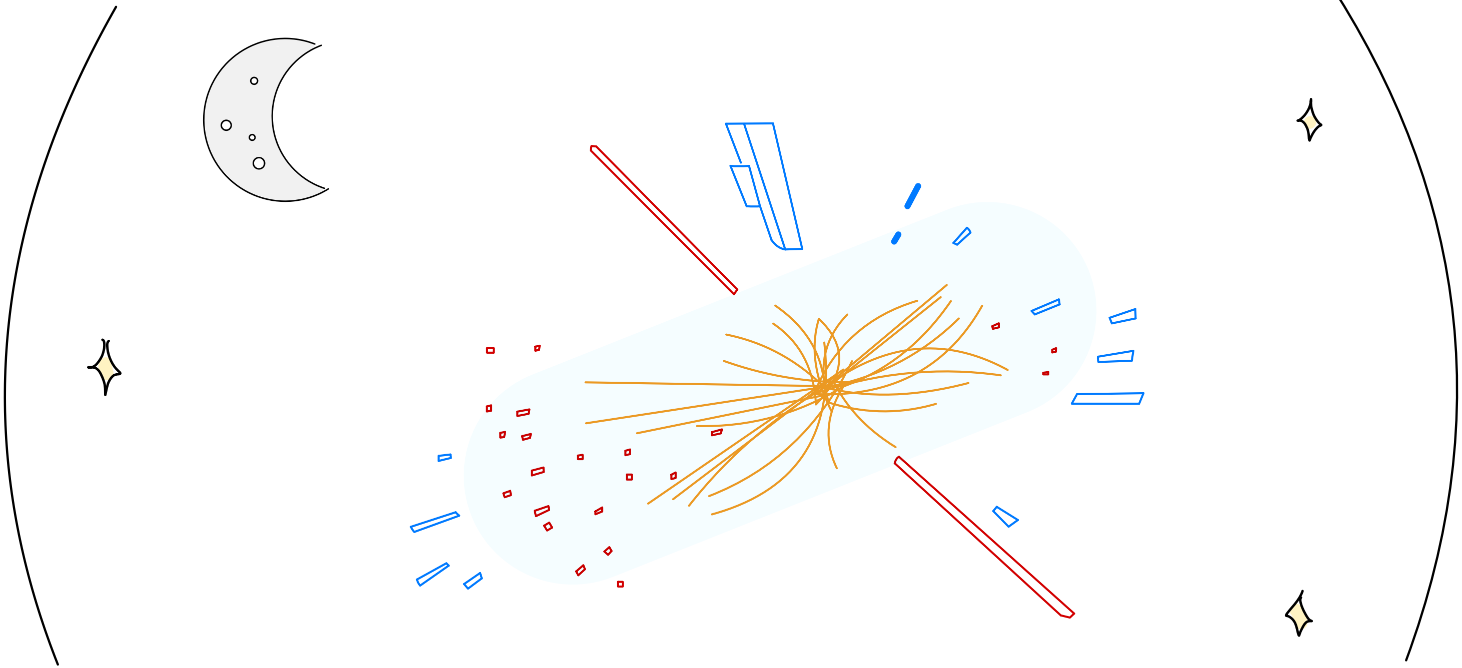
Strings 2021, ICTP-SAIFR, São Paulo

Celestial Amplitudes

Sabrina Gonzalez Pasterski, Princeton PCTS

- Celestial Holography purports a duality between gravitational scattering in asymptotically flat spacetimes and a CFT living on the celestial sphere.
- Celestial Amplitudes are S-matrix elements in a basis where they transform as conformal correlators in D-2.



Today we will review the current status of this dictionary.

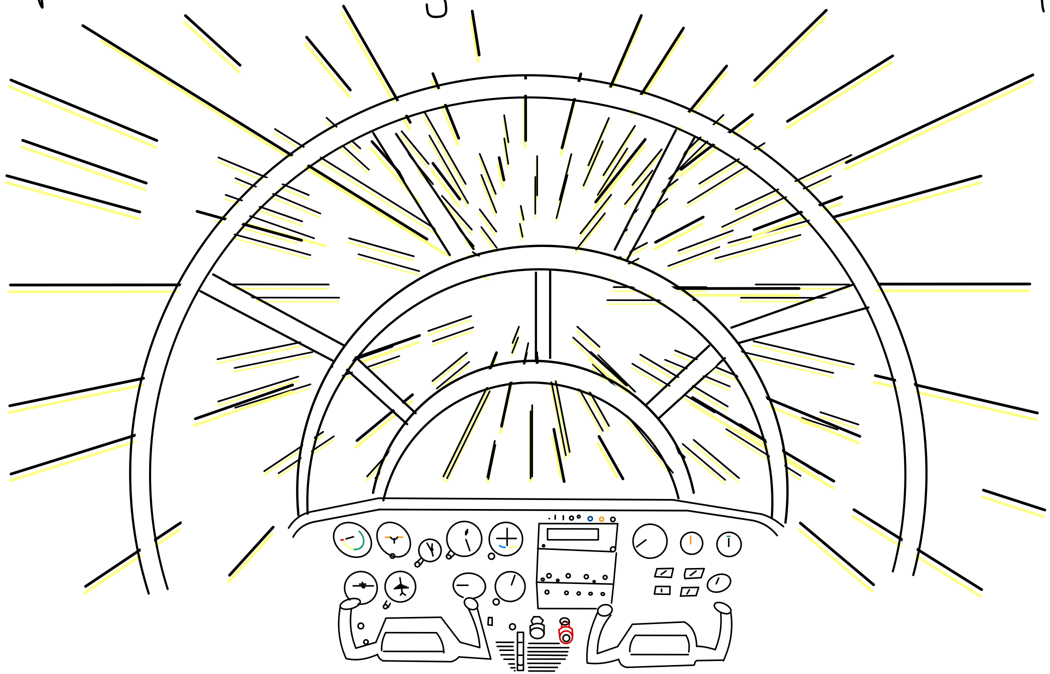
- Part I Asymptotic Symmetries
- Part II Celestial Amplitudes

The results presented are thanks to:

Adamo, Arkani-Hamed, Ashtekar, Atanasov, Athira, Avery, Ball, Banerjee, Barnich, Bhatakar, Cachazo, Campiglia, Casali, Chang, Cheung, Coito, Compere, Crawley, de la Fuente, Donnay, Dumitrescu, Fan, Fiorucci, Fotopoulos, Giribet, Gonzalez, Gosh, Guevara, Haco, Hamada, Hawking, He, Himwich, Huang, Kapec, Laddha, Lam, Law, Li, LippStreu, Liu, Long, Lysov, Magnea, Manu, Mao, Mason, Melton, Miller, Mirzaiyan, Mitra, Mizera, Nandan, Nande, Narayanan, Nguyen, Nichols, Oblak, Oliveri, Pandey, S.P., Pate, Paul, Perry, Porfyriadis, Prabhu, Puhm, Raclariu, Raju, Rojas, Rosso, Ruzziconi, Saha, Salzer, Samal, Schreiber, Schwab, Sen, Seo, Seraj, Shao, Sharma, Shiu, Shrivastava, Stieberger, Strominger, Sundrum, Suskind, Taylor, Trevisani, Troessaert, Venugopalan, Verlinde, Volovich, Wald, Wen, Yuan, Zhiboedov, Zhu, Zlotnikov ...

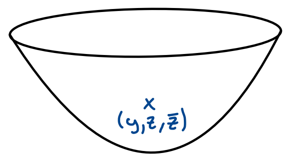
12:20 – 13:10 Discussion session w/ Andy & Tomasz!

The first step is to match the symmetries on both sides of the proposed duality...

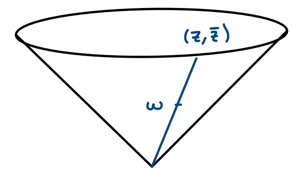


Lorentz transformations of Minkowski space act as global conformal transformations on the celestial sphere.

For what follows we will want to keep in mind how to connect position and momentum space descriptions of the scattering problem



$p^2 = -m^2$



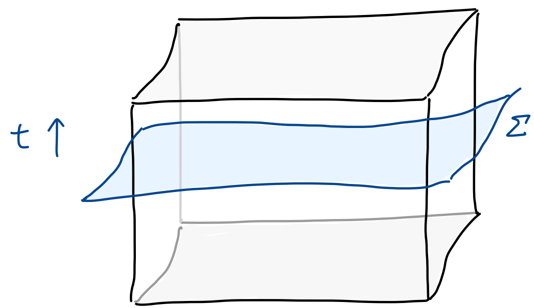
$p^2 = 0$

$$p^\mu = \frac{m}{2y}\left(1+y^2+z\bar{z},\, z+\bar{z},\, i(\bar{z}-z),\, 1-y^2-z\bar{z}\right)$$

$$p^\mu = \omega\left(1+z\bar{z},\, z+\bar{z},\, i(\bar{z}-z),\, 1-z\bar{z}\right)$$

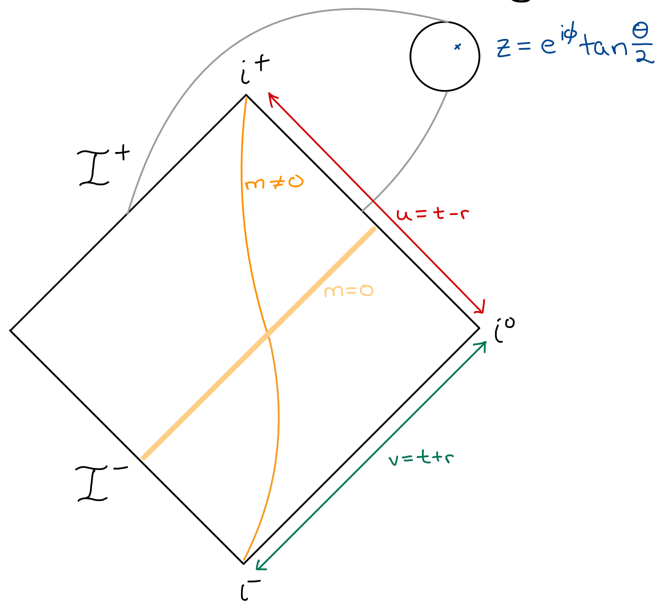
For S-matrix elements we would typically specify a set of on-shell momenta for the in and out particles.

While in position space we need to specify the field configurations on early and late Cauchy slices.



For gravitational scattering we want to allow fluctuations of the bulk geometry and will specify the free data at the conformal boundary.

Let's look at the Penrose diagram for Mink_4. The causal structure of the boundary will be the same for asymptotically flat spacetimes.



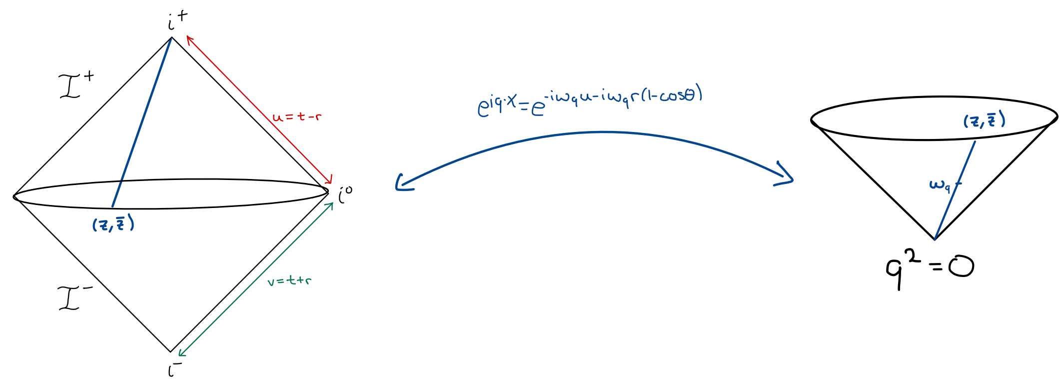
Massless excitations enter and exit along null hypersurfaces $\mathcal{I}^{\pm} \cong \mathbb{R} \times S^2$. We call this S^2 cross-section the celestial sphere.

Let's compare this to the free mode expansion of the field operators:

$$h_{\mu\nu} = \sum_{\alpha=\pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[\varepsilon^{\alpha *}_{\mu\nu} a_\alpha e^{iq\cdot x} + \varepsilon^{\alpha}_{\mu\nu} a^\dagger_\alpha e^{-iq\cdot x} \right]$$

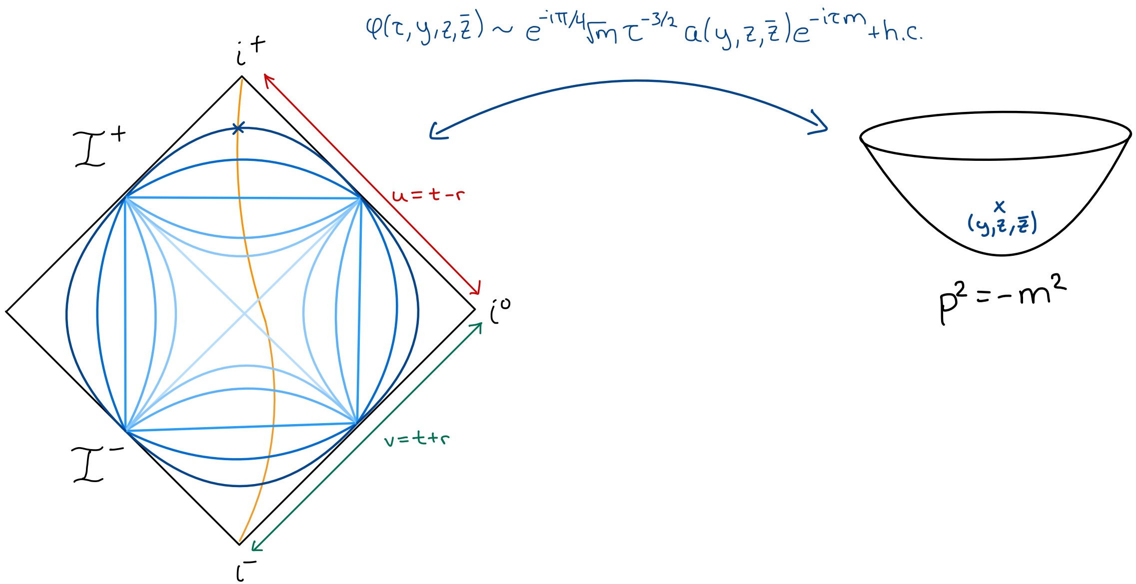
The saddle point approximation localizes the momentum direction to align with the corresponding point on the celestial sphere.

$$\lim_{r\to\infty} \frac{1}{r} h_{\bar z \bar z} = \frac{-i}{4\pi^2} \frac{2}{(1+z\bar z)^2} \int_0^\infty d\omega_q \left[a_-(\omega_q \hat{x}) e^{-i\omega_q u} - a_+^\dagger(\omega_q \hat{x}) e^{i\omega_q u} \right]$$



He, Lysov, Mitra, Strominger '14

For massive fields we have a similar identification between the late time momentum and a point on the hyperboloid resolving timelike infinity.



de Boer, Solodukhin '03 Campiglia, Laddha '15

We're interested in gravitational scattering in spacetimes with $\Lambda = 0$. The outgoing radiation is captured by the behavior of the metric at large r, fixed u.

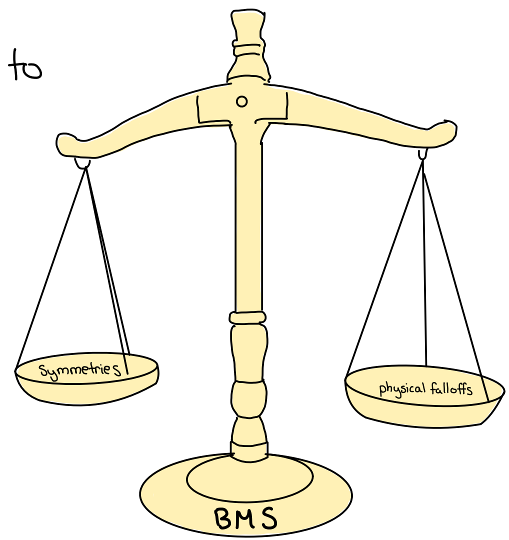
flat

$\frac{1}{r}$# corrections

$$ds^2 = -du^2 - 2du\,dr + 2r^2\gamma_{z\bar{z}}\,dz\,d\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + D^zC_{zz}\,du\,dz + D^{\bar{z}}C_{\bar{z}\bar{z}}\,du\,d\bar{z} + \ldots$$

To study the phase space & symmetries one needs to

- pick a convenient gauge
- specify physical falloffs



Bondi, van der Burg, Metzner '62 Sachs '62

Residual diffeomorphisms that preserve the falloffs and act non-trivially on the asymptotic data are part of the Asymptotic Symmetry Group.

$$\text{ASG} = \frac{\text{Allowed Symmetries}}{\text{Trivial Symmetries}}$$

The ASG will be much larger then the group of isometries of any given spacetime within this class.

$$\text{BMS} \supset \text{Poincaré}$$

	BMS	Poincaré
# generators:	∞	10

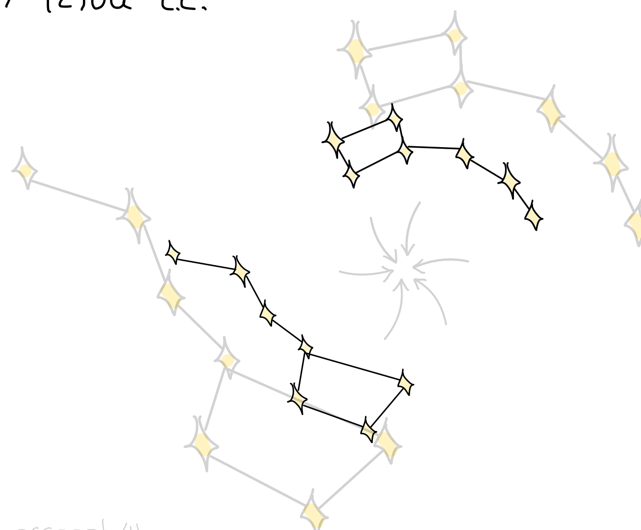
Bondi, van der Burg, Metzner '62 Sachs '62

✧ Supertranslations induce angle-dependent shifts in the time coordinate

$$\xi\big|_{\mathcal{I}^+} = f(z,\bar{z})\,\partial_u$$

✧ Superrotations extend global conformal transformations to local CKVs

$$\xi\big|_{\mathcal{I}^+} = Y^z(z)\,\partial_z + \frac{u}{2} D_z Y^z(z)\,\partial_u + c.c.$$



Bondi, van der Burg, Metzner '62 Sachs '62 Barnich, Troessaert '11

These asymptotic symmetries manifest in scattering amplitudes as soft theorems.

$$\langle \text{out} | QS - SQ | \text{in} \rangle \longleftrightarrow \lim_{\omega \to 0} \omega \langle \text{out} | a(\omega \hat{x}) S | \text{in} \rangle \propto \langle \text{out} | S | \text{in} \rangle$$

- The canonical charges are co-dim 2 and can be written as an integral of data on $\mathcal{I}^{+/-}$.

$$\int du \, \partial_u (\cdot) \longleftrightarrow \lim_{\omega \to 0} \omega$$

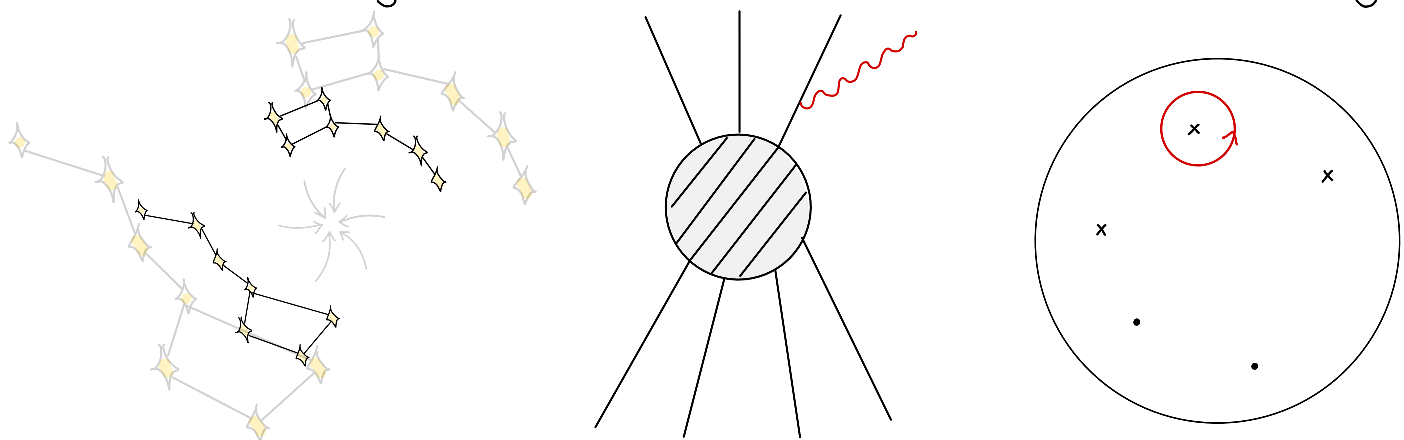
- Most of the symmetries are spontaneously broken by the vacuum and the term that generates inhomogeneous shifts is linear in $h_{\mu\nu}$

$$Q \ni Q_S \sim \int du \, \partial_u \int d\omega \, e^{-i\omega u} a(\omega \hat{x})$$

Weinberg '65 Strominger '13 He, Lysov, Mitra, Strominger '14

Moreover, these Ward identities are naturally organized in terms of currents in a 2D CFT.

4D ASG Ward Identity ⟷ Soft Theorem ⟷ 2D CFT Ward Identity

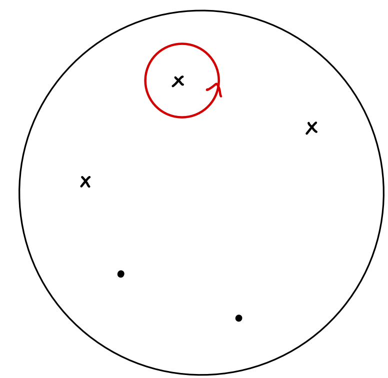


In particular, the subleading soft graviton theorem gives us a candidate stress tensor.

$$T_{zz} = \frac{i}{8\pi G}\int du\, u\partial_u \int d^2w \sqrt{\gamma}\, \frac{1}{z-w} D_w^3 C^{ww}$$

However, to construct operators with definite 2D conformal weights we will need to change our scattering basis.

$$\langle T_{zz}\, \mathcal{O}_1^+ \dots \mathcal{O}_n^- \rangle = \sum_k \left[\frac{h_k}{(z-z_k)^2} + \frac{\partial_{z_k}}{z-z_k} \right] \langle \mathcal{O}_1^+ \dots \mathcal{O}_n^- \rangle$$



$$h_k = \frac{1}{2}\left(-\omega_k \partial_{\omega_k} + s_k\right), \quad \bar{h}_k = \frac{1}{2}\left(-\omega_k \partial_{\omega_k} - s_k\right)$$

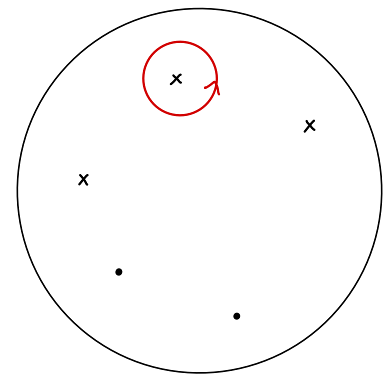
Cachazo, Strominger '14 Kapec, Lysov, S.P., Strominger '14 Kapec, Mitra, Raclariu, Strominger '16

In particular, the subleading soft graviton theorem gives us a candidate stress tensor.

$$T_{zz} = \frac{i}{8\pi G}\int du\, u\partial_u \int d^2w\sqrt{\gamma}\,\frac{1}{z-w}\,D_w^3 C^{ww}$$

However, to construct operators with definite 2D conformal weights we will need to change our scattering basis.

$$\langle T_{zz}\mathcal{O}_1^+ \cdots \mathcal{O}_n^-\rangle \Leftrightarrow \lim_{\omega\to 0}(1+\omega\partial_\omega)\langle out|a_-(\omega\hat{x})S|in\rangle$$



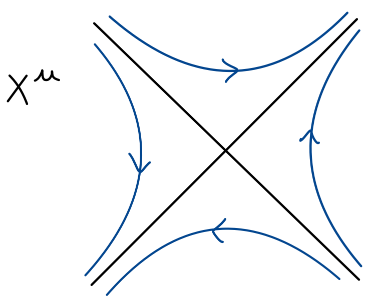
$$h_k = \frac{1}{2}\left(-\omega_k\partial_{\omega_k} + s_k\right),\quad \bar{h}_k = \frac{1}{2}\left(-\omega_k\partial_{\omega_k} - s_k\right)$$

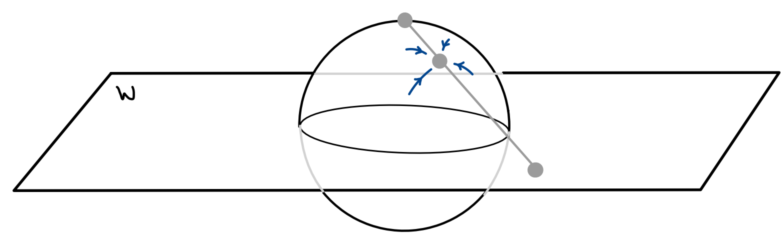
Cachazo, Strominger '14 Kapec, Lysov, S.P., Strominger '14 Kapec, Mitra, Raclariu, Strominger '16

A conformal primary wavefunction is a function of a bulk point X^μ and a reference point $w \in \mathbb{C}$ which transforms as follows

$$\Phi^{s}_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu ; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J} D_s(\Lambda)\, \Phi^{s}_{\Delta,J}(X^\mu ; w, \bar{w})$$

Where D_s is the spin-s rep. of the Lorentz group. Here we will consider $\Phi^{s}_{\Delta,J}$ with $s=|J|$ that solve the appropriate source free linearized eom.



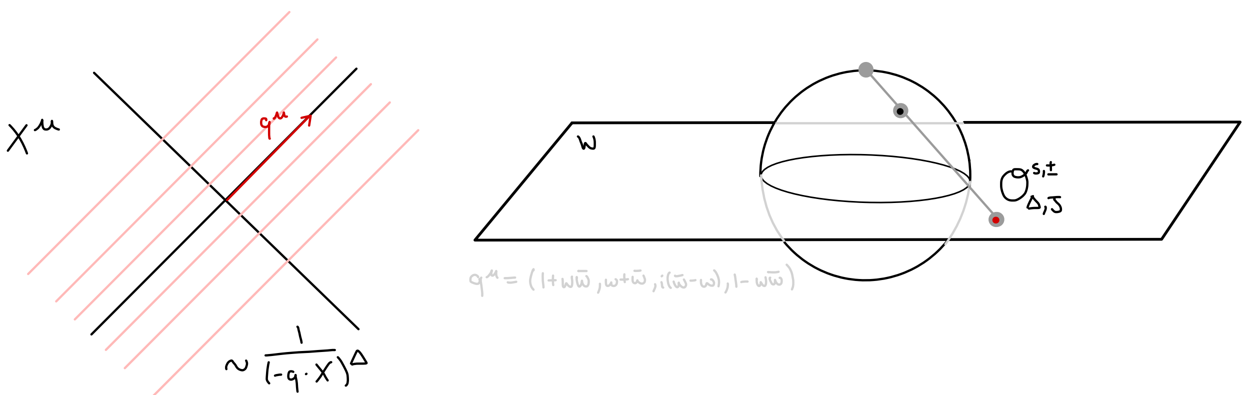


S.P., Shao '17

Taking an inner product of such a wavefunction with the field operator gives a (quasi)-primary operator with 2D conformal dimension Δ and spin J.

$$\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) = i\left(\hat{\mathcal{O}}^{s}(X^\mu),\, \Phi^{s}_{\Delta^*,-J}(X^\mu_\mp; w,\bar{w})\right)_\Sigma$$

These operators carry a $\pm$ label indicating in versus out, selected by taking $u \mapsto u \mp i\varepsilon$.

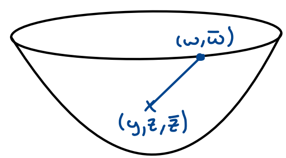


S.P., Shao '17 Donnay, S.P., Puhm '20

For $m=0$ these wavefunctions are gauge equivalent to a Mellin transform of on-shell plane waves.

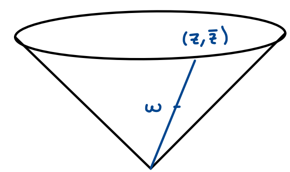
$$\Phi^{|J|}_{\Delta,J} \sim \int d\omega\, \omega^{\Delta-1}\, \varepsilon_{\mu_1\ldots\mu_{|J|}} e^{\pm i\omega q X_\pm}$$

A similar transform exists when $m\neq 0$. In either case, we can apply this transform directly to S-matrix elements to land on Celestial Amplitudes.



$p^2=-m^2$

$$\int_0^\infty \frac{dy}{y^3}\int d^2z\, G_\Delta(y,z,\bar{z};w,\bar{w})(\,\cdot\,)$$



$p^2=0$

$$\int_0^\infty d\omega\, \omega^{\Delta-1}(\cdot)$$

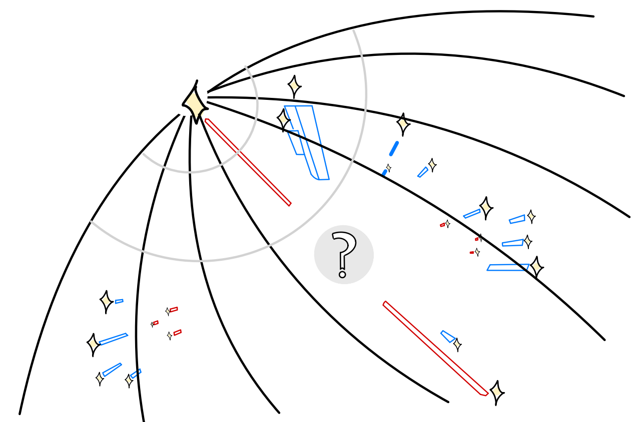
de Boer, Solodukhin '03 Cheung, de la Fuente, Sundrum '16 S.P., Shao, Strominger '17

By construction, S-matrix elements in this basis

$$\tilde{A}(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \left(\int_0^{\infty} d\omega\, \omega^{\Delta_i - 1} \right) A(\omega_i, z_i, \bar{z}_i)$$

will transform like correlators of quasi-primaries because the external particles are in boost eigenstates.

$$U(\Lambda) | h, \bar{h}; z, \bar{z} \rangle = (cz+d)^{-2h} (\bar{c}\bar{z}+\bar{d})^{-2\bar{h}} \, | h, \bar{h}; \frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}} \rangle$$



Joos '62 Banerjee '18

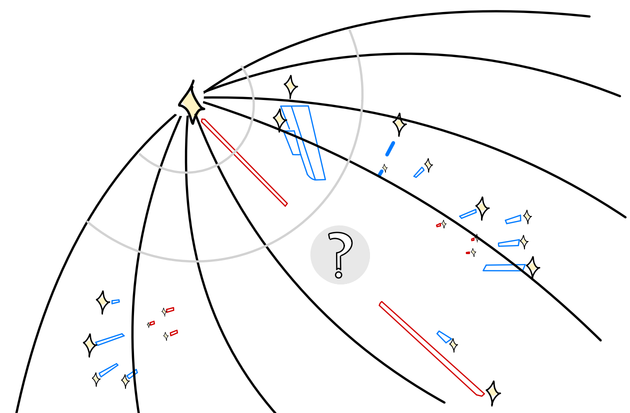
By construction, S-matrix elements in this basis

$$\widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \left(\int_0^\infty d\omega\, \omega^{\Delta_i - 1} \right) A(\omega_i, z_i, \bar{z}_i)$$

will transform like correlators of quasi-primaries because the external particles are in boost eigenstates.

J_j = jth helicity

$$\widetilde{\mathcal{A}}\left(\Delta_i, \frac{a z_i + b}{c z_i + d}, \frac{\bar{a}\bar{z}_i + \bar{b}}{\bar{c}\bar{z}_i + \bar{d}_i}\right) = \prod_{j=1}^{n} \left[(c z_j + d)^{\Delta_j + J_j} (\bar{c}\bar{z}_j + \bar{d})^{\Delta_j - J_j} \right] \widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i)$$



Joos '62 Banerjee '18

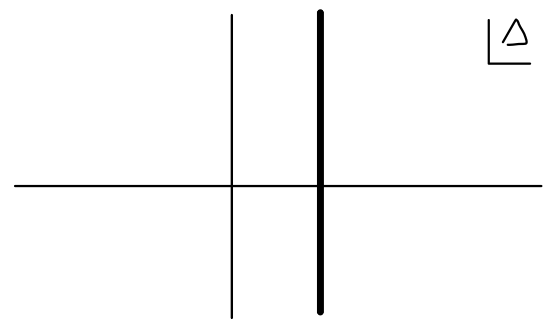
This transformation is easily inverted for Δ_i on the principal series which capture finite energy radiation.

$$\Delta = 1 + i\lambda, \qquad \lambda \in \mathbb{R}$$

However translations shift the conformal dimension

$$p^\mu = q^\mu e^{\partial_\Delta} \qquad \Longleftrightarrow \qquad \Delta \mapsto \Delta + 1$$

and we will want to analytically continue to $\Delta \in \mathbb{C}$



S.P., Shao '17 Donnay, Puhm, Strominger '18 Stieberger, Taylor '18 Donnay, S.P., Puhm '20

It is straightforward to do this transform for low point amplitudes.

$$\mathcal{A}(\omega_i, z_i, \bar{z}_i) = M \times \delta^4\Big(\sum p_i\Big), \qquad \lambda_i = \sqrt{\omega_i}\begin{pmatrix} z_i \\ 1 \end{pmatrix}$$

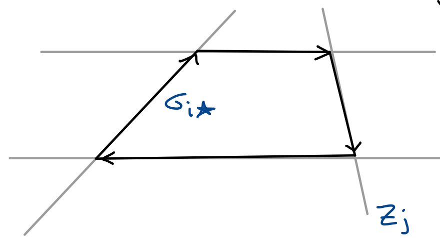
← fixed

integrated

Letting $s = \sum \omega_i$, $\sigma_i = s^{-1}\omega_i$

$$\prod_{i=1}^{n}\int_0^\infty d\omega_i\, \omega_i^{\Delta_i - 1}(\cdot) = \int_0^\infty ds\, s^{-1+\sum\Delta_i}\prod_{i=1}^{n}\int_0^1 d\sigma_i\, \sigma_i^{\Delta_i - 1}\,\delta\Big(\sum\sigma_i - 1\Big)(\cdot)$$

So that for $n \leq 5$ the σ_i are localized by the $\delta^{(4)}(\sum p_i)$ and $\delta(\sum \sigma_i - 1)$



The remaining δ-functions imply singular low-point correlators.

S.P., Shao, Strominger '17   Schreiber, Volovich, Zlotnikov '17

The aim of our map from 4D S-matrix elements to 2D correlators

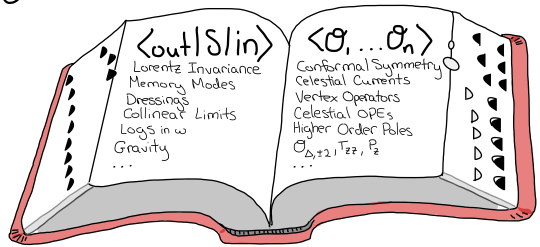
$$\langle p_{out,i}|S|p_{in,j}\rangle \longmapsto \langle \mathcal{O}^{+}_{\Delta_i,J_i} \cdots \mathcal{O}^{-}_{\Delta_j,J_j} \cdots \rangle$$

is to be able to use CFT techniques to learn about amplitudes

? We are already seeing that this dual CFT is exotic, both in its complex spectrum, and in the singular behavior at $n \leq 4$pt.

? Meanwhile, the quantity we want to look at in 4D is also unusual because we are probing scattering at all energy scales.

To understand how to effectively use this framework we must build up our dictionary!



Let us now examine how celestial amplitudes encode the behavior of scattering in various limits

- Infrared – soft thm's, currents, dressings, null states
- Collinear – celestial OPEs, conformal block decomposition
- Ultraviolet – convergence $\leftrightarrow$ ultrasoft, anti-Wilsonian paradigm

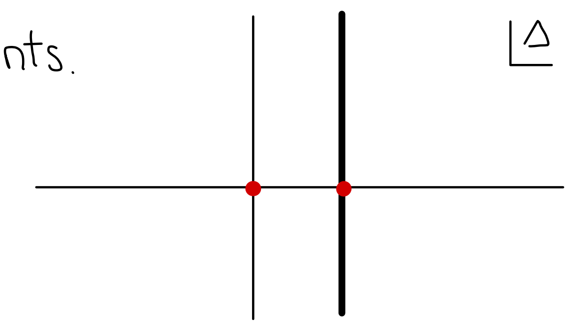
Since soft theorems motivated this program we should understand what they translate to in the celestial basis.

$$\int_0^{\omega_*} d\omega\, \omega^{a-1} \sim \frac{1}{a}$$

So that factorizations at various orders in $\omega \sim 0$ turn into factorizations of the residues as $\Delta \to$ special values

$$\langle \text{out} | a_-(\omega q) S | \text{in} \rangle = \sum_k \Big(\underbrace{\frac{1}{\omega} \frac{(p_k \cdot \varepsilon^-)^2}{p_k \cdot q}}_{\hookrightarrow \frac{1}{\Delta - 1}} - i \underbrace{\frac{p_k \cdot \varepsilon^- \varepsilon^{-\nu} q^\lambda J_{k\lambda\nu}}{p_k \cdot q}}_{\hookrightarrow \frac{1}{\Delta}} \Big) \langle \text{out} | S | \text{in} \rangle + \mathcal{O}(\omega)$$

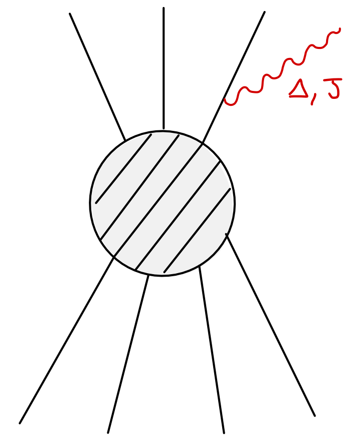
These residues correspond to celestial currents.



The familiar universal soft theorems correspond to Δ for which the conformal primary wavefunctions are pure gauge.

$$\mathcal{O}_{\Delta,J} = i\left(\hat{\mathcal{O}},\ \underbrace{\Phi^{G}_{\Delta^*,-J}}_{\text{pure gauge}}\right)_\Sigma$$

$\lvert J \rvert$	Δ	Soft Thm.	Current	Asym. Sym.
1	1	ω^{-1}	J	large $U(1)$
$3/2$	$1/2$	$\omega^{-1/2}$	S	large SUSY
2	1	ω^{-1}	P	supertranslations
	0	ω^{0}	$\tilde{T}$	superrotations / $\mathrm{Diff}(S^2)$



Cheung, de la Fuente, Sundrum '16 S.P. Shao '17 Donnay, Puhm, Strominger '18 Donnay, S.P., Puhm '20 S.P., Puhm, Trevisani '21

These (Δ, J) are also special from the point of view of the global conformal multiplets. Primary states

$$L_1|h,\bar{h}\rangle = \bar{L}_1|h,\bar{h}\rangle = 0$$

have primary descendants when

$$L_1(L_{-1})^k|h,\bar{h}\rangle = -k(2h+k-1)(L_{-1})^{k-1}|h,\bar{h}\rangle = 0$$

and ditto for $\bar{L}_{-1}$ When both conditions are met we get nested primaries.

$$\begin{tikzcd}
& \left|\frac{1-k}{2},\frac{1-\bar{k}}{2}\right\rangle \arrow[dl, "(\bar{L}_{-1})^{\bar{k}}"'] \arrow[dr, "(L_{-1})^{k}"] & \\
\left|\frac{1-k}{2},\frac{1+\bar{k}}{2}\right\rangle \arrow[dr] & & \left|\frac{1+k}{2},\frac{1-\bar{k}}{2}\right\rangle \arrow[dl] \\
& \left|\frac{1+k}{2},\frac{1+\bar{k}}{2}\right\rangle &
\end{tikzcd}$$

These correspond to an infinite tower of 'conformally soft' theorems at $\Delta \in 1 - \mathbb{Z}_{\geq 0}$.

$$
\begin{tikzcd}
 & \mathcal{O}_{\frac{1-k}{2}, \frac{1-\bar{k}}{2}} \arrow[dl, "\partial_{\bar{w}}^{\bar{k}}"'] \arrow[dr, "\partial_w^k"] & \\
0 \arrow[dr] & & 0 \arrow[dl] \\
 & &
\end{tikzcd}
$$

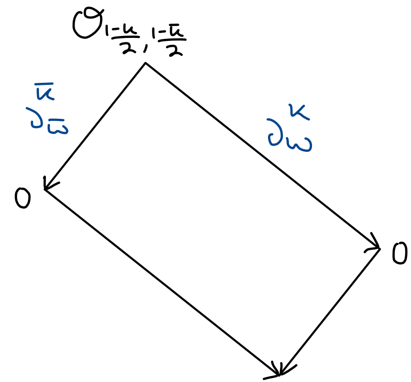
While the soft theorems of the previous table descend to operators generating asymptotic symmetry transformations

$$
\begin{tikzcd}
 & \arrow[dl] \arrow[dr] & \\
\mathcal{O}_{\Delta, J} \arrow[dr] & & \tilde{\mathcal{O}}_{2-\Delta, -J} \arrow[dl] \\
 & \mathcal{O}_{soft} &
\end{tikzcd}
$$

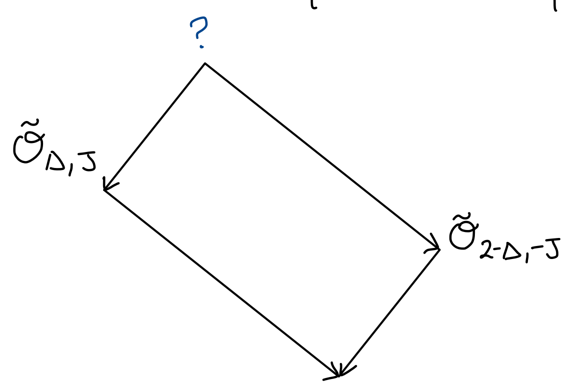
$$\mathcal{O}_{soft} \longleftarrow Q[Y] = \int d^2z\, \mathcal{O}_{soft} \cdot Y$$

Banerjee, Pandey, Paul '19 Guevara, Himwich, Pate, Strominger '21 S.P., Puhm, Trevisani '21

These correspond to an infinite tower of 'conformally soft' theorems at $\Delta \in 1-\mathbb{Z}_{\geq 0}$.



Whose partners can themselves be expressed as primary descendants of dressing modes.



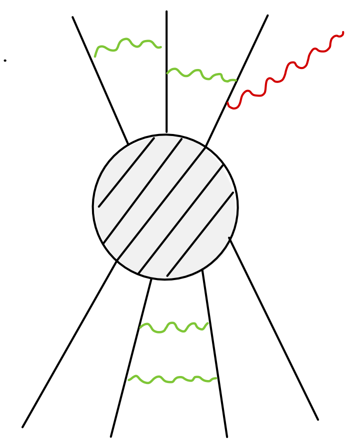
Banerjee, Pandey, Paul '19 Guevara, Himwich, Pate, Strominger '21 S.P., Puhm, Trevisani '21

Soft factors relate amplitudes with and without an extra single particle emission/absorption.

Exchanges between the charged external legs exponentiate so that amplitudes without soft radiation vanish.

This vanishing can be interpreted as non-conservation for the charges generating asymptotic symmetry transformations.

One can avoid this vanishing by dressing the operators.



Chung '65 Faddeev, Kulish '70 Kapec, Perry, Raclariu, Strominger '17

These dressings can be adapted to the conformal basis. For E&M

$$W_j = \exp\left[-eQ_j \int \frac{d^3k}{(2\pi)^3} \underbrace{\frac{1}{2k^0}}_{\Delta=1} \frac{p_j^{\mu}}{p_j \cdot k} \left(\varepsilon^{*}_{\alpha\mu} a - \varepsilon_{\alpha\mu} a^{\dagger} \right)\right] = e^{iQ_j \Phi(z_j, \bar{z}_j)}$$

$SL(2,\mathbb{C})$ ascendant

and we see that the dressing takes the form of a vertex operator. Moreover the celestial amplitudes factorize

$$\mathcal{A} = \mathcal{A}_{soft} \mathcal{A}_{hard}$$

where

$$\mathcal{A}_{soft} = \langle e^{iQ_1 \Phi(z_1, \bar{z}_1)} \cdots e^{iQ_n \Phi(z_n, \bar{z}_n)} \rangle$$

while $\mathcal{A}_{hard}$ equals the amplitude for dressed operators.

Arkani-Hamed, Pate, Raclariu, Strominger '20

The spontaneous symmetry breaking dynamics for the 4D asymptotic symmetries is captured by simple 2D models.

Large - U(1) $\Longleftrightarrow$ free boson

With the important observation that the levels of the 2D current algebra are set by the cusp anomalous dimension in 4D.

$$\langle \Phi(z,\bar{z})\Phi(w,\bar{w})\rangle = \frac{e^2}{4\pi^2}\ln\Lambda_{IR}\ln|z-w|^2$$

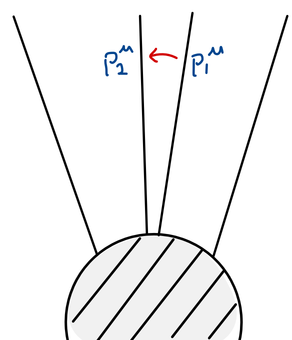
Nande, Pate, Strominger '17 Himwich, Narayanan, Pate, Paul, Strominger '20 Nguyen, Salzer '20-'21

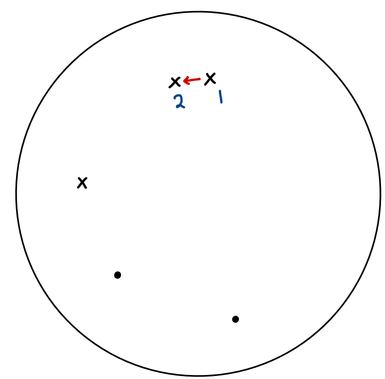
Currents arise from tuning Δ_i to special values. Rather than tuning Δ_i, we can tune $z_{ij} = z_i - z_j$.

Collinear limits in 4D should be captured by a celestial OPE. Consider taking two gluons collinear

$$\mathcal{O}^a_{\Delta_1,+1}(z_1,\bar{z}_1)\,\mathcal{O}^b_{\Delta_2,+1}(z_2,\bar{z}_2) \sim -\frac{i f^{ab}{}_c}{z_{12}}\, C(\Delta_1,\Delta_2)\,\mathcal{O}^c_{\Delta_1+\Delta_2-1,+1}(z_2,\bar{z}_2)$$

A very interesting observation is that we can use symmetries to find $C(\Delta_1,\Delta_2)$.





Fan, Fotopoulos, Taylor '19 Pate, Raclariu, Strominger, Yuan '19

From translation invariance

$$C(\Delta_1,\Delta_2) = C(\Delta_1+1,\Delta_2) + C(\Delta_1,\Delta_2+1)$$

while from the leading soft gluon theorem

$$\lim_{\Delta_1\to 1}(\Delta_1-1)C(\Delta_1,\Delta_2) = 1.$$

Moreover, the kernel of the descendancy relation for the subleading soft gluon gives an additional global symmetry that imposes

$$(\Delta_1-2)C(\Delta_1-1,\Delta_2) = (\Delta_1+\Delta_2-3)C(\Delta_1,\Delta_2)$$

This recursion can be solved, giving

$$C(\Delta_1,\Delta_2) = B(\Delta_1-1,\Delta_2-1), \quad B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

which matches what we get from transforming the collinear limit.

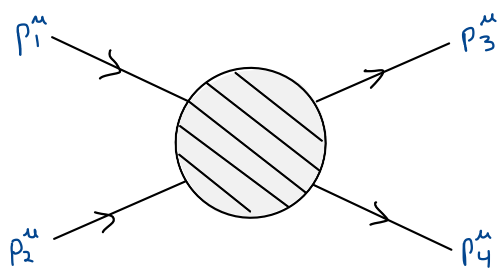
Pate, Raclariu, Strominger, Yuan '19

Despite the singular behavior of low point functions, collinear limits let us extract the C_{ijk}.

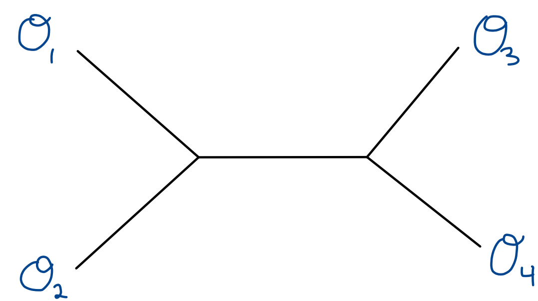
With this data and our understanding of the spectrum, we can apply CFT machinery to celestial amplitudes. For example:

- Using symmetries to go beyond the leading singular terms of the OPE and constrain correlators.

- Examining conformal block decompositions to interpret intermediate exchanges and radial quantization in CCFT.



vs.



Lam, Shao '17 Nandan, Schreiber, Volovich, Zlotnikov '19 Banerjee, Gosh, Gonzo '20 Law, Zlotnikov '20 Fan, Fotopoulos, Stieberger, Taylor, Zhu '21 Atanasov, Melton, Raclariu, Strominger '21

Let's take a closer look at $2\to 2$ scattering. Starting from the momentum space amplitude

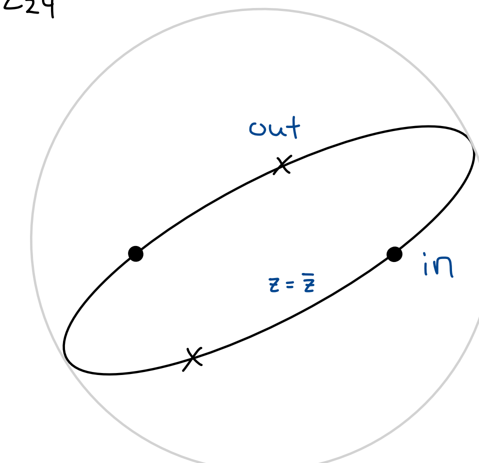
$$\mathcal{A} = M(s,t) \times \delta^{(4)}\left(\sum p_i\right)$$

the corresponding celestial amplitude can be written as

$$\tilde{\mathcal{A}} = X \mathcal{A}(\beta, z), \quad \beta = \sum(\Delta_i - 1), \quad z = \frac{z_{12} z_{23}}{z_{13} z_{24}}$$

where for massless external states

$$X \propto \prod_{i<j}^{4} z_{ij}^{\,h/3 - h_i - h_j}\, \bar{z}_{ij}^{\,\bar{h}/3 - \bar{h}_i - \bar{h}_j}\, \delta\big(i(z - \bar{z})\big).$$



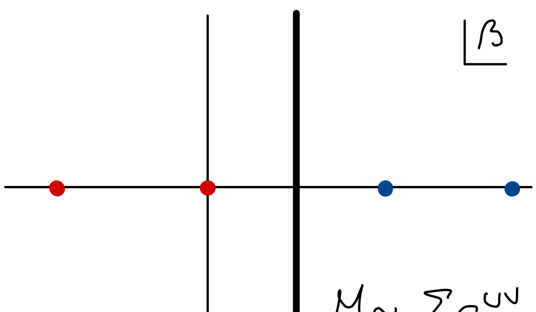
Arkani-Hamed, Pate, Raclariu, Strominger '20

The stripped amplitude is probed at all energy scales

$$\mathcal{A}(\beta, z) = \int_0^\infty d\omega\, \omega^{\beta-1} M(\omega^2, -z\omega^2)$$

- The convergence of this integral for external for external dimensions on the principal series is tied to the UV behavior.

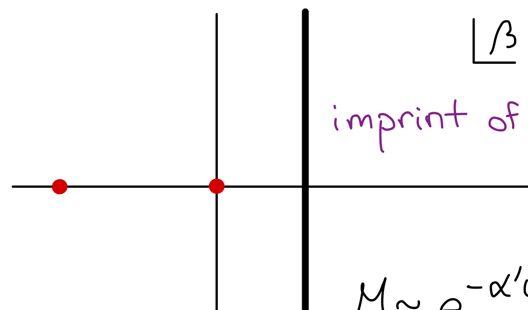
- Poles at $\beta \in 2\mathbb{Z}_>$ present in field theory are absent in quantum gravity.



$M \sim \sum a_m^{UV} \omega^{-2m}$

vs

imprint of UV completion



$M \sim e^{-\alpha' \omega^2}$

Arkani-Hamed, Pate, Raclariu, Strominger '20

Meanwhile, expanding M around $\omega=0$ gives

$$M = \sum a_{n,m,r}(z)\,\omega^{2n}(G_N\omega^2)^m \log^r(\omega/\Lambda_{UV})$$

- The logs coming from running couplings give higher order poles in β.

$$\int_0^{\omega_*}\frac{d\omega}{\omega}\,\omega^a\log^b\omega = \frac{\partial^b}{\partial a^b}\int_0^\infty \frac{d\omega}{\omega}\,\omega^a \sim \frac{1}{a^{1+b}}$$

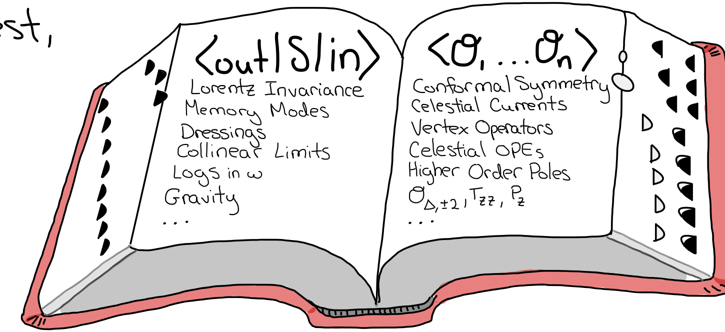
- Positivity constraints from amplitudes translate to positivity constraints on the residues of the simple poles at $\beta = 2\mathbb{Z}_{\leq}$.

imprint of causality

Lam, Shao '17 Arkani-Hamed, Pate, Raclariu, Strominger '20 Chang, Huang, Huang, Li '21

We have a framework that

- makes symmetry enhancements manifest,
- reorganizes soft and collinear limits,
- and is sensitive to the deep UV.

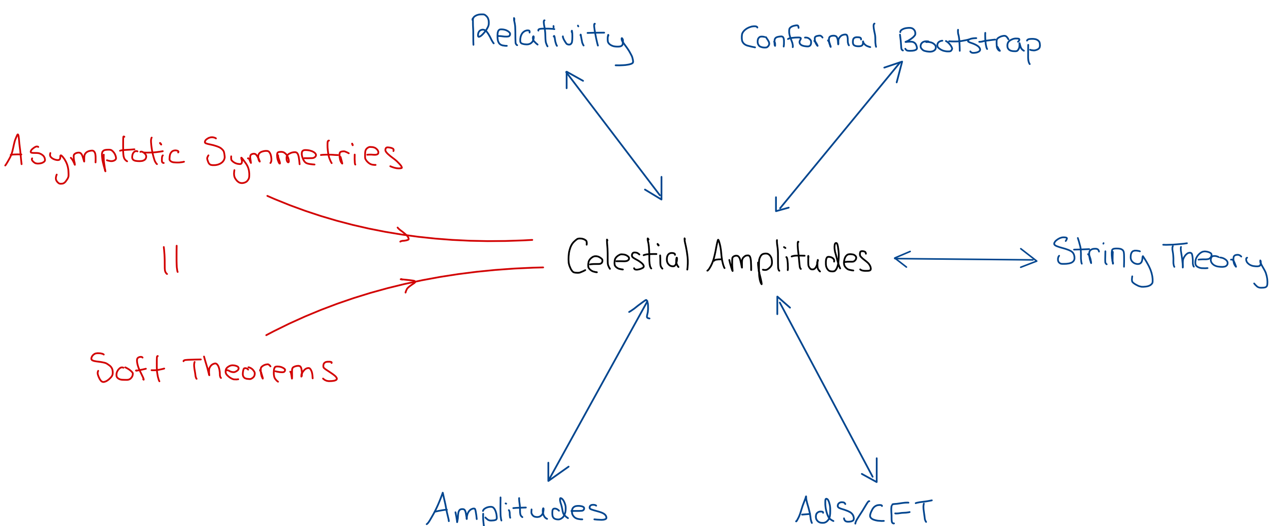


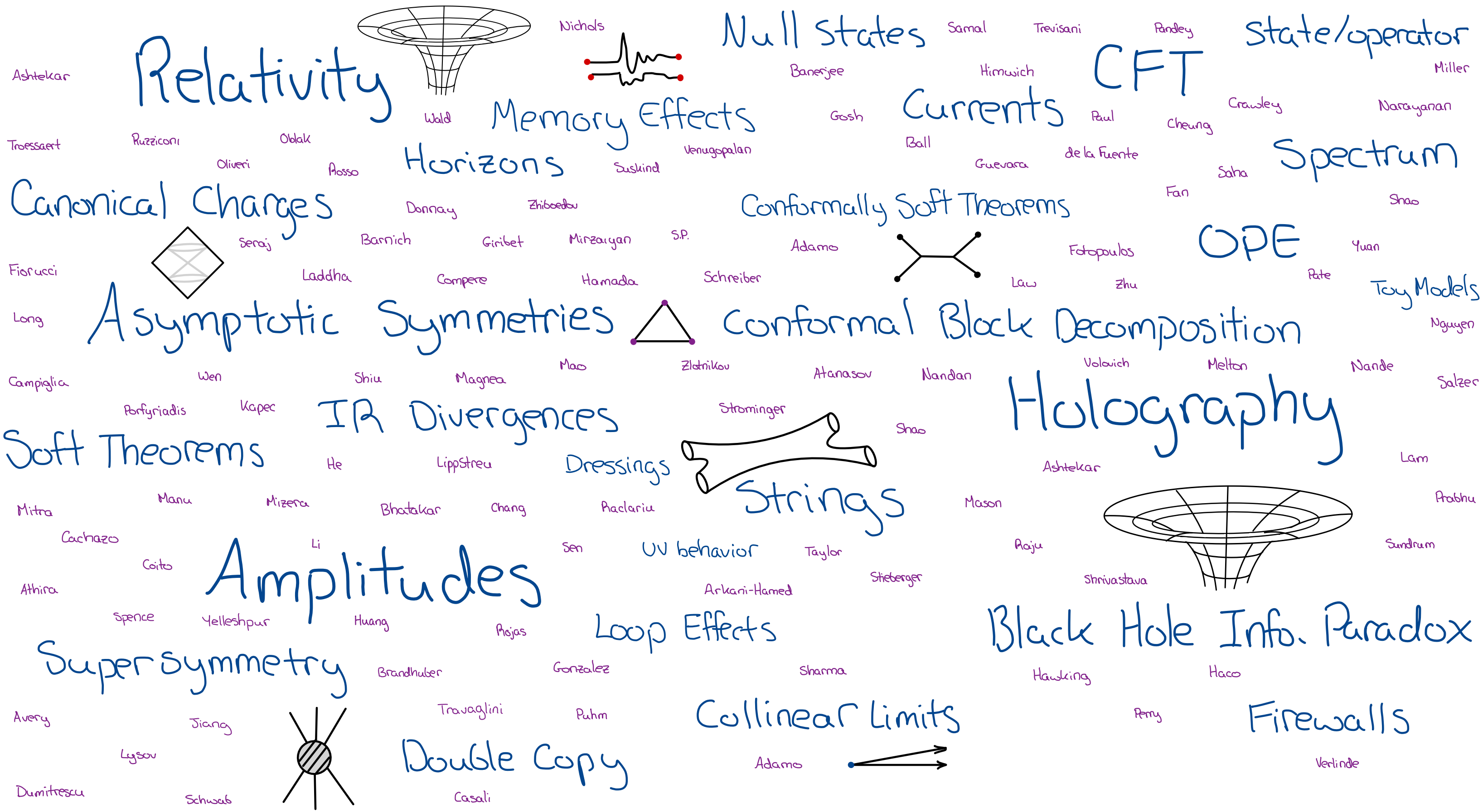
We have given examples of

- how to translate features of amplitudes into objects within the celestial CFT
- and what kind of properties we can demand of CCFTs.

The next steps forward are to

- Expand our dictionary
- Connect to adjacent subfields
- Look for an intrinsic construction





Relativity
Nichols
Null States
Samal
Trevisani
Pandey
State/operator
CFT
Ashtekar
Banerjee
Himwich
Miller
Memory Effects
Wald
Gosh
Currents
Paul
Crawley
Narayanan
Cheung
Troessaert
Ruzziconi
Oblak
Venugopalan
Ball
Oliveri
Horizons
Susskind
Guevara
de la Fuente
Spectrum
Saha
Fan
Shao
Canonical Charges
Donnay
Zhiboedov
Conformally Soft Theorems
Seraj
Barnich
Giribet
Mirzaiyan
S.P.
Adamo
Fotopoulos
OPE
Yuan
Fiorucci
Laddha
Compere
Hamada
Schreiber
Law
Zhu
Pate
Toy Models
Long
Asymptotic Symmetries
Conformal Block Decomposition
Nguyen
Campiglia
Wen
Shiu
Magnea
Mao
Zlotnikov
Atanasov
Nandan
Volovich
Melton
Nande
Salzer
Porfyriadis
Kapec
IR Divergences
Strominger
Holography
Soft Theorems
Shao
He
Lippstreu
Dressings
Ashtekar
Lam
Strings
Mitra
Manu
Mizera
Bhatkar
Chang
Pasterski
Mason
Prabhu
Cachazo
Li
Sen
UV behavior
Taylor
Raju
Sundrum
Coito
Amplitudes
Stieberger
Shrivastava
Athira
Arkani-Hamed
Spence
Yelleshpur
Huang
Rojas
Loop Effects
Black Hole Info. Paradox
Supersymmetry
Brandhuber
Gonzalez
Sharma
Hawking
Haco
Travaglini
Puhm
Collinear Limits
Avery
Jiang
Perry
Firewalls
Lysov
Double Copy
Adamo
Verlinde
Dumitrescu
Schwab
Casali