Higher-Form Symmetries from String Theory

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2102.01693, 2106.10265 with Lakshya Bhardwaj and Max Hübner
Motivation

Higher-form symmetries [Gaiotto, Kapustin, Seiberg, Willett, 2014] are by now very well established in QFT.

What’s the added benefit of studying them in string theory?

Access to strongly-coupled QFTs, non-Lagrangians:

- Geometric engineering: 5d and 6d theories and SCFTs
- Strongly coupled theories via holography
- Non-Lagrangian theories, e.g. class S and generalizations
Motivation

Access to strongly-coupled QFTs, non-Lagrangians: some examples

- Geometric engineering of 5d theories and SCFTs: (all in the past year)
  Higher-form symmetries, ’t Hooft anomalies, 2-group structures
  [Morrison, SSN, Willett][Albertini, del Zotto, Garcia Etxebarria, Hosseini]
  [Closset, SSN, Yi-Nan Wang][Apruzzi, Dierigl, Lin][Benetti Genolini, Tizzano][Bhardwaj, SSN][Cvetic, Dierigl, Lin, Zhang][Apruzzi, Bhardwaj, Oh, SSN]

- Holography: class S, ABJM and revisiting Klebanov-Strassler confining cascade [Witten][Bah, Bonetti, Minasian][Hofman, Iqbal][Bergman, Tachikawa, Zafrir][Apruzzi, van Beest, Gould, SSN]

- Non-Lagrangian theories: from 6d such as class S, class R
  [Tachikawa][Cordova, Dumitrescu, Intriligator][Eckhard, Kim, SSN, Willett][Gukov, Pei, Hsin][Bharwaj, Hubner, SSN]\(^2\).
4d $\mathcal{N} = 1$ SYM 1-form symmetry, is a diagnostic for confinement: Wilson lines with area law indicate confining vacua, where the 1-form symmetry is unbroken. Perimeter law implies deconfining phase and 1-form symmetry is spontaneously broken.

The goal of this talk is to generalize this insight to theories that may not have a simple underlying UV Lagrangian description, such as $\mathcal{N} = 1$ deformations of 4d $\mathcal{N} = 2$ Class S theories.

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For this, we first cast the standard case of SYM into this framework and show how to determine the line operators and 1-form symmetry in each vacuum.
Plan

1. Line operators, polarization and 1-form symmetry
2. 1-form Symmetry in 4d $\mathcal{N} = 2$ Class S
3. Confinement in $\mathcal{N} = 1$ deformations of class S
   (i) Super Yang-Mills
   (ii) Confinement index for theories with adjoint chirals
   (iii) Non-Lagrangian theories
1. Line Operators, Polarization and 1-form Symmetry

Pure gauge theory in 4d, gauge algebra $\mathfrak{g}$ and simply-connected group $G$. The set of line operators are

\[ \mathcal{L} = \Lambda_w/\Lambda_r \oplus \Lambda_{mw}/\Lambda_{cr} = Z_G \oplus Z_G \quad Z_G = \text{center of } G. \]

Not all lines are mutually local (relative theory): Dirac pairing

\[ L_\alpha L_\beta = L_\beta L_\alpha e^{2\pi i \langle L_\alpha, L_\beta \rangle}. \]

Polarization $\Lambda \subset \mathcal{L}$:
choice of a maximal set of mutually local line operators (absolute theory).

E.g. for $\mathfrak{su}(N)$: $\mathcal{L} = \mathbb{Z}_N \oplus \mathbb{Z}_N$ with

\[ \langle W, H \rangle = \frac{1}{N}. \]

$\Lambda = \langle W \rangle$: gauge group $G = SU(N)$
$\Lambda = \langle H \rangle$: gauge group $G = PSU(N)$
1-Form Symmetry

$\Lambda \subset \mathcal{L}$ choice of polarization (absolute theory). Then the 1-form symmetry is the Pontryagin dual group to $\Lambda$

$$\Gamma^{(1)} = \hat{\Lambda} = \text{Hom}(\Lambda, U(1)).$$

E.g. $\mathfrak{su}(N)$: $\Lambda = \mathbb{Z}_N$ and $\Gamma^{(1)} = \mathbb{Z}_N$.

In a given vacuum $r$ let

$$\Lambda_r = \{ L \in \Lambda; L \text{ has perimeter law in vacuum } r \}$$

Then the 1-form symmetry that is preserved in this vacuum is

$$\Gamma_r^{(1)} = \frac{\hat{\Lambda}}{\Lambda_r} \subset \Gamma^{(1)}.$$ 

If this is non-trivial, then the vacuum $r$ is confining.
2. 1-form symmetry of 4d $\mathcal{N} = 2$ Class S

[Tachikawa][Bhardwaj, Hubner, Schafer-Nameki]

6d (2,0) of type $g=$ADE is a relative theory, with 2-form symmetry $\mathcal{Z}$.
Compactify on $\mathcal{C}_{g,n} \subset T^*\mathcal{C}$ to 4d $\mathcal{N} = 2$ class S [Gaiotto].

6d surface operators wrapped on $H_1(\mathcal{C}_{g,n}, \mathbb{Z})$ give line operators.
Consider $\mathcal{C}_{g,\emptyset}$:

$$\mathcal{L} = H_1(\mathcal{C}_g, \mathcal{Z}) \cong \mathcal{Z}_A \oplus \mathcal{Z}_B$$

from A-/B-cycles. There is a pairing on these line operators

$$a_i \otimes \alpha_i \in H_1(\mathcal{C}_g, \mathbb{Z}) \otimes \mathcal{Z} : \quad \langle a_1 \otimes \alpha_1, a_2 \otimes \alpha_2 \rangle = (a_1 \cdot a_2) \langle \alpha_1, \alpha_2 \rangle .$$

$\Rightarrow$ relative theory

Canonical polarization choices:

$$\mathcal{L} \supset \Lambda = \mathcal{Z}_A \quad \text{or} \quad \mathcal{Z}_B \quad \Rightarrow \quad \Gamma^{(1)} = \hat{\Lambda}.$$  

Extra class S data:

twist lines and punctures i.e. $g$ with outer autos [Bhardwaj, Hubner, SSN, 2021].
Interlude: Geometric Engineering

Construct 4d $\mathcal{N} = 2$ theories from IIB on singular, non-compact CY3 $X$.

Line operators are D3s on non-compact 3-cycles, modulo screening by D3s on compact 3-cycles:

$$\mathcal{L} = \frac{H_3(X, \partial X, \mathbb{Z})}{H_3(X, \mathbb{Z})} \cong \{ L \in H_2(\partial X, \mathbb{Z}); L \text{ extends trivially to } X \}$$

Computable for non-Lagrangian theories like Argyres-Douglas. [Closset, SSN, Wang][Albertini, del Zotto, Garcia Etxebarria, Hosseini].

A subset of class S theories: $\mathbb{C}^2/\Gamma_{ADE} \rightarrow \mathcal{C}_g$.

ALE-fibration is governed by the Higgs field $\phi$, which satisfies the Hitchin equations [Gaiotto, Moore, Neitzke]

$$\bar{D}\phi = 0, \quad F_{zz} + [\phi, \phi^*] = 0.$$ 

Spectral curve of this Higgs field is the Seiberg-Witten curve $\det(v - \phi) = 0$, which is an $N$-fold cover of the Gaiotto curve $\mathcal{C}$. Then

$$\mathcal{L} = \text{Ab}(\Gamma_{ADE})^{2g}.$$
3. $\mathcal{N} = 1$ Deformation

4d $\mathcal{N} = 1$ from 6d (2,0) on $C_{g,n}$ in $V_1 \oplus V_2 \to C_{g,n}$, with sections $(\phi, \psi)$.

Simple class of configurations: $\phi = (1, 0)$- and $\psi = (0, 0)$-forms on $C$, with BPS equations [Xie], corresponding to a generalized Hitchin system

$$\bar{D}\phi = \bar{D}\psi = 0$$

$$[\phi, \psi] = 0$$

$$F + [\psi, \psi^*] + [\phi, \phi^*] = 0.$$  

$(\phi, \psi)$ each defines an $N$-sheeted covers of $C$.

Strategy: start with $\mathcal{N} = 2$ Higgs field $\phi$, and then "rotate" to $\mathcal{N} = 1$ [Barbon][Witten][Hori, Ooguri, Oz][Bonelli, Giacomelli, Maruyoshi, Tanzini] (related [Dijkgraaf, Vafa] curve).
4d $\mathcal{N} = 2$ pure SYM from 6d

Focus at first on $\mathfrak{su}(2)$ for clarity. The Seiberg-Witten curve is

$$v^2 = \frac{\Lambda^2}{t} + u + \Lambda^2 t.$$ 

$t=\text{coordinate on the Gaiotto curve } \mathcal{C} = S^2$.

Class S construction:

g = 0, n = 2 with two $\mathcal{P}_0$ irregular punctures:

$$\text{Tr } \phi^2 \equiv \phi_2 = \frac{v^2}{t^2} dt^2 = \left( \frac{\Lambda^2}{t^3} + \frac{u}{t^2} + \frac{\Lambda^2}{t} \right) dt^2$$

$\phi_2$ has poles of order 3 at $t = 0, \infty$. 
Rotating to $\mathcal{N} = 1$ SYM

Turn on $\psi$, where we rotate to $\mathcal{N} = 1$ at $t = \infty$:

\[
t \to \infty : \quad \psi \to \mu \phi \zeta, \quad \phi \zeta \frac{dt}{t} = \phi
\]

\[
t \to 0 : \quad \psi \to c
\]

Furthermore for diagonalizable Higgs fields, the BPS equation $[\phi, \psi] = 0$ implies simultaneous diagonalizability. For generic eigenvalue spectrum, this implies that one is a function of the other, and thus the branch-cuts must match.

→ solving for the curve

→ topological matching of branch-cuts
The $v$-curve for $\phi$ and $w$-curve for $\psi$ are:

$$v : \quad v^2 = \frac{1}{t}(\Lambda^2 t^2 + ut + \Lambda^2)$$

$$w : \quad w^2 = \mu^2 \Lambda^2 t + c$$

Tune CB moduli to combine into single cover, i.e. move branchpoints $\times$:

$$v^2 = \Lambda^2 \left( \frac{1}{t} \pm 2 + t \right), \quad w^2 = \mu^2 \Lambda^2 t, \quad vw = \mu \Lambda^2 (t \pm 1).$$
Curves describing the two vacua

Pure $\mathcal{N} = 1$ SYM: take $\mu \to \infty$, while $\Lambda_{N=1}^3 = \mu \Lambda^2$ fixed, and $\mu t = \tilde{t}$:

$$v^2 = \frac{\Lambda_{N=1}^3}{\tilde{t}}, \quad w^2 = \Lambda_{N=1}^3 \tilde{t}, \quad vw = \pm \Lambda_{N=1}^3$$

The two vacua differ by asymptotics of $w$, which can be changed by encircling $t = 0$:

$\Sigma_+$:

$\Sigma_-$:

\[ t = \infty \quad t = 0 \]
Line Operators and 1-form Symmetry of the Vacua

\( \mathcal{L} \) and \( \Lambda \) = lines and a polarization of the \( \mathcal{N} = 2 \) class S theory.
Define for each \( \mathcal{N} = 1 \) vacuum with curve \( \Sigma_r \):

\[ \mathcal{I}_r = \{ \text{projections of 1-cycles on the } \mathcal{N} = 1 \text{ curve } \Sigma_r \text{ onto } \mathcal{C} \} \subset H_1(\mathcal{C}, \hat{\mathbb{Z}}). \]

Then lines with perimeter law are \( \Lambda_r = \Lambda \cap \mathcal{I}_r \) and the 1-form symmetry \( \Gamma_r^{(1)} \) preserved in the vacuum \( r \) is

\[ \Gamma_r^{(1)} = \left( \frac{\Lambda}{\Lambda \cap \mathcal{I}_r} \right) \subset \hat{\Lambda} \]

If \( \Gamma_r^{(1)} \neq \emptyset \) then the vacuum \( r \) is confining.

Why? Confining strings arise from from membranes on relative 1-cycles of the local CY3 and \( \Sigma_r \)[Witten].
Confinement for $\mathfrak{su}(2)$ SYM

$\mathfrak{su}(2)$: Line operators are $W$ and $H$.
For both vacua $2W = 0$ because of the branch-cut crossing (only going twice around the puncture gives a 1-cycle on the curves). The lines with perimeter law for each vacuum are:

$$\mathcal{I}_+ = < 2W, H >, \quad \mathcal{I}_- = < 2W, H + W > .$$

Preserved 1-form symmetry for each polarization:

<table>
<thead>
<tr>
<th>$\Lambda$</th>
<th>$G$</th>
<th>$\Gamma_+^{(1)}$</th>
<th>$\Gamma_-^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; W &gt;$</td>
<td>$SU(2)$</td>
<td>$\mathbb{Z}_2$</td>
<td>$\mathbb{Z}_2$</td>
</tr>
<tr>
<td>$&lt; H &gt;$</td>
<td>$SO(3)_+$</td>
<td>$\emptyset$</td>
<td>$\mathbb{Z}_2$</td>
</tr>
<tr>
<td>$&lt; H + W &gt;$</td>
<td>$SO(3)_-$</td>
<td>$\mathbb{Z}_2$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

Of course well-known in general [Aharony, Seiberg, Tachikawa], but we derive it purely from the curve.
Generalization: Pure $\mathcal{N} = 1$ SYM $\mathfrak{su}(N)$

Match Branchcuts:
The $N$ vacua of the $\mathfrak{su}(N)$ SYM theories have curves

$$\Sigma_r : \begin{array}{c} t = \infty \\ \mathbb{Z}_N \\ t = 0 \end{array}$$

Again choosing polarizations, we can determine the 1-form symmetries in each vacuum for all global forms.

Similarly we can determine the confinement index of $\mathfrak{su}(N)$ SYM with adjoint chirals with superpotential

[Elitzur, Forge, Giveon, Intriligator, Rabinovici][Cachazo, Seiberg, Witten]

→ see [Bhardwaj, Hubner, SSN]
3. Confinement in Non-Lagrangian 4d $\mathcal{N} = 1$ Theories

The main reason for developing this class S based framework was to be able to apply it to theories without UV Lagrangians. In [Bhardwaj, Hubner, SSN] we determined a family of theories with no UV Lagrangian, which have confining vacua:

$$\mathcal{P}_{n,\alpha} = 6d \ (2,0) \ su(n) \ \text{theory on a sphere with } \alpha \ \text{irregular } \mathcal{P}_0 \ \text{punctures}$$

where $\mathcal{P}_0$ has pole of order $1 + 1/n$.

Simplest case $\alpha = 3$: $T_n$ theory with $su(n)^3$ flavor gauged:
Line operators:

\[
\mathcal{L} = \langle W_i, H_{jk} \rangle / \langle W_1 + W_2 + W_3 = 0; H_{21} + H_{32} + H_{13} = 0 \rangle.
\]

The pairing on these line operators is

\[
\langle W_i, H_{ij} \rangle = 1/n, \quad \langle W_j, H_{ij} \rangle = -1/n
\]

Again, many choices of polarization. A simple one is

\[
\Lambda = \langle W_i \rangle / \langle W_1 + W_2 + W_3 = 0 \rangle.
\]
$\mathcal{N} = 1$ Curve for $\mathcal{P}_{3,3}$

The $\mathcal{N} = 1$ curve before collision of branch-points is, with punctures rotated at $t = 0, \infty$:

The branch-cuts in $v$-curve can be collided to agree with the cut-structure on the $w$-curve.

In the electric polarization we find in this vacuum

$$\Gamma^{(1)}_{\Sigma} = \mathbb{Z}_3 \times \mathbb{Z}_3,$$

which means this vacuum is confining.
A Family of Confining Theories with no UV Lagrangian

\( \mathcal{P}_{n,n} = 6d \ (2,0) \ su(n) \) on sphere with \( n \ \mathcal{P}_0 \)s. Line ops \( \mathcal{L} = \mathcal{L}_W \times \mathcal{L}_H \):

\[
\mathcal{L}_W = \frac{\langle W_i \rangle}{\langle \sum W_i = 0 \rangle} \sim \mathbb{Z}_{n}^{n-1}, \quad \mathcal{L}_H = \frac{\langle H_{i+1,i}, H_{1,n} \rangle}{\langle \sum H_{i+1,i} + H_{1,n} = 0 \rangle} \sim \mathbb{Z}_{n}^{n-1}
\]

with pairing \( \langle W_i, H_{ij} \rangle = \frac{1}{n}, \langle W_j, H_{ij} \rangle = -\frac{1}{n} \). Again, on special locus of CB match the branch-cut structure, resulting in the vacuum with curve:

\[
\Sigma : \infty \rightarrow p_1 \rightarrow p_2 \rightarrow \ldots \rightarrow p_{n-3} \rightarrow 1 \rightarrow 0
\]

With \( \Lambda = \mathcal{L}_W \) polarization this vacuum preserves 1-form symmetry and is confining for \( n \) prime:

\[
\Gamma^{(1)}_{\Sigma} = \mathbb{Z}_{n}^{n-1}.
\]
Conclusion and Outlook

There’s a strong case for studying higher-form symmetries in string theory constructions of QFTs.

Today we focused on 4d $\mathcal{N} = 1$ from class S, and determined the 1-form symmetry in known cases such as SYM, but we also predict confinement in theories without 4d UV Lagrangian.

Future directions: compute anomalies, and TQFTs governing the IR of the confining vacua from 6d.

In higher dimensional gauge theories/SCFTs: higher-form symmetries can provide important sets of ’t Hooft anomalies in 5d and 6d theories [Benetti-Genolini, Tizzano], constrain the global, generalized and higher-group symmetry structure [Apruzzi, Bhardwaj, Oh, SSN].

Holography: using b.c. à la [Witten], the global forms of gauge groups can be determined e.g. in ABJM like theories in [Bergman, Tachikawa, Zafrir], and confinement in Klebanov-Strassler theories revisited and the IR TQFT derived from supergravity [Apruzzi, van Beest, Gould, SSN].