Entropic analysis of an opinion formation model presenting a spontaneous third position emergence

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Motivation

Third position Emergence.

Argentina from 1900 to 1940

U.C.R Party

Conservative Party

Social Kinetics

Argentina from 1940...

U.C.R. Party

Justicialist Party

Conservative Party
Third position Emergence.

How could we explain this fact?
Third position Emergence.

**The model**

Configuration map

- Square discreet arrangement composed by $\lambda^2$ pixels.
- Each pixel represents an idea associated to a person.
- There are three different ideas or ideological positions, Y, L y B.
- Positions Y y L are active ones. B is a passive position.
- Initially, each ideological position has a given number of supporters.
- Let $r_i$ (i= Y, L, B) be the initial fraction of supporters.
Third position Emergence.

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The Sznajd model

Let $M_{ij}$ be the matrix element of the configuration map that represents the idea of the person $(i, j)$.

$$M_{ij}(t + \Delta t) = M_{i,j}(t), \quad M_{\bar{i}, \bar{j}} \neq 0$$

$(n, m)$ is the first active neighbor $(i, j)$ and $(\bar{i}, \bar{j})$ is the second active neighbor of $(i, j)$. 

$t = 0$
Third position Emergence.

**The model**

Initial conditions
- a – random
- b – structured

At long times
- Chessboard-like structure
- Emerging party

Sznajd model.

$t = 0$  
$t \rightarrow \infty$
Third position Emergence. **Structured initial condition**

Idea structures are characterized by means of **fractal dimension D**.

In our case: \( 0 \leq D \leq 2 \)

An entropical characterization for complex systems becoming out of control.

The initial configuration is generated by using the **box counting** method.

Configuration map

Initial pattern: \( \lambda = 64 \) side
\( r_Y = 0.15 \quad D_Y = 1.70 \)
\( r_L = 0.15 \quad D_L = 1.60 \)
Third position Emergence

**Random initial conditions**

Assumption: \( r_Y = r_L \)

\( p(J) \): probability of winning for J party

\[ p(Y) \geq \frac{r_Y}{r_Y + r_L} \]
Third position Emergence

**Structured initial conditions**

$r_Y = r_L$

Initial pattern: $\lambda = 64$
- $r_Y = 0.15, D_Y = 1.70$
- $r_L = 0.15, D_L = 1.60$

$\mathbf{p(Y) = 0.45 > 0.25 = p(L)}$
$\mathbf{p(G) = 0.30}$

**Random initial conditions**

$p(Y) = p(L) = 0.25$
$p(G) = 0$
Non monotonous regime is associated to the maximum unpredictability.
Third position Emergence

Entropic analysis

\[ J_r = \{ D : D_{\text{min}} \leq D \leq D_{\text{max}} \} \]

\[ D_r = \frac{D_{\text{min}} + D_{\text{max}}}{2} \]

\[ D_{\text{min}} \leq D_Y, D_L \leq D_{\text{max}}, \]

\[ D_{\text{min}} = \max \left( 0, 2 + \frac{\log r}{m} \right) \]

\[ D_{\text{max}} = \min \left( 2, 2 + \frac{\log(r\lambda^2)}{m} \right) \]

\[ S(D) \approx \left( \frac{\lambda}{2^{m-1}} \right)^N H(2^{D-N}) \frac{2^{mD} - 1}{2^D - 1} \]

An entropical characterization for complex systems becoming out of control.
There exists an intrinsic weakness inside the initial structures of the Y and L parties that favours and maximizes the emergence of a third party.

\[ D_Y = D_L + 0.1 \]
\[ r = 0.15 \]

\[ \frac{D_L + D_Y}{2} = 1.63 \]
\[ \frac{D_1 + D_2}{2} \approx 1.64 \]

\[ ||\nabla S_Y|| > ||\nabla S_L|| \]

maximum unpredictability
Conclussions

➢ Initial conditions matter.

➢ Hierarchically structured systems have an anomalous entropy production.
Published related Works

- Spontaneous emergence of a third position in an opinion formation model

- Entropical analysis of an opinion formation model presenting a spontaneous third position emergence

- On the role of structured initial conditions in the Schelling model.
  Physca A. in press.

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I invite you to participate in Marcos’s talk

Why are the borders of Palestine/Israel and Wallonia/Flanders so different?:
Entropic Analysis of a Schelling model with hierarchically structured initial conditions.

It will be held on Tuesday 19 October at 11:30 hs.

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Thank you for your attention!!!