Thermodynamics applied to game theory: study of the risk associated with cooperation in the Public Goods Games

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Laboratório de Modelagem de Sistemas Complexos.

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- Stress this evidence in evolutive potential games
- Set up a natural connection with the thermodynamics, only with cooperation risk.

State equation: partial information.

Ideal gas

Equipartion law

State equation: partial information.

Fundamental equation: all information, but not handy.

The most handy and optimized way to keep all the system information.

Two-player game

Conflict: global mininum is better (cooperation), but player chose local ones (competition)

Card players, Paul Cezanne.

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What game to play?

$R > T > P > S$	Stag hunt
$T > R > P > S$	Prisoner's dilemma
$T > R > S > P$	Chicken (hawk-dove)

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Payo matrix:

	C	D
C	R; P	S; T
D	T ; S	P; P

;

Solution to these dilemmas: Nash equilibrium (local minima)!!!!

Two-player game

Player 1 payoff:

$$G_1 = R\, b_1 b_2 + T\,(1 - b_1) b_2 + S\, b_1 (1 - b_2) + P\,(1 - b_1)(1 - b_2)$$

with $b_i = \{0; 1\}$.
Calling: $t = (T - P)/R$, $s = (S - P)/R$ and
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$$g_1 = \frac{G_1 - P}{R} = (1 - t - s)\, b_1 b_2 + s\, b_1 + t\, b_2$$

$$g_2 = \frac{G_2 - P}{P} = (1 - t - s)\, b_1 b_2 + t\, b_1 + s\, b_2 :$$

Examples of two-player games.

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Total payo : $g = g_1 + g_2 = 2(1 \quad t \quad s) b_1 b_2 + (t + s)(b_1 + b_2)$.

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Changing to Ising variables $s_i = \{-1; 1\}$: $b_i = (s_i + 1)/2$:

$$\underbrace{2g - 1 - t - s}_{E_2} = \underbrace{(1 - t - s)}_{J}\, s_1 s_2 + s_1 + s_2$$

Two-player game and the Ising model

Two-player game Hamiltonian:

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To compare to the Ising model

$$E_2^{(I)} = \quad J\, s_1 s_2 + H\, (s_1 + s_2) :$$

with J and H being independent parameters.

$$E_N = \frac{J}{\langle k \rangle} \sum_{i>j=1}^{N} A_{i,j} s_i s_j \qquad \sum_{i=1}^{N} s_i \,,$$

mean degree: $\langle k \rangle = \sum_{i=1}^{N} \langle k_i / N \rangle$ with $k_i = \sum_{j=1}^{N} A_{i,j}$, i-th node degree and no self-interation: $A_{i,i} = 0$.

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$$E_N = -\frac{J}{\langle k \rangle} \sum_{i>j=1}^{N} A_{i,j} s_i s_j - \sum_{i=1}^{N} s_i \, ;$$

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Symmetric case: $A_{i,j} = A_{j,i}$:

$$E_N = \sum_{i=1}^{N} s_i \left(1 - \frac{J}{2\langle k \rangle} \sum_{j=1}^{n} A_{i,j} s_j \right)$$

$$E_N^{(I)} = \sum_{i=1}^{N} s_i \left(H - \frac{J}{2\langle k \rangle} \sum_{j=1}^{n} A_{i,j} s_j \right) .$$

Game Hamiltonian: $E_N = \sum_{i=1}^{N} s_i \quad 1 \quad \frac{J}{2hki} \sum_{j=1}^{n} A_{i;j} s_j$. No way to have vanishing external field (only one parameter)!

N PLAYER ON A GRAPH

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But what is that stuff that brings us additional information of game theory? THE COOPERATION RISK.

Cooperation risk

The risk of one agent to cooperate and the other does not.

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The risk of being a sucker.

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It

- has been introduced by Nash, in a qualitative way;
- remained only been intuitive, for decades;
- shows up in evolutive potential games.

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Evolutive potential games

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Adding a chemical potential to $U(s)$ solves the problem.

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We define the **cooperation risk** as:

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This is the constant we have to add up to make to game Hamiltonian equivalent to the Ising one.

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- we would like to understand its role in other games!!!