# Multifractal Study of Mobility in Argentina During the Pandemic

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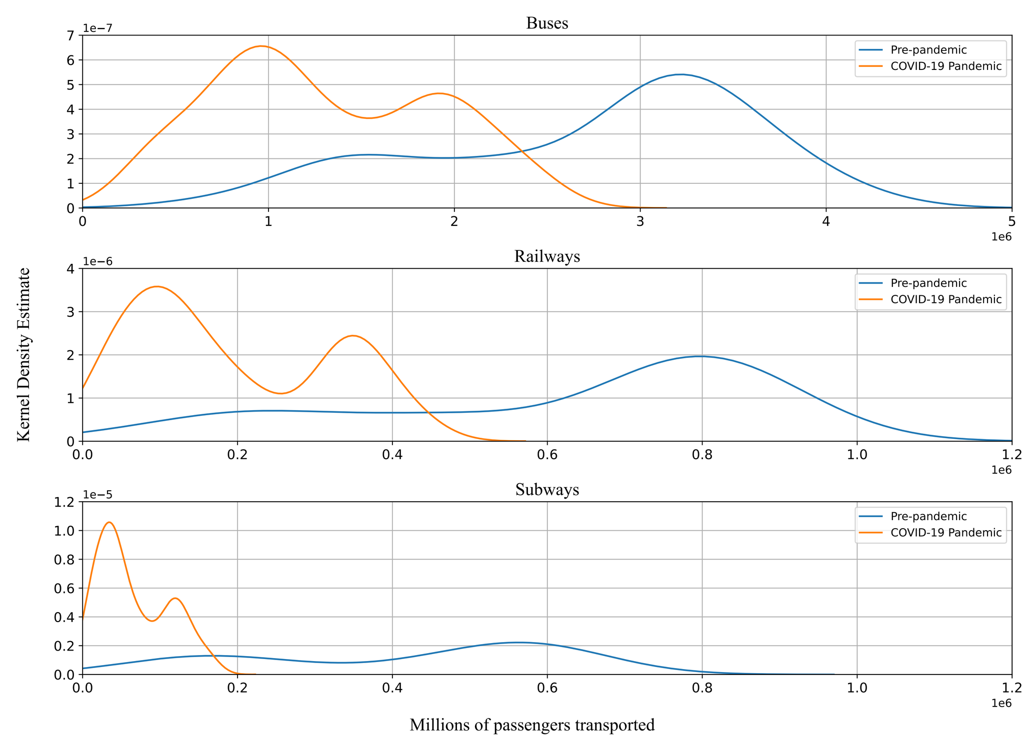
## Abstract

In Argentina, there is a great need for mobility mainly between the Autonomous City of Buenos Aires and its surroundings due to the distribution of population density in residential areas and the territorial distribution of jobs. The public transport system plays an important role to satisfy this need. For example, we can mention the bus lines, surface and underground railways. With the arrival of the COVID-19 pandemic, the demand was strongly modified, due to the restrictions on circulation imposed. This had a considerable impact on public transport in the region. In this work we apply the Detrended Cross Correlation Analysis (DCCA) [2] and the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) [1] to study the cross-correlation behavior between the number of users of public transport and the new cases of COVID-19 in Argentina during the pandemic. The analysis has verified the existence of cross-correlation between the two data series. The change of degree of cross correlation with time was studied and the results are interpreted qualitatively.

## Univariate Analysis

Figure shows the kernel density estimation plot for each of the means of transport studied.




## DCCA and MF-DCCA

MF-DCCA consists of dividing the previously integrated time series $X(k), Y(k)$ into $N_n$ segments of equal length $n$, and in each of them applying an ordinary linear regression to capture the local trend. The integrate series are then detrended by substracting the local trend from the data in each box and the detrended covariance is calculated as

$$F^2_{s,\nu}(n) = \frac{1}{s}\sum_{i=1}^{s}\left\{X\left[(\nu-1)s+i\right]-\tilde{X}_\nu(i)\right\}\left\{Y\left[(\nu-1)s+i\right]-\tilde{Y}_\nu(i)\right\} \quad (1)$$

While considering the average of all segments, the $q$th order fluctuation function of the detrended covariance is calculated:

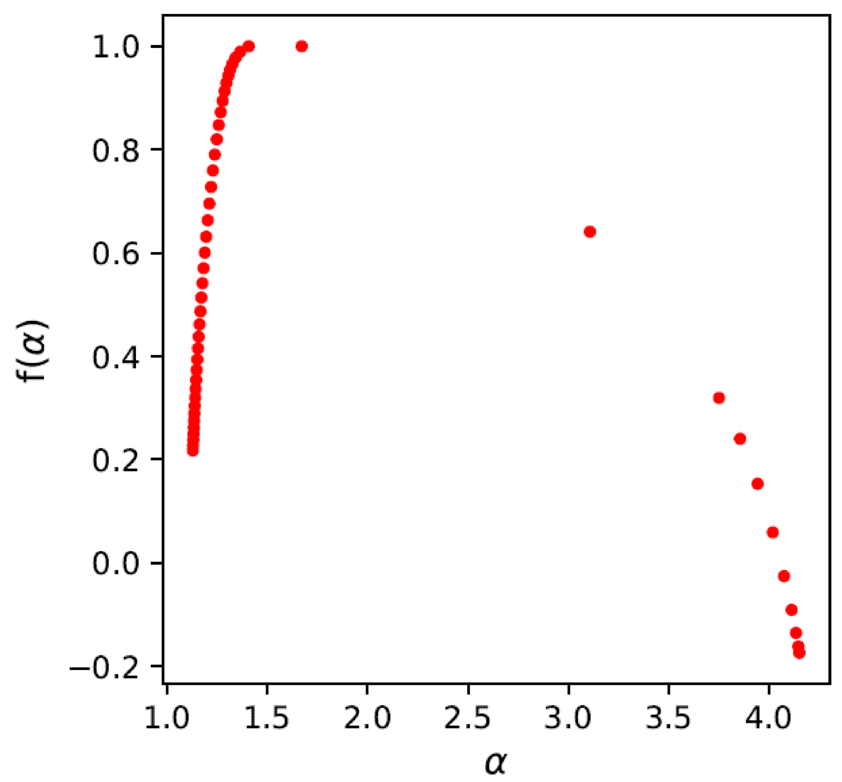
$$F_q(s) = \left\{\frac{1}{N_s}\sum_{v=1}^{N_s}\left[F(s,\nu)\right]^{q/2}\right\}^{1/q} \quad (2)$$

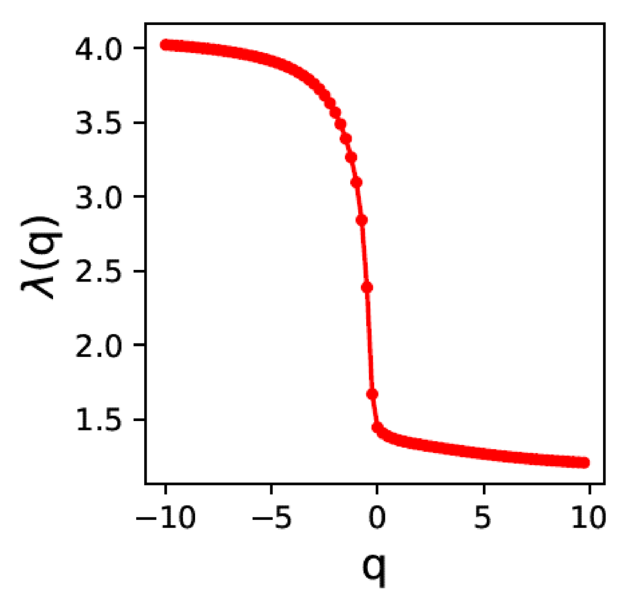
For $q = 2$, the standard DCCA is retrieved. A long-range correlation is inferred if the fluctuation function of the covariance is related to time scale via a power law, i.e.,

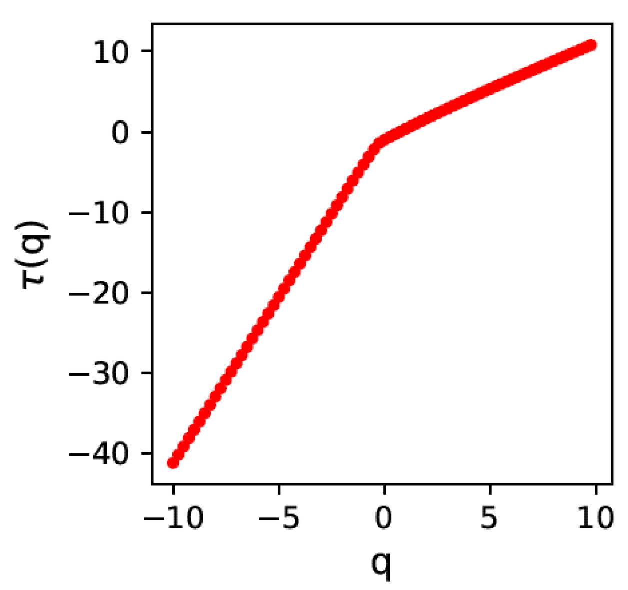
$$F_q(s) \sim s^h(q) \quad (3)$$

## Results: MF-DCCA

The plot of multifractal spectrum shows the well-known single-humped shape that characterizes multifractal behavior. The width of the spectrum ($\omega = 3.02$) confirms that the cross-correlation between transport and covid-19 is multifractal. The value of The asymmetry ($a_s = 0.15$) less than 1 indicates that the multifractal spectrum is right-skewed which, as we have already seen, is related to a relative dominance of the small fluctuations.

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The relationship of $\lambda(q)$ with $q$ is decreasing with values ranging from 4 to just over 1. That shows that cross-correlations between both series are multifractal. Furthermore, the fact that the exponents corresponding to negative values of $q$ are larger than those for positive values of $q$ indicates that the cross-correlated behavior of small fluctuations is more persistent than that of large fluctuations.

## Results: DCCA

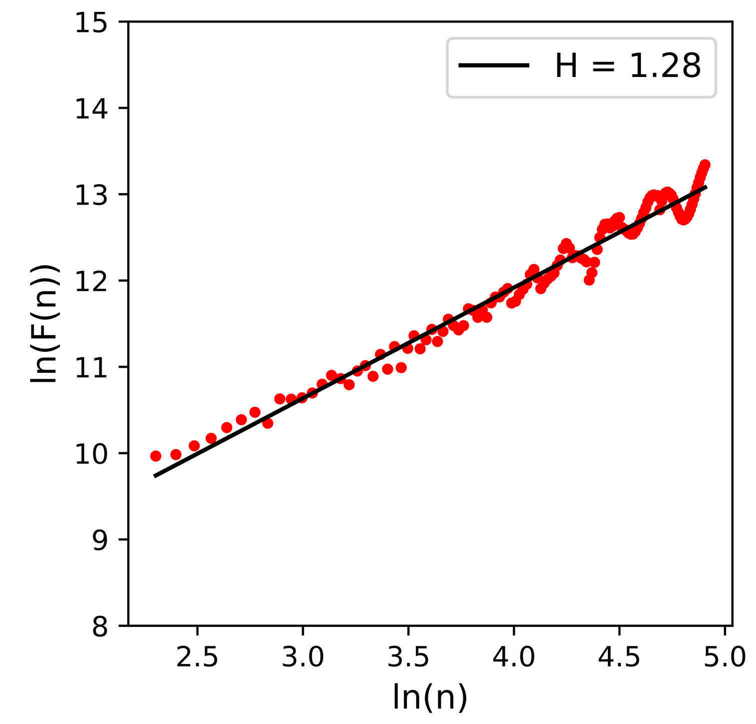


**Figure 1:** Fluctuation function given by DCCA

As can be seen in the figure the scaling behavior of $F_{DCCA}$ shows the presence of cross-correlations between the number of passengers and reported cases. The cross-correlation exponent $\lambda = 1.28$ reflects the presence of long-term cross-correlation between both time series.

## Conclusion

- The statistical analysis of the transport series revealed that during the pandemic their binomial character was accentuated, a fact that we attribute to the closure policies.
- DCCA allowed us to determine the presence of cross-correlations between both series-
- The MF-DCCA method shows that the correlation between the series has a strong multifractality. Moreover, the multifractal spectrum shows the dominance of the small fluctuations.

## References

[1] Wei-Xing Zhou. *Phys. Rev. E*, 77:066211, 2008.

[2] B. Podobnik and H. E.. Stanley. Article Title. *Phys. Rev. Lett.*, 100:1–11, 2008.