Contrasting the fuzzball and wormhole paradigms

Samir D. Mathur

The Ohio State University
The black hole information paradox

Small corrections theorem: The entanglement must keep growing as

\[ S_{N+1} > S_N + \log 2 - 2\epsilon \]

The Page curve cannot come down if (i) horizon dynamics can be approximated in any way by semiclassical evolution (ii) we assume there are no long distance nonlocal effects

\[ \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) + O(\epsilon) \]
The fuzzball paradigm


weak coupling

strong coupling

The size of the bound state grows with the number of branes, and a horizon never forms

Bound states of D1, D5, P charges:

\[ D \sim \left[ \frac{\sqrt{n_1 n_5 n_p g^2 \alpha'^4}}{V L} \right]^{\frac{1}{3}} \sim R_H \]

Many many examples of string bound states have been constructed, and in each case one finds a fuzzball instead of a horizon
small corrections theorem
(If not fuzzball, then one of (i) nonlocality (ii) remnants (iii) nonunitarity)

With all the things we know, it is hard to imagine that there is any alternative to the fuzzball paradigm in string theory...
Recently there has been interest in the "Wormhole paradigm" ...

Maintain semiclassical horizon dynamics in some approximation

Invoke nonlocal effects in some way ...

I do not find myself in agreement with (what I understand) of these ideas ...

In this context, it is important to understand what constraints we get from the small corrections theorem
Consider a scenario that is tempting but where the Page curve will NOT come down

(A) The EXACT description of the black hole is some very complicated string theory dynamics in the region of the hole (say $r \lesssim 4M$)

All we assume is that the dynamics in this region is unitary

(B) Far from the hole (say $r \gtrsim 100M$) the EXACT dynamics is given by standard quantization of string theory around flat spacetime (no novel nonlocal effects)
We can extract an **EFFECTIVE SEMICLASSICAL DYNAMICS** from these EXACT degrees of freedom.

Let these **semiclassical degrees of freedom** describe a scalar field

\[ \phi = \sum_k \left( \hat{a}_k f_k(x) + \hat{a}_k^\dagger f_k^*(x) \right) \]

\[ \Box \hat{\phi} = 0 \]
We will be forced to get the creation of entangled pairs in the effective description

(i) We only ask that this **effective field** work for low energy modes, say wavelengths
\[ \lambda \gtrsim 1 \text{ fermi} \]

(ii) The dynamics of \[ \Box \hat{\phi} = 0 \] needs to be reproduced only upto some approximation, specified by some a parameter \[ \epsilon \ll 1 \]

(iii) We do not even assume that this map work for all times: it just works over the spacetime region required to emit a few Hawking modes, so that we can use it for the information puzzle
(D) The following depicts what will happen with the above assumptions:

\[ r = 4M \]

\[ \frac{1}{\sqrt{2}} (|0\rangle_b |0\rangle_c + |1\rangle_b |1\rangle_c) + O(\epsilon) \]
This situation will evolve as follows:

Since $b$ was an approximate construct, these graviton sets can differ by $O(\epsilon)$.

$b$ travels to far region and must be written as normal excitations of the exact string theory.
The small corrections theorem:

Under these assumptions, the entanglement graph in the EXACT quantum gravity theory will keep rising

(It is also rising in the approximate semiclassical theory)

The entanglement at step $N$ must keep growing as

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$

(SDM 2009)

The result is not an obvious one since it uses the power of the strong subadditivity inequality of quantum entanglement entropy, which is nontrivial to prove
Thus the small corrections theorem leaves us with two sharply different possibilities

(1) The fuzzball paradigm:

Just like a piece of coal; no effective description with semiclassical physics at horizon

(2) The wormhole paradigm:

An effective semiclassical description can be obtained, at least for describing the emission of a few quanta

Then the EXACT theory MUST have nonlocal effects
Fuzzball paradigm:  
Many exciting things to do !!

Good agreements of emission rates, scrambling, energy gaps, 3 and 4 point functions, observational signatures …

Approximate universality in $E \gg T$ limit?  
(Fuzzball complementarity)

Constraints from causality: The VECRO hypothesis for the nature of the gravitational vacuum

Cosmology: Source of energy in inflation, Dark energy …

Wormhole paradigm:  I find that different approaches have one or more of the following features, which I find myself not willing to reconcile to:

- Non-unitarity evolution in black hole interior in \textit{EXACT} theory
- Long-distance nonlocality of Hamiltonian interactions in \textit{EXACT} theory
- Remnants, baby universes in \textit{EXACT} theory
- Altered dynamics at infinity in \textit{EXACT} theory: non-standard laboratory dynamics, nonlocal identification of bits, ensemble averaged theory …
- No map between \textit{EXACT} theory and semiclassical approximation

Many exciting things to do !!
THANK YOU !!