

# THERMODYNAMICS APPLIED TO GAME THEORY: STUDY OF THE RISK ASSOCIATED WITH COOPERATION IN THE PUBLIC GOODS GAMES

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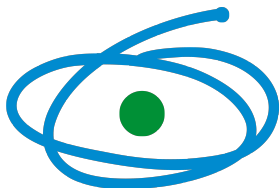
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# ACKNOWLEDGEMENT



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**CAPES**



**CNPq**  
*Conselho Nacional de Desenvolvimento  
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**Inct-SC**  
*instituto nacional de ciência e tecnologia  
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Laboratório de Modelagem de Sistemas Complexos.

Gilberto and Guilherme.

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Our paper: *Physica A* 515 (2019) 102–111

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Set up a natural connection with the thermodynamics, only with cooperation risk.

# Ideal gas

Clayperon equation

State equation: partial information.

# Ideal gas

## Equipartition law

State equation: partial information.



# Ideal gas

## Entropy

Fundamental equation: all information, but not handy.

# Ideal gas

## Thermodynamic potentials

The most handy and optimized way to keep all the system information.

# Two-player game

**Conflict** : global minimum is better  
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Card players, Paul Gézanne.

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**Values** : T : temptation, P:  
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What game to play?

$R > T > P > S$  Stag hunt

$T > R > P > S$  Prisoner's dilemma

$T > R > S > P$  Chicken (hawk-dove)

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Payo matrix:

	C	D	
C	R;P	S;T	;
D	T;S	P;P	

Solution to these dilemmas: **Nash equilibrium (local minima)!!!!**

# Two-player game

Player 1 payo :

$$G_1 = R b_1 b_2 + T (1 - b_1) b_2 + S b_1 (1 - b_2) + P (1 - b_1)(1 - b_2)$$

with  $b_i = f 0; 1g$ .

Calling:  $t = (T - P)/R$ ,  $s = (S - P)/R$  and

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$$g_1 = \frac{G_1 - P}{R} = (1 - t - s) b_1 b_2 + s b_1 + t b_2$$

$$g_2 = \frac{G_2 - P}{P} = (1 - t - s) b_1 b_2 + t b_1 + s b_2 :$$

Examples of two-player games.



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Total payo :  $g = g_1 + g_2 = 2(1 - t - s) b_1 b_2 + (t + s)(b_1 + b_2)$ .

Changing to **using variables**,  $s = f - 1; 1g$ :  $b_i = (s_i + 1)/2$ :

$$2g = \underbrace{1}_{E_2} \{ \underbrace{t}_{J} \} s = \left( \underbrace{1}_{J} \{ \underbrace{t}_{E_2} \} \right) s_1 s_2 + s_1 + s_2$$

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To compare to the Ising model

$$E_2^{(I)} = -J s_1 s_2 + H (s_1 + s_2) :$$

with  $J$  and  $H$  being independent parameters.

# N PLAYER ON A GRAPH

$$E_N = \frac{J}{\overline{hki}} \sum_{i>j=1}^N A_{i,j} s_i s_j$$

mean degree:  $\overline{hki} = \frac{1}{N} \sum_{i=1}^N hki = Ni$  with  $k_i = \sum_{j=1}^N A_{i,j}$ ,  $i$ -th node degree and no self-interaction:  $A_{i,i} = 0$ .



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Symmetric case:  $A_{i,j} = A_{j,i}$ :

$$E_N = \sum_{i=1}^N s_i @ 1 \left[ \frac{J}{2hki} \sum_{j=1}^N A_{i,j} s_j \right] A$$

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Game Hamiltonian:  $E_N = \prod_{i=1}^N s_i - \frac{J}{2\hbar k_i} \prod_{j=1}^n A_{ij} s_j$  . No way to have vanishing external field (**only one parameter**)!

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Game Hamiltonian:  $E_N = \prod_{i=1}^N s_i \left[ 1 - \frac{J}{2\hbar k_i} \prod_{j=1}^n A_{i;j} s_j \right]$  . No way to have vanishing external field (**only one parameter**)!

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**THE COOPERATION RISK.**



The risk of one agent to cooperate and the other does not.

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The risk of being a sucker.

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It

has been introduced by Nash, in a qualitative way;

remained only been intuitive, for decades;

shows up in **evolutive potential games**.

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Adding a **chemical potential** to  $U(s)$  solves the problem.

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Information about the cooperation risk is missing.

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Strategies are not correlated:  $h b_k b_{k+1} i \neq h b_k i h b_{k+1} i$ .

We define the **cooperation risk** as:

$$\frac{h H_k i}{h b_k i}$$

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For each player  $k$ :  $\langle h g_k \rangle$ .

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We define the **cooperation risk** as:

$$\frac{\langle h H_k \rangle}{\langle h b_k \rangle}$$

This is the constant we have to add up to make to game Hamiltonian equivalent to the Ising one.

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we would like to understand its role in other games!!!