Adaptive Network Approach for Emergence of Societal Bubbles

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Introduction

- **Social Interactions**: An almost unavoidable part of living in a society. It is through social interactions that information spreads.
- **Complex Networks**: Essentially a way of representing and studying systems by breaking them down to components and connections.
- **Opinion Dynamics**: Social interactions are not homogeneous nor static, people tend to have different opinions and also change them depending on the situation.
- **Consensus or Polarization**: When a decision needs to be made, the general wish is to please most of the population either by getting everyone to agree with each other or by ignoring whatever the lesser part of the population wants.

Models and Methods

Networks

Consider a social network, where an individual is represented by a node. If two nodes interact then they are connected by an edge. Each node has a degree k, defined by the number of neighbors it has.

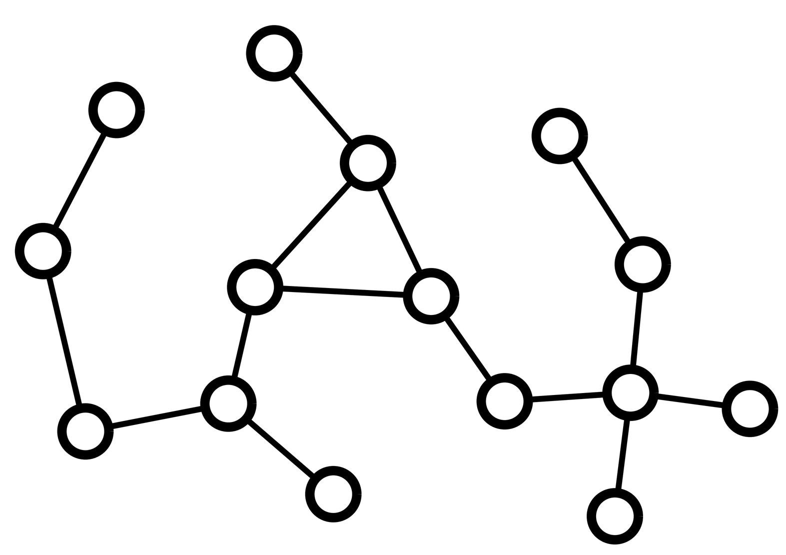


Fig. 1: Example of a simple network.

Empirical data suggests that most social networks are scale-free having a power-law degree distribution as

$$P(k) \approx k^{-\gamma} \tag{1}$$

Some other relevant network properties are the clustering coefficient and the average path length, defined as:

$$C_i = \frac{2L_i}{k_i(k_i-1)} \quad \textit{and} \quad \langle D \rangle = \frac{1}{N(N-1)}\sum_{i \neq j} D_{ij} \tag{2}$$

Here, L_i is the total number of trios formed by i and two of his neighbors, and D_{ij} is the smallest number of links between i and j.

Rules for Changes in Opinion

- ✓ A network of N individuals is built. A continuous value o_i between 0 and 1 is assigned to each node representing their opinion.
- ✓ For every time step, the individuals verify their neighbors' average opinion $\langle o_i(t) \rangle = \frac{1}{k_i}\sum_{j \in v_i} o_j(t)$, and computes $\Delta o_i(t) = \langle o_i(t) \rangle - o_i(t)$.
- ✓ The new opinion given by:

$$o_i(t+1) = \begin{cases} o_i(t) + \mu \Delta o_i j(t) & \text{if } \ \Delta o_i(t) \leq \varepsilon \\ o_i(t) & \text{if } \ \Delta o_{ij}(t) > \varepsilon, \end{cases}$$

There is a tendency for individuals to approach their opinion to that of his group at a rate $\mu \in (0,1)$ only if it is within a range ε of tolerance.

Rules for Changes in Topology

After the opinions o_i are updated, the connections between i and his/her k_i neighbors can be broken or redirected.

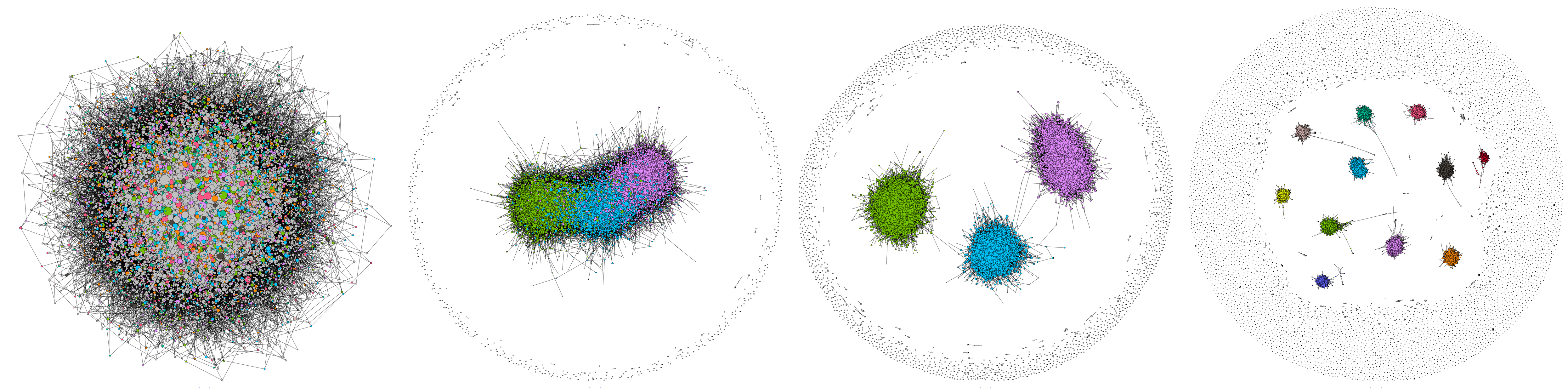
- ✓ A connection between any pair of nodes i and j is kept if $\Delta o_{ij} = |o_i(t) - o_j(t)| < \varepsilon$, otherwise, the connection is broken with probability $p = 1 - e^{-\kappa \Delta o_{ij}}$
- ✓ After breaking a connection, a node can create a new one with any other node k if their opinion differs by less than ε. The probability is proportional to the distance between them given by:

$$q = \begin{cases} e^{-d_{ik}/d_o}, & \text{if } d_{ik} < d_{max} \\ 0, & \text{otherwise} \end{cases}$$

- ✓ If a connection is broken but not replaced at a specific time step, it can be replaced at a future time step following the above rules.

Here, κ and d_o are control parameters representing the tendency for breaking connections and reconnection characteristic distance respectively. The parameter d_{max} is an upper cutoff.

Results



(a) (b) (c) (d)

Fig. 2: Typical networks exhibiting (a) single community (the full consensus regime for $\varepsilon \geq 0.5$ in which $m = 0$), (b) a single connected component concomitant with small isolated groups (majority consensus regime for $\varepsilon_c < \varepsilon < 0.5$), (c) multiple modules connected by bridges (the radicalization regime for $\varepsilon \lesssim \varepsilon_c$), and (d) disconnected modules (still in the radicalization regime, but for $\varepsilon \ll \varepsilon_c$). In this range, the rupture of several bridges between communities generates isolated clusters. These networks have $N = 12,800$ nodes and tolerance thresholds are (a) $\varepsilon = 0.5$, (b) $\varepsilon = 0.145$, (c) $\varepsilon = 0.085$, and (d) $\varepsilon = 0.025$. Different colors represents communities detected by the Louvain algorithm.

From the initial state, the opinion distribution changes drastically according to the given value of ε, resulting in one of the 4 categories (a, b, c or d) shown in Figs. 2 and 3.

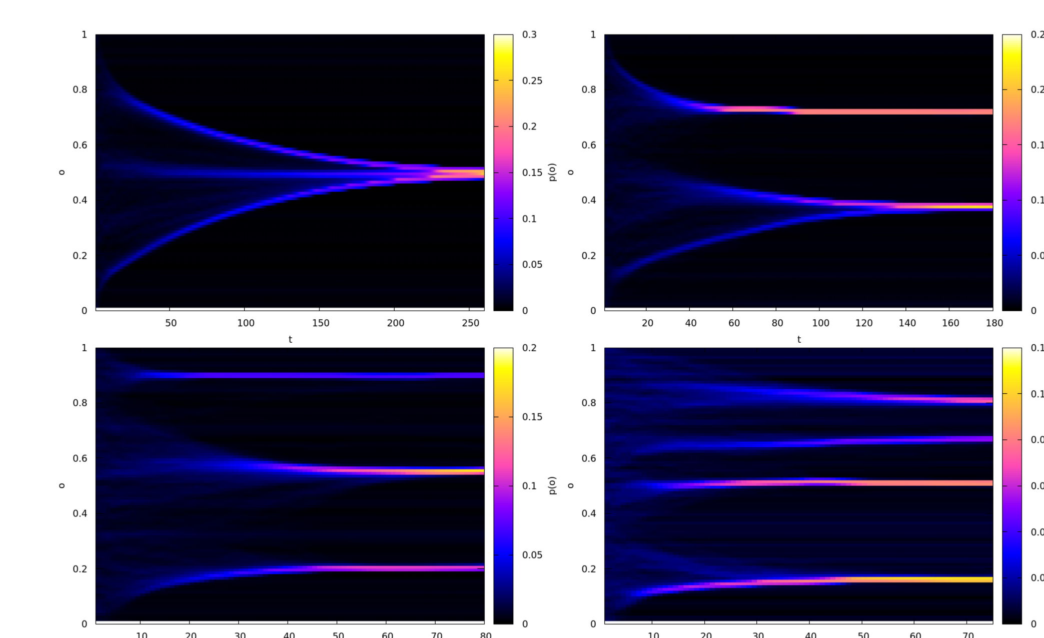
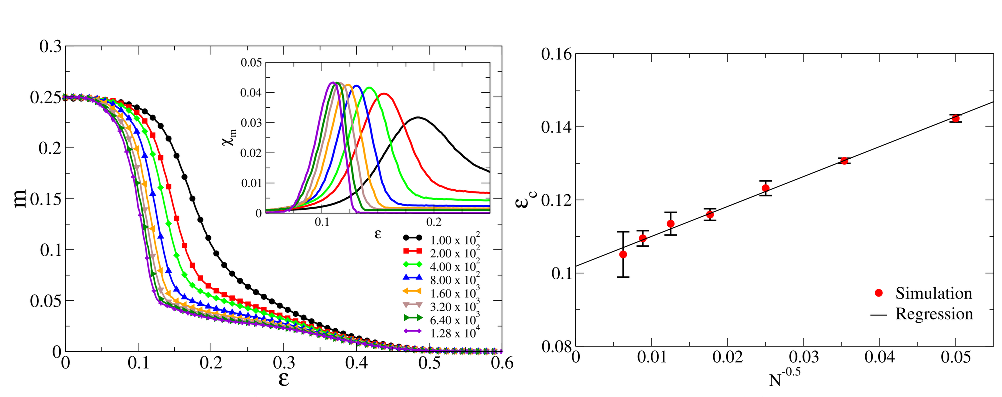


Fig. 3: Temporal evolution of opinion distribution on a network of 1000 nodes. Each one with the same initial state but with different values for tolerance, $\varepsilon = 0.120$, $\varepsilon = 0.115$, $\varepsilon = 0.090$ e $\varepsilon = 0.075$ respectively.

We define a measure of consensus $m = \frac{1}{N}\sum_i |o_i - \langle o \rangle|$ as an order parameter. The associated variability is defined as the variance of m computed over the ensemble of samples for $t \to \infty$, $\chi_m = \langle o^2 \rangle_{\text{en}} - \langle o \rangle_{\text{en}}^2$ can be used to estimate the transition point ε_c. At asymptotically large N, a value $\varepsilon_c \approx 0.10$ was found using a finite size scaling of the form $\varepsilon_c(N) = \varepsilon_c(\infty) + \text{const.} \times N^{-1/2}$



(a) (b)

Fig. 4: (a) Order parameter m at the stationary state as a function of the tolerance threshold ε and the stationary state opinions' variability for different system sizes N. (b) Finite-size scaling of ε_c, the critical tolerance threshold

Changes of topology can be detected by the average shortest path length $\langle D \rangle$ of the largest connected component. The average distances present local maxima and minima as ε is increased from zero. These regions are related to fragmentation points.

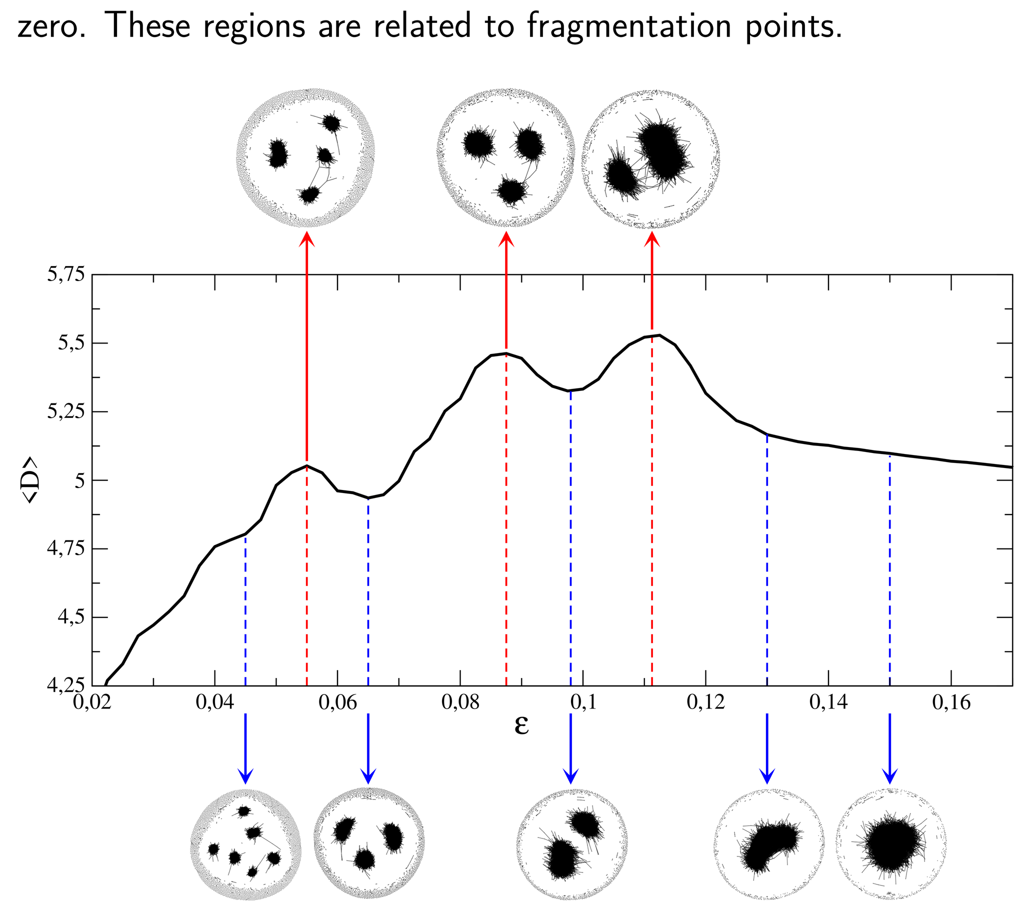
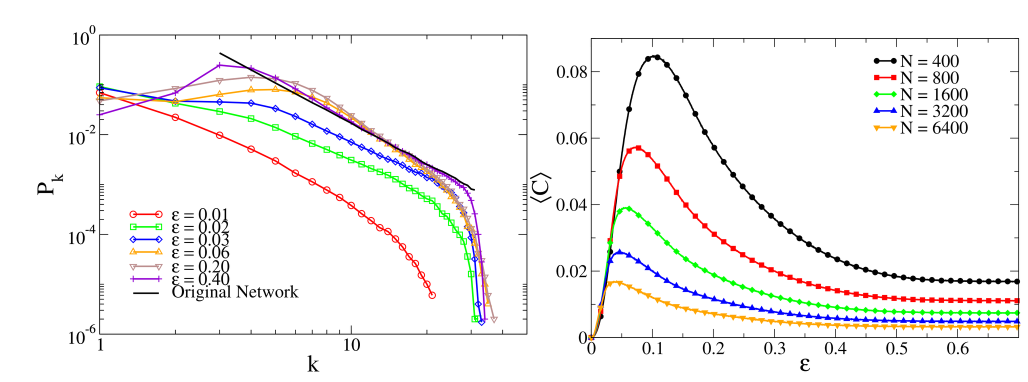


Fig. 5: Average distance between the nodes belonging to the largest connected component as a function of ε. The network size was fixed as $N = 12,800$. Top and bottom snapshots represent typical network structures at minima and maxima of curve.

Due to changes in topology, the network degree distribution deviates from the initial UCM power-law distribution. Due to homophily, individuals tend to form ties with others of the same group, this can be quantified by the average clustering coefficient $\langle C \rangle$.



(a) (b)

Fig. 6: (a) Degree distributions for stationary networks generated at distinct ε values. $N = 1000$ nodes was fixed. (b) Clustering coefficient as a function of ε for various network sizes N.

Concerning the structure of the adaptive social network, a measurement of the largest connected component indicates different regimes in relation to ε.

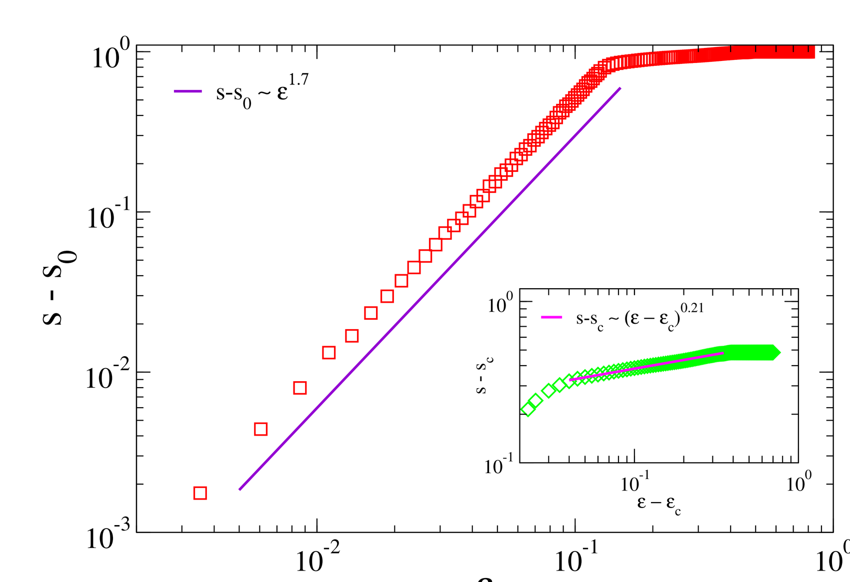


Fig. 7: Fractional size of the largest connected component as a function of the tolerance threshold ε. The constant $s_0 = 0.00135$ represents the average component fractional size as $\varepsilon \to 0$. Inset shows the scaling above the critical threshold. The network has $N = 1,600$ nodes.

Conclusions

- This approach replicates different sorts of polarization regimes caused only by opinion difference among the nodes.
- Through order parameters, transition points between said regimes becomes evident as shown in Fig. 4. Such transition points are backed by other measurements.
- Polarized networks show sometimes drastically different properties when compared to regular scale-free unpolarized networks.

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Financial Support: CNPq, FAPEMIG e CAPES