THERMODYNAMICS APPLIED TO GAME THEORY: STUDY OF THE RISK ASSOCIATED WITH COOPERATION IN THE PUBLIC GOODS GAMES

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COOPERATION RISK

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Cooperation risk and Nash equilibrium: Quantitative description for realistic players



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HIGHLIGHTS

- · Mathematical description of cooperation risk to restore Nash equilibrium in potential games.
- · First- and second-order phase transitions between cooperative and other behaviors.
- · Punishments and cooperation risk in public goods games with algebraic operators.

ARTICLE INFO

ABSTRACT

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Keywords: Complex system Game theory Potential games Phase transitions The emergence of cooperation figures among the main goal of game theory in competitivcooperative environments. Potential games have long been hinder as value laternatives to study realistic player behavior. Here, we expand the potential games approach by taking into account the internet risks of cooperation. We show the Potilic Goods game reduces to a Hamiltonian with one-body operators, with the correct Nath Equilibrium as the ground has the. The inclusion of punishments to the Public Coods game reduces cooperation risks, creating two-body interactions with a rich plase diagram, in which plase transitions supergates cooperative from competitive regimes.

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- Stress this evidence in evolutive potential games
- Set up a natural connection with the thermodynamics, only with cooperation risk.



State equation: partial information.

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COOPERATION RISK



For three translational degrees of freedom, such as in an ideal monoatomic gas.

State equation: partial information.



 $f \Rightarrow$ 3 (monatomic), 5 (diatomic), 6 (polyatomic)

Fundamental equation: all information, but not handy.

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COOPERATION RISK

IDEAL GAS

THERMODYNAMIC POTENTIALS

$$T \, \mathrm{d}S = \mathrm{d}U + p \, \mathrm{d}V - \sum_{i=1}^{k} \mu_i \, \mathrm{d}N_i + \sum_{i=1}^{n} X_i \, \mathrm{d}a_i + \cdots$$
$$\mathrm{d}(TS) - S \, \mathrm{d}T = \mathrm{d}U + \mathrm{d}(pV) - V \, \mathrm{d}p - \sum_{i=1}^{k} \mu_i \, \mathrm{d}N_i + \sum_{i=1}^{n} X_i \, \mathrm{d}a_i + \cdots$$
$$\mathrm{d}(U - TS + pV) = V \, \mathrm{d}p - S \, \mathrm{d}T + \sum_{i=1}^{k} \mu_i \, \mathrm{d}N_i - \sum_{i=1}^{n} X_i \, \mathrm{d}a_i + \cdots$$
$$\mathrm{d}G = V \, \mathrm{d}p - S \, \mathrm{d}T + \sum_{i=1}^{k} \mu_i \, \mathrm{d}N_i - \sum_{i=1}^{n} X_i \, \mathrm{d}a_i + \cdots$$

The most handy and optimized way to keep all the system information.



CONFLICT : global mininum is better (cooperation), but player chose local ones (competition)

Card players, Paul Cézanne.



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punishment, *R*: reward and *P*: punishment.

What game to play?

R > T > P > S Stag hunt T > R > P > S Prisoner's dilemma T > R > S > P Chicken (hawk-dove)



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Payoff matrix:

 $\begin{array}{cc} C & D \\ C & \begin{pmatrix} R, P & S, T \\ D & T, S & P, P \end{pmatrix}, \end{array}$

Solution to these dilemmas: Nash equilibrium (local minima)!!!!



Examples of two-player games.

Player 1 payoff:

$$G_1 = R \ b_1 b_2 + T \ (1-b_1) b_2 + S \ b_1 (1-b_2) + P \ (1-b_1) (1-b_2)$$

with
$$b_i = \{0, 1\}$$
.
Calling: $t = (T - P)/R$, $s = (S - P)/R$ and $t = (T - P)/R$

$$g_1 = \frac{G_1 - P}{R} = (1 - t - s) \ b_1 b_2 + s \ b_1 + t \ b_2$$

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Total payoff:
$$g = g_1 + g_2 = 2(1 - t - s)b_1b_2 + (t + s)(b_1 + b_2).$$



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Total payoff: $g = g_1 + g_2 = 2(1 - t - s)b_1b_2 + (t + s)(b_1 + b_2)$. Changing to Ising variables, $s_i = \{-1, 1\}$: $b_i = (s_i + 1)/2$:

$$\underbrace{2g - 1 - t - s}_{-E_2} = \underbrace{(1 - t - s)}_{J} s_1 s_2 + s_1 + s_2$$

Two-player game Hamiltonian:

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Prisoner's dilemma can be both!

To compare to the Ising model

$$E_2^{(I)} = -J s_1 s_2 + H (s_1 + s_2).$$

with J and H being independent parameters.

N player on a graph

$$E_N = -rac{J}{\langle k
angle} \sum_{i>j=1}^N A_{i,j} s_i s_j - \sum_{i=1}^N s_i \; ,$$

mean degree: $\langle k \rangle = \sum_{i=1}^{N} \langle k_i / N \rangle$ with $k_i = \sum_{j=1}^{N} A_{i,j}$, *i*-th node degree and no self-interation: $A_{i,i} = 0$.

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Symmetric case: $A_{i,j} = A_{j,i}$:

$$E_{N} = \sum_{i=1}^{N} s_{i} \left(-1 - \frac{J}{2\langle k \rangle} \sum_{j=1}^{n} A_{i,j} s_{j} \right)$$
$$E_{N}^{(I)} = \sum_{i=1}^{N} s_{i} \left(H - \frac{J}{2\langle k \rangle} \sum_{j=1}^{n} A_{i,j} s_{j} \right)$$

.

Game Hamiltonian: $E_N = \sum_{i=1}^N s_i \left(-1 - \frac{J}{2\langle k \rangle} \sum_{j=1}^n A_{i,j} s_j \right)$. No way to have vanishing external field (only one parameter)!

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- A simple way to fix thing up and make the two systems equivalent is to add, by hand, something that depends on N to the game hamiltonian.
- But what is that stuff that brings us additional information of game theory? THE COOPERATION RISK.

The risk of one agent to cooperate and the other does not.

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The risk of being a sucker.

The risk of one agent to cooperate and the other does not.



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lt

- has been introduced by Nash, in a qualitative way;
- remained only been intuitive, for decades;
- shows up in evolutive potential games.

EVOLUTIVE POTENTIAL GAMES

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 with $Z = \sum_{s'} e^{-eta U(s')}$

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Nash equilibrium configurations are not the potential minima

Something is missing to make Nash configurations to correspond to potential minima.

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Adding a chemical potential μ to U(s) solves the problem.

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This is the constant we have to add up to make to game Hamiltonian equivalent to the Ising one.

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- in evolutive potential public game with punishment a cooperative transition occurs;
- independently of the game played, the cooperation risk is an one-body quantity that each player carries along (often neglected because it has not been quantified) and
- we would like to understand its role in other games!!!