Coevolution dynamics
of opinion and social network

Meghdad Saeedian

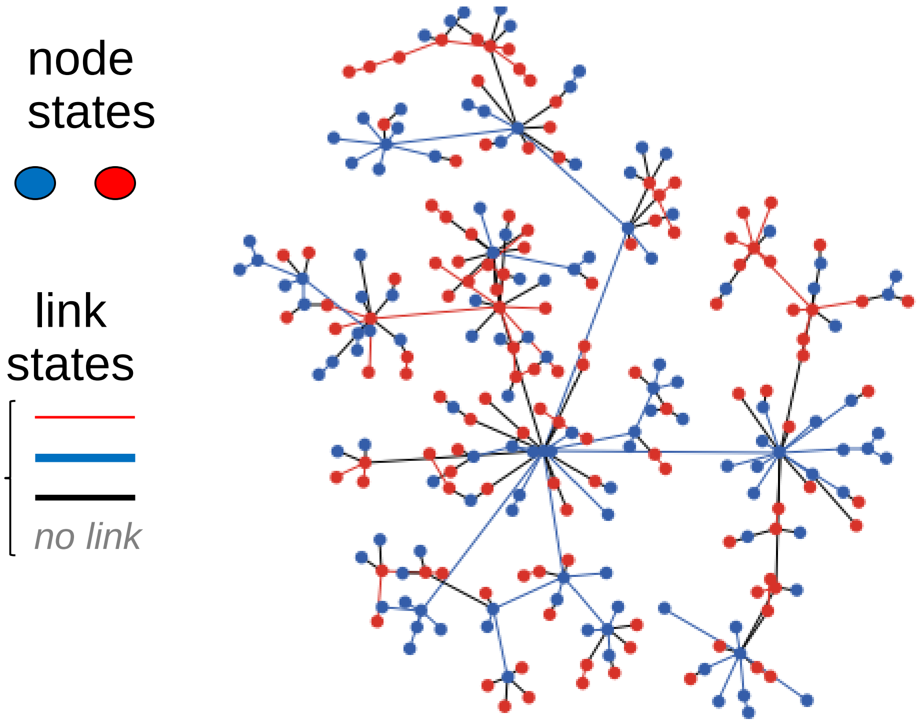
Raúl Toral

Maxi San Miguel

maxi@ifisc.uib-csic.es

M. Saeedian et al, New J. Phys. ***22****, 113001 (2020); Sci. Rep.* ***9****, 1 (2019)*

What is Co-evolution?



Opinion dynamics

Agents (nodes) in a network of interactions

State of the node: opinion

Characteristics of the link:
Existence, Weight,
State of link: type of interaction (homophily...)

Step I

No state of links
Links are not persistent

Coupled dynamics of node states and network topology

F. Vázquez, et al, Phys. Rev. Lett. 100, 108702 (2008)

Step II

Fixed network

Coupled dynamics of node states and link states

*A. Carro et al, New Journal of Physics **18**, 113056 (2016)*

Step III

Coupled dynamics of node states, link states and network topology

CO-EVOLUTION: Step I

Dynamics of Networks:

1. Dynamics **OF** network formation: Structure created by individual choices/actions

2. Dynamics **ON** the network: Actions of individuals constrained by the social network

3. Co-evolution of agents and network :
Circumstances make men as much as men make circumstances

..new research agenda in which the structure of the network is no longer a given but a variable.....explore how a social structure might evolve in tandem with the collective action it makes possible (Macy, Am. J. Soc. 97, 808 (1991))

Final Goals:

Understanding dynamical processes of group formation / social differentiation

Opinion dynamics: Emergence of POLARIZATION and ECO-CHAMBERS

Early papers on co-evolution:

M. Zimmerman, V. M. Eguíluz and M. San Miguel Lecture Notes in Economics and Mathematical Systems N°503,73 (2001)

M. Zimmerman, V. M. Eguíluz and M. San Miguel, Phys. Rev. E. 69, 065102-6 (2004)

F. Vázquez, et al, Phys. Rev. Lett. 100, 108702 (2008)

Coevolving voter model: Non-persisting ties

Dynamics **on** the network coupled with dynamics **of** the network

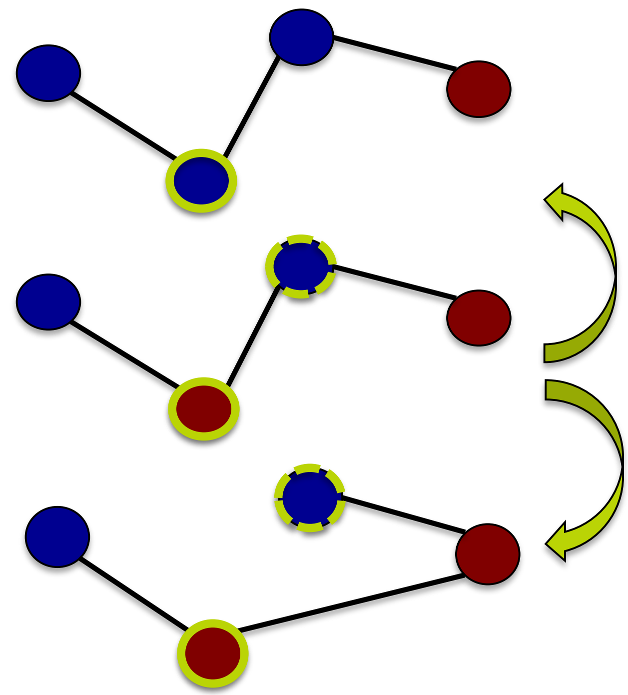
Social Imitation

Voter Model

Breaking and..

..establishing ties

Rewiring



changing state

rewiring

Imitating vs Choosing neighbors

Co-evolving Voter Model

F. Vázquez, et al, Phys. Rev. Lett. 100, 108702 (2008)

Imitation

Choosing neighbors

Network Fragmentation Transition

Fragmentation due to

competition of time scales:

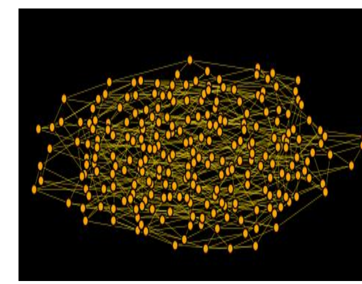
- evolution **of** the network

 (link dynamics)

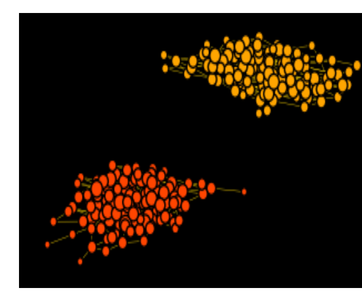
- evolution **on** the network

(node state dynamics)

Critical value of plasticity p_c



Transition



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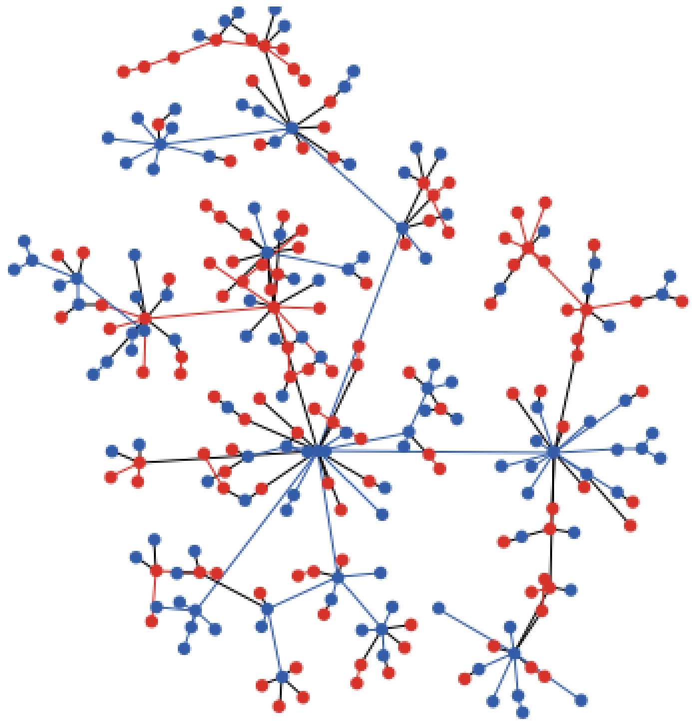
What is Co-evolution?

node states

link states

no link



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Step II Fixed network

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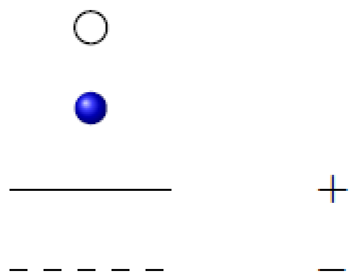
Step III *Coupled dynamics of node states, link states and network topology*

Step III: general co-evolution model

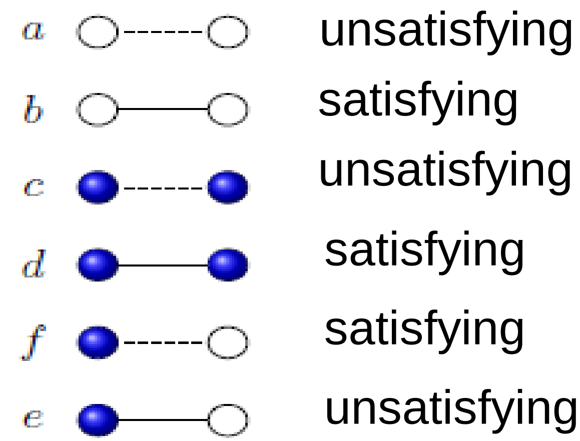
*M. Saeedian et al, New J. Phys. **22**, 113001 (2020)*
*Sci. Rep. **9**, 1 (2019)*

Node states

Link states



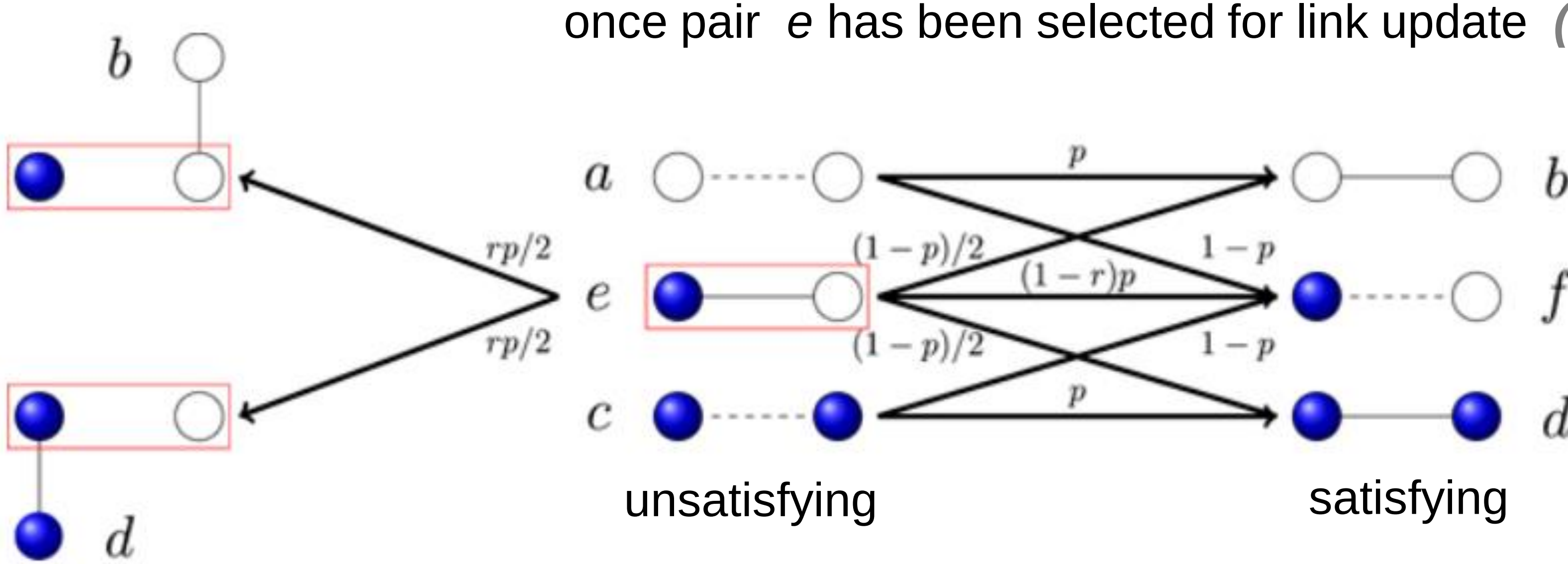
Pair relations



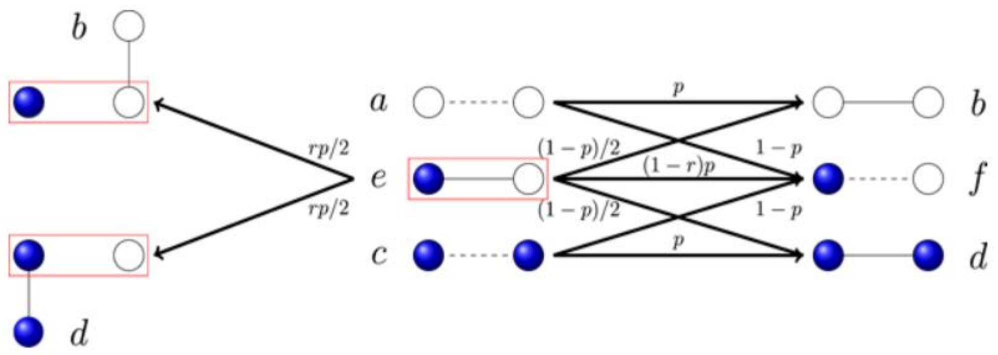
Dynamics towards satisfaction: i) Change of link state with probability *p* ***(local)***

ii) Change of node state with probability *1-p* ***(local)***

iii) Link rewiring with prob. r
once pair *e* has been selected for link update ***(nonlocal)***



Absorbing Transition



Random network mean degree μ

Parameters: N, μ , r, p

Variables: Link densities $\{\rho_a, \rho_b, \rho_c, \rho_d, \rho_e, \rho_f\}$

Absorbing state: No unsatisfying pairs $\rho_a = \rho_c = \rho_e = 0$

Absorbing transition: Dynamically active state $\Leftrightarrow$ Absorbing frozen configuration

Rate equation analysis N → ∞

Dynamically active state: Non vanishing $\rho_a, \rho_b, \rho_c, \rho_d, \rho_e, \rho_f$ *as functions of* ρ_e

r=1 $\Rightarrow$ $\rho_a = \rho_c = \rho_f = 0$. No negative links

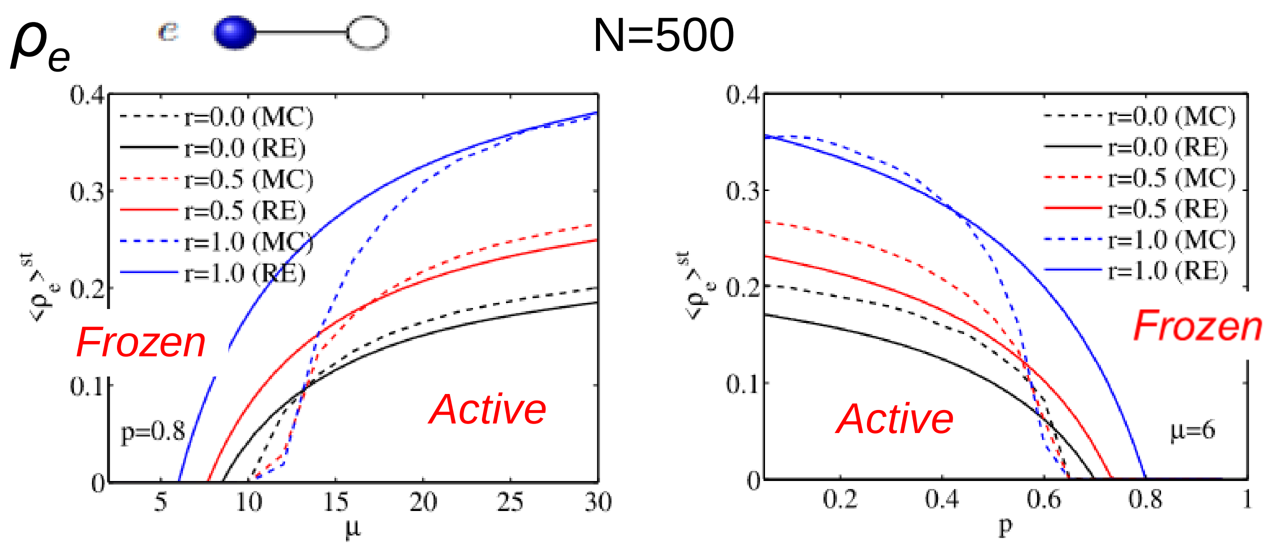
Absorbing frozen state:: $\rho_a = \rho_c = \rho_e = 0$ (ρ_b, ρ_d, ρ_f) arbitrary

Absorbing Transition line:

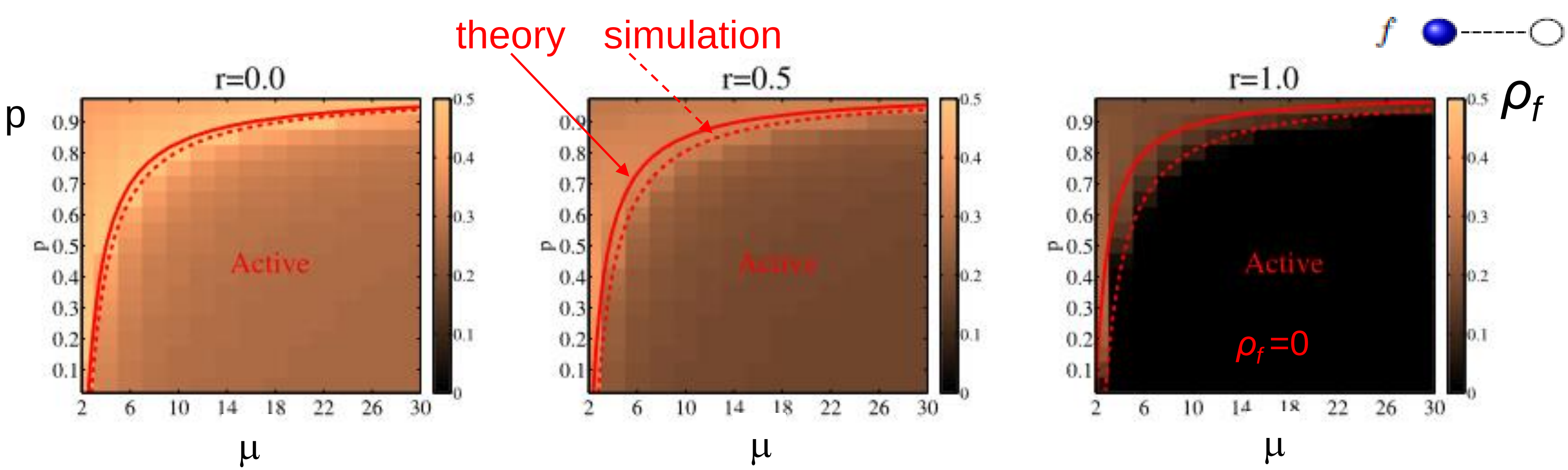
$$p_c(\mu, r) = 1 - \frac{3-2r}{(2-r)(\mu-1)}.$$

ρ_f at criticality $\rho_f^c(r) = \dfrac{1-r}{2-r}.$

Absorbing Transition

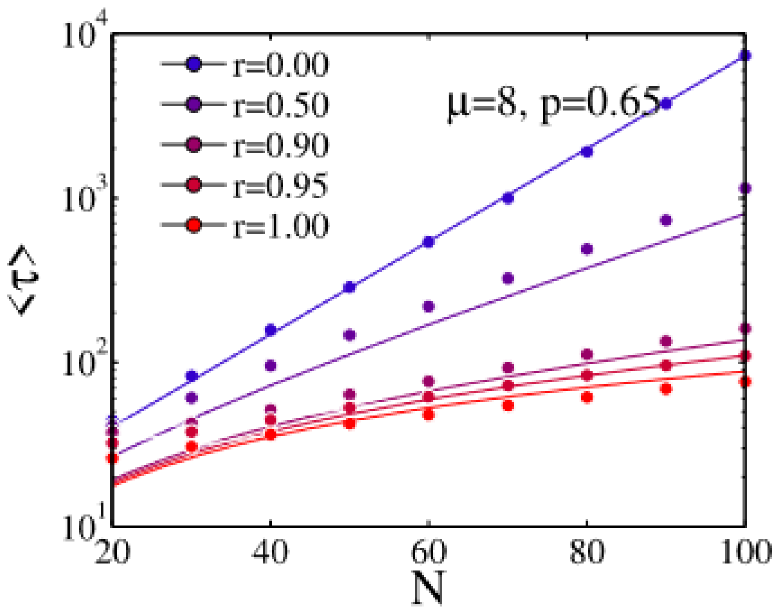


$$p_c(\mu, r) = 1 - \frac{3 - 2r}{(2 - r)(\mu - 1)}.$$



Active phase:

Finite size fluctuations take the system to an absorbing state



r=0 $\langle\tau\rangle \sim e^{\alpha N}$;

$$\langle\tau\rangle = \alpha_1(\alpha_2 N)^r \, e^{(1-r)\alpha_3 N}$$

r=1 $\langle\tau\rangle \sim \beta N$

Adaptive network (rewiring):

Reduces exponentially the lifetime of the active unsatisfying state

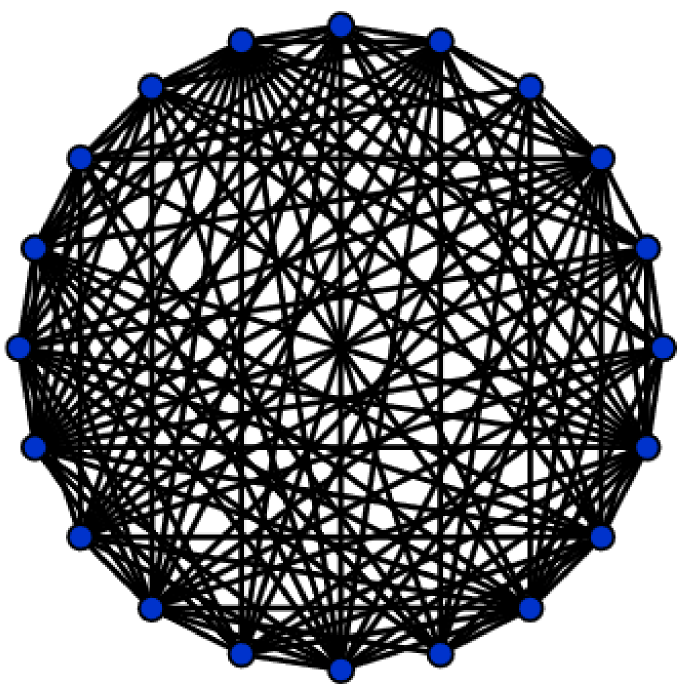
Topological transitions

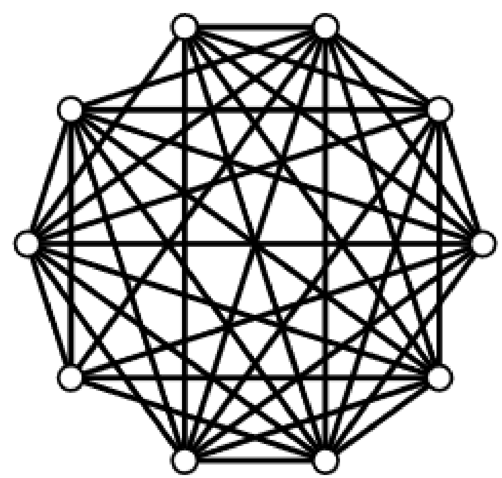
For finite N absorbing satisfying state is always reached

$$\rho_a = \rho_c = \rho_e = 0$$

FRAGMENTED $\rho_f = 0, \rho_b \neq 0, \rho_d \neq 0$

- a unsatisfying
- b satisfying
- c unsatisfying
- d satisfying
- f satisfying
- e unsatisfying



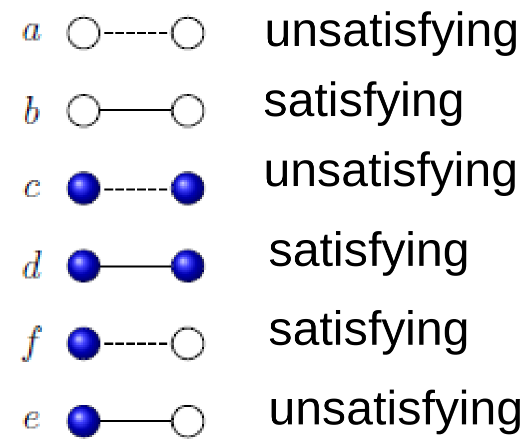


$p = 0.80, \mu = 12, r = 1$

Topological transitions

For finite N absorbing satisfying state is reached

$$\rho_a = \rho_c = \rho_e = 0$$

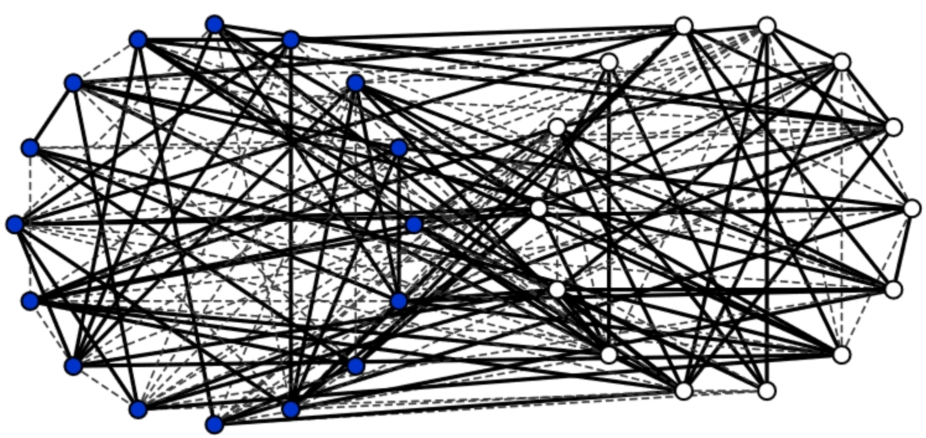


FRAGMENTED: $\rho_f = 0, \rho_b \neq 0, \rho_d \neq 0$

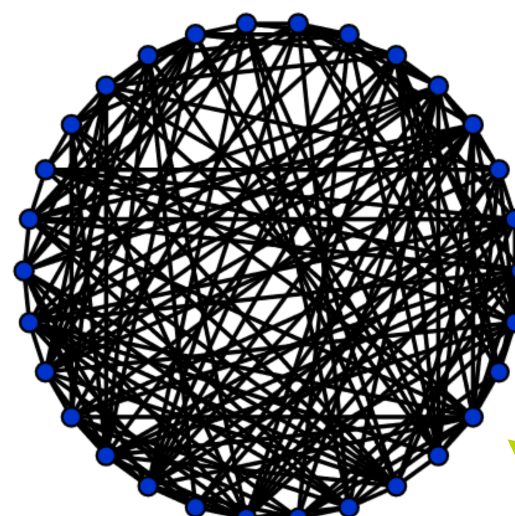
CONNECTED:

i) Consensus: $\rho_f = 0, \quad \rho_b = 0 \text{ or } \rho_d = 0$

CONSENSUS



$p = 0.30, \mu = 12, r = 1$

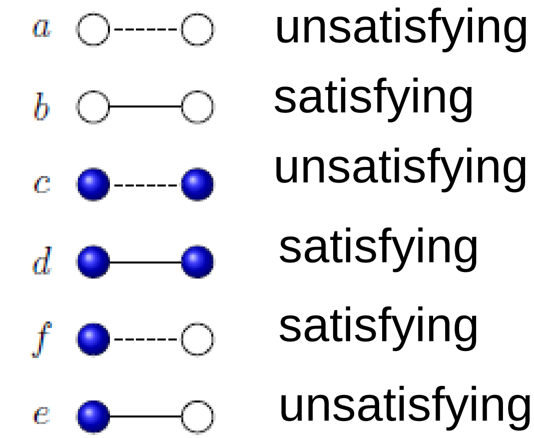


Snapshot of dynamically active state

Finite size absorbing configuration

For finite N absorbing satisfying state is reached

$$\rho_a = \rho_c = \rho_e = 0$$



FRAGMENTED: $\rho_f = 0, \rho_b \neq 0, \rho_d \neq 0$

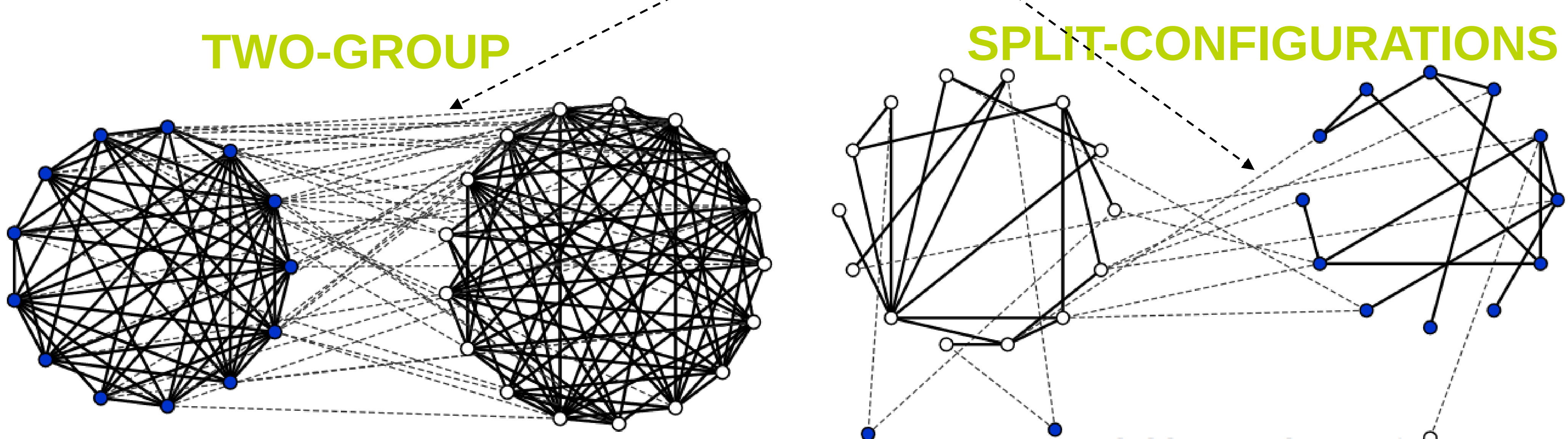
CONNECTED:

i) Consensus: $\rho_f = 0, \quad \rho_b = 0 \text{ or } \rho_d = 0$

ii) Two-group:

$\rho_b \neq 0, \rho_d \neq 0, \rho_f \neq 0.$

iii) Split configurations:



$p = 0.90, \mu = 12, r = 1$ $\qquad$ $p = 0.30, \mu = 3, r = 1$

The case r=1

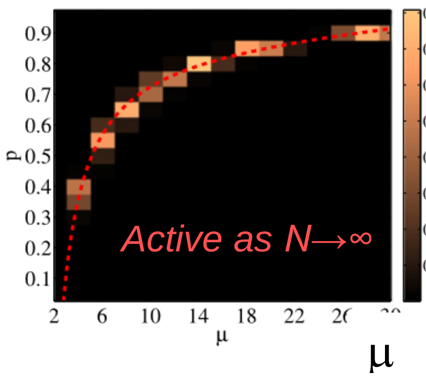
e ●—○ unsatisfying

No change of link state, only link rewiring

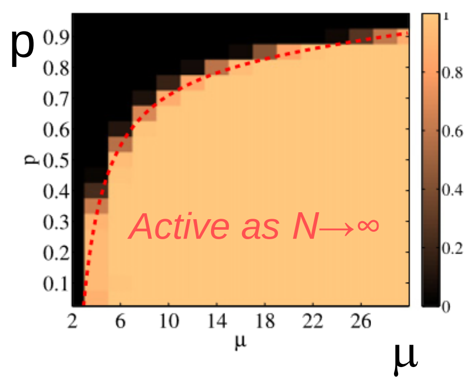
- No negative links in active state
- $\rho_f = 0$ in active state and at criticality

f ●------○

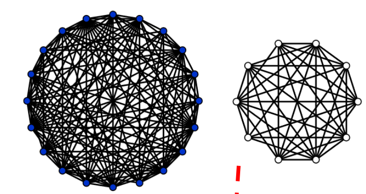
Fragmentation $\rho_f = 0$



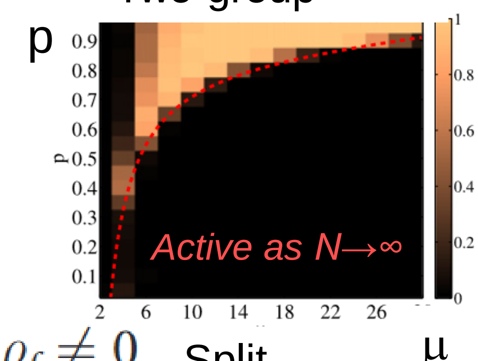
Consensus

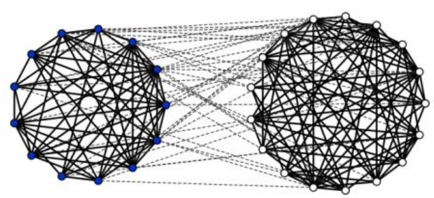


Fragmentation as a manifestation of criticality

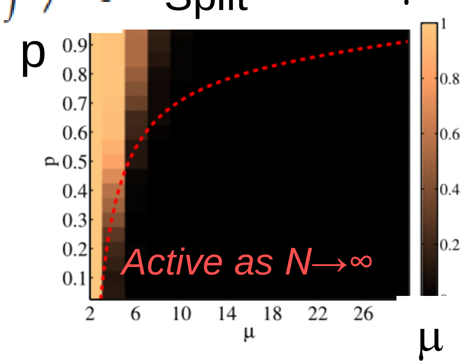


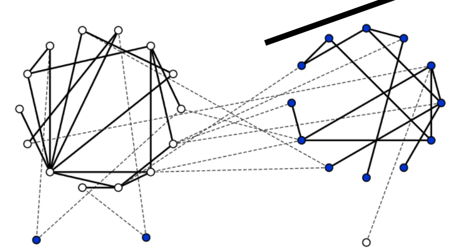
Two-group

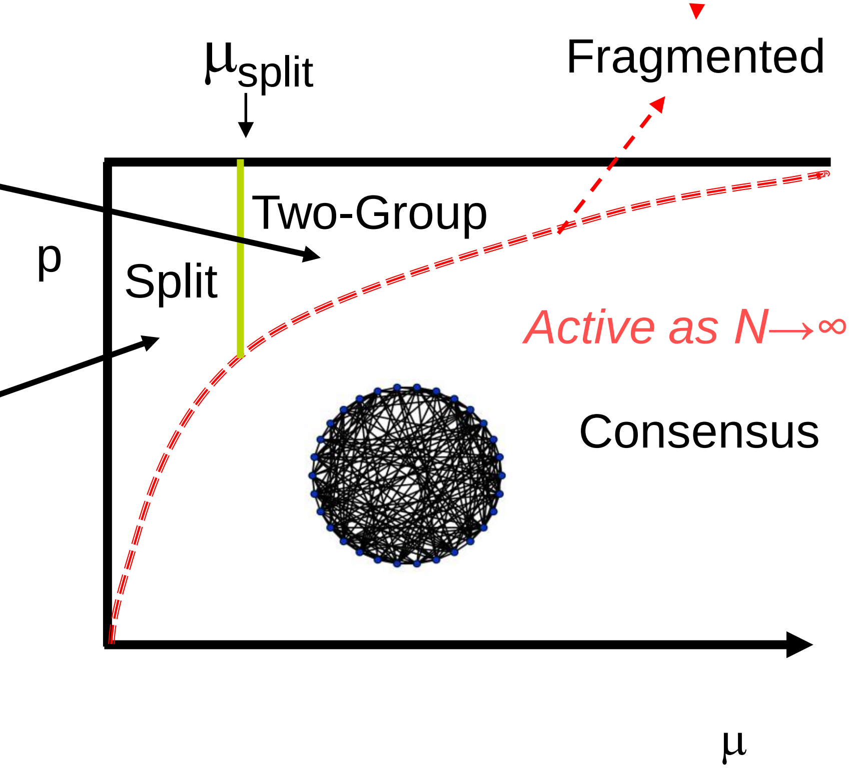




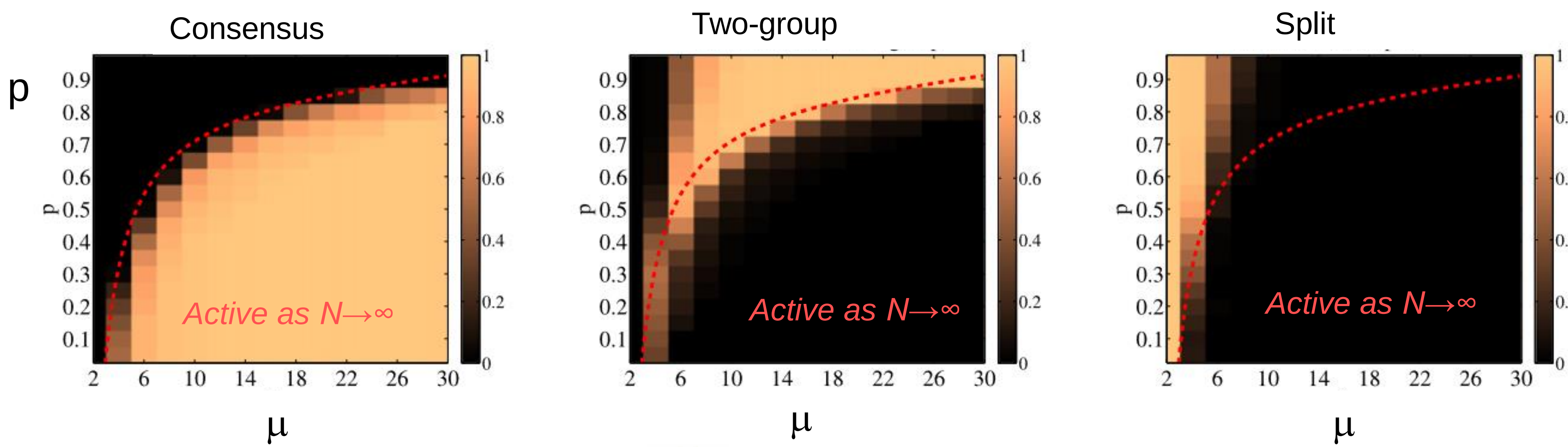
$\rho_f \neq 0$ Split



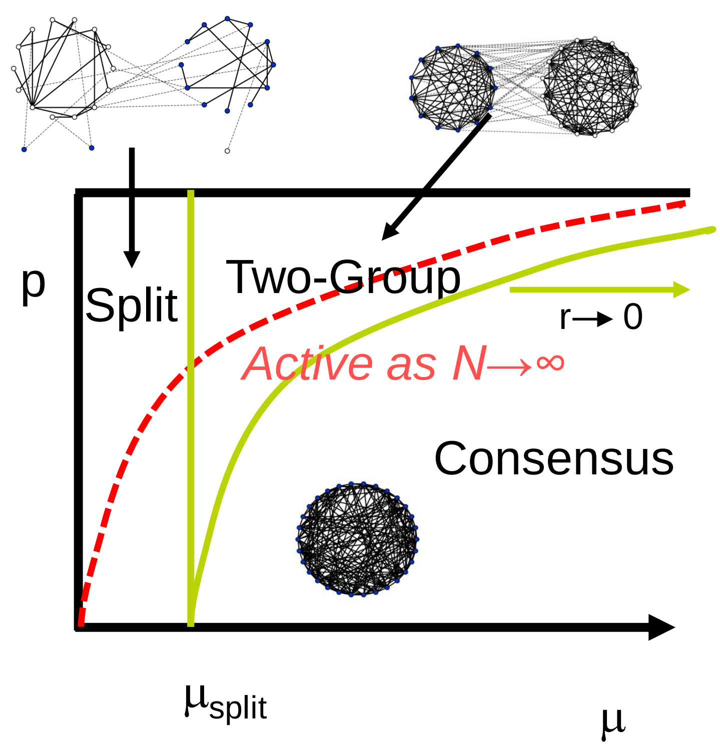




Topological transitions r<1

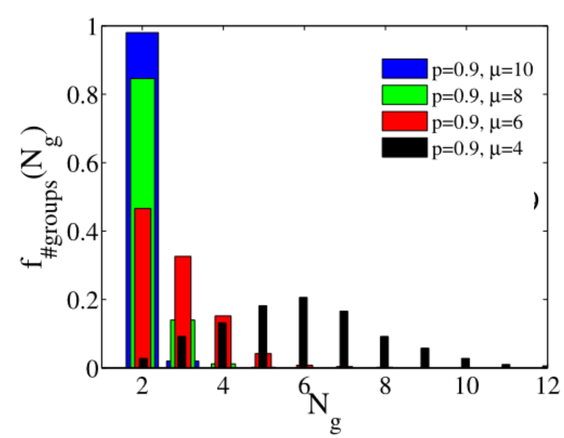


r=0.9

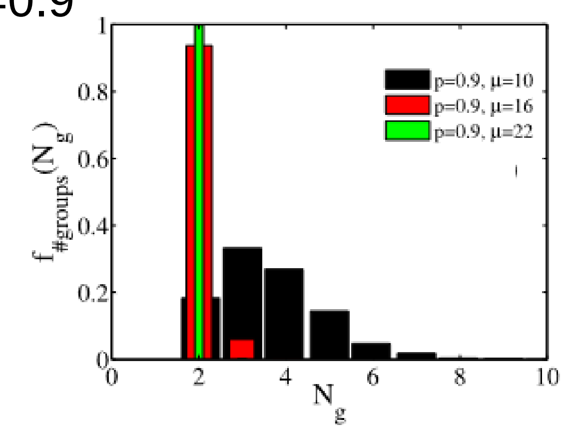


- No fragmentation found
- Consensus phase disappears as r ⟶0
- Finite size topological transition: μ_{split} decreases with r

p=0.9



r=0.9, $\mu_{\rm split} \sim 9\text{–}10$



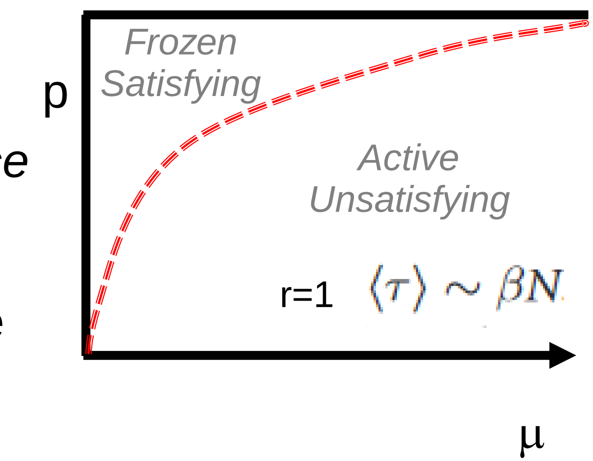
r=0, $\mu_{\rm split} \simeq 22$

DISCUSSION

Active (unsatisfying) - Frozen (satisfying) transition

p= rate of change of link state vs. node state
global unsatisfaction in spite of local mechanism of convergence towards satisfaction for small p

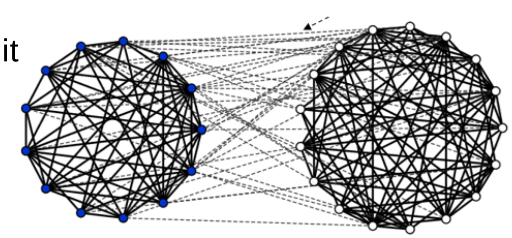
r=rewiring Exponential reduction of lifetime of unsatisfying state



Final Configurations:

SOCIAL POLARIZATION and ECHO-CHAMBERS: Two-Group and Fragmentation

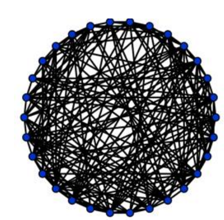
-**Globalization** (large connectivity) leads to Two-Group $\mu > \mu_{split}$
Smaller connectivity needed in adaptive networks (rewiring)



-Negative (heterophilic) interactions promote polarization

-**Fragmentation** is a manifestation of criticality

CONSENSUS only possible with rewiring:
Needs choosing positive satisfactory relations opting out of disagreement with our positive relations:



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