Bulk causal features + boundary correlation in AdS/CFT

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Based on: 2105.08094 AM
2101.08855 AM

See also: 1912.05649 AM, Penington, Source
1902.06845 AM
Bulk causal features

- A classic, fruitful, set-up in AdS/CFT:

  can “scatter” in bulk

  no place to scatter

- HPPS understood that for the boundary to reproduce bulk physics, boundary theory must be large $N$ gapped

This talk: What do bulk causal features tell us about the boundary state?
This talk

How do bulk causal features constrain the boundary state?

1. Recall ways to measure correlation and relationship (holographically) to extremal surfaces

2. State the "Privacy-duality" theorem and give argument

3. State connected wedge theorem

4. Final comments

Follow from dynamics of quantum information in spacetime
Measuring correlations in quantum states
Q. I, Measures of correlation

-we will be concerned with:

① Mutual information: \( I(A:B) = S(A) + S(B) - S(AB) \)
   - Amount of correlation between A and B

② Conditional mutual info: \( I(A:C|B) = I(A:B|C) - I(A:B) \)
   - Interpret as amount of correlation between between A and BC not already present between A and B
RT formula + the entanglement wedge

- Given a boundary region $V$:

$$S(V) = \frac{\text{Area}[\text{Ext}]}{4G_N} + S[E_v]$$

- $Pr$ records all the bulk data inside the entanglement wedge $E_v$

\( \text{“entanglement wedge reconstruction”} \)
Correlation and extremal surfaces

- The order of these correlation measures records qualitative features of extremal surfaces

E.g.

\[ I(A:B) = O(1) \]

\[ \downarrow \]

\[ E_{AB} = E_A U E_B \]

\[ \downarrow \]

\[ I(A:B) = O(\sqrt{m}) \]

\[ \downarrow \]

\[ E_{AB} \neq E_A U E_B \]
Privacy-duality theorem
If there is a causal curve from $E_c$ to $E_R$ that avoids $E_0$, then

$$I(V_1: V_2 \mid U) = O(1/6w)$$
Privacy-Duality theorem

- Pick $e, R, U$ in boundary, and a Cauchy surface $\Sigma$ "through" $U$.

Define: $V_1 = \hat{D}(J^+(e) \cap \Sigma \cap Nu')$

$V_2 = \hat{D}(J^-(R) \cap \Sigma \cap Nu')$
Privacy-Duality theorem

- Pick $E, R, U$ in boundary, and a Cauchy surface $\Sigma$ "through" $U$

\[
V_1 = \hat{D}(J^+(E) \cap \Sigma \cap U')
\]
\[
V_2 = \hat{D}(J^-(R) \cap \Sigma \cap U')
\]

Then: If $\exists$ a causal curve from $E_c$ to $E_R$ that avoids $E_U$, then $I(V_1; V_2|U) = O(1/\sqrt{n})$
Proof strategy (general)

1. Introduce a probe system $Q$ (or systems $Q_i$) to the bulk, and exploit causal feature to have $Q$ evolve in some interesting way.

2. Translate evolution of $Q$ to a boundary statement.

3. Argue that for boundary to realize this evolution, while lacking same causal feature, appropriate correlations are necessary.
Q.I. argument for Privacy-duality

1. Introduce a "probe" system $Q$, holding state $|\psi\rangle_Q$, and send it along $\Gamma$.

2. Sends message from $E_e$ to $E_R$, while keeping message secret from $E_u$.
   Sends message from $E_e$ to $R$, while keeping message secret from $U$.

3. In boundary, there is no private curve.
   Instead, exploit correlations among subsystems to hide state on $Q$. 

$\overrightarrow{E_R}$
$\overrightarrow{E_u}$
$\overrightarrow{E_c}$
$\overrightarrow{R}$
$\overrightarrow{U}$
$\overrightarrow{V_1}$
$\overrightarrow{V_2}$
$\overrightarrow{C}$
How to hide the state on $Q$

- Use the "one-time pad"

1) Prepare a string of bits, called $k$, the key and make a copy $k'$
2) Send $k$ through $V_2$
3) Encode $|\psi\rangle$ using $k'$ (do $P_k$)
4) Send "encoded" $Q$ through $U$
5) Undo $P_k$ using $k$, recover $|\psi\rangle$

- Can show any such procedure must have $I(V_1;V_2|U)$ large

\[ P_U \propto \sum_k P_k |\psi\rangle |4\rangle |4\rangle P_k = \mathbb{I} \]
(Improved)

Connected wedge theorem
Connected wedge theorem

- Pick for wedges, define:

\[ \mathcal{J}_{\ell_2 \to \ell_2}^E = \mathcal{J}^+(E_{e_1}) \cap J^+(E_{e_2}) \cap J^-(E_{e_1}) \cap J^-(E_{e_2}) \]

And:

\[ V_1 = \mathcal{J}^+(E_{e_1}) \cap \mathcal{J}^-(E_{e_2}) \]
\[ V_2 = \mathcal{J}^+(E_{e_2}) \cap \mathcal{J}^-(E_{e_1}) \]

If \( \mathcal{J}_{\ell_2 \to \ell_2}^E \) is non-empty, then \( I(V_1; V_2) = O(1/\ell_0) \)
Connected wedge theorem

Use same strategy...

For the boundary to reproduce the bulk unitary $U_{Q_1Q_2}$, $V_1$ and $V_2$ must share lots of entanglement $\Rightarrow I(V_1:V_2) = O(\sqrt{N})$
Final Remarks
Two theorems together suggest a general causal feature \$\sim\$ entanglement connection.

Complements usual extremal surface \$\sim\$ entanglement connection given by RT.

- Interesting to revisit problems studied using extremal surfaces, and look for a causal perspective.

E.g., in the BH + islands context, found causal condition for island formation (at least in toy models).
Thanks! Questions?
(also, available on Slack)
or may@phas.ubc.ca)
Improved connected wedge theorem

- Pick $C_1, C_2, R_1, R_2$:

Define: $V_1 = \hat{\mathcal{J}}^+(C_1) \cap \hat{\mathcal{J}}^-(R_1) \cap \hat{\mathcal{J}}^- (C_2)$

$V_2 = \hat{\mathcal{J}}^+(C_2) \cap \hat{\mathcal{J}}^-(R_1) \cap \hat{\mathcal{J}}^- (R_2)$, $\mathcal{J}_{12 \to 12}^E = \hat{\mathcal{J}}^+(C_{E_1}) \cap \hat{\mathcal{J}}^+(C_{E_2}) \cap \hat{\mathcal{J}}^- (E_{E_1}) \cap \hat{\mathcal{J}}^- (E_{E_2})$

Then: $\mathcal{J}_{12 \to 12}^E \neq \emptyset$

$\downarrow$

$\text{I}(V_1; V_2) = O(1/\sqrt{N})$

i.e. $E_{V_1 V_2} \neq E_{V_1 U E V_2}$, $V_1 \nsubseteq V_2$
**Privacy-Duality theorem**

- Pick \( E, R, U \) in boundary, and a Cauchy surface \( \Sigma \) "through" \( U \)

Define:
\[
V_1 = \hat{D}(J^+(c) \cap \Sigma \cap U')
\]
\[
V_2 = \hat{D}(J^-(R) \cap \Sigma \cap U')
\]

Then: **If \( \exists \) a causal curve from \( E_c \) to \( E_R \)**
that avoids \( E_U \), then \( I(V_1 : V_2 | U) = 0 \)