The subject is well motivated by data. An Occam’s razor interpretation of cosmological data: we live with a positive cosmological constant.

In the context of string theory, the subject then splits into two interesting pieces (which may enjoy interplay):

1. How do we arrange the ingredients the theory provides to make lower dimensional dS space? This involves engineering in extra dimensions.
2. How do we formulate quantum gravity in dS space? No precise analogue of flat space S-matrix or CFT correlators...

I’ll focus on the first question, and describe how I think about it here.

Basic point: The question reduces to finding four-dimensional effective theories with potential functions that have minima at $V>0$. As the theory has constrained ingredients with known energetics, one can get intuition for whether this is possible. E.g. critical strings on a compact manifold of size $R$:

— Fluxes:

$$\Delta V = \int \sqrt{-g} g^{\mu_1 \nu_1} \cdots g^{\mu_p \nu_p} F_{\mu_1 \cdots \mu_p} F_{\nu_1 \cdots \nu_p} \sim \frac{1}{R^{6+2p}}$$
— D-brane or O-plane wrapping \( q \) internal dimensions and filling “our” space:

\[ \Delta V \sim \pm \frac{1}{R^{12-q}} \]

There are other potentially important sources of potentials, e.g.:

— The leading term in the beta function:

\[ \Delta V \sim (d - d_{\text{crit}}) \times \ldots \]

— curvature, if any, at the KK scale

— in theories with supersymmetry beneath the KK scale

- my subject henceforth - non-perturbative superpotentials:

\[ \Delta W \sim \sum_D e^{-\text{vol}(D)} + \ldots \]
One quickly convinces oneself that randomly mixing & matching these qualitative ingredients, $V>0$ minima occur in toy models. Global models don't allow random mixing and matching, so more to do.

Slightly more thought reveals a variety of — limited — classical no-go theorems:

— de Wit / Smith/ Hari Dass and Maldacena / Nunez type theorems (purely classical supergravity)

— arguments including limited classical stringy ingredients, like prescribed classes of planes, branes and fluxes

e.g. Hertzberg, S.K., Taylor, Tegmark ’07
However, the limits are obvious, and indeed have (to my mind) been overcome already in work originating in the early 2000s.

— There are classical constructions involving additional contributions from geometry and branes.

c.f. Silverstein ’04 TASI lectures; Torroba talk

— And of course, physics isn’t classical.

This last point is crucial for various macroscopic objects in our Universe (atoms, white dwarfs, neutron stars, most mass!).

It would be striking if it were crucial to the existence of the basic solution we inhabit.
One class of constructions use, as a base, flux vacua of IIB string theory on Calabi-Yau spaces (orientifolds).

The tree level effective theory is a no-scale supergravity with

$$ W = \int (F - \tau H) \wedge \Omega. $$

This depends on complex, but not Kahler, moduli.

$\alpha'$ corrections to $K$ are always present and play a key role in some ideas.

Depending on the geometry, there are also contributions to the superpotential from certain divisors

$$ W_{np} = \sum_{D} e^{-\text{Vol}(D)} + \ldots $$

Strong dynamics on branes can lead to similar contributions, with differing prefactor for $\text{Vol}(D)$. 

Giddings, S.K., Polchinski '01

c.f. Balasubramanian, Berglund, Conlon, Quevedo '05; Quevedo discussion
There are various ways supersymmetry can be spontaneously broken in a model like this. One that I like, for reasons that will become apparent, is supersymmetry breaking by dynamics of branes.

— much studied: anti-branes in warped throat

For small $p$ (say 1), there is a stable vacuum visible in the non-compact throat model. Approximations can be made arbitrarily reliable in the non-compact model by choosing a large flux throat (“large N”).

— but there are other ideas that tie beautifully to older literature on dynamical supersymmetry breaking as well:
Some use “stringy instantons” to break SUSY in models like Fayet or O’Raifeartaigh models.

Others engineer models of more conventional field theoretic DSB.

In a paper in ’03, we suggested a particular scenario for constructing de Sitter vacua in string theory (trivially generalizable to a similar class of ideas) that seems well suited for this setting. Ingredients were:

a) Classical flux vacua with
\[ e^K |W|^2 \ll 1. \]
b) strong dynamics or instantons to stabilize volume.
c) small (dynamical) SUSY-breaking (shifts V>0).
The merit of the resulting class of models is that they reflect a (potential) separation of scales, allowing analysis of simple subproblems in the spirit of effective field theory. The small |W| justifies aspects of the analysis.

\[
\begin{align*}
M_{Pl} & \\
M_{st} & \\
M_{KK} & \sim \frac{1}{R} \\
M_{cs} & \sim \frac{1}{M_s^2 R^3} \\
\Lambda & \\
M_\rho & \sim m_\tilde{g}(a\sigma_0) \\
m_\tilde{g} & \sim \frac{a\Lambda^3}{M_{pl}^2 \sqrt{\sigma_0}} \\
m_{soft} & \sim \frac{m_\tilde{g}}{(a\sigma_0)}
\end{align*}
\]

Figure 1: Various energy scales in a class of IIB flux vacua. $M_{Pl}$: 4 dimensional Planck scale, $M_{st}$: string scale, $M_{KK}$: Kaluza-Klein mass, $R$: radius of compactification, $M_{cs}$: mass of complex structure moduli and dilaton, $\Lambda$: scale of non-perturbative dynamics, $M_\rho$: mass of $\rho$ modulus, $m_\tilde{g}$ mass of gravitino, $m_{soft}$: soft suSy breaking mass. $\sigma_0 = R^4 M_{st}^4$: a given in eq.(29).
(Incidentally, the small $|W|$ to offset energy from DSB was always an ingredient of SUSY model building, though not widely discussed — here it serves a role for control.)

Global constructions are obviously work intensive but not evidently beyond reach. Many partial problems solved:

— Statistics of classical flux vacua show convincingly that small $|W|$ is attainable.

Explicit examples recently provided.

Ashok, Douglas '03; Denef, Douglas'04; many checks…

Demirtas, Kim, McAllister, Moritz ’19, ’20
— global models with instantons in right configurations to stabilize all moduli

The construction of full dS models in this framework hasn’t been a direction of much effort. It wasn’t clear why to do this, since specific global models will not teach new lessons, but will require careful control of factors of pi. It has been clear what to try and do (with no obvious obstructions) for many years.

My own list of good problems in this area would emphasize “technology development” over construction:

— The classification of local DSB models is an active subject, and should be amenable to progress.

— techniques to compute exact superpotentials should morally be within reach, and would presumably have rich ties to mathematics.

— further explore rich mathematics of string models beyond CYs.

— formulating dS quantum gravity is interesting! Metastable state in formulatable theory? dS/dS?

These seem like good problems to me.