Coevolution dynamics of opinion and social network

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What is Co-evolution?

**Node states**
- Blue
- Red

**Link states**
- Existence, Weight
- No link

**Opinion dynamics**
- Agents (nodes) in a network of interactions
- State of the node: opinion
- Characteristics of the link:
  - Existence, Weight
  - State of link: type of interaction (homophily...)

**Step I**
- No state of links
- Links are not persistent

**Step II**
- Fixed network

**Step III**
- Coupled dynamics of node states, link states and network topology

*Coupled dynamics of node states and network topology*

*Coupled dynamics of node states and link states*
A. Carro et al, New Journal of Physics 18, 113056 (2016)
**Dynamics of Networks:**

1. Dynamics **OF** network formation: Structure created by individual choices/actions
2. Dynamics **ON** the network: Actions of individuals constrained by the social network
3. **Co-evolution of agents and network:**
   
   *Circumstances make men as much as men make circumstances*

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**Final Goals:**

Understanding dynamical processes of group formation / social differentiation

**Opinion dynamics:** Emergence of POLARIZATION and ECO-CHAMBERS

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**Early papers on co-evolution:**


Coevolving voter model: Non-persisting ties

Dynamics on the network coupled with dynamics of the network

Social Imitation

Breaking and...

..establishing ties

Voter Model

Rewiring

Imitating vs Choosing neighbors
Co-evolving Voter Model


**Imitation**

Choosing neighbors

Network Fragmentation Transition

**Fragmentation** due to **competition** of time scales:
- evolution of the network (link dynamics)
- evolution on the network (node state dynamics)

**Critical value of plasticity** $p_c$
What is Co-evolution?

Opinion dynamics

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Step I
No state of links
Links are not persistent
Coupled dynamics of node states and network topology

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Step III
Coupled dynamics of node states, link states and network topology
Dynamics towards satisfaction:

i) Change of link state with probability $p$ (local)

ii) Change of node state with probability $1-p$ (local)

iii) Link rewiring with prob. $r$

once pair $e$ has been selected for link update (nonlocal)
Parameters: $N, \mu, r, p$

Variables: Link densities $\{\rho_a, \rho_b, \rho_c, \rho_d, \rho_e, \rho_f\}$

Absorbing state: No unsatisfying pairs $\rho_a = \rho_c = \rho_e = 0$

Absorbing transition: Dynamically active state $\iff$ Absorbing frozen configuration

Rate equation analysis $N \to \infty$

Dynamically active state: Non vanishing $\rho_a, \rho_b, \rho_c, \rho_d, \rho_e, \rho_f$ as functions of $\rho_e$

\[ r = 1 \quad \rho_a = \rho_c = \rho_f = 0. \quad \text{No negative links} \]

Absorbing frozen state:: $\rho_a = \rho_c = \rho_e = 0$  \quad ($\rho_b, \rho_d, \rho_f$) arbitrary

Absorbing Transition line:

\[ p_c(\mu, r) = 1 - \frac{3 - 2r}{(2 - r)(\mu - 1)}. \]

$\rho_f$ at criticality \quad $\rho_f^c(r) = \frac{1 - r}{2 - r}$. 

Random network mean degree $\mu$
Absorbing Transition

$p_c(\mu, r) = 1 - \frac{3 - 2r}{(2 - r)(\mu - 1)}$.

theory simulation
Active phase:

Finite size fluctuations take the system to an absorbing state

Adaptive network (rewiring):

Reduces exponentially the lifetime of the active unsatisfying state
For finite N absorbing satisfying state is always reached

\[ \rho_a = \rho_c = \rho_e = 0 \]

\[ \rho_f = 0, \rho_b \neq 0, \rho_d \neq 0 \]

\[ p = 0.80, \mu = 12, r = 1 \]
For finite $N$ absorbing satisfying state is reached

$$\rho_a = \rho_c = \rho_e = 0$$

**FRAGMENTED:**

$$\rho_f = 0, \; \rho_b \neq 0, \; \rho_d \neq 0$$

**CONNECTED:**

i) Consensus:

$$\rho_f = 0, \; \rho_b = 0 \text{ or } \rho_d = 0$$

$\rho = 0.30, \mu = 12, r = 1$

Snapshot of dynamically active state

Finite size absorbing configuration
For finite $N$ absorbing satisfying state is reached

$$\rho_a = \rho_c = \rho_e = 0$$

**FRAGMENTED:**

$$\rho_f = 0, \rho_b \neq 0, \rho_d \neq 0$$

**CONNECTED:**

i) Consensus:

$$\rho_f = 0, \rho_b = 0 \text{ or } \rho_d = 0$$

ii) Two-group:

$$\rho_b \neq 0, \rho_d \neq 0, \rho_f \neq 0$$

iii) Split configurations:

**TWO-GROUP**

$$p = 0.90, \mu = 12, r = 1$$

**SPLIT-CONFIGURATIONS**

$$p = 0.30, \mu = 3, r = 1$$
Topological transitions

The case $r=1$

- Unsatisfying
- No change of link state, only link rewiring
- No negative links in active state
- $\rho_f = 0$ in active state and at criticality

*Fragmentation as a manifestation of criticality*

- Fragmentation
- $\rho_f = 0$
- Consensus

- Two-group
- $\rho_f \neq 0$
- Split

Active as $N \to \infty$
Topological transitions $r<1$:

- **Consensus**: Active as $N \rightarrow \infty$.
- **Two-group**: Active as $N \rightarrow \infty$.
- **Split**: Active as $N \rightarrow \infty$.

- **No fragmentation found**
- **Consensus phase disappears as** $r \rightarrow 0$.
- **Finite size topological transition**: $\mu_{\text{split}}$ decreases with $r$.

$r=0.9$

$p=0.9$

$r=0, \mu_{\text{split}} \sim 22$
**DISCUSSION**

Active (unsatisfying) - Frozen (satisfying) transition

\[ p = \text{rate of change of link state vs. node state} \]

*global unsatisfaction in spite of local mechanism of convergence towards satisfaction* for small \( p \)

\[ r = \text{rewiring} \quad \text{Exponential reduction of lifetime of unsatisfying state} \]

**Final Configurations:**

- **SOCIAL POLARIZATION** and ECHO-CHAMBERS: Two-Group and Fragmentation

  - **Globalization** (large connectivity) leads to Two-Group \( \mu > \mu_{\text{split}} \)
    Smaller connectivity needed in adaptive networks (rewiring)

  - **Negative (heterophilic) interactions promote polarization**

  - **Fragmentation** is a manifestation of criticality

- **CONSENSUS only possible with rewiring:**
  Needs choosing positive satisfactory relations opting out of disagreement with our positive relations:
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