Dynamics of the threshold q-voter model with independence in random networks

Allan Vieira*, Antonio Peralta, Raul Toral, Maxi San Miguel and Celia Anteneodo

October 20th of 2021

Summary

1. Introduction
2. Threshold q-voter model
3. Results
4. Conclusion

Introduction

- Through agent-based models one can study collective phenomena in social systems.
- Microscopic interaction rules produce macroscopic behaviors.
- To model realistic features.
- Networks.

Introduction

Scenario with only two opinions:

- Although there are some real situations that require us to contemplate several discrete and continuous opinions, the binary case constitutes the minimal setting to study opinion dynamics.

- Binary opinion dynamics:

  being favorable or unfavorable to a given proposal or buying one of two similar products that compete in a market.

The model

In a group of N agents

Be x the fraction of individuals with opinion +1 (○):

$$x = \frac{n}{N}$$

Be (1 - x) the fraction of individuals with opinion -1 (●):

$$1 - x = \frac{N - n}{N}$$

Rate equation

+1 (○) → -1 (●) $$w(n \rightarrow n-1) = nG(1-x)$$

-1 (●) → +1 (○) $$w(n \rightarrow n+1) = (N-n)G(x)$$

Rate equation

The time evolution of the fraction individuals with opinion +1 (○), in a fully connected network is:

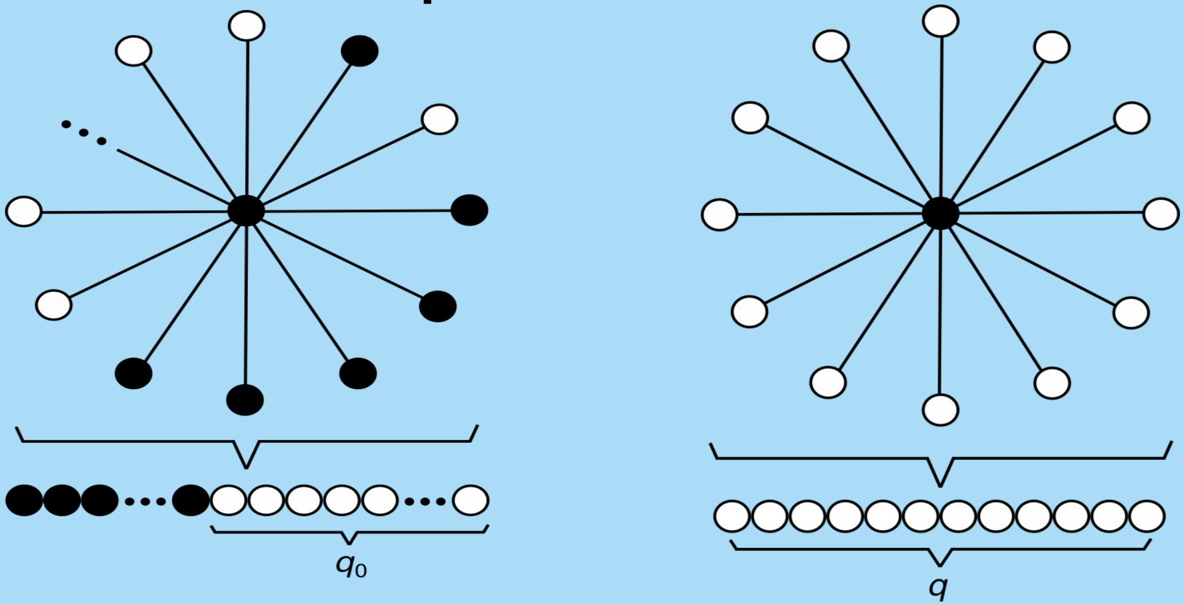
$$\frac{dx}{dt}=(1-x)G(x)-xG(1-x)$$

Threshold q-voter model

- **<u>Voter Model:</u>** One of the simplest binary opinion dynamics, where each agent can flip its opinion by imitation (or contagion) of a randomly chosen neighbor.
- **<u>q-Voter Model</u>**: a influence group of q neighbors persuades the agent. In case of unanimity in the q-panel, the agent agrees. On the other hand, with probability ε the agent also can change its opinion.

[1] Claudio Castellano, Miguel A. Muñoz, and Romualdo Pastor-Satorras, Nonlinear q-voter model, Phys. Rev. E 80, 041129 (2009)

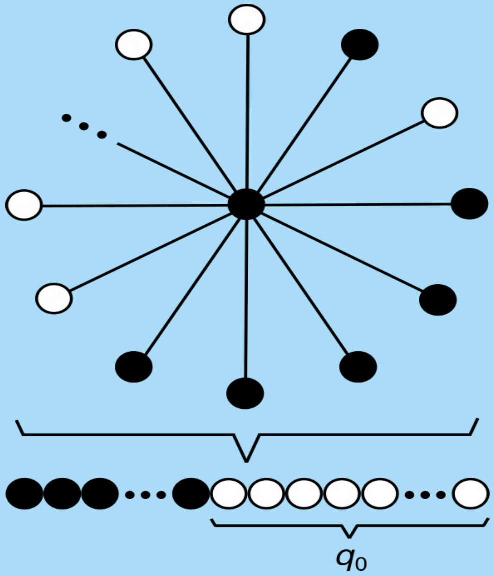
Threshold q-voter model

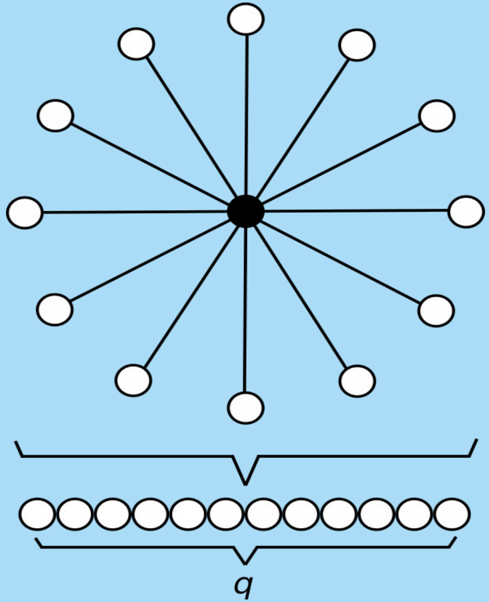


[2] P. Nyczka, K. Sznajd-Weron, Anticonformity or Independence?—Insights from Statistical Physics, J. Stat. Phys. 151, 174–202 (2013)

[3] A. R. Vieira, C. Anteneodo, Threshold q-voter model, Phys. Rev. E 97, 052106 (2018)

Threshold q-voter model





$$\sum_{j=q_0}^{q} \binom{q}{j} x^j (1-x)^{(q-j)}$$

$$x^q$$

Threshold q-voter model with independence

$$g(x,q,q_0)=\sum_{j=q_0}^{q}\binom{q}{j}x^j(1-x)^{(q-j)}$$

$$G(x)=(1-p)\,g(x,q,q_0)+p/2$$

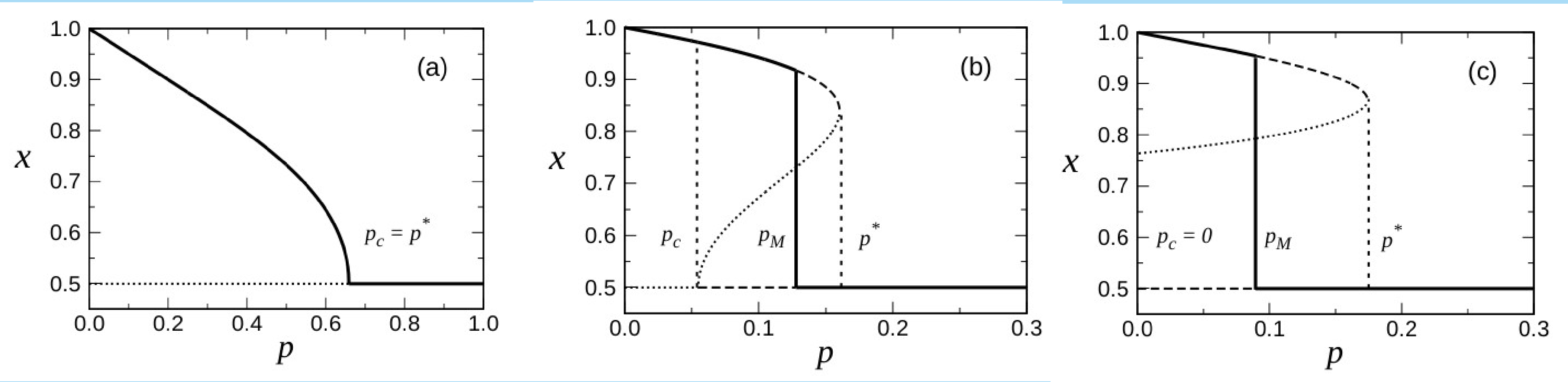
Stationary state of x: $\left(\frac{dx}{dt}=(1-x)G(x)-xG(1-x)=0\right)$

$$p=\frac{xg(1-x)-(1-x)g(x)}{xg(1-x)-(1-x)g(x)+(1/2-x)}$$

Threshold q-voter model with independence

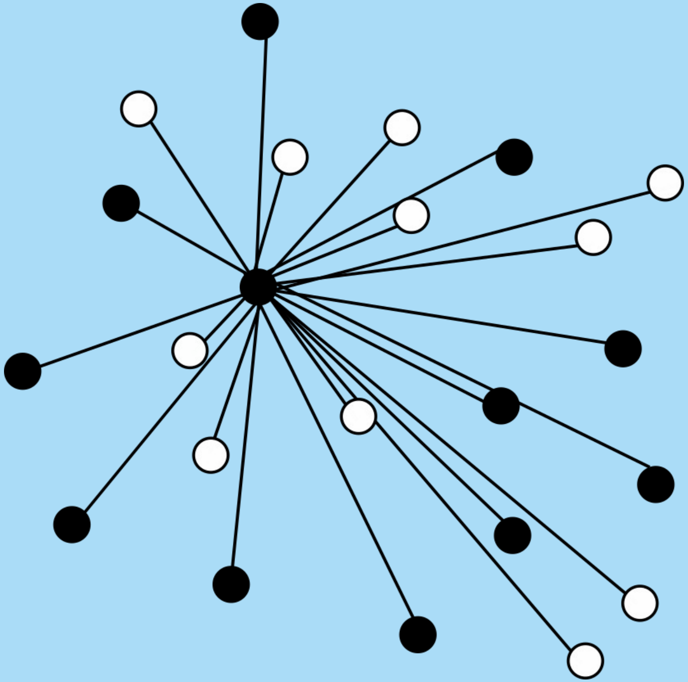
- x → ½ :

$$p_c^{-1} = 1 + \frac{2^{q-1}\Gamma(q_0+1)\Gamma(q-q_0+1)}{\Gamma(q+1)[q_0 - {}_2F_1(1, q_0-q, q_0+1, -1)]}$$



[4] A. R. Vieira, A. F. Peralta , R. Toral, M. San Miguel, C. Anteneodo,PHYSICAL REVIEW E 101, 052131 (2020)

With (without) repetition in networks



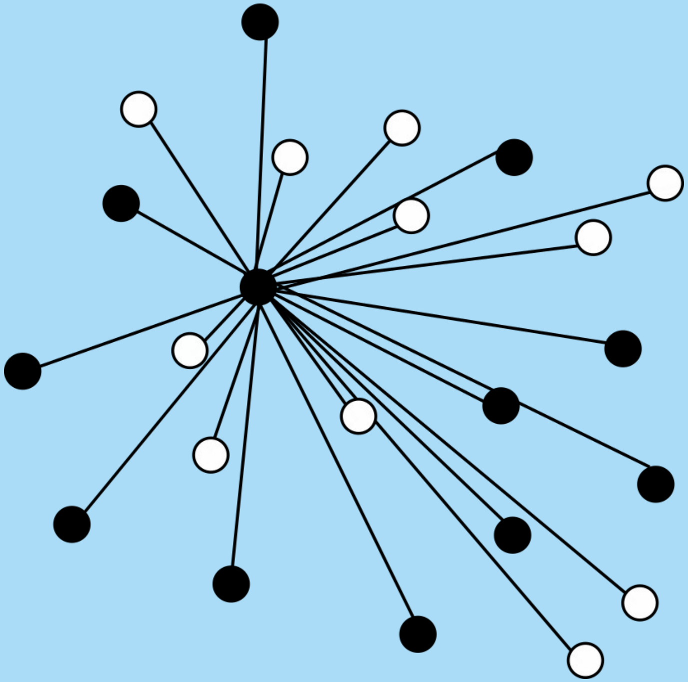
Fraction of active links (MF)

$$\rho = 2x(1-x)$$

P(○|●) = $\rho / 2(1-x)$

P(●|○) = $\rho / 2x$

With (without) repetition in networks

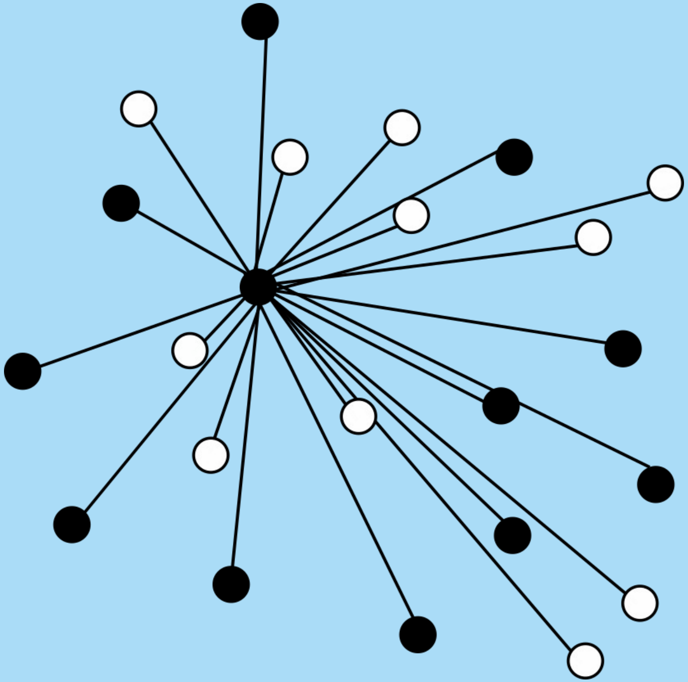


For a given site with degree k:

$$F(\ell;k,q,q_0,p) = (1-p)f(\ell;k,q,q_0) + p/2.$$

$$f(\ell;k,q,q_0) \equiv \begin{cases} \sum_{j=q_0}^{q} \binom{q}{j}\binom{k-q}{\ell-j} \Big/ \binom{k}{\ell} \\ \sum_{j=q_0}^{q} \binom{q}{j}\left(\frac{\ell}{k}\right)^{j}\left(1-\frac{\ell}{k}\right)^{q-j} \end{cases}$$

With (without) repetition in networks



For a given site with degree k:

$$\frac{dx}{dt} = -\sum_k \sum_{i=\oplus,\ominus} \frac{k}{\mu} P(k)\, S_i\, P_i \langle F(\ell; k, q, q_0, p)\rangle_{\rho_i}$$

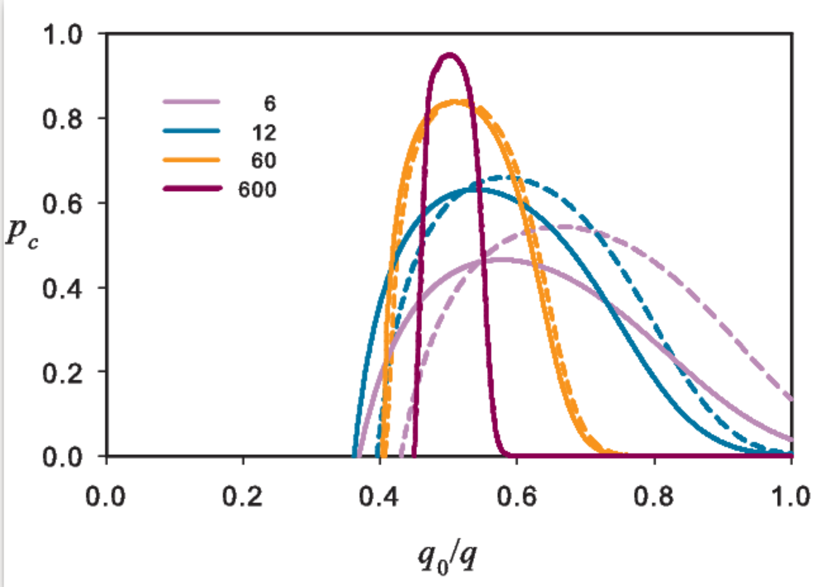
$$\frac{d\rho}{dt} = \frac{2}{\mu} \sum_k \sum_{i=\oplus,\ominus} P(k)\, P_i \langle (k - 2\ell) F(\ell; k, q, q_0, p)\rangle_{\rho_i}$$

Pair Approximation in Random Regular Networks (RRN)

- P(k) = δ(k – *μ*)

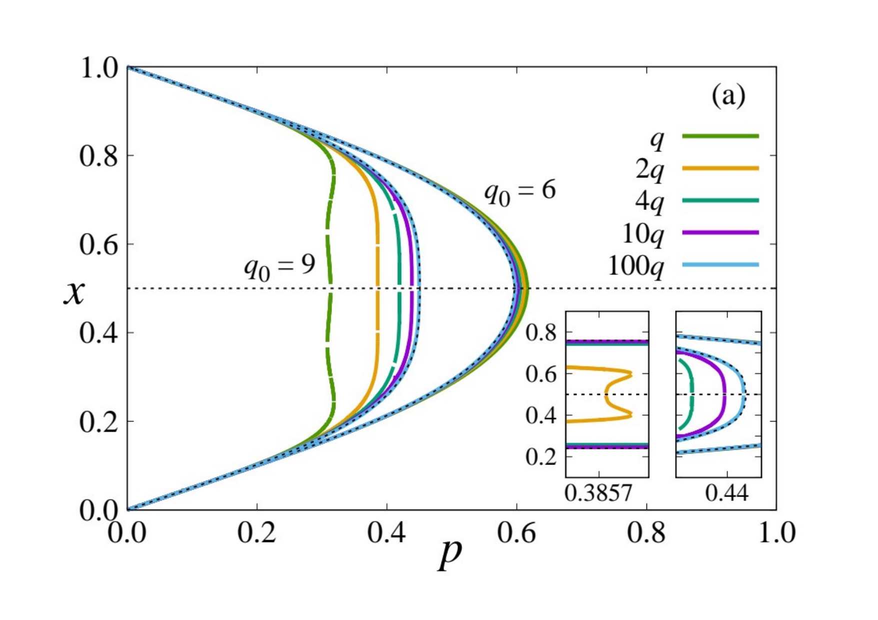
Without repetition

$$p_c^{-1} = 1 + \frac{2^{q-1}\left(\frac{\mu}{\mu-2}\right)^{q_0}\left(\frac{\mu-1}{\mu}\right)^{q}\Gamma(q_0+1)\Gamma(q-q_0+1)}{\Gamma(q+1)[q_0 - {}_2F_1(1, q_0-q, q_0+1, 2/\mu-1)]}$$

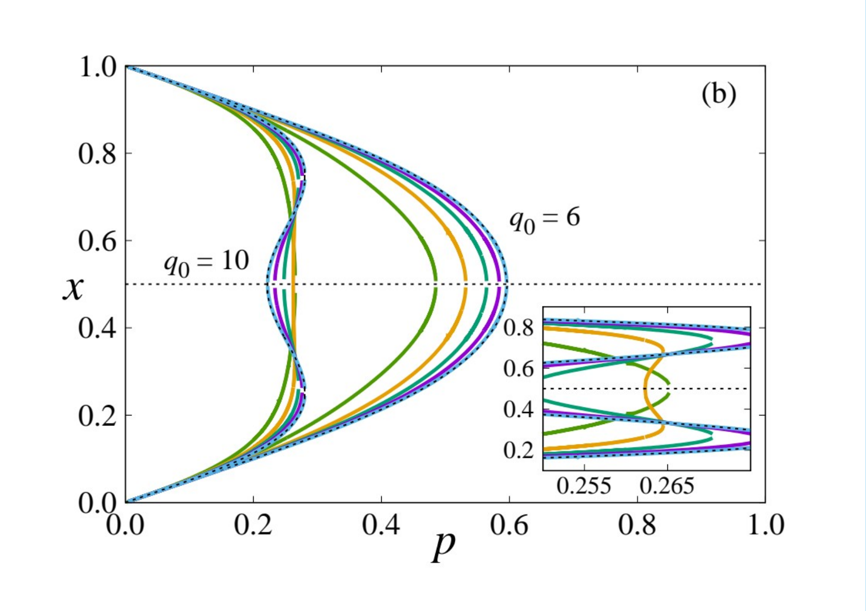


Pair Approximation in Random Regular Networks (RRN)

q = 12



Without repetition



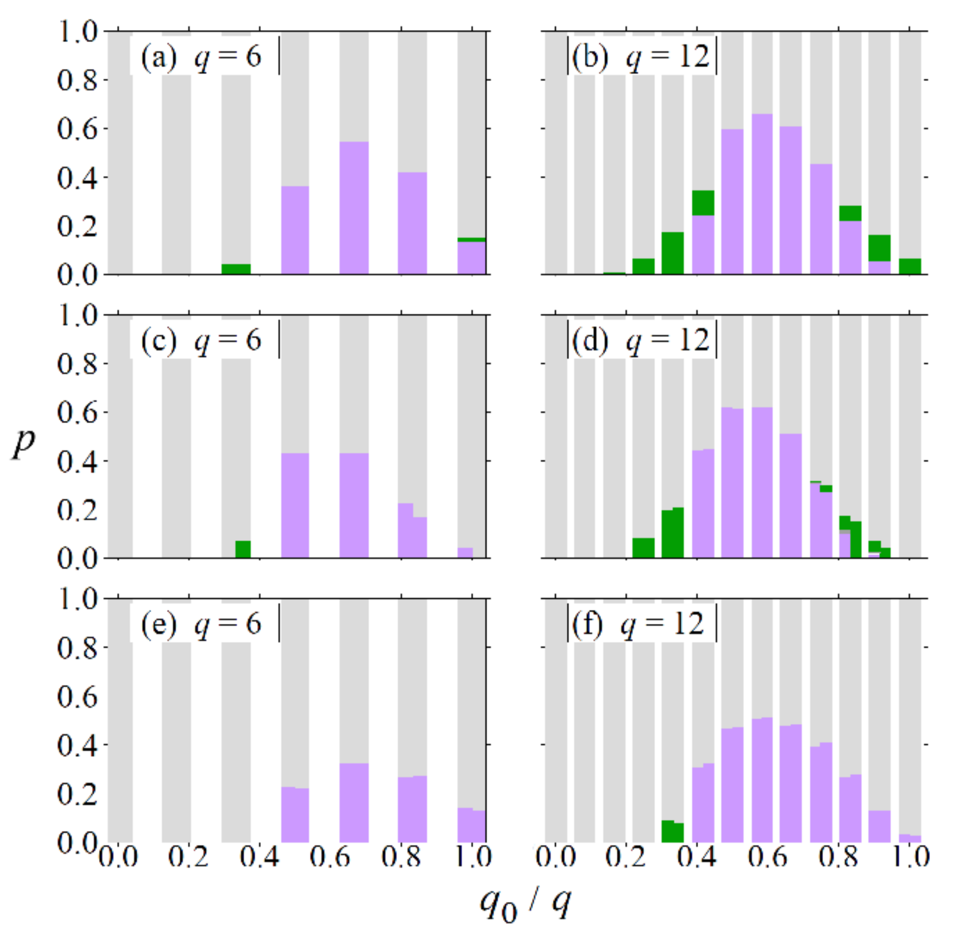
Allowing repetition

Mean Field - RRN

MF →

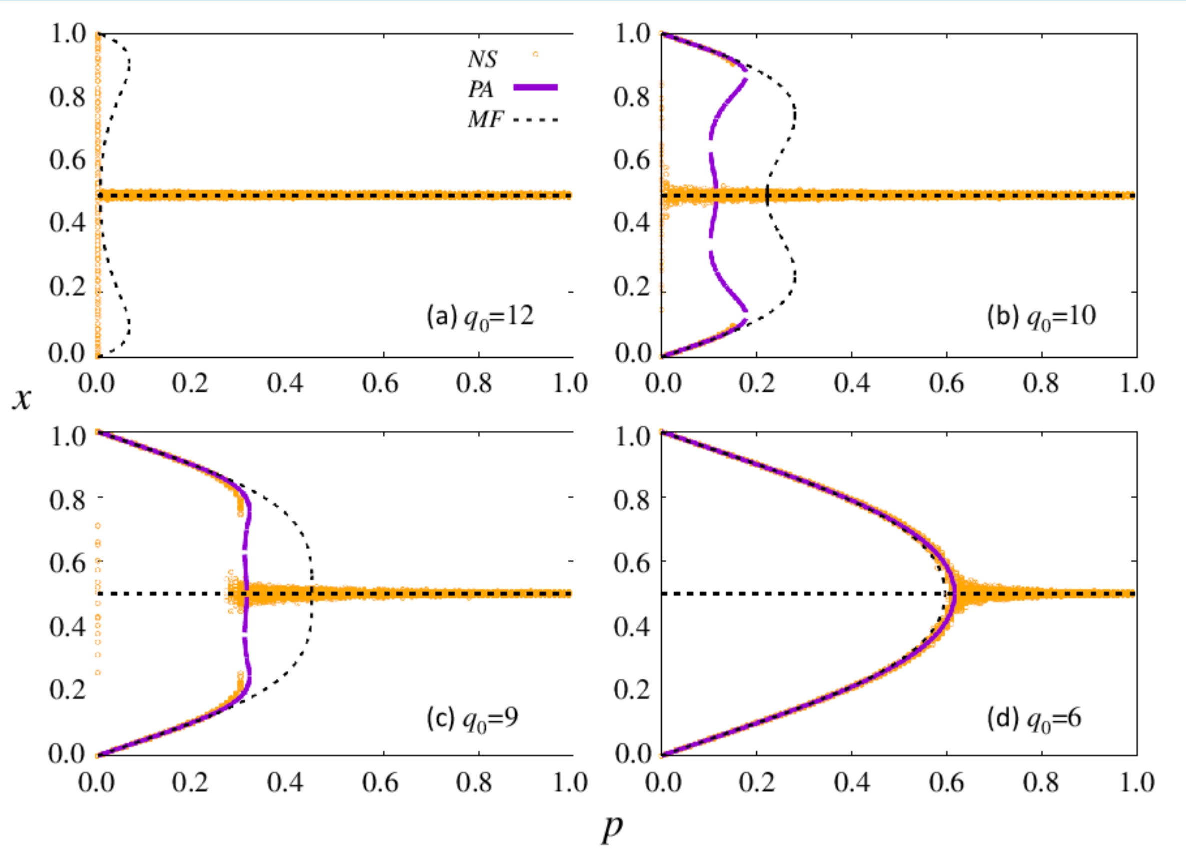
Without → repetition

Allowing → repetition



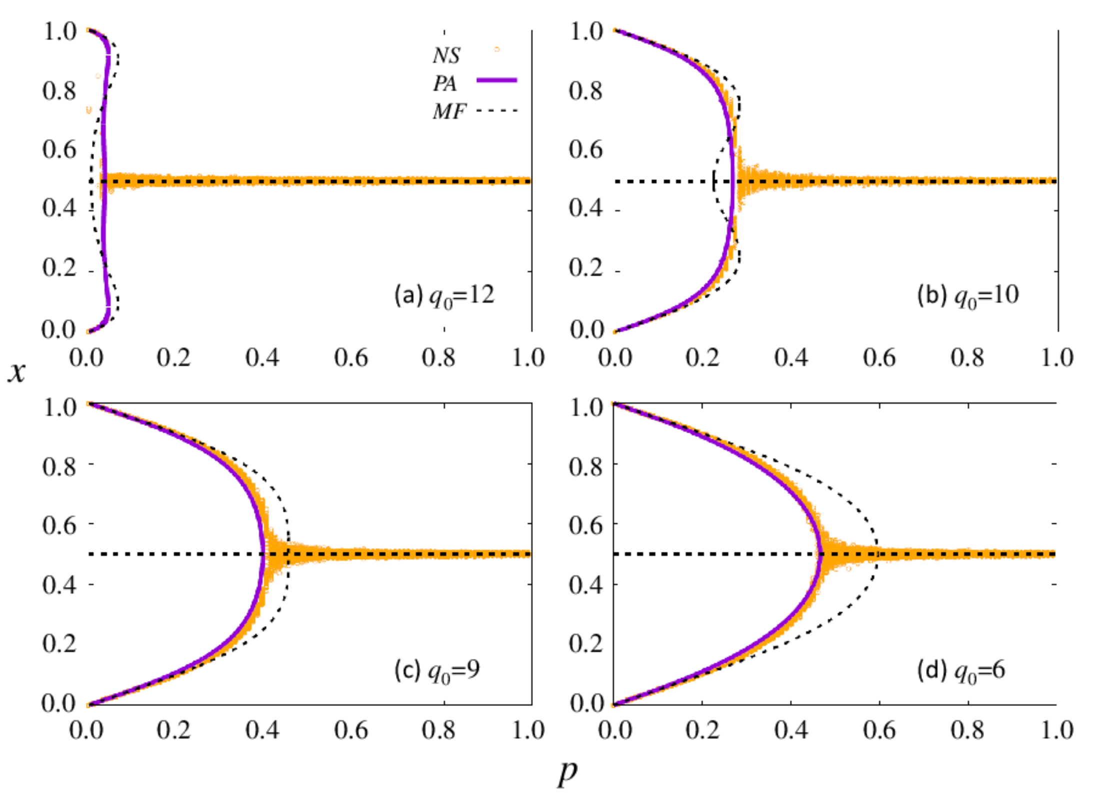
Analytical and numerical results

- RRN (without repetition)



Analytical and numerical results

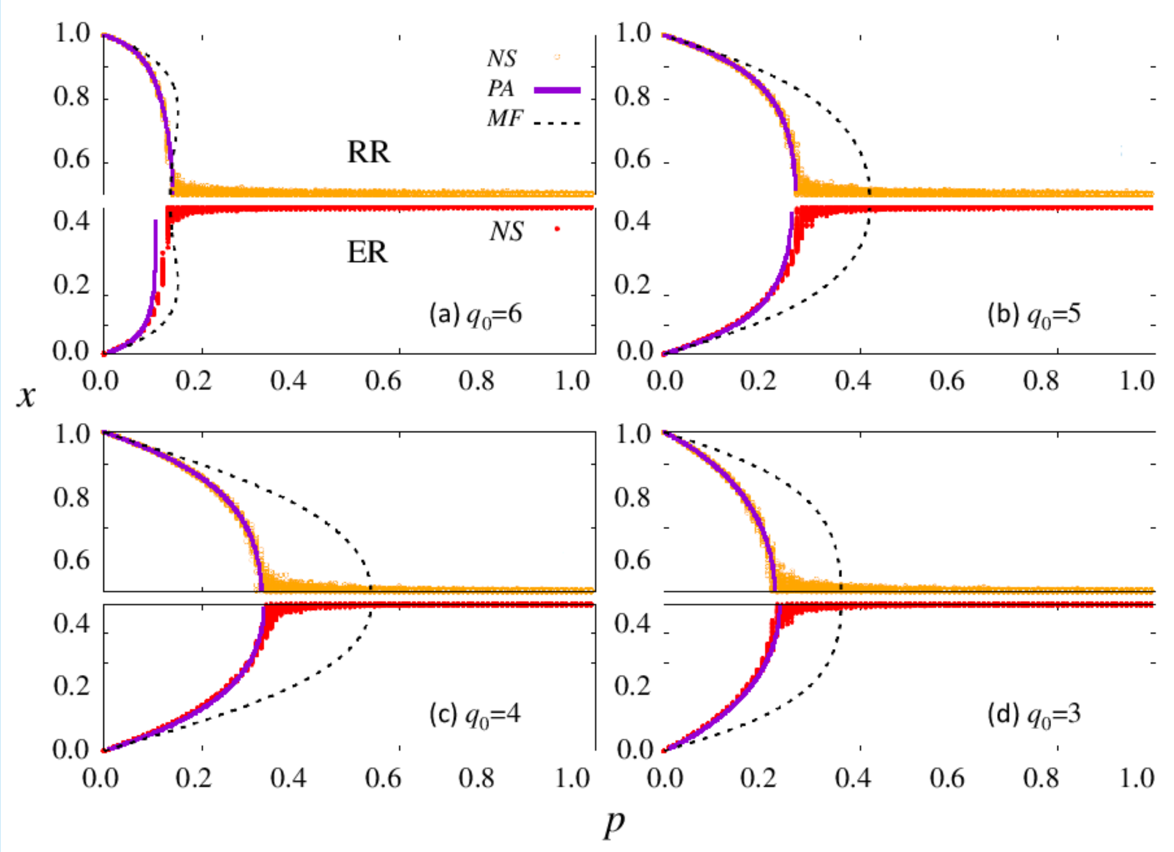
- RRN (allowing repetition)



Analytical and numerical results

- Erdos-Renyi

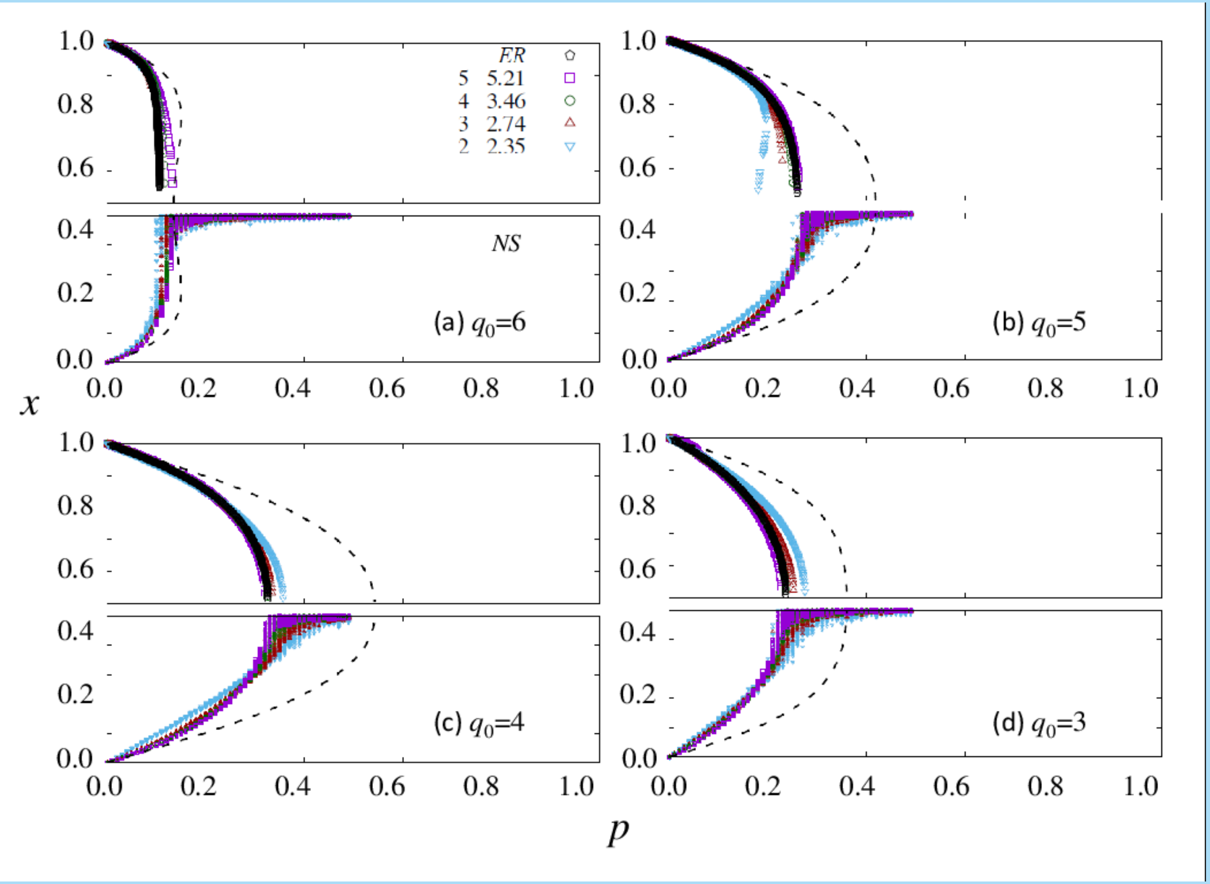
  (allowing repetition)



Analytical and numerical results

- $P(k) \sim k^{-\alpha}$

  (allowing repetition)



Conclusions

- The results for Pair Approximation in RRN, Erdos-Renyi networks and for power-laws distributions show a good agreement with numerical simulations.
- RRN are more suitable for the dynamics of this model.
- In RRN, allowing repetition makes discontinuous transitions less frequent.

Conclusions

- Also in RRN: the results for the model without repetition show a bigger disagreement compared with mean field for $q_0 \sim q$; mean while, allowing repetition, the disagreement is more pronounced for $q_0 \sim q/2$. Moreover, in both cases were observed a changing in the feature of the transition comparatively with mean field results.

References

- [1] Claudio Castellano, Miguel A. Muñoz, and Romualdo Pastor-Satorras, Nonlinear q-voter model, Phys. Rev. E 80, 041129 (2009)
- [2] P. Nyczka, K. Sznajd-Weron, Anticonformity or Independence?—Insights from Statistical Physics, J. Stat. Phys. 151, 174–202 (2013)
- [3] A. R. Vieira, C. Anteneodo, Threshold q-voter model, Phys. Rev. E 97, 052106 (2018)
- [4] A. R. Vieira, A. F. Peralta , R. Toral, M. San Miguel, C. Anteneodo,Phys. Rev. E 101, 052131 (2020)