Entropic analysis of an opinion formation model presenting a spontaneous third position emergence

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Workshop on Sociophysics:
Social Phenomena from a Physics Perspective
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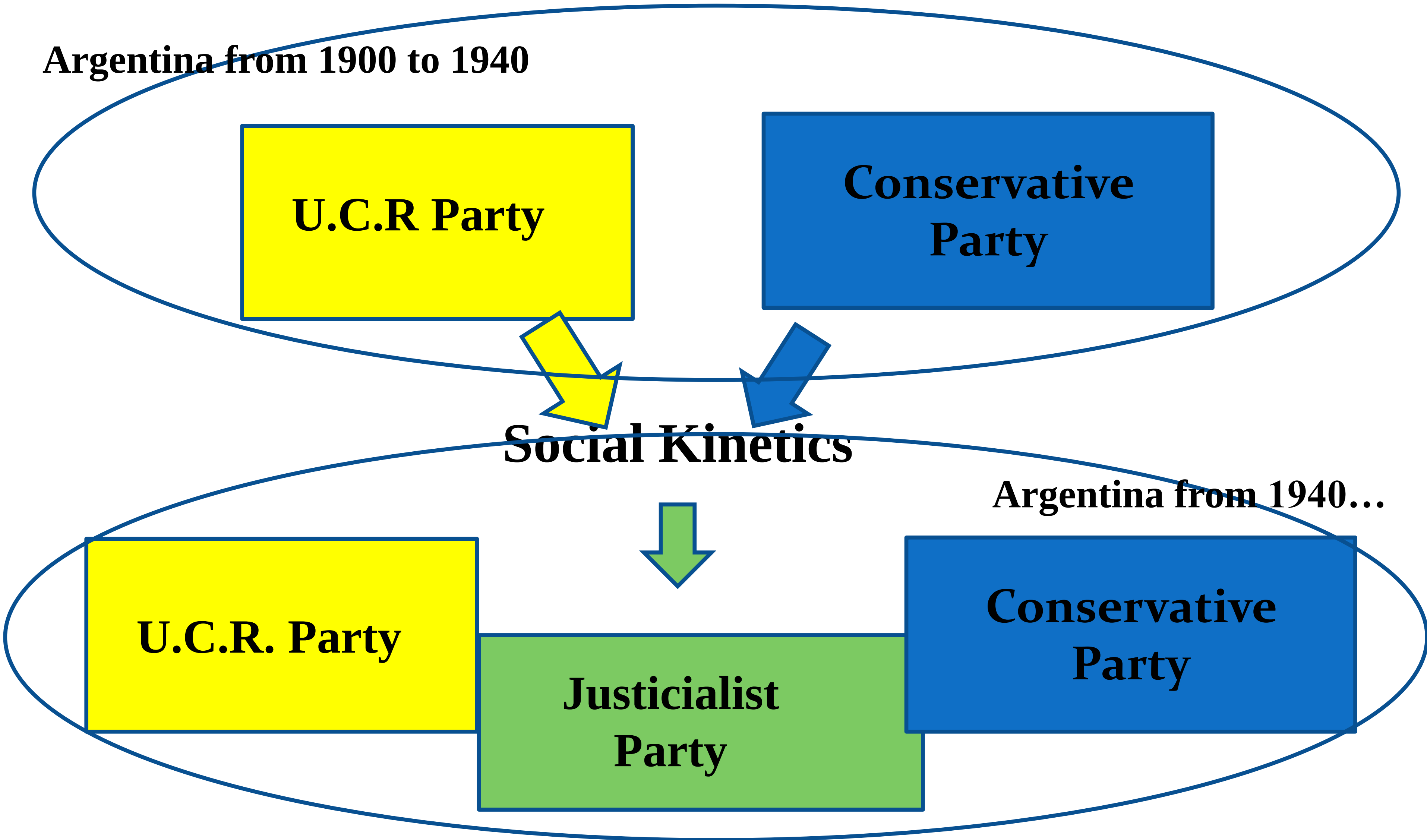
Motivation

Third position Emergence.



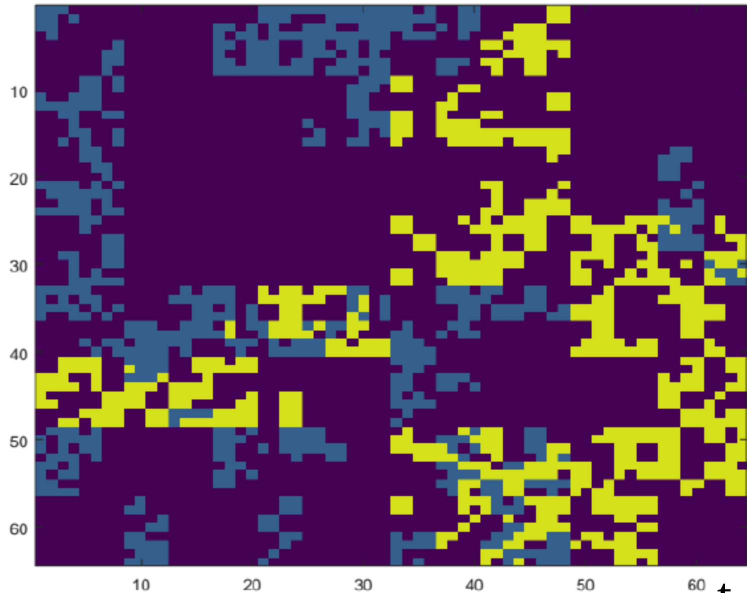
Third position Emergence.

How could we explain this fact?

Third position Emergence.

The model

Configuration map

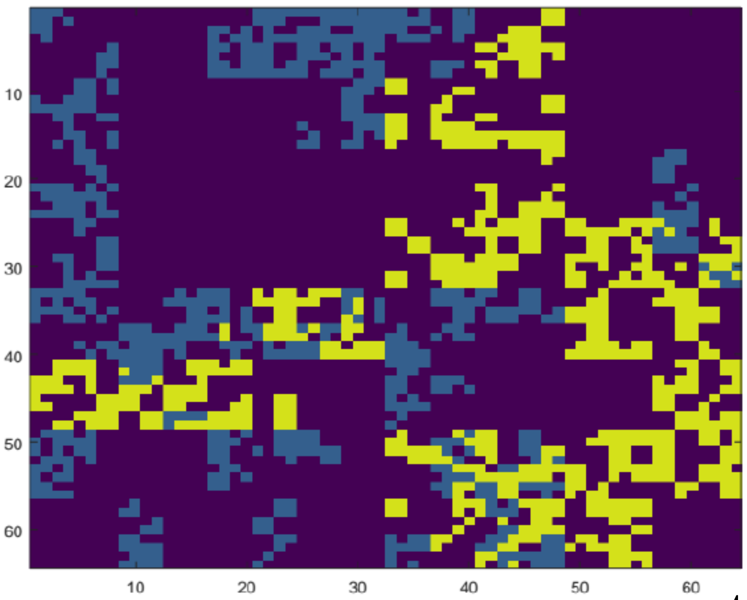


t = 0

- Square discreet arrangement composed by λ^2 **pixels**.
- Each pixel represents an **idea** associated to a person.
- There are **three different ideas or ideological positions**, Y, L y B.
- Positions Y y L are **active ones**. B is a **passive position**.
- Initially, each ideological position has a given **number of supporters.**
- Let r_i (i= Y, L, B) be the initial fraction of supporters.

Third position Emergence.

The model



t = 0

The Sznajd model

Let M_{ij} be the matrix element of the configuration map that represents The idea of the person (i,j).

$$M_{ij}(t+\Delta t) = M_{\bar{i},\bar{j}}(t), \quad M_{\bar{i},\bar{j}} \neq 0$$

(n, m) is the first active neighb (i, j)

$(\bar{i}, \bar{j})$ is the second active neighbor of (i, j)

- Square discreet arrangement composed by **λ^2 pixels**.
- Each pixel represents an **idea** associated to a person.
- There are **three different ideas or ideological positions**, Y, L y B.
- Positions Y y L are **active ones**. B is a **passive position**.
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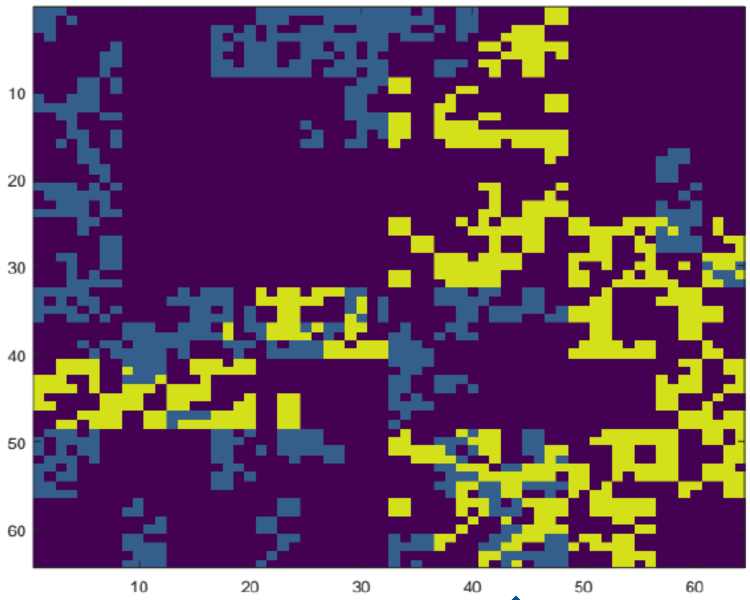
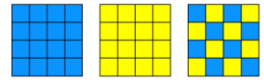
Third position Emergence.

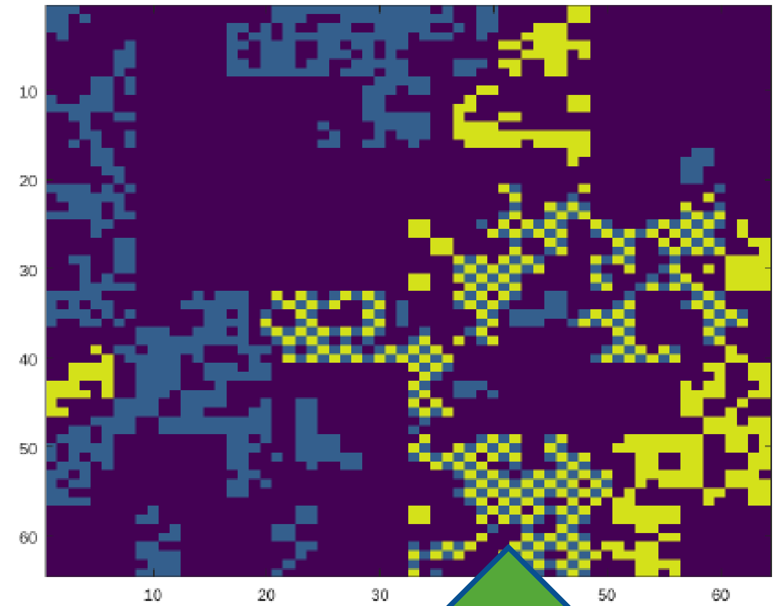
The model



At long times



Sznajd model.



t = 0

$t \to \infty$

Initial conditions

a – random

b – **structured** → **Afinity and organization**

Chessboard-like structure

emerging party

Third position Emergence.
Structured initial condition

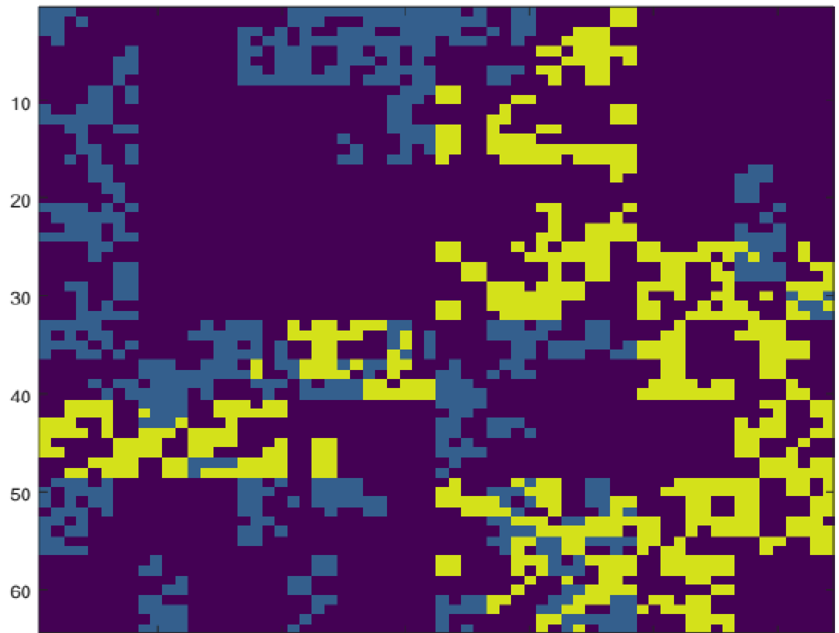
Idea structures are characterized by means of **fractal dimensioin D.**

In our case: $0 \leq D \leq 2$

An entropical characteization for complex systems becoming out of control.
M. Gaudiano, Physica A, **440**, 185, (2015).

The initial configuration is generated by using the´´ box counting´´ method.

Configuration map



Initial pattern: λ=64 side

r_Y=0.15 D_Y=1.70

r_L=0.15 D_L=1.60

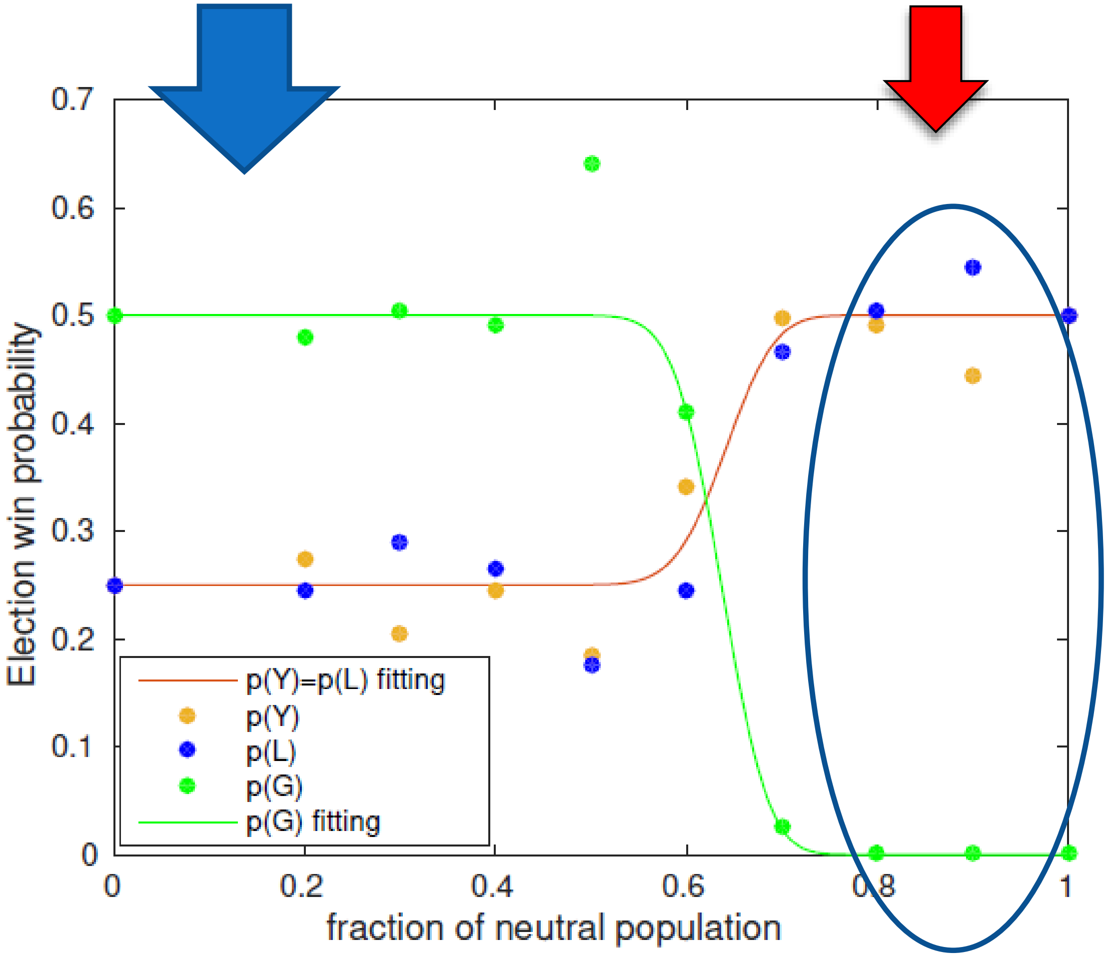
Third position Emergence
Random *initial conditions*

Assumption: $r_Y = r_L$

p(J): probability of winning for J party

Low Apathy **High Apathy**

$$p(Y) \geq \frac{r_Y}{r_Y + r_L}$$



Third position Emergence

Structured *initial conditions*

$r_Y = r_L$

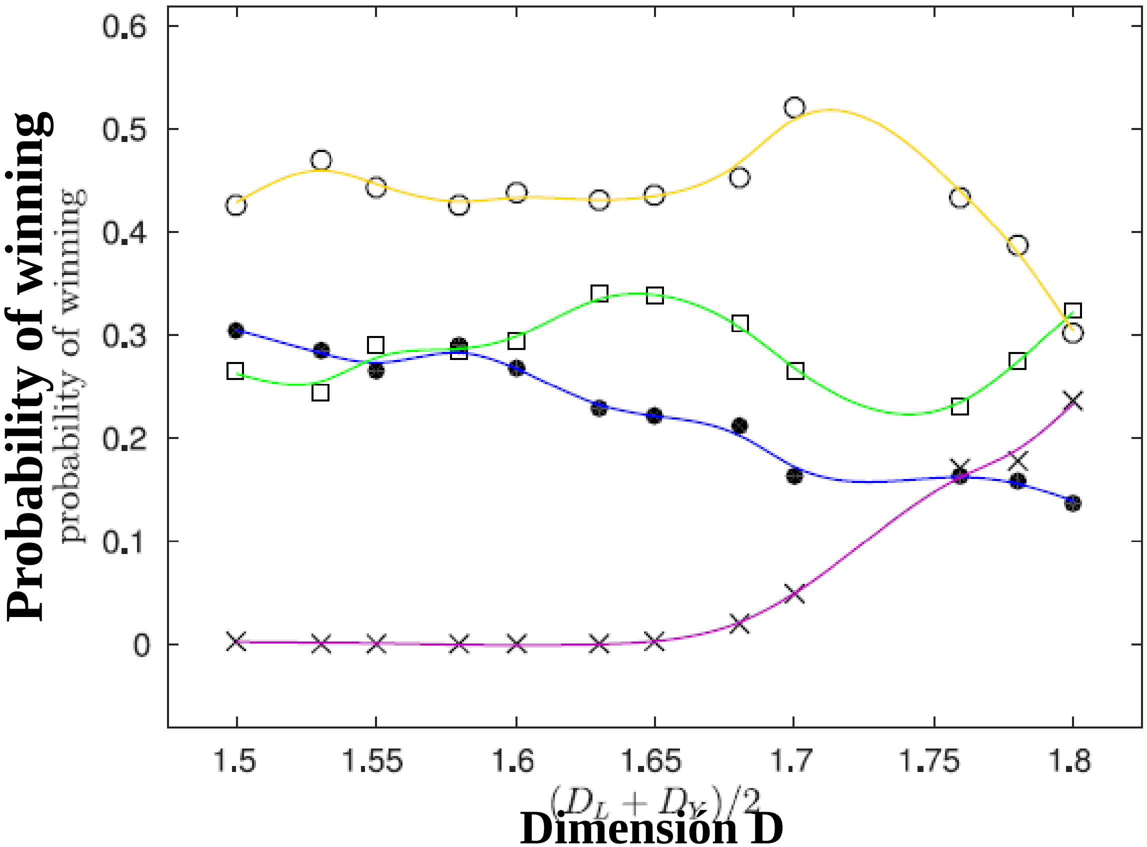
Initial pattern: λ=64

r_Y=0.15, D_Y=1.70

r_L=0.15, D_L=1.60

p(Y) = 0.45 > 0.25 = p(L)

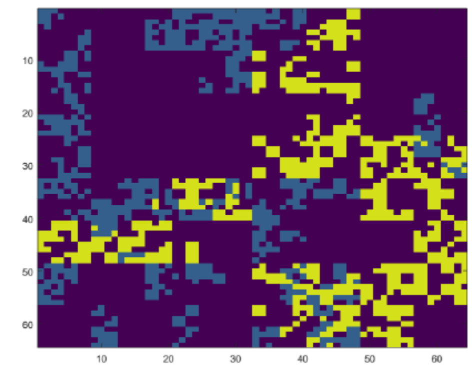
p(G)=0.30

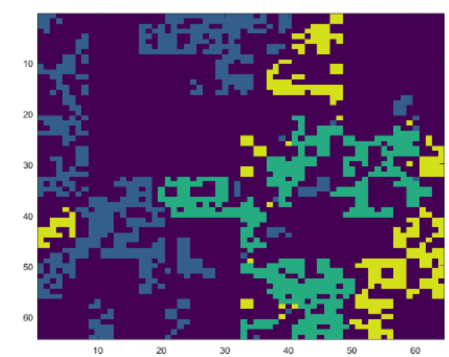


Random ***initial conditions***

$p(Y) = p(L) = 0.25,$

$p(G) = 0$

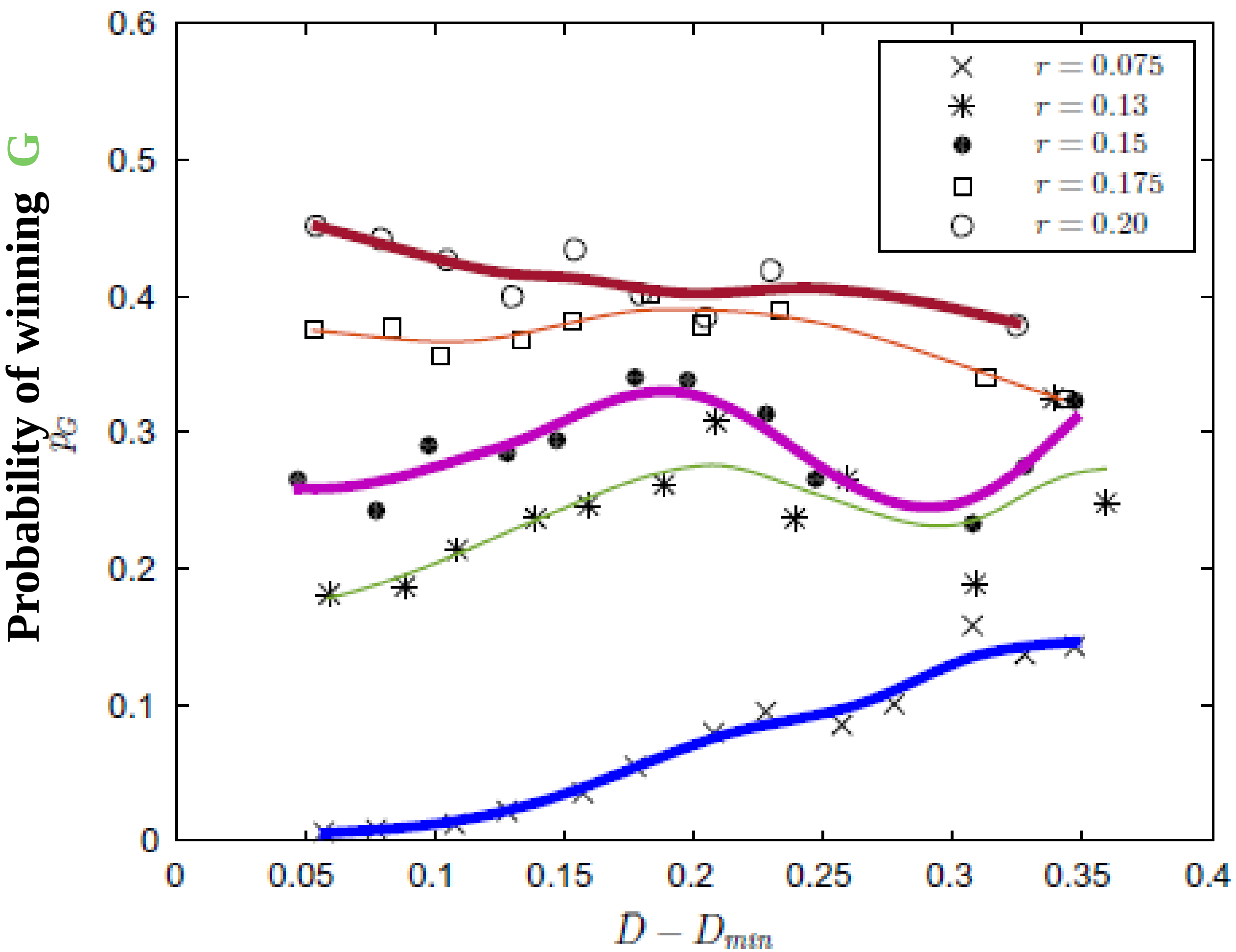




Third position Emergence

:

Probability of winning for **the emerging party**



Monotonous regime

Non monotonous regime

Monotonous regime

Non monotonous regime is associated to **the maximum unpredictability**

Third position Emergence

Entropic analysis

An entropical characterization for complex systems becoming out of control.
M. Gaudiano, PhysicaA, **440**, 185, (2015).

$$J_r = \{D : D_{\min} \le D \le D_{\max}\}$$

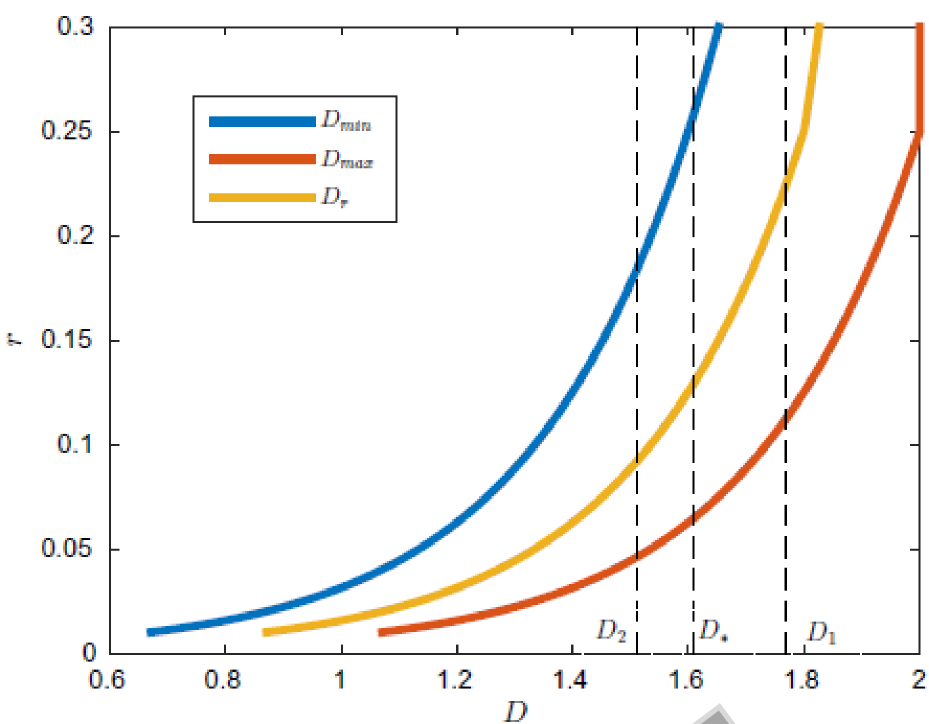
$$D_r \doteq \frac{D_{\min} + D_{\max}}{2}.$$

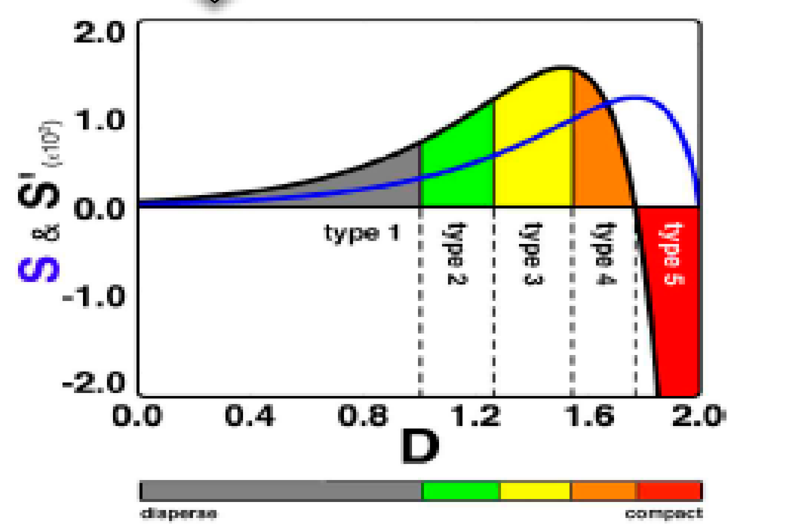
$$D_{min} \le D_Y, D_L \le D_{max},$$

$$D_{min} = \max\left(0, 2 + \frac{\log r}{m}\right)$$

$$D_{max} = \min\left(2, 2 + \frac{\log(r\lambda^2)}{m}\right)$$

$$S(D) \approx \left(\frac{\lambda}{2^{m-1}}\right)^N H(2^{D-N}) \frac{2^{mD} - 1}{2^D - 1}$$





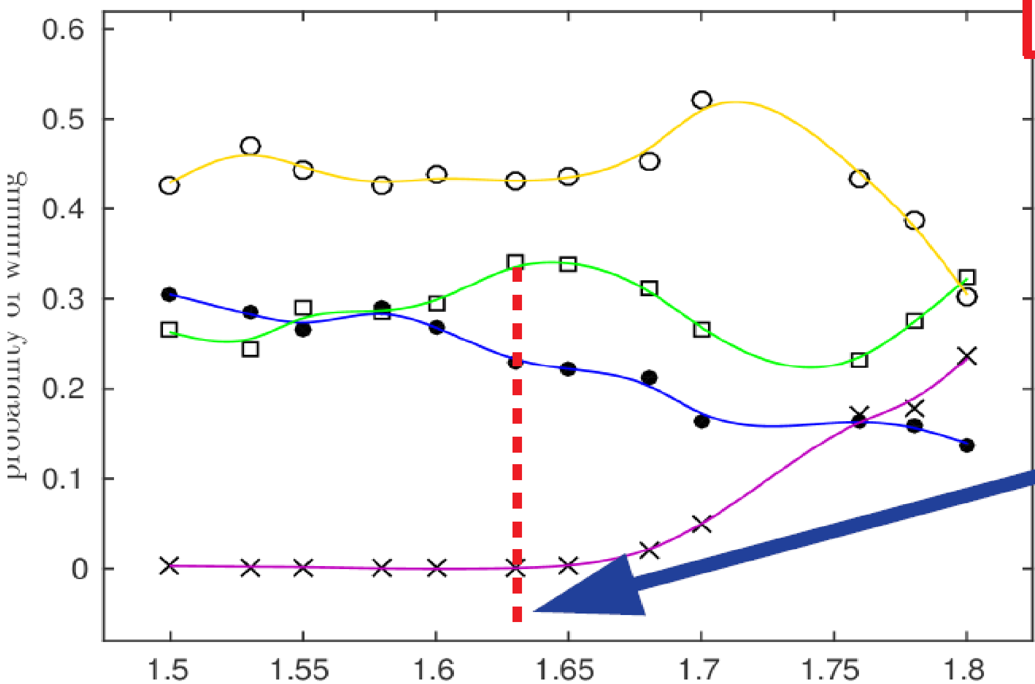
$$I_{21} = \{D : D_2 \le D \le D_1\},$$

Third position Emergence

Entropic analysis

$$D_Y = D_L + 0.1$$

r = 0.15

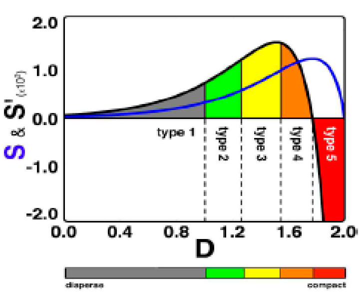


$$\frac{D_L + D_Y}{2} = 1.63$$

$$\frac{D_1 + D_2}{2} \approx 1.64$$

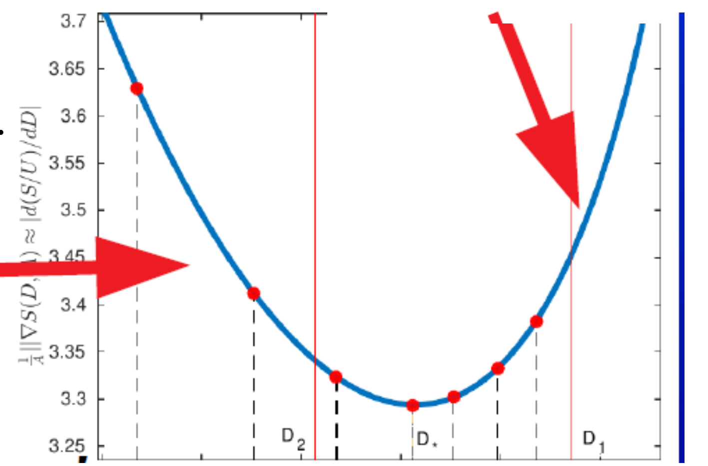
$$||\nabla S_Y|| > ||\nabla S_L)||$$

There exists **an intrinsic weakeness** inside
the initial structures of hte Y and L parties
that **favours and maximizes the emergence of a third party**.



$$||\nabla S_Y|| < ||\nabla S_L)||$$

maximum unpredictability



Conclussions

- **Initial conditions matter.**

- Hierarchically structured systems have an **anomalous entropy production.**

Published related Works

- Spontaneous emergence of a third position in an opinion formation model
Physica A. (2019) **521**, 501.

- Entropical analysis of an opinion formation model presenting a spontaneous third position emergence
Eur. Phys. J. B (2021), **94**, 89

- **On the role of structured initial conditions in the Schelling model.**
Physca A. **in press.**

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Marcos E. Gaudiano : marcosgaudiano@gmail.com

I invite you to participate in Marcos´s talk

Why are the borders of Palestine/Israel and Wallonia/Flanders so different?:
Entropic Analysis of a Schelling model with hierarchically structured initial conditions.

It will be held **on Tuesday 19 October at 11:30 hs.**

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Thank you for your attention!!!

Entropic analysis…

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