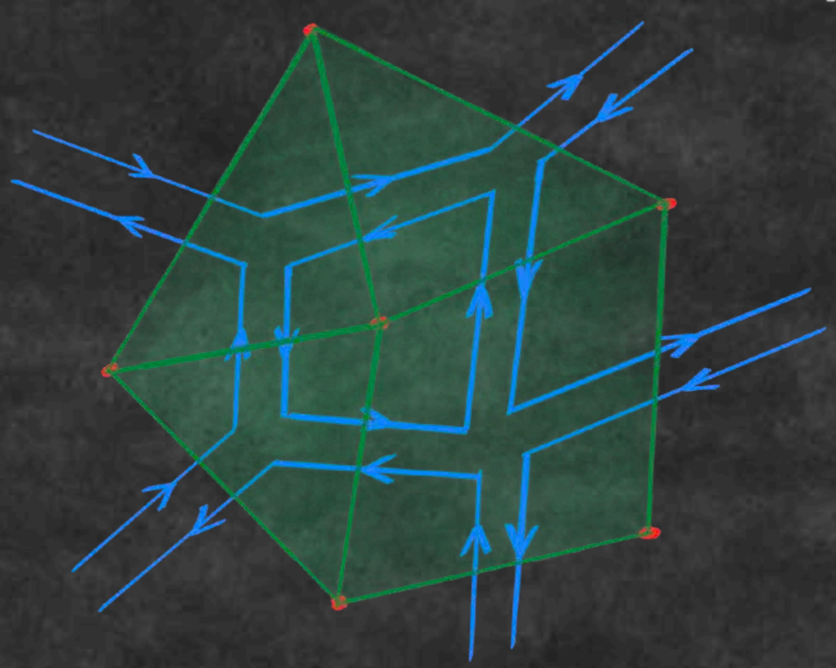
e.g. $Z(\beta) \Longleftrightarrow \langle \mathrm{Tr}\, e^{\beta M} \rangle_{DSL}$

$$\tilde{Z} = \int dM\, e^{-\mathrm{Tr}\, V(M)}$$

- $V(M) = \frac{1}{2} M^2 + \sum_p g_p M^p$
- smooth surface interpretation as $N \to \infty$ and $\{g_p\} \to \{g_p^c\}$

Comments:

- Inherently perturbative, using "topological recursion" properties (Mirzakhani, Eynard-Orantin)
- $V(M)$ not explicitly known; recursion seeded by $\rho_0(E)$
- Generalization (+classification) to other JT (Stanford-Witten '19)
- Some partial non-pert insights, but...



RMM prototype: Gaussian Unitary Ensemble

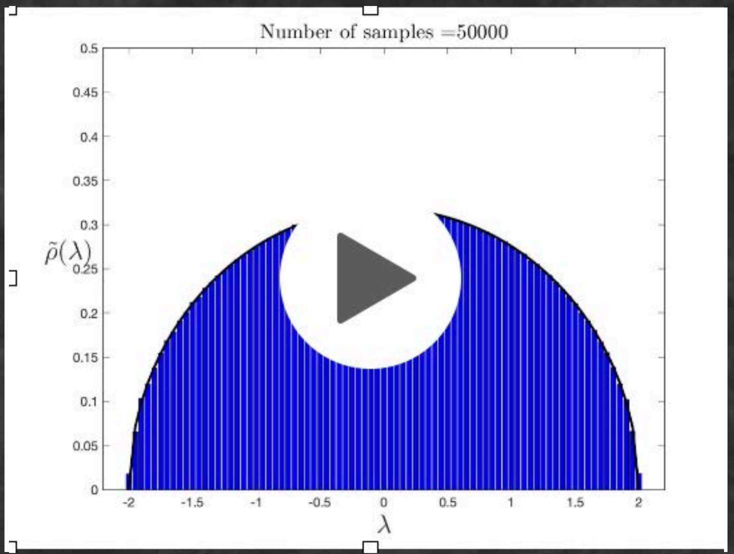
Try this at home!

- sample hermitian matrices M with probability

$$p(M) = e^{-\frac{\mathrm{Tr}\,M^2}{2}}$$

- compute eigenvalues, λ_i $i=1\dots N$; histogram them:

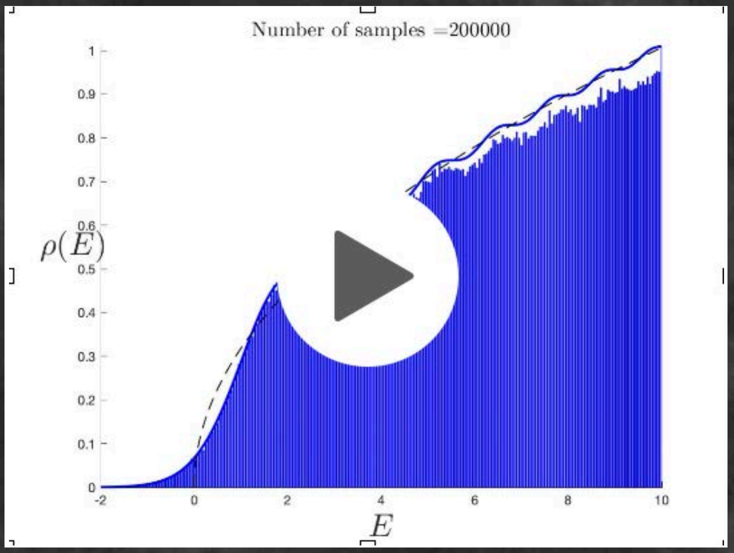
$$\lambda \to \frac{\lambda}{\sqrt{N}} \;;\; \hat{\rho}_0(\lambda) = \frac{\sqrt{4-\lambda^2}}{2\pi}$$ Wigner semi-circle



- focus on an endpoint and zoom in:

$$\lambda = -2 + \underbrace{\delta^2}_{N^{2/3}} E \qquad \boxed{\begin{aligned}\hat{\rho}_0(\lambda) &\to \frac{1}{N^2}\frac{E^{1/2}}{\pi\hbar} \\ &= \frac{1}{N^2}\rho_0(E)\end{aligned}} \qquad \hbar = \frac{1}{N\delta^3}$$

$\delta \to 0$ as $N \to \infty$



- the magnification reveals undulations/wiggles:

$$\rho(E) = \hbar^{-2/3}\left(\mathrm{Ai}'(\zeta)^2 - \zeta\,\mathrm{Ai}(\zeta)^2\right) \text{ for } \zeta = -\hbar^{-2/3} E$$

$$= \rho_0(E) + \rho_1(E) + \rho_2(E) + \dots \text{ (non-pert.)}$$

This is now the "Airy model". Why? Orthogonal polynomials

The Airy Model

- Rewrite matrix computations in terms of $P_n(\lambda)$, orthogonal wrt $d\lambda\, e^{-V(\lambda)}$

$$\int P_n(\lambda) P_m(\lambda) e^{-V(\lambda)} d\lambda = h_n \delta_{nm} \; ; \quad |n\rangle = \frac{|P_n\rangle}{h_n} \qquad \int \to \langle n|m\rangle = \delta_{nm}$$

- $\lambda P_n(\lambda) = P_{n+1}(\lambda) + R_n P_{n-1}(\lambda)$ recursion relation (even V)

$$\left(\begin{array}{c} \text{Gaussian case: } P_n \text{ are Hermite polynomials} \\ \lambda H_n = H_{n+1} + n H_{n-1} \qquad R_n = n \end{array} \right)$$

(beware convention)

- Everything can be computed from the R_n.
- At large N: $\frac{n}{N} \to X$; $P_n(\lambda) \to P(X,\lambda)$; $R_n \to R(X)$
- In DSL (zoom an endpoint) scaling pieces survive:

$$\lambda \to E \qquad \hat{\lambda} \to -\hbar^2 \frac{\partial^2}{\partial x^2} + u(x) \equiv \mathcal{H}$$

$$X \to x \in \mathbb{R}$$

$$\boxed{\mathcal{H}\psi(x,E) = E\psi(x,E)}$$

$$\frac{1}{N} \to \hbar$$

$$P \to \psi(E,x)$$

$$R \to u(x)$$

$$\boxed{\rho(E) = \int_{-\infty}^{0} |\psi(E,x)|^2 dx}$$

'89, '90
Brezin-Kazakov
Douglas-Shenker
Gross-Migdal

Banks et al.
Moore

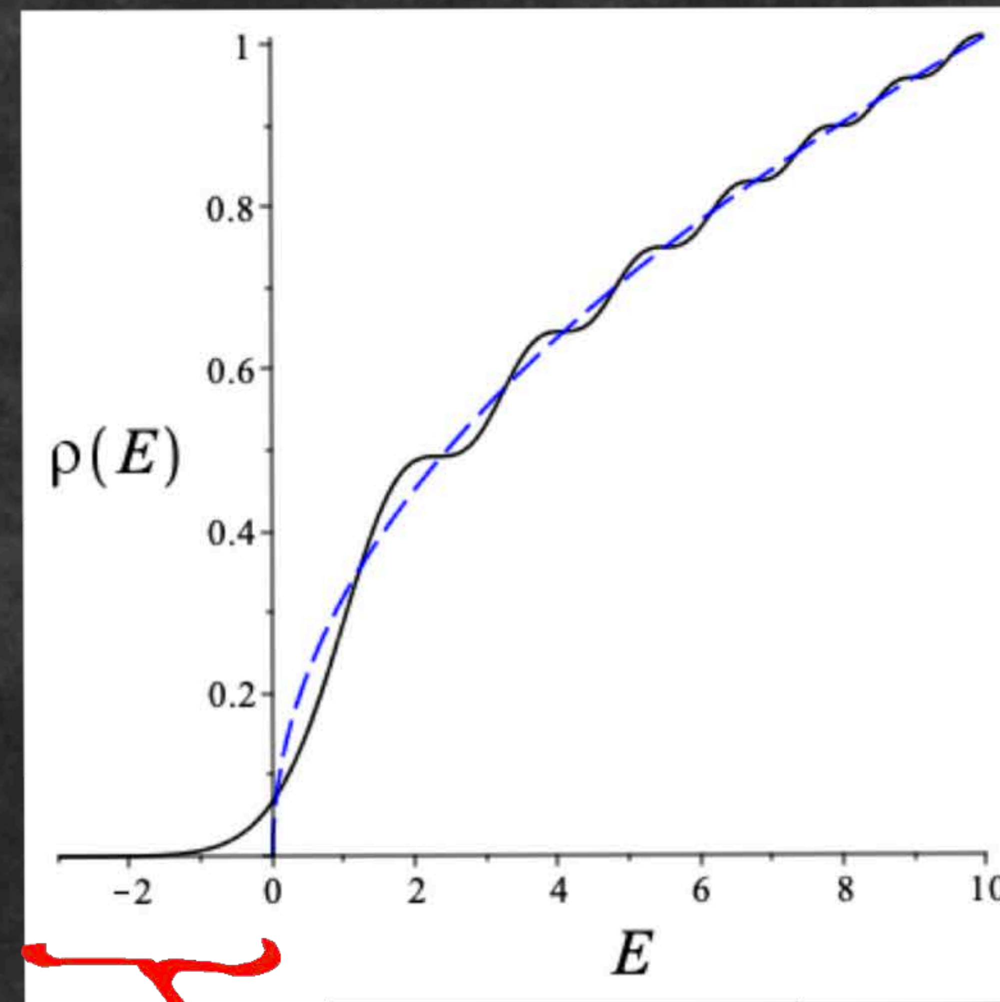
Gaussian: $u(x) = -x$: $\psi(E,x) = \hbar^{-2/3} \mathrm{Ai}\left(-(E+x)\hbar^{-2/3}\right)$

$u(x) = -x$

$$\psi(E,x) = \hbar^{-2/3}\, Ai\left(-(E+x)\hbar^{-2/3}\right)$$

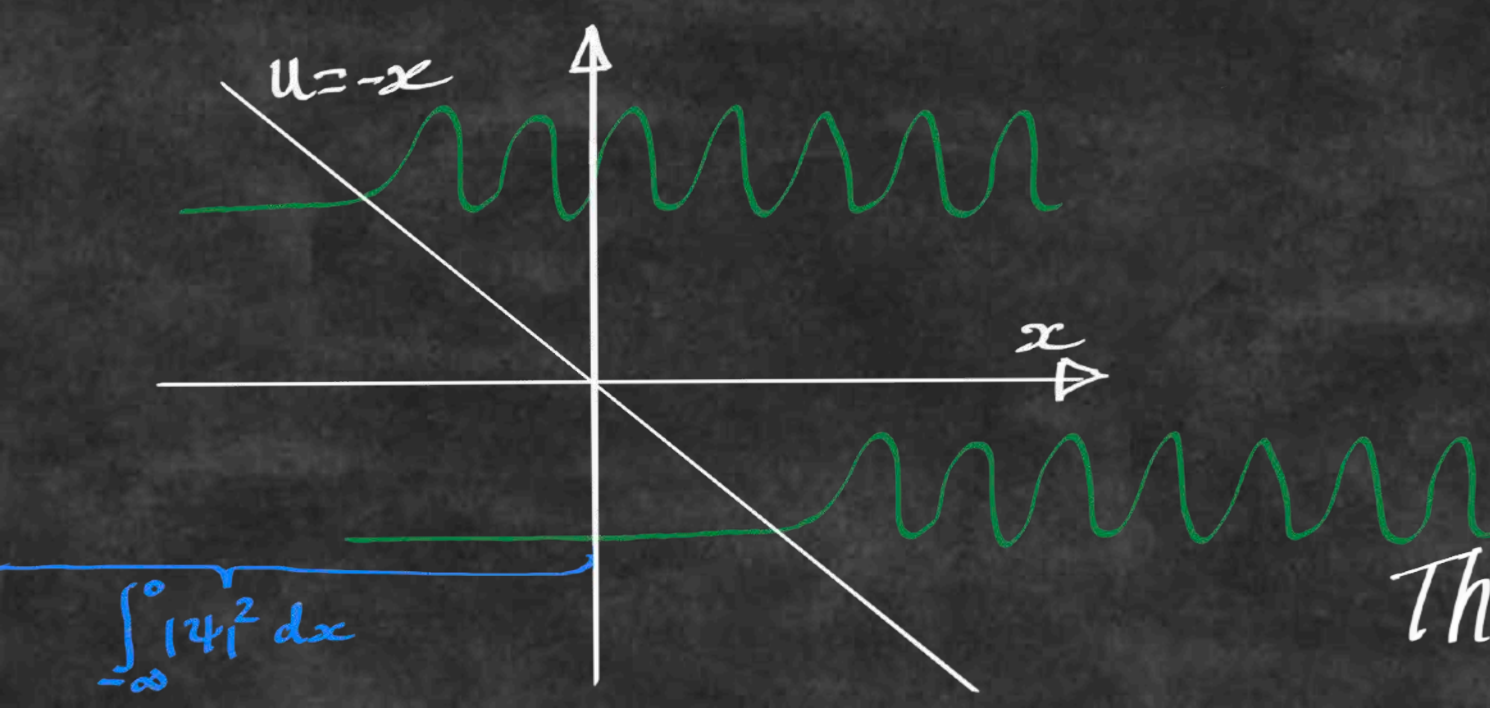
$$\rho(E) = \int_{-\infty}^{0} |\psi(E,x)|^2\, dx$$

$$= \hbar^{-2/3}\left(Ai'(\zeta)^2 - \zeta Ai(\zeta)^2\right) \text{ for } \zeta = -\hbar^{-2/3}E$$



Comments:

- Not a theory of surfaces, but illustrates the power of this approach.
- Airy has precursor of problem for SSS model:



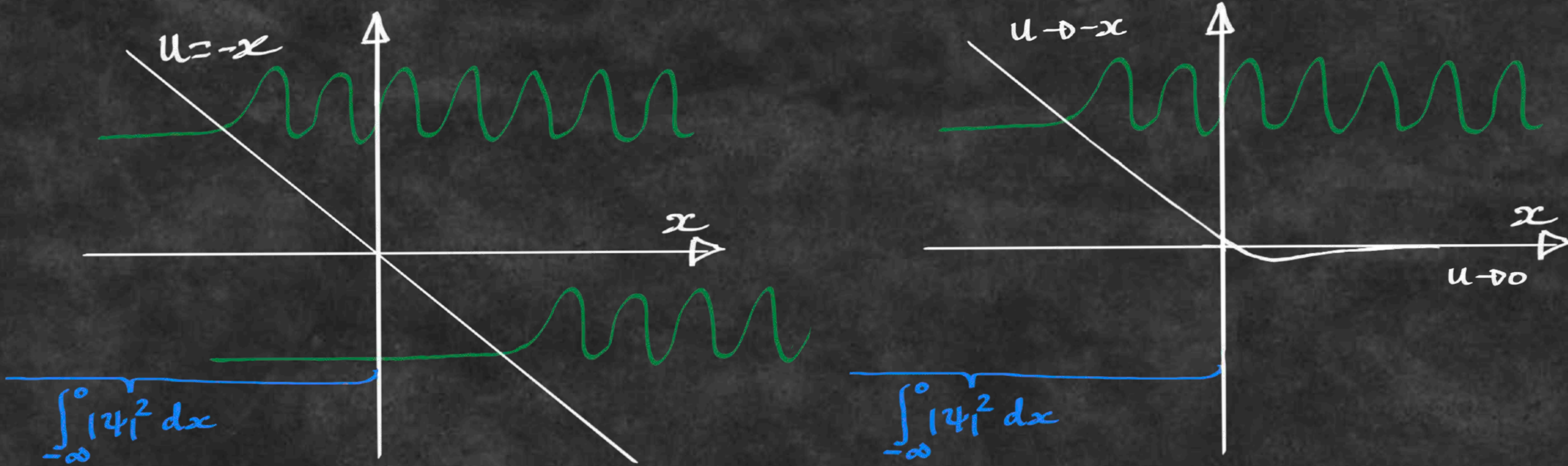
states at $E<0$

$\Downarrow$

non-perturbative instability for full SSS

The picture suggests the cure...

Achieving Non-Perturbative Stability



- Identical perturbation theory (large E regime)
- Acheivable with model of complex matrices M
- Potential is $V(MM^\dagger)$, so hermitian matrices with $\lambda \geqslant 0$
- $u(x)$ instead solves: $uR^2 - \frac{\hbar^2}{2}RR'' + \frac{\hbar^2}{4}(R')^2 = 0$

  (R is polynomial in $x, u, u', u'', \ldots$)
- Resulting $\rho(E)$ similar to Airy, but ends at $E=0$

Morris; 90/91
Dalley, CVJ, Morris
CVJ, 1912.03637

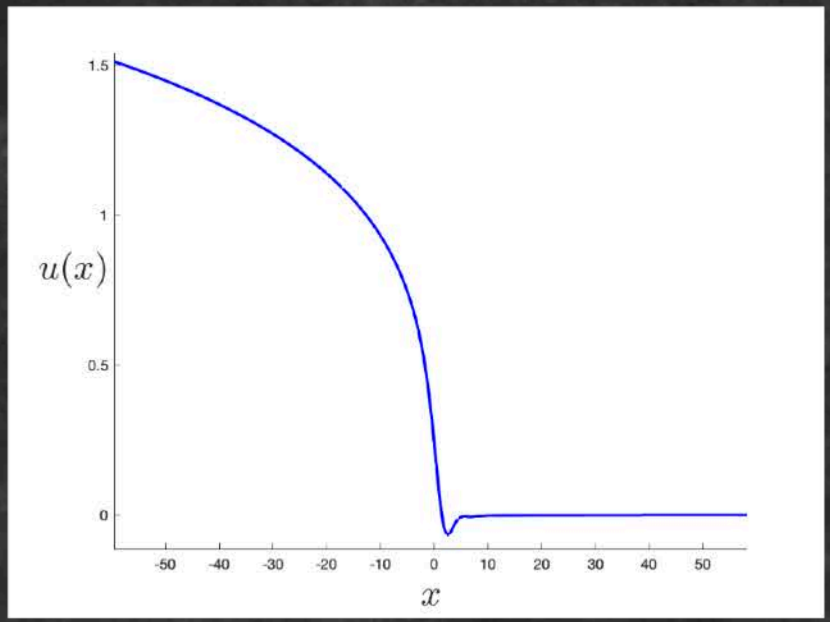
Challenge: Extend this to full JT gravity!

$$\mathcal{H} = -\hbar^2\frac{\partial^2}{\partial x^2} + u(x)$$

find eqn for this

⋮

solve it



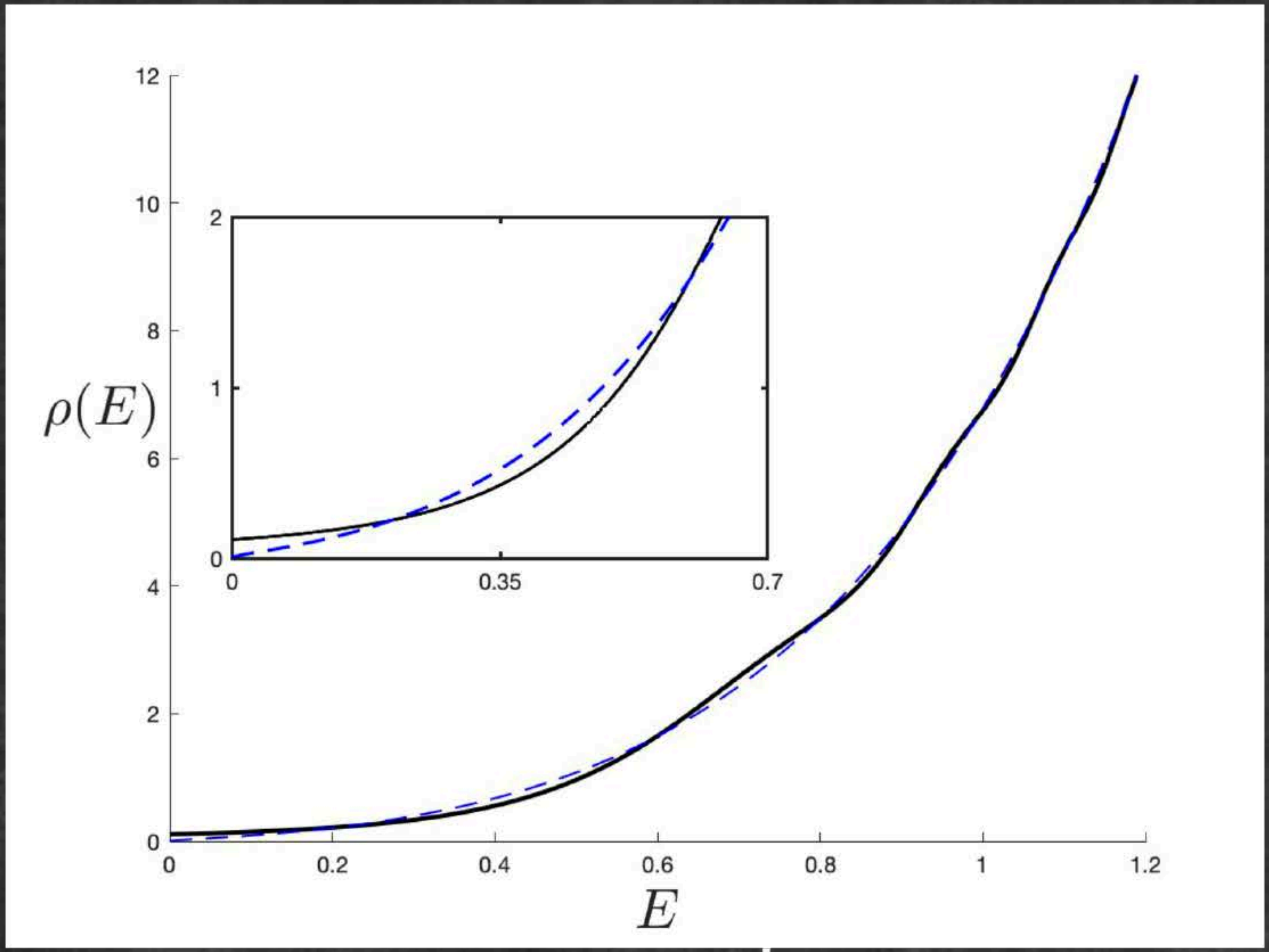
$$\mathcal{H}\psi(E,x) = E\psi(E,x)$$

solve this

⋮

then compute this

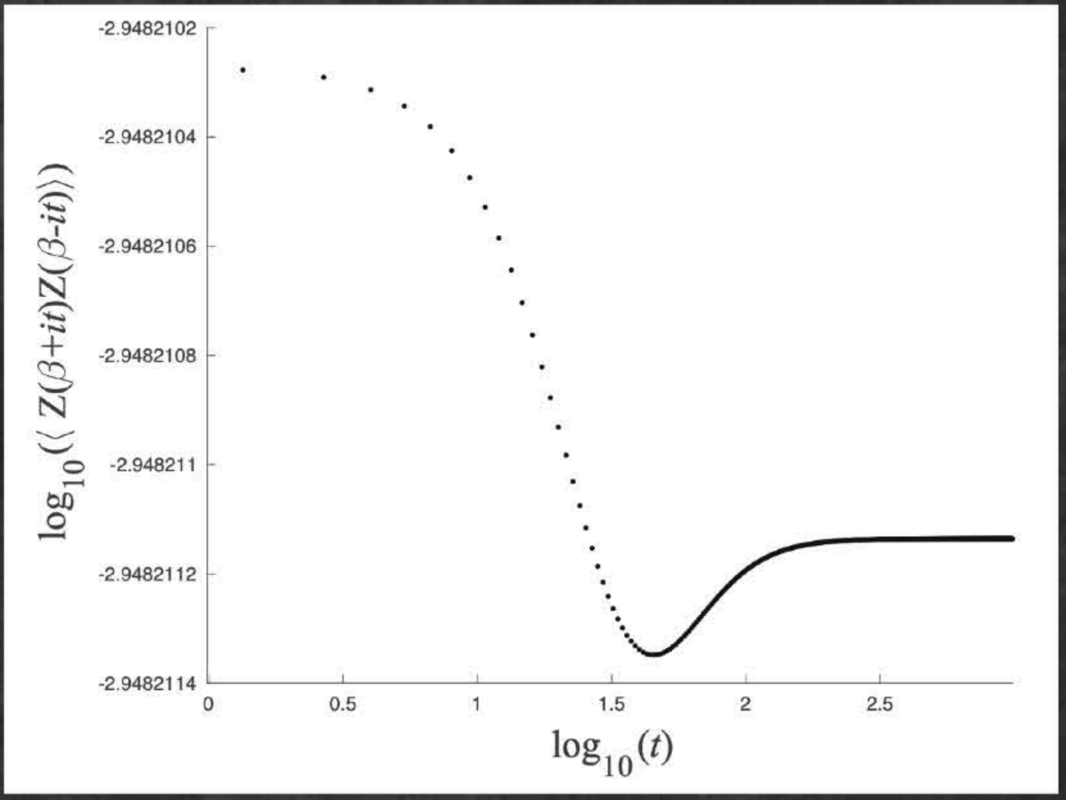
$$\rho(E) = \int_{-\infty}^{0} |\psi(E,x)|^2\, dx$$



CVJ 1912.03637 & 2006.10959

Comments:

- This is a full non-perturbative completion of JT gravity
- Explicit results, not merely formal
- Can compute useful things with it:



- Same methodology used to tackle many SJT Models of Stanford-Witten.

CVJ 2005.01893
CVJ 2008.13120

... and study the JT deformations of Witten, and Maxfield-Turiaci

CVJ + Rosso 2011.06026

... and complete a different SJT model using a two-cut matrix model

CVJ + Rosso + Svesko 2102.02227

Also note: connections with minimal string + Liouville work of Okuyama et al.
Mertens et. al.
Betsios et. al.
⋮

Note: other NP definitions disussed in SSS and Gao-Jafferis-Kolchmeyer 2104.01184 although so far difficult to work with...

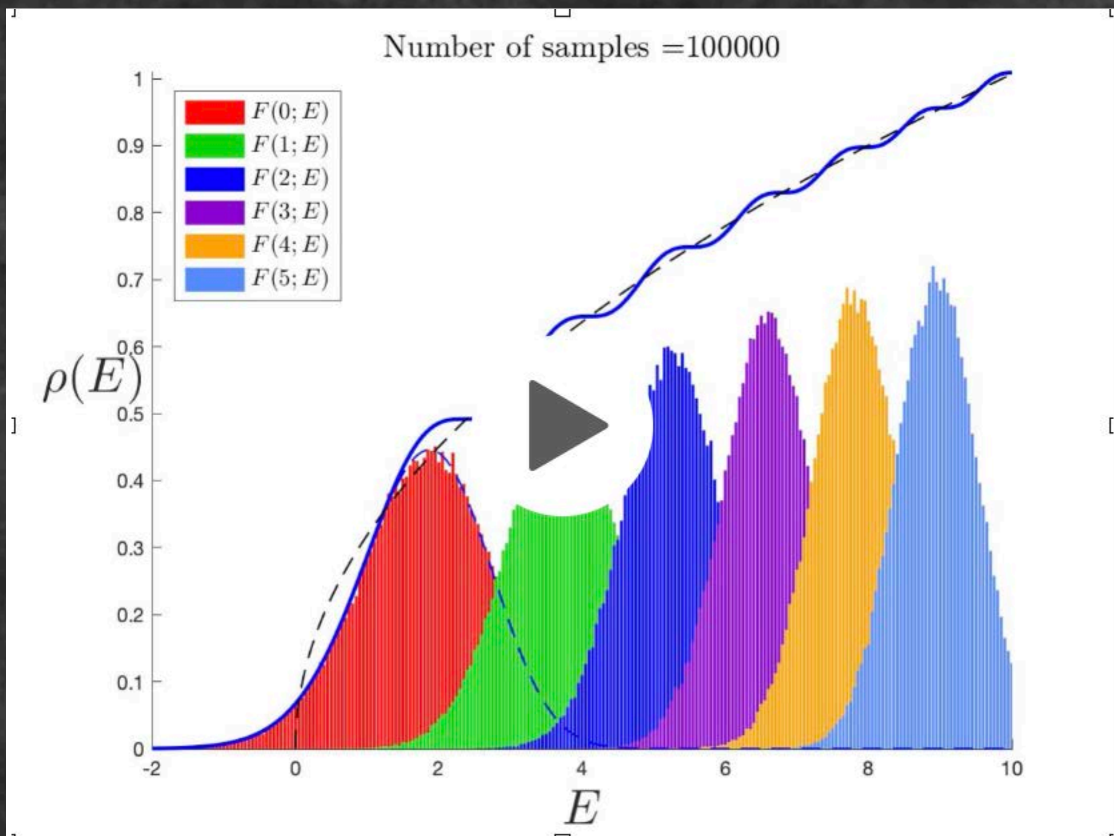
Is that it?

No!

Revisit the Gaussian/Airy sampling ...

Try this at home!

- If also keep track of the order of the λ_i for each sample:



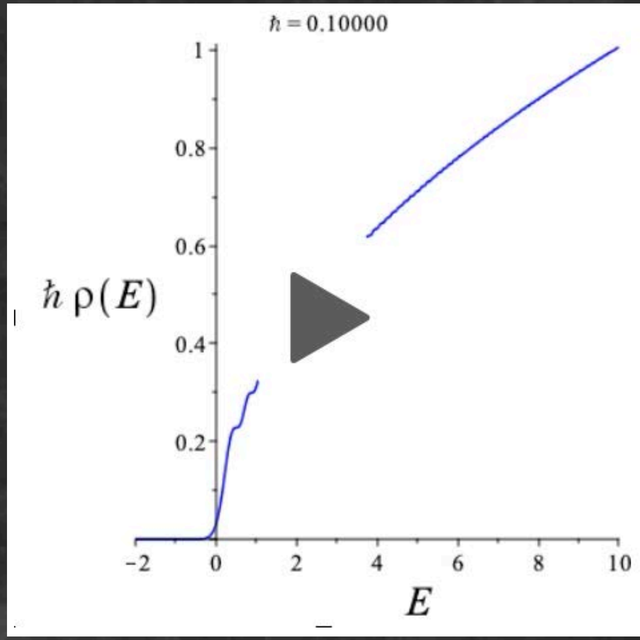
Tracy-Widom '94

- Undulations in $\rho(E)$ actually built from individual underlying energy levels' statistics!
- Sharper and more dense as $E \to \infty$ $\boxed{\text{A continuum forms}}$
- In the full gravity model, these are the microstates!

- Recall $e^{-S_0} = \hbar \sim \frac{1}{N}$

  i.e., $S_0 \sim \ln N$

Smaller $\hbar$, more "classical", larger S_0



New tool: Fredholm Determinant

- Already have the components: $\psi(E,x) = \hbar^{-2/3} Ai(-(E+x)\hbar^{-2/3})$
- Assemble into "Kernel" $K(E',E) = \int_{-\infty}^{0} \psi(x,E)\psi(x,E')\,dx$

$$= \frac{\psi(E)\psi'(E') - \psi(E')\psi'(E)}{E-E'}$$

- "Airy Kernel" for Airy case.
- JT and variants give new Kernels!
- $f(E) - \int_a^b dE'\, K(E,E') f(E') = g(E)$ (Fredholm 1903)

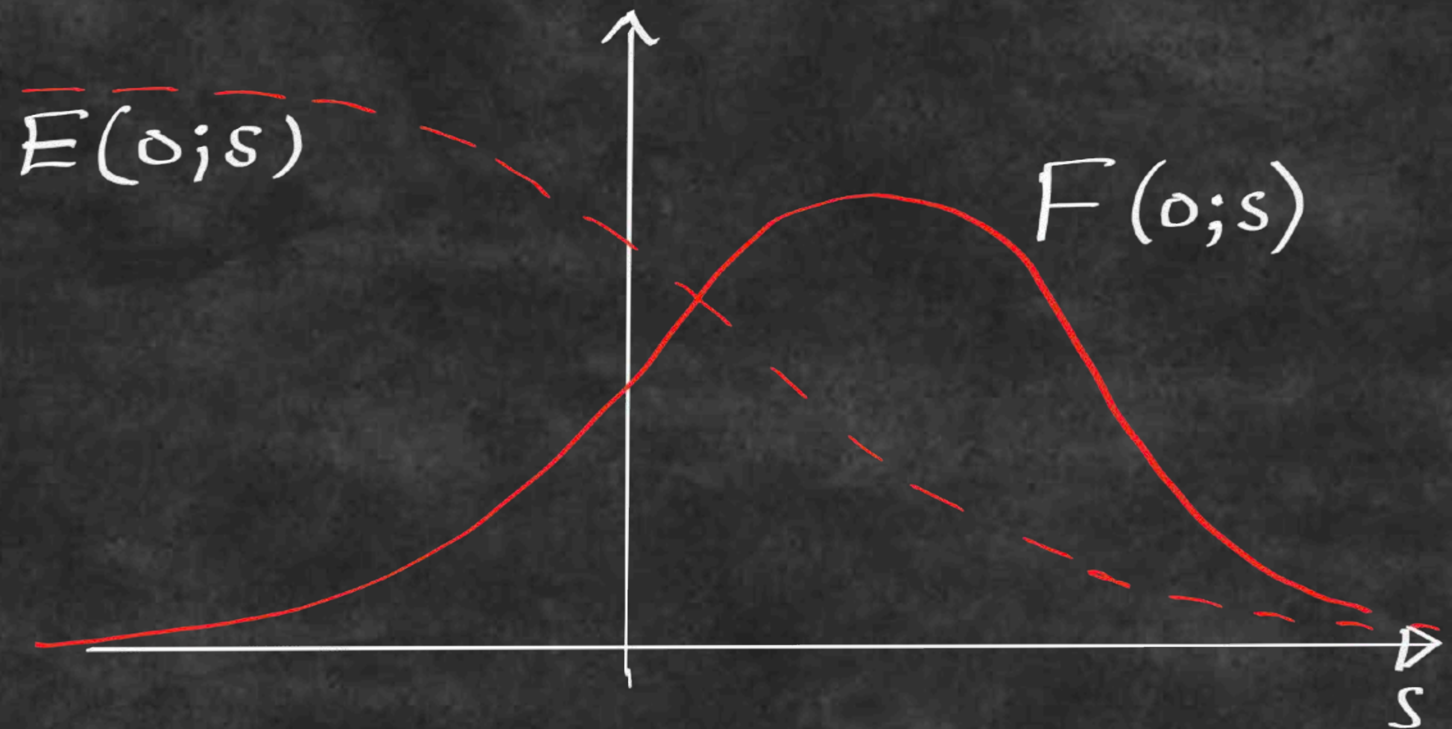
$\det(\mathbb{1} - K)$ is a natural object

RMM Literature:

Probability of not finding 1st level in $(-\infty, s)$ is:

$$\boxed{E(0;s) = \det\left[\mathbb{1} - K|_{(-\infty,s)}\right]}$$

Probability distribution: $\quad F(0;s) = -\frac{dE(0;s)}{ds}$

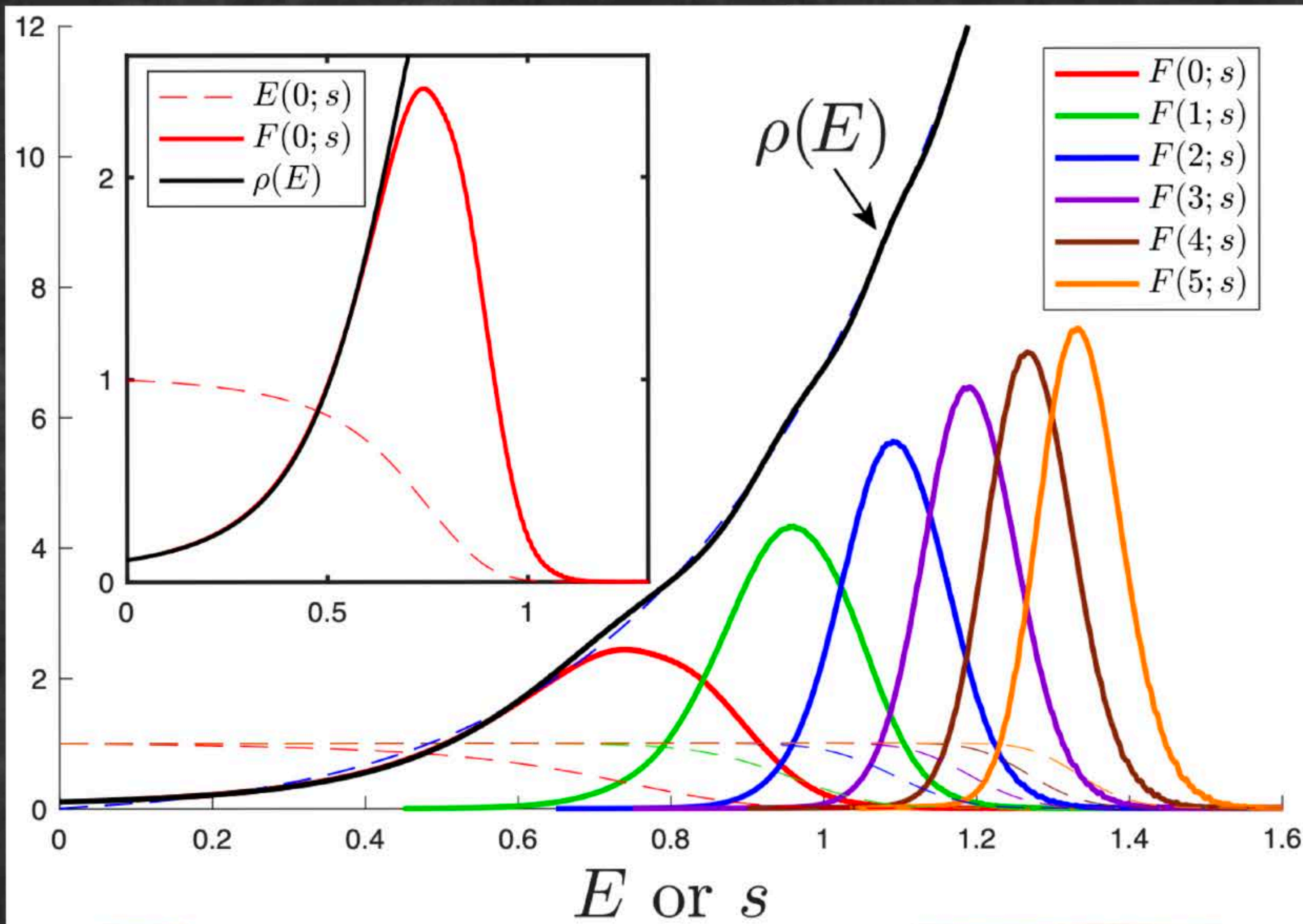


- Other levels found recursively from this.
- Challenging problem, det of ∞-dim^l object.
- Bornemann '10 helpful.

$\Rightarrow$ Can do this for JT gravity!

JT Gravity Microstates!

CVJ 2106.09048



(Also, higher D black hole microstates!)

- If have all microstates and their statistics, can compute anything about the model...

geometrical description bad

Individual microstates emerge

geometrical description good

Microstates merge into continuum

JT Gravity Quenched Free Energy!

CVJ 2106.09048

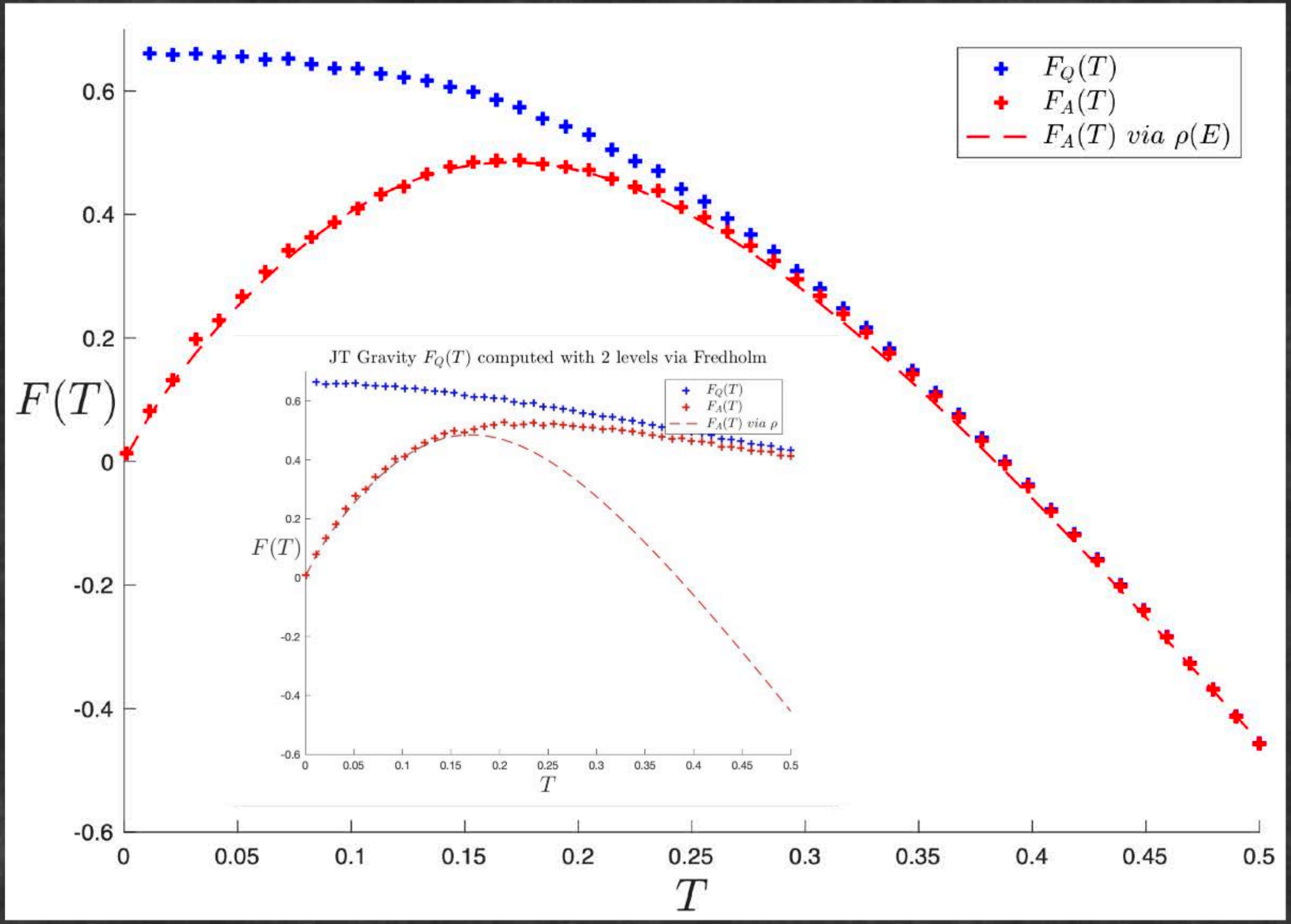
- Need $F_Q(T) = -\beta^{-1}\langle \log Z(\beta)\rangle$ to follow thermo to low T
- Computation needs wormholes. (Engelhardt, Fischetti Maloney 2007.07444)
- Should need NP physics too. (Matrix model?) (CVJ 2008.13120)
- Partial results at low T... (Okuyama 2009.02840, 2101.05990 Janssen + Mirbabayi 2103.03896 CVJ 2104.02733)
- But can now just reverse engineer the spectrum data to compute by direct ensemble computation...

JT Gravity Quenched Free Energy!

CVJ 2106.09048

Comments:

- $F_Q(0) = \langle E_0 \rangle$
  Anticapated in Okuyama 2009.02840
- $F_Q(T) \approx -\# T^4 + \ldots$
  Low T prediction
  Janssen + Mirbabayi 2103.03896
- No replica symmetry breaking. cvj 2008.13120



Final Remarks

Non-perturbative sector captures JT/Black hole microstates!

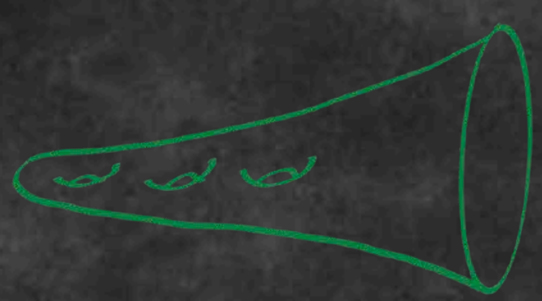
This is an explicit illustration of transition from geometry to non-geometry in quantum gravity.

Fredholm determinant is a D-brane probe...

What other Random Matrix Model tools might be useful here?

What other quantum gravity questions can be answered with this remarkable tool box?

Thank You!



— cvj