Some new developments in Fractional Quantum Hall Effect

Dam T. Son (University of Chicago)

Strings 2021
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Plan

- Formulation of the problem
- Nature of the $\nu = 5/2$ state
- Magnetoroton: a spin-2 excitation
- (Two gravitons in Jain's sequences around $\nu = \frac{1}{4}$)

The Theory of Everything

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Contributed by David Pines, November 18, 1999

The Theory of Everything

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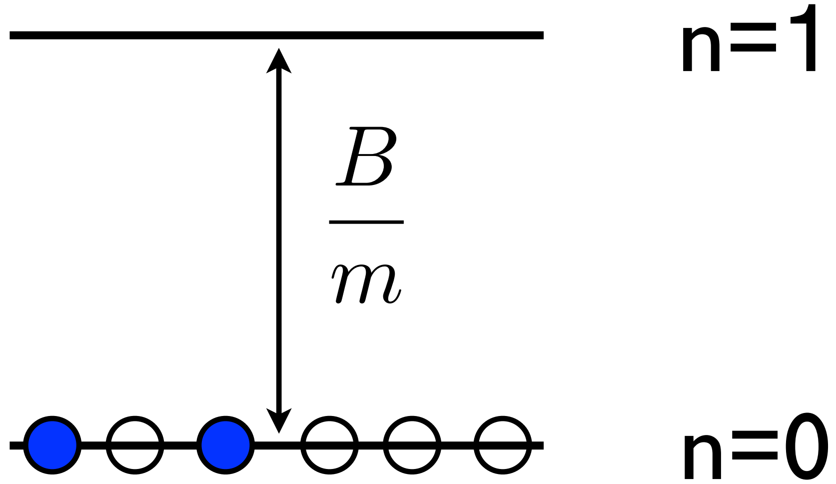
$$\mathcal{H} = -\sum_{j}^{N_e} \frac{\hbar^2}{2m} \nabla_j^2 - \sum_{\alpha}^{N_i} \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2 - \sum_{j}^{N_e} \sum_{\alpha}^{N_i} \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j \ll k}^{N_e} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + \sum_{\alpha \ll \beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|}. \quad [2]$$

Lowest Landau level limit

$$H = \sum_a \frac{(\mathbf{p}_a + e\mathbf{A}_a)^2}{2m} + \sum_{\langle a,b \rangle} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$



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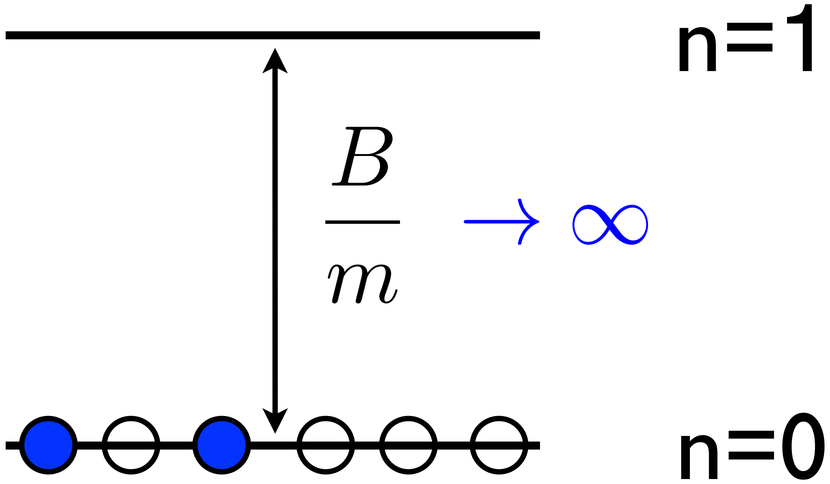
$m \to 0$



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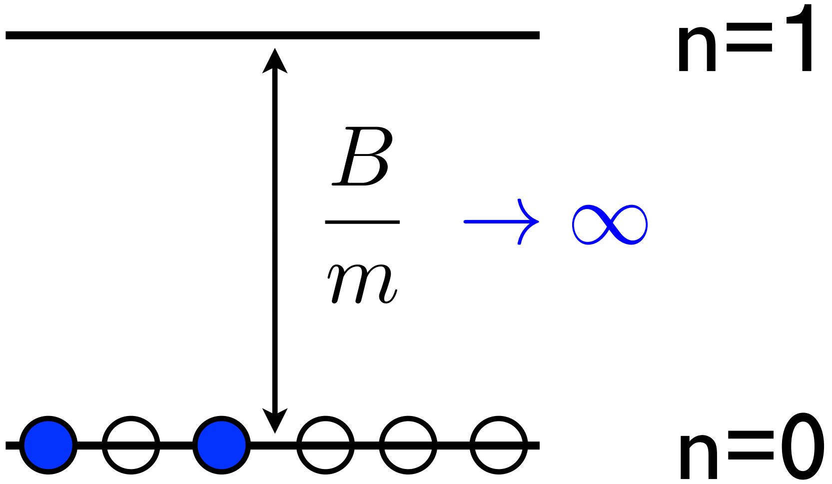


$$H = P_{\mathrm{LLL}} \sum_{a,b} \frac{e^2}{|\mathbf{x}_a - \mathbf{x}_b|}$$

Projection to lowest Landau level

Microscopic theory

$$S = \int dt d^2x \left(i\psi^\dagger(\partial_t - iA_0)\psi - \frac{1}{2m} |(\partial_i - iA_i)\psi|^2 + \frac{gB}{2m}\psi^\dagger\psi \right)$$

$$-\frac{1}{2}\int dt\, d^2x\, d^2y\, \psi^\dagger(x)\psi^\dagger(y)V(x-y)\psi(y)\psi(x)$$

Background magnetic field $B \neq 0$

$g = 2$: Schroedinger equation has $N_\phi = \frac{1}{2\pi}\int d^2x\, B$ zero modes

Problem: what is the effective theory of the LLL?

$$\lim_{m\to 0} Z[A_0, A_i] = ?$$

Chern-Simons theory

filling factor

$$\nu = \frac{\text{number of electrons}}{\text{degeneracy of a LL}}$$

- For gapped states, EFT below the gap is typically a CS theory, i.e., for $\nu = 1/3$ state

$$L = \frac{3}{4\pi} ada + \frac{1}{2\pi} Ada \to \frac{1}{3}\frac{1}{4\pi} AdA \quad + \nu \mathcal{S}\, \omega dA$$

- More difficult questions: gapless or states with small gap ($\ll$ natural energy scale)
  for example $\nu = 1/2$ or $1/4$

EFT near half filling

- Near half-filling the low-energy effective theory is that of a "Dirac composite fermion"

$$L = i\psi^\dagger_{\mathrm{cf}}\gamma^\mu(\partial_\mu - ia_\mu)\psi_{\mathrm{cf}} - \frac{1}{4\pi}Ada + \frac{1}{8\pi}AdA + \cdots$$

Particle-vortex duality: $\rho_{\mathrm{cf}} = \frac{B}{4\pi}$ $\rho_{\mathrm{e}} = \frac{B}{4\pi} - \frac{b}{4\pi}$

Half-filled Landau level of electrons = Fermi liquid of CFs
FQHE with

An experimental realized example of duality

$$L[\psi, a] - \frac{1}{2}\frac{1}{4\pi}ada + \frac{1}{2\pi}adb - \frac{2}{4\pi}bdb + \frac{1}{2\pi}Adb$$

DTS 2015
Metlitskii, Viswanath 2015
Wang, Senthil 2015
Karch, Tong 2016
Seiberg, Senthil, Wang, Witten 2016

Nature of $\nu = 5/2$ state

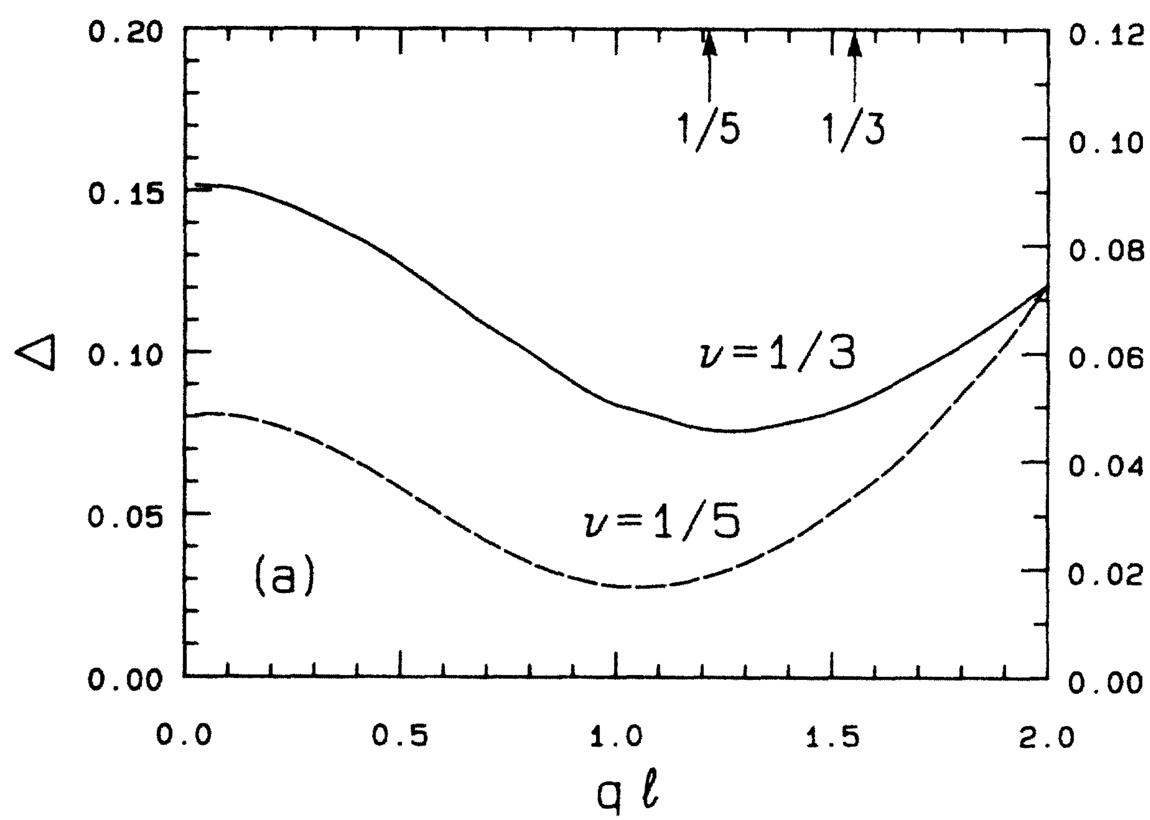
- $\nu = 5/2$: the only even-dominator gapped quantum Hall state
- a half-filled Landau level $\nu = 2 + \frac{1}{2}$
- Most well-known proposal: Moore-Read (Pfaffian)
  alternative: anti-Pfaffian state
- From the point of view of the composite fermion theory: BCS pairing of composite fermions

Pairing channels

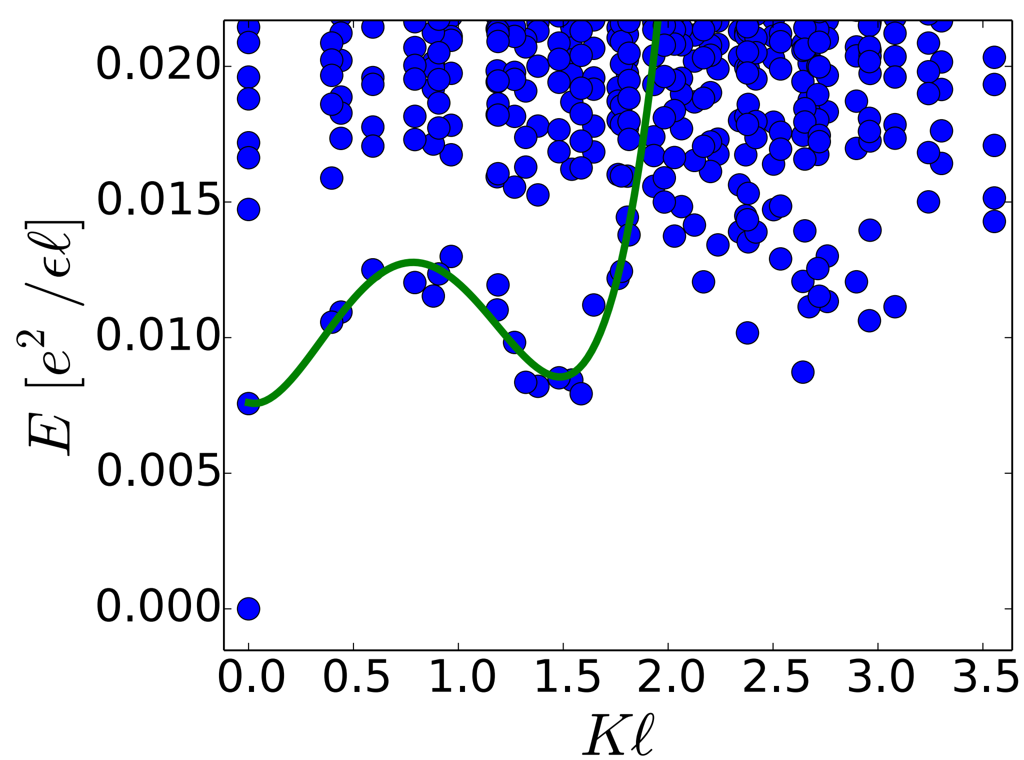
- Simplest pairing: "s-wave" $\langle \varepsilon^{\alpha\beta} \psi_\alpha \psi_\beta \rangle \neq 0$ corresponds to the PH-Pfaffian state
- "d-wave" pairing channels $\langle \varepsilon^{\alpha\beta} \psi_\alpha (\partial_x \pm i\partial_y)^2 \psi_\beta \rangle \neq 0$: Pfaffian and anti-Pfaffian
- Numerical simulations: favor anti-Pfaffian or Pfaffian
- but recent experiments prefer PH-Pfaffian (edge thermal conductivity)
- Tension between numerics and experiment has not been resolved

Magnetoroton

- Lowest neutral excitation of a gapped FQH state is the magnetoroton
- studied variationally by Girvin, MacDonald, Platzman 1986, also in numerics



$\nu = 1/3$



$\nu = 7/3$

Jolicoeur, 2017

- Q: what operator creates the magnetoroton?
- Magnetoroton: pole in the density-density correlation function, but the residue at the pole goes to 0 at small q

$$\langle \rho\rho \rangle_{\omega,q} \sim \frac{q^4}{\omega^2 - \Delta^2(q)}$$

- We will now see that q^4 is a consequence of a higher-rank conservation law
  (gauge invariance requires only q^2)

Conservation laws

- Conservation of particle number and momentum

$$\partial_t \rho + \overrightarrow{\nabla} \cdot \vec{j} = 0$$

$$\partial_t (m j_i) + \partial_k T_{ik} = (j \times B)_i$$

$m = 0$: force balance

$$\mathbf{j} \sim \frac{1}{B} \partial T$$

$$\dot{\rho} + \partial^2 T = 0$$

a higher-rank symmetry

Higher-rank symmetry

- A FQH system in fixed background B field can be coupled to A_0 and g_{ij}
- LLL physics invariant under volume-preserving diff

$$g_{ij} \to g_{ij} + \partial_i \xi_j + \partial_j \xi_j, \quad \xi^i = \varepsilon^{ij} \partial_j \lambda$$

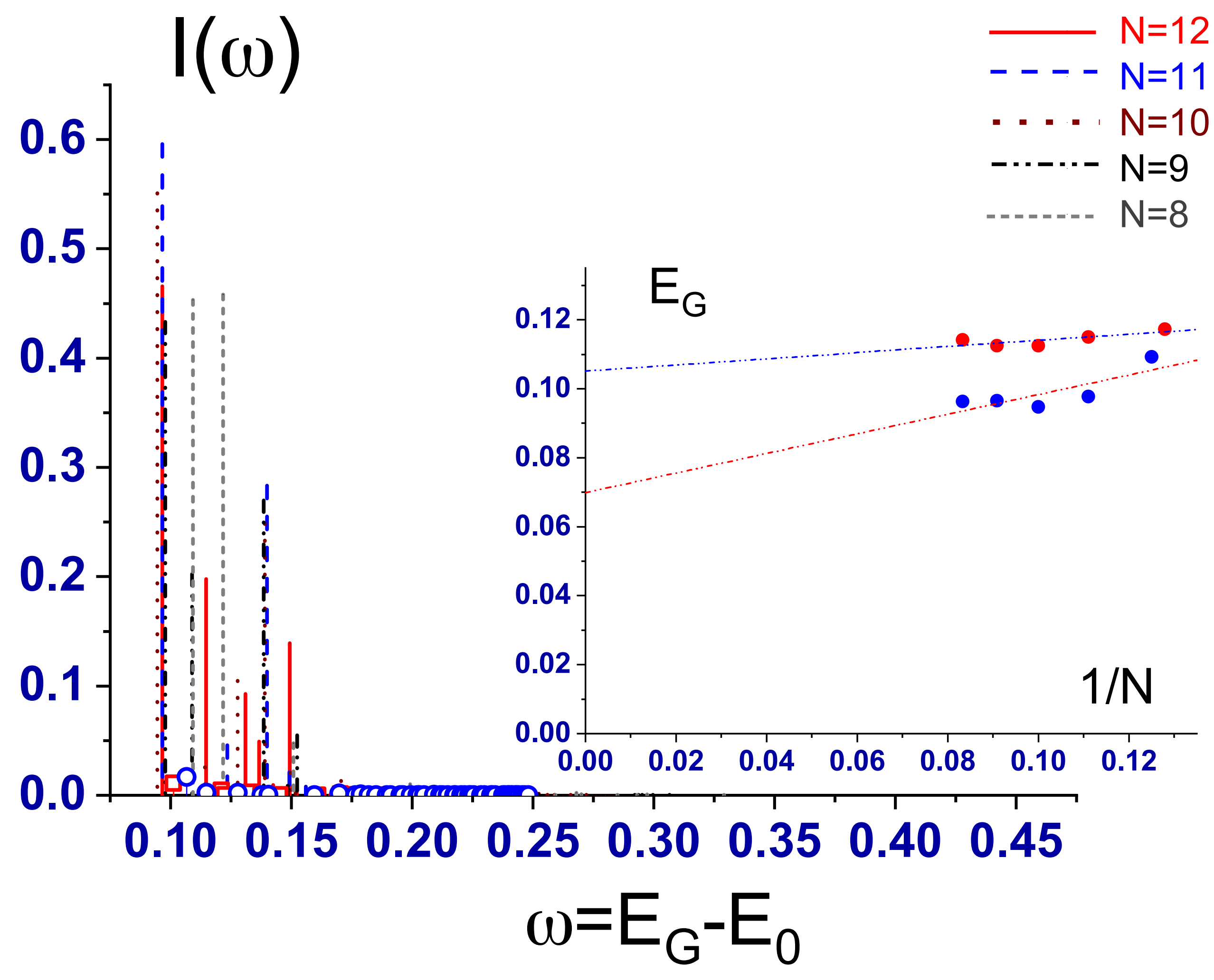
$$A_0 \to A_0 + \dot{\lambda}$$

- The Ward-Takahashi identity is the higher-rank conservation law $\partial_t \rho + \partial^2 T = 0$ Yi-Hsien Du, Umang Mehta, Dung Nguyen, DTS, 2103.09826

Operator creating magnetoroton

- ρ is not efficient in creating magnetoroton with $q = 0$
- The operators that can create $q = 0$ magnetoroton is the stress tensor
  T_{zz} , $T_{\bar{z}\bar{z}}$
- spin of the magnetoroton is either 2 or -2
  - which one?

- $\nu = 1/3$ state: strong suppression of spin-(-2) spectral density compared to that of spin-2



Liou et al. PRL 2019

A spectral sum rule

- $\rho(\omega) = N_e^{-1} \sum_n |\langle n | T | 0 \rangle|^2 \delta(\omega_n - \omega)$
- $\bar{\rho}(\omega) = N_e^{-1} \sum_n |\langle n | \bar{T} | 0 \rangle|^2 \delta(\omega_n - \omega)$

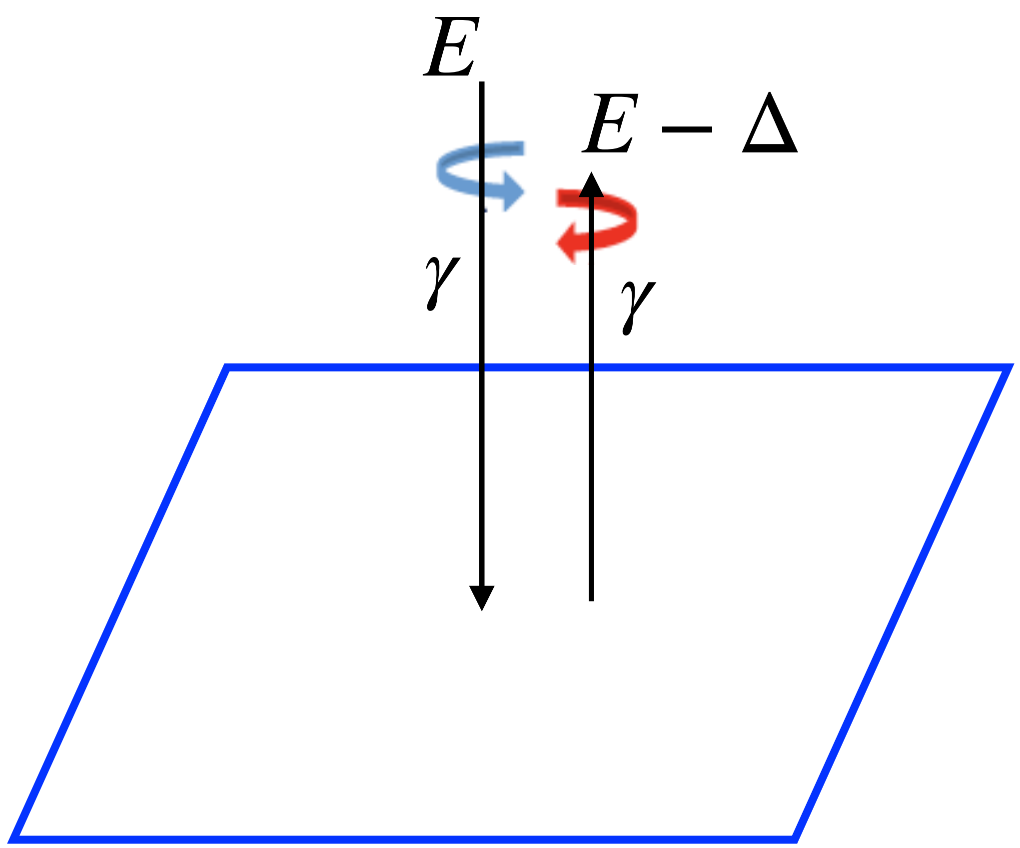
$T \equiv \int d\mathbf{x}\, T_{zz}(\mathbf{x})$

$\bar{T} \equiv \int d\mathbf{x}\, T_{\bar{z}\bar{z}}(\mathbf{x})$

- $\displaystyle\int_0^\infty \frac{d\omega}{\omega^2}[(\rho(\omega) - \bar{\rho}(\omega)] = \frac{\mathcal{S} - 1}{8},$ $\mathcal{S}$=shift
- If the integral is dominated by one mode:
  $\mathcal{S} > 1$: spin-2
  $\mathcal{S} < 1$: spin-(-2)

Golkar, Dung Nguyen, DTS, 2013

Polarized Raman scattering



can in principle be used to determine the spin of the magnetoroton experimentally

Dung Nguyen and DTS, 2101.02213
also Haldane, Rezayi, Kun Yang

Distinguishing $\nu = 5/2$ states by polarized Raman scattering

- Argument based on sum rule suggests that
  - Pfaffian state $\mathcal{S} = 3$: $s = 2$
  - anti-Pfaffian state $\mathcal{S} = -1$: $s = -2$
  - PH-Pfaffian $\mathcal{S} = 1$: both $s = 2$ and $s = -2$ magnetorotons
- Polarized Raman scattering: a bulk probe that complements boundary probes

SSF and Haldane bound

- Static structure factor $S(q) = \int e^{iqx} \langle \rho(0,x) \rho(0,0) \rangle$
- $S(q) = s_4 q^4 + \cdots$
- Haldane bound:
  - $s_4 \geq \frac{s}{4} = \frac{\mathcal{S} - 1}{8}$ (shift → $\mathcal{S}$)
  - saturated in Dirac CF theory near $\nu = 1/2$

Jain's states near $\nu = 1/4$

- Near $\nu = \frac{1}{4}$: CF = electron + 4 flux quanta
- Effective field theory: CF coupled to dynamical CS gauge field
- Fails to satisfy the Haldane bound!

$$s_4 \geq \frac{\mathcal{S} - 1}{8}$$

$$\nu = \frac{N}{4N+1} \qquad \frac{N+1}{8} \geq \frac{N+3}{8}$$

Solution to the puzzle

- To solve the problem with the Haldane bound for Jain's states near $\nu = 1/4$, one requires at least one additional magnetoroton
- For $\nu = N/(4N \pm 1)$: one magnetoroton with energy $O(1/N)$, one with energy $O(1)$
- opposite chiralities for $\nu = N/(4N - 1)$, the same chirality for $\nu = N/(4N + 1)$
- can be in principle verified numerically and hopefully, experimentally

Dung Nguyen, DTS, 2105.02092

Conclusion

- FQHE is an important theoretical problem
- Nature of $\nu = 5/2$ state: still an open question
- $q = 0$ magnetoroton has spin 2 or -2 depending on the QH state
- Polarized Raman scattering can distinguish different FQH states, in particular different $\nu = 5/2$ candidates
- Extra magnetoroton mode(s) at and near $\nu = 1/4$

Thank you