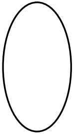
Fine-grained quantum data (energy levels, operator matrix elements) are complicated. But one can imagine a simple effective description where these quantities are drawn from random distributions [Wigner,. . .].

Perhaps "simple gravity" is such a theory, describing a (fake) ensemble of boundary theories with different fine-grained quantum data.

In special cases (e.g. pure dilaton gravity [SSS, Witten, Maxfield, Turiaci]) both the bulk "simple gravity" theory and the boundary ensemble are separately well-defined and dual to each other.

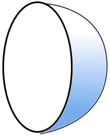
More generally, both the simple bulk theory and the boundary ensemble are probably incomplete. But they seem to be fairly detailed, going beyond just RMT universality and ETH statistics. How far do they go?

In AdS/CFT, the thermal partition function of the boundary theory is represented by a boundary condition consisting of a circle of length β:



$$= \int \mathrm{d}E\ \rho(E)\, e^{-\beta E}$$

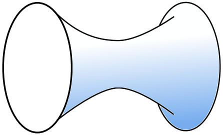
The Euclidean black hole geometry can fill this in, and it gives the leading gravitational answer for the density of states



$$= \int \mathrm{d}E\ \mathbf{G}[\rho(E)]\, e^{-\beta E}$$

Here $\mathbf{G}[\cdot]$ means "evaluation in simple gravity description" and we will interpret it as "average over ensemble."

Is the ensemble nontrivial? A wormhole like this

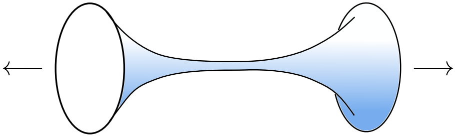


$$= \int \mathrm{d}E_L \mathrm{d}E_R e^{-\beta_L E_L - \beta_R E_R} \mathbf{G}_c[\rho(E_L)\rho(E_R)] \qquad (1)$$

would tell us about the variance in the ensemble:

$$\mathbf{G}_c[\rho(E_L)\rho(E_R)] = \mathbf{G}[\rho(E_L)\rho(E_R)] - \mathbf{G}[\rho(E_L)]\mathbf{G}[\rho(E_R)].$$

However, there is actually no solution [Witten,Yau], because the wormhole wants to get long and narrow:



The boundary interpretation of this is that $\mathbf{G}_c[\rho(E)\rho(E)]$ is not growing with energy, so there is no saddle point in (1).

If there *were* a solution, analytic continuation to Lorentzian signature would give an eternal traversable wormhole with no interaction between the two sides, violating boundary causality.

But there are several ways to tweak this so there *is* a classical solution:

1. Couple the two sides [Maldacena, Qi], see also replica wormholes
2. Insert heavy operators on the two sides [Saad,DS]
3. Put spatially-varying sources on the boundary [Maldacena, Maoz, Marolf, Santos]
4. Put imaginary sources [Maldacena, Qi, Garcia-Garcia, Godet]
5. Study $\beta_L + \beta_R = 0$ [Saad, Shenker, DS]

The boundary conditions in cases 2,3,4,5 remain factorized, so the wormhole seems to require an ensemble interpretation in those cases. (And none of the above run into a contradiction with causality, but for different reasons!)

5. Suppose we set $\beta_L = \mathrm{i}T$ and $\beta_R = -\mathrm{i}T$, so we are computing

$$\int_{\text{window}} \mathrm{d}E_L \mathrm{d}E_R e^{\mathrm{i}T(E_L - E_R)} \mathbf{G}_c[\rho(E_L)\rho(E_R)] \tag{2}$$

Since $\beta_L + \beta_R = 0$, the pressure on the average energy vanishes and a classical solution exists. The wormhole has a periodic direction with geodesic length proportional to T.

For small T, there is a short circle and the answer is UV sensitive. We need a more complete theory for this region!

For large T, the matter fields are projected into the vacuum so the calculation is controlled. Also, in this limit, random matrix universality predicts an answer for (2). The wormhole reproduces this [Shenker's talk].

For intermediate T, there are exponentially decaying corrections from matter loops that wind around the periodic direction — a prediction for the energy statistics of the boundary theory that goes beyond RMT.

Questions

1. What do we have to add to simple gravity to get a specific boundary quantum system?

2. How detailed is the "fictitious ensemble" of theories that simple gravity seems to describe? What is the input to this theory, and what is the output?