Optimal risk in a wealth exchange model: Agent dynamics from a microscopic perspective

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Optimal risk in wealth exchange models: Agent dynamics from a microscopic perspective

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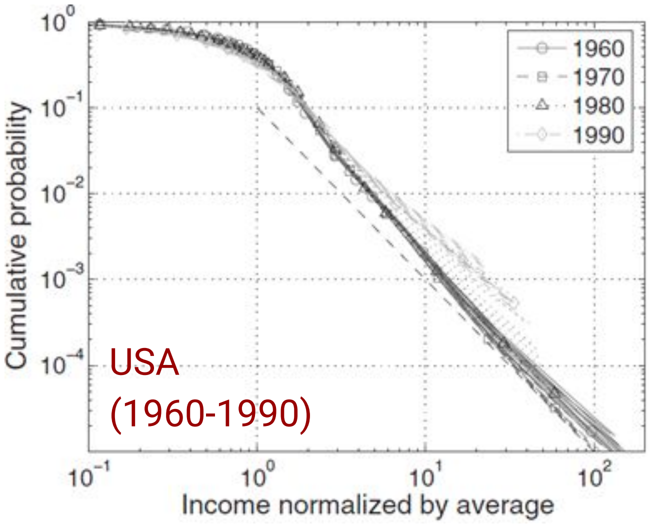
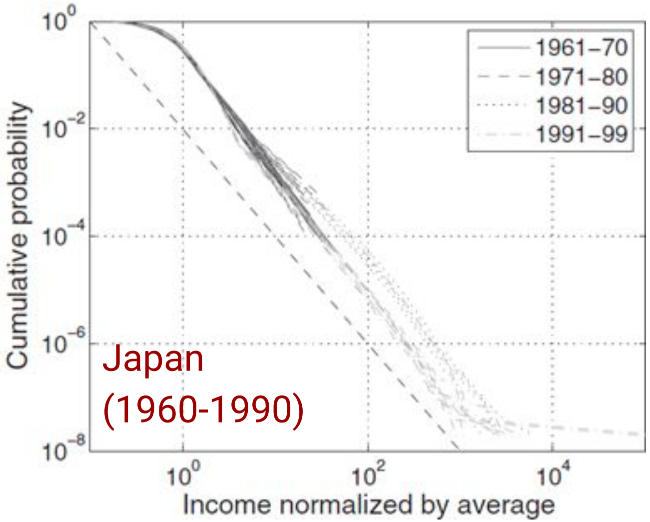
A B S T R A C T

In this work we present a new type of microscopic analysis for two well-known wealth exchange models that have been deeply studied at the macroscopic level for the past decades. This approach allowed us to study and classify the individual strategies carried out by the agents undergoing transactions. We analyze the role of their interaction parameter, the risk propensity, and find a critical risk such that agents with risk above that value always end up losing everything when the system approaches equilibrium. Moreover, we find that the wealth of the agents is maximum for a range of risk values that depend on particular characteristics of the model, such as the social protection factor. Our findings allow to determine a region of parameters for which the strategies of the economic agents are successful.

Wealth distributions

Pareto, V. Cours d'Economie Politique (1897)



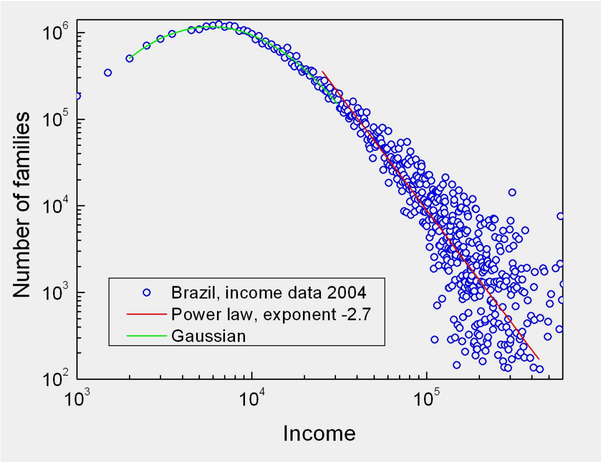


$$f_X(x) \propto \frac{1}{x^{\alpha+1}}$$

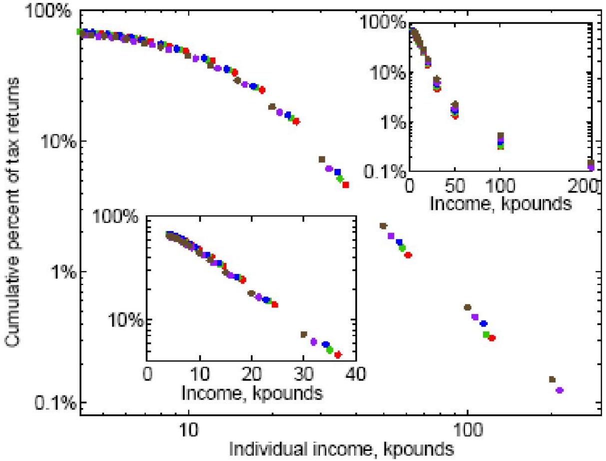
$\boldsymbol{\alpha}$ = Pareto exponent

Power laws only for the higher classes

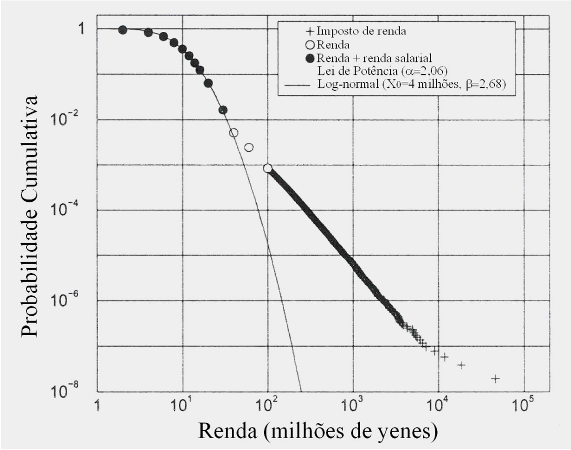
Brazil, 2004:
Gaussian + power law



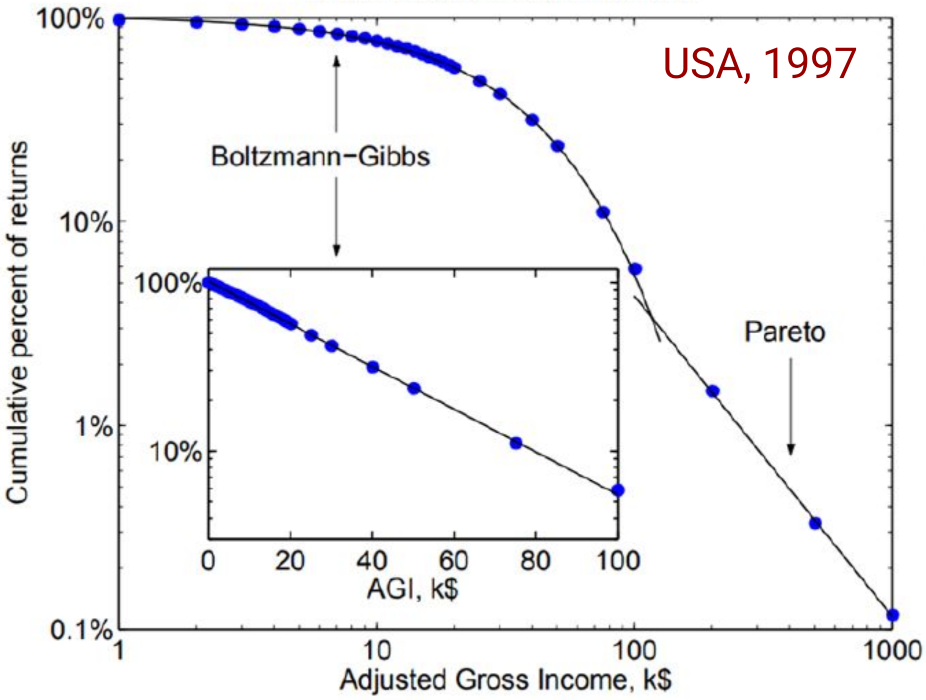
UK, 1994-1998:
Exponential + power law

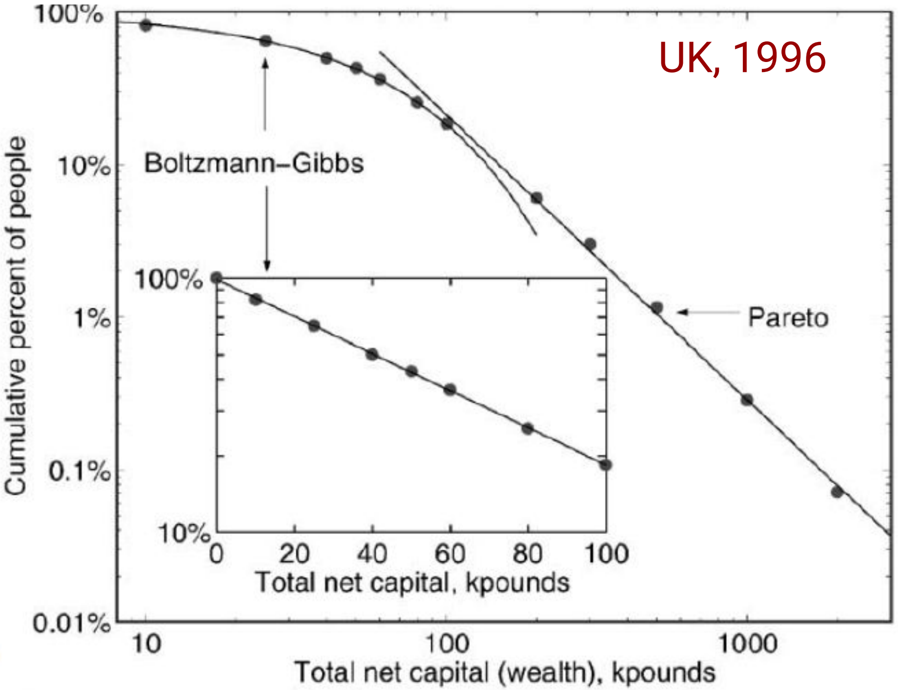


Japan, 1998:
Log-normal + power law



Power laws only for the higher classes





Several simple models based on ensembles of economic agents have been proposed for the past decades.

- ➢ Based on stochastic wealth flow processes between entities.
- ➢ Time evolution is given by certain trading rules.

Our purpose was to address a study regarding the microscopic aspects of multi-agent models,
in order to analyze whether the success or failure of economic agents are determined by the model parameters.

Yard-Sale Model

A. Chakraborti, Int. J. M Phys. C 13, 1315 (2002)
B. Hayes, Am. Sci. 90, 400 (2002)

$$\boxed{\begin{aligned} w_i(t+1) &= w_i(t) + (2\eta_{i,j} - 1)\Delta w_{i,j} \\ w_j(t+1) &= w_j(t) - (2\eta_{i,j} - 1)\Delta w_{i,j} \end{aligned}}$$

$\eta_{i,j} \in \{0, 1\}$

binary random variable

$$\boxed{\Delta w_{i,j} = \min(r_i w_i, r_j w_j)}$$

$w \in (0, 1)$

wealth (change)

$r \in (0, 1)$

risk factor (fixed)

determines agent behavior

Favoring the poorest: the social protection factor f

$$\eta_{i,j} \in \{0, 1\} \longleftarrow p_{i,j} = \frac{1}{2} + f \frac{|w_i - w_j|}{w_i + w_j}$$

$$f \in [0, \tfrac{1}{2}]$$

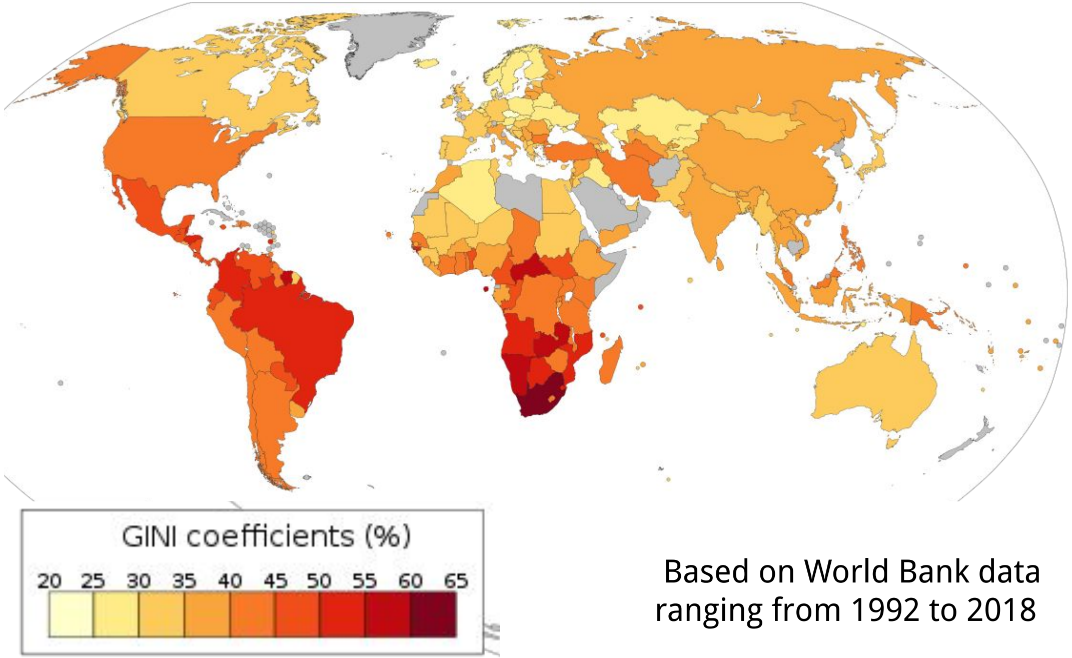
macroscopic equilibrium
highly dependent on f

Gini index: a measure of inequality

G → 0 everyone has the same wealth

G → 1 a single individual has the wealth of the whole system

$$G = \frac{1}{2N \sum_i w_i} \sum_{i=1}^{N} \sum_{j=1}^{N} |w_i - w_j|$$



Based on World Bank data ranging from 1992 to 2018

Simulations details

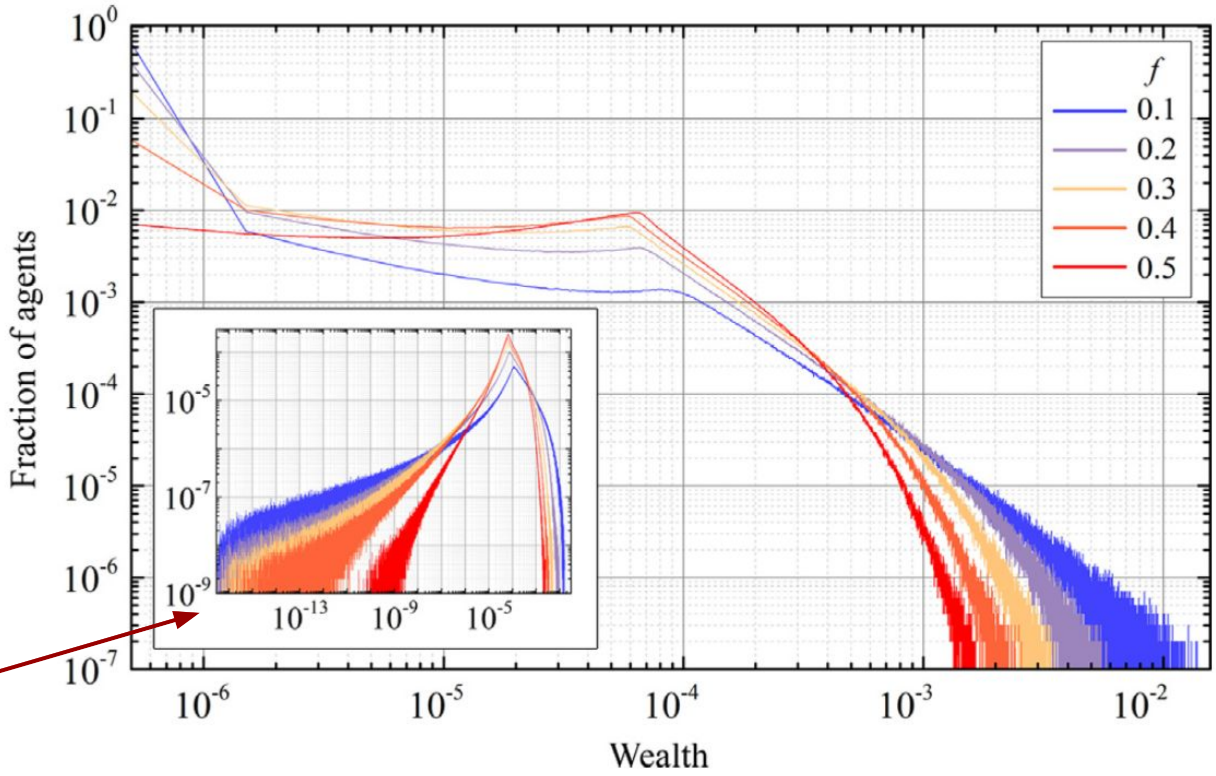
- Systems of $N = 10^4$ agents.
- Initial wealths and risks uniformly distributed in the interval [0, 1].
- Total amount of wealth fixed at $W_{tot} = 1$.
- Bankruptcy at $W_{min} = 3 \times 10^{-17}$ (agents cannot recover).
- Monte Carlo step (MCS) defined as N/2 transactions.
- 10^3 systems were evolved in parallel by using CUDA (GPU).
- The system evolves until a macroscopic equilibrium is reached.
- Equilibrium defined as the time needed for the Gini index to become constant (~ 10^4 MCS).

Wealth distributions in the stationary state

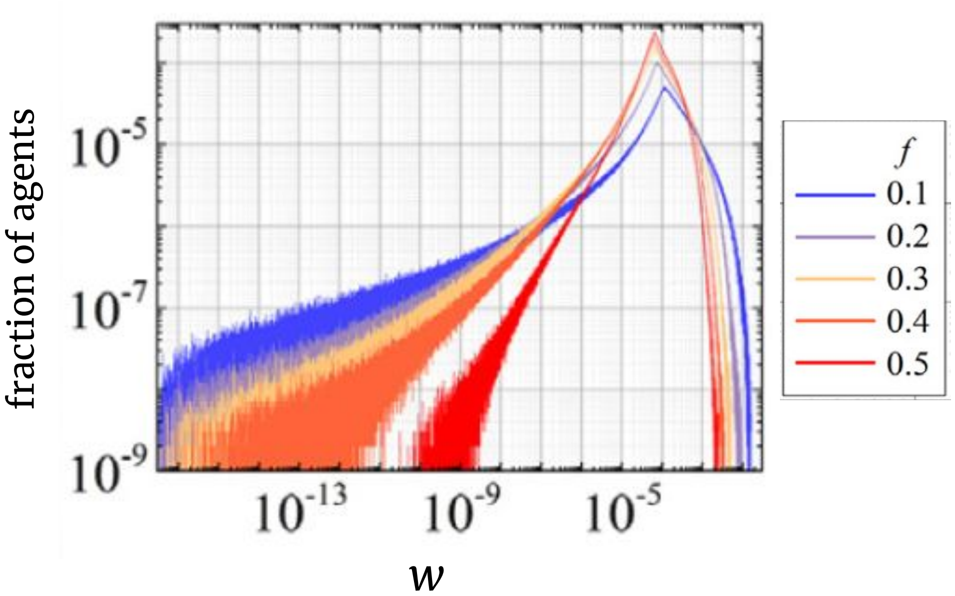
- Wealth distributions with exponential tails.
- Pareto distributions not observed.

The model replicates the behavior of the low and middle classes.

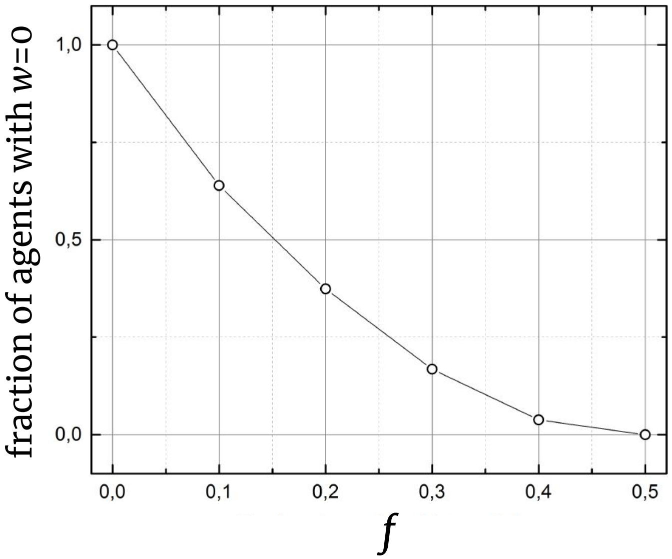
The gap between the rich and poor becomes smaller with increasing f



Agents out of the system and social protection factor



The gap between the rich and poor becomes smaller with increasing f

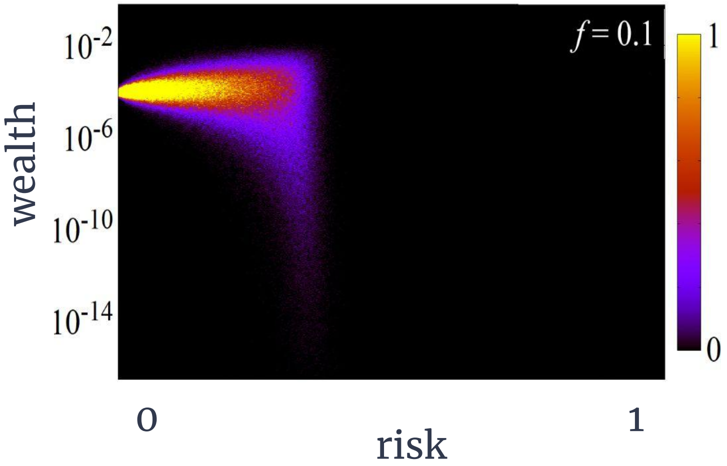


f = 0.5 no agents out of the system: the poorest have $w \gg W_{min} = 3 \times 10^{-17}$

Microscopic study: correlation between r and w

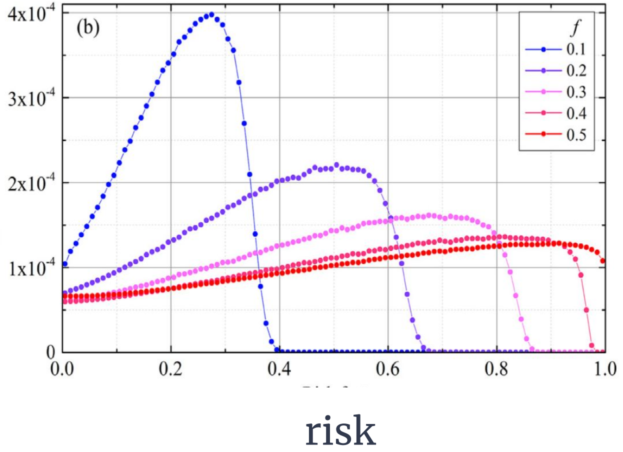
The critical risk

density maps at equilibrium

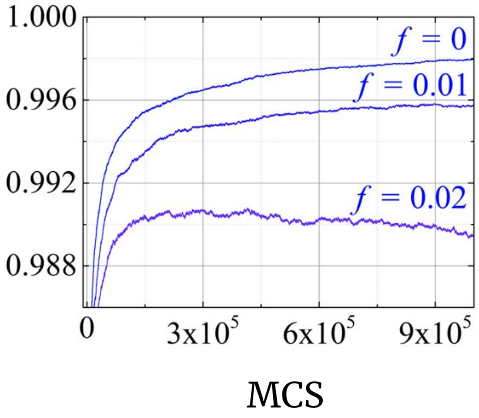


The optimal risk

best strategies



The maximum in the Gini index



Density plots in the r–w plane

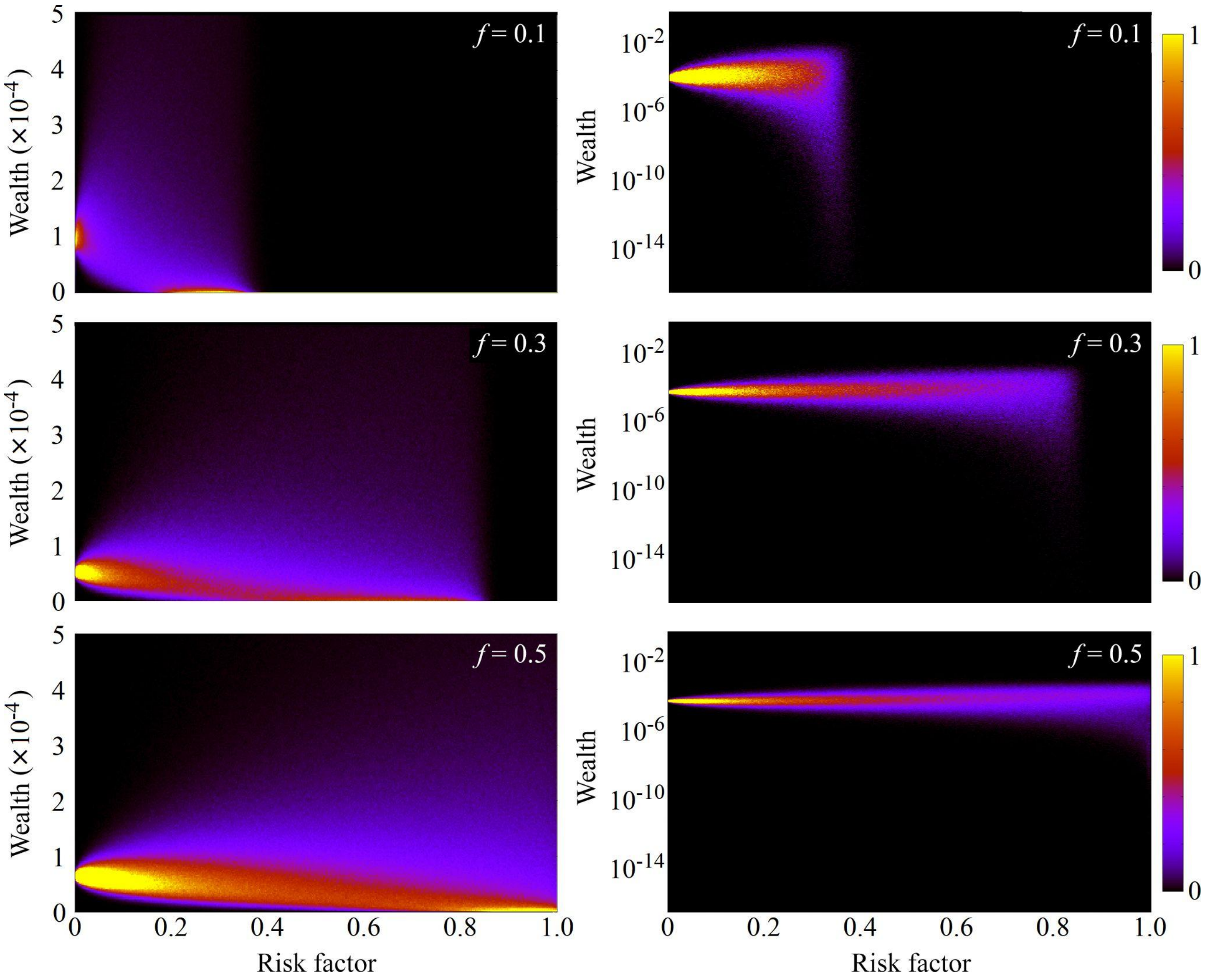
final state of 10^7 agents

Left: linear scale in the w axis.

Two regions of maximum density are observed.

Right: logarithmic scale in the w axis.

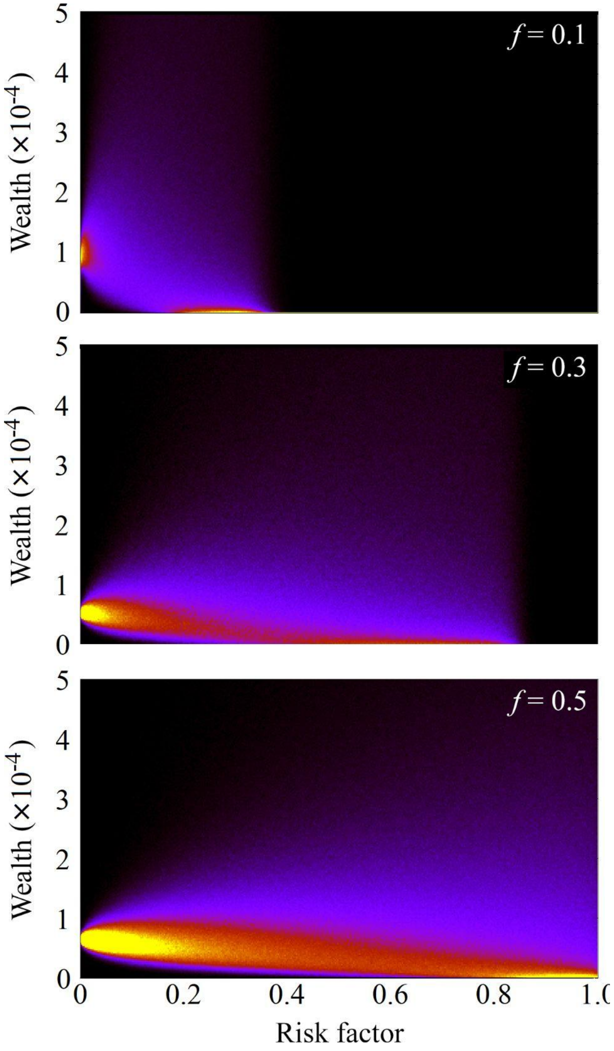
A critical risk $\langle r_{crit} \rangle$ is seen: $w = 0$ for any r > $\langle r_{crit} \rangle$



Density plots in the r–w plane

Left: linear scale in the w axis.

Two regions of maximum density are observed.



- agents with very low risk always preserve a considerable amount of wealth, but they will never become rich.
- agents with higher risks preserve some of their wealth, but not make good transactions.

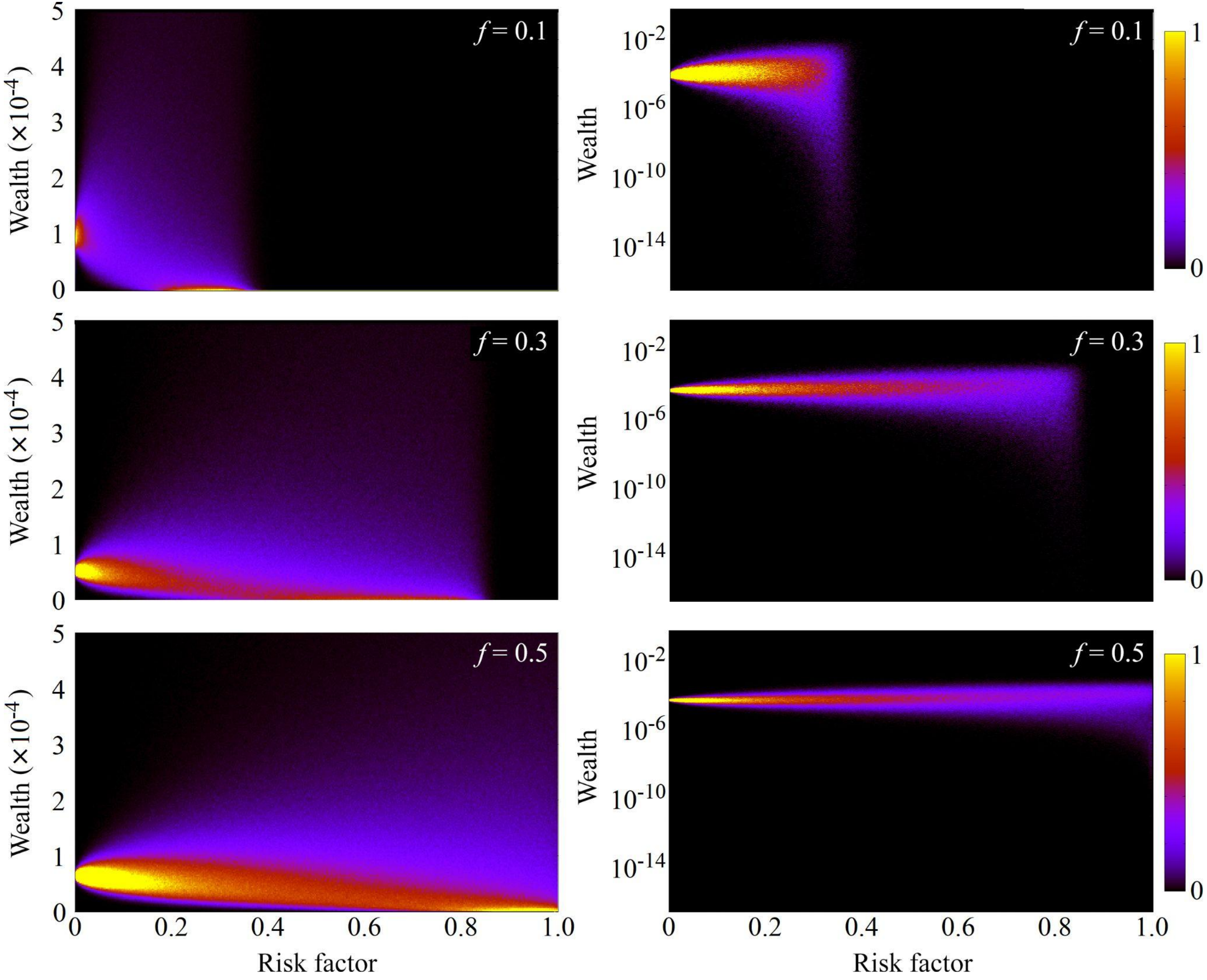
A few will be rich, but most will lose almost everything.

Density plots in the r–w plane

Right: logarithmic scale in the w axis.

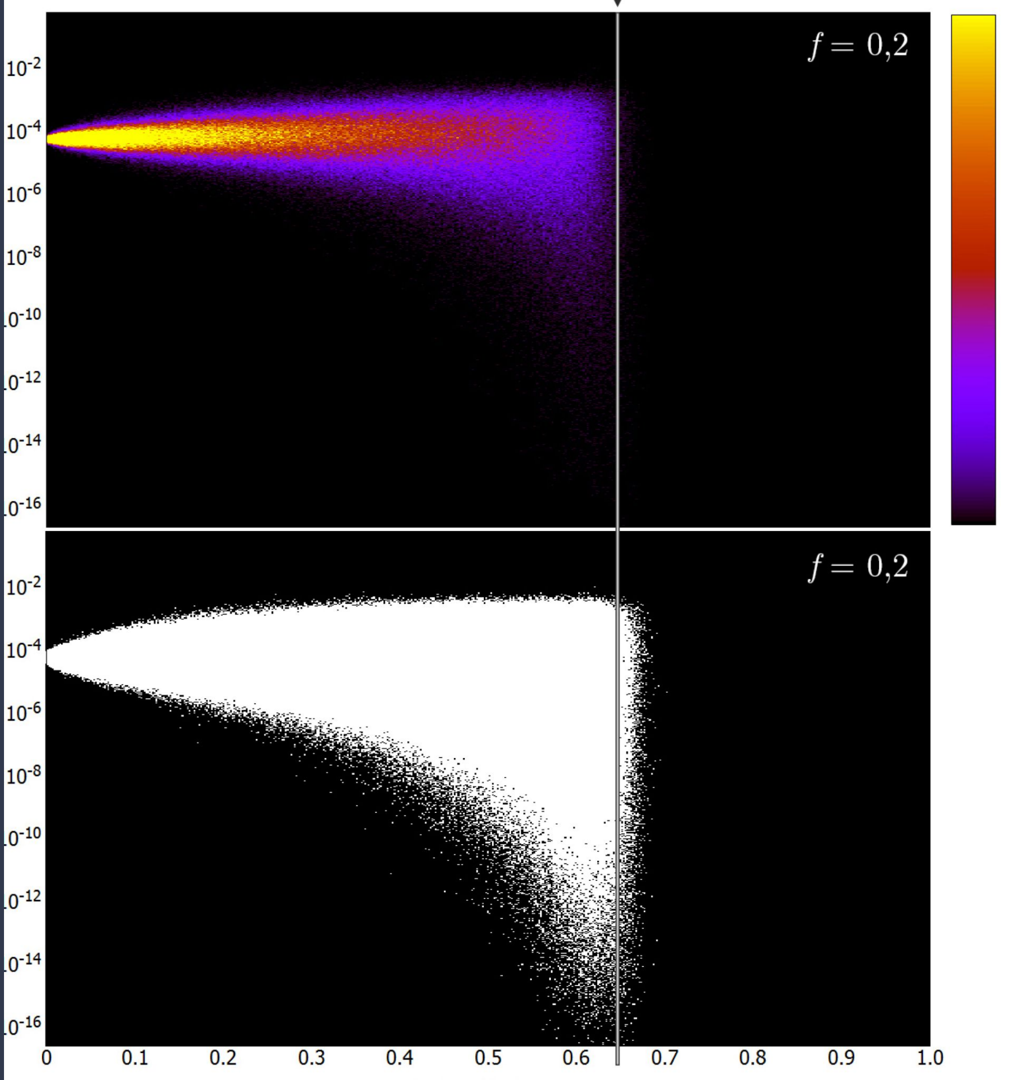
A critical risk $\langle r_{crit} \rangle$ is observed:

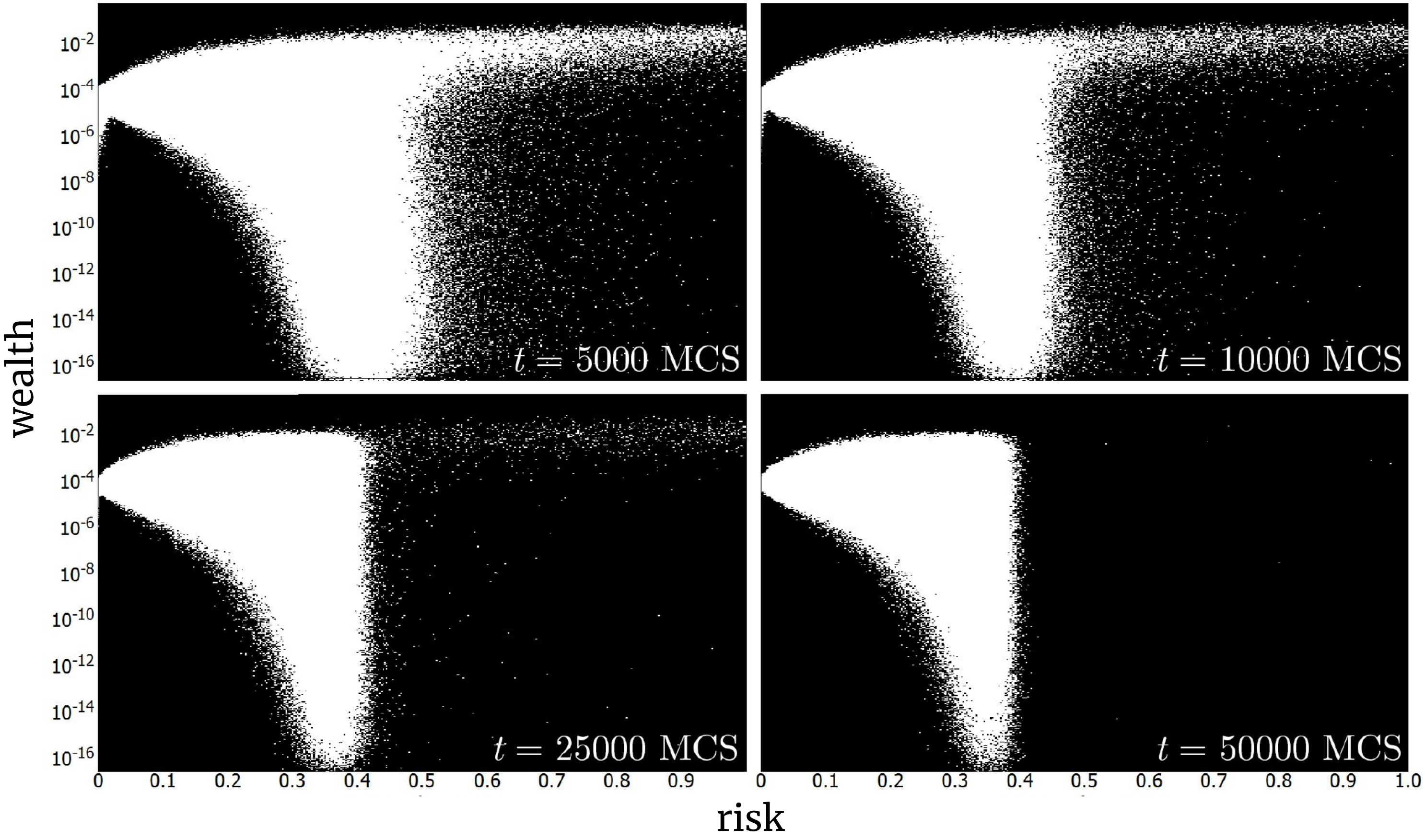
$w = 0$ for any $r > \langle r_{crit} \rangle$



For all *f*, there is a critical risk above which agents always end up losing all their wealth

for $f = 0.2$, $\langle r_{crit} \rangle = 0.64$

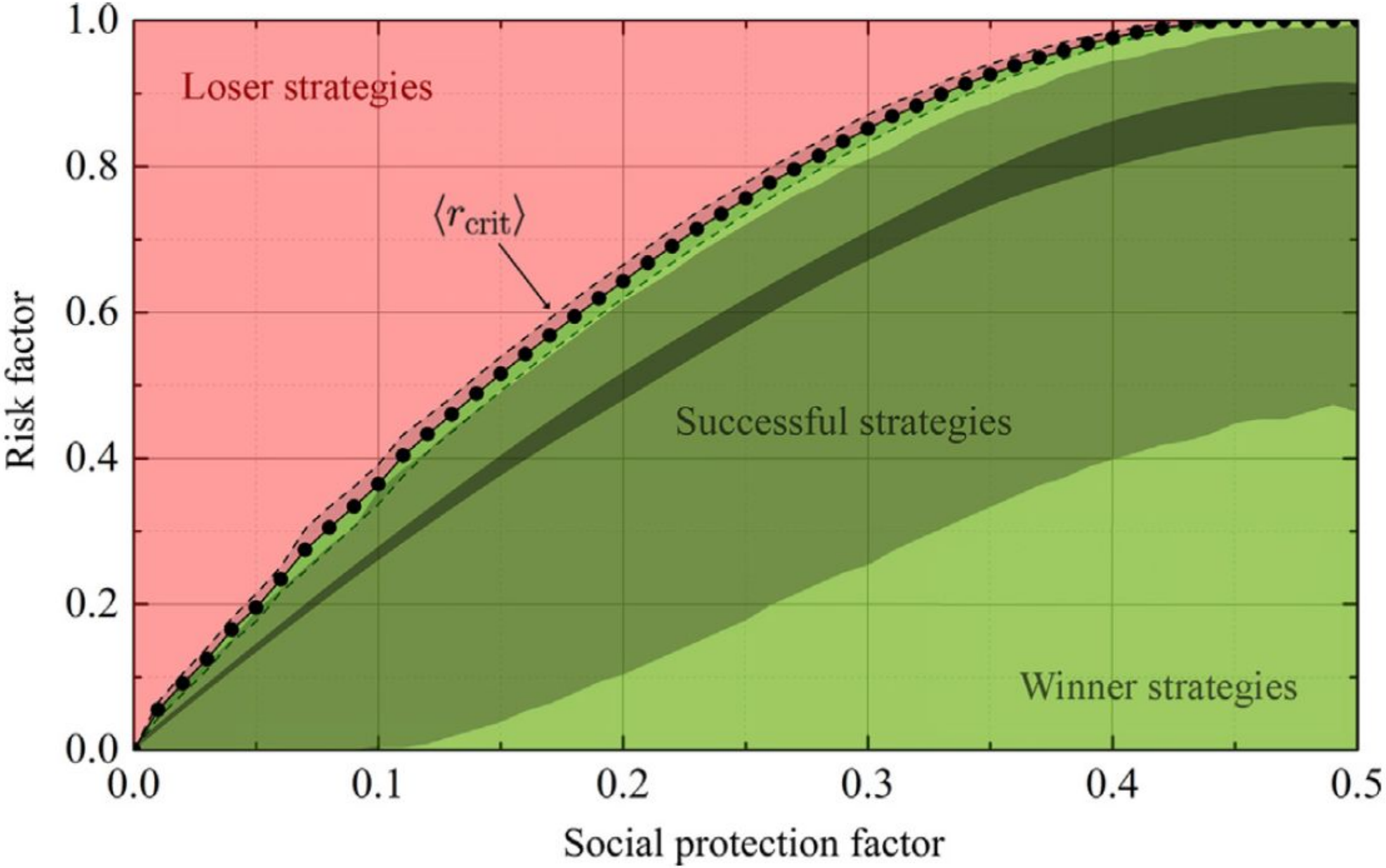




Agents with risk factors that are too high end up being left out of the system:

critical risk becomes apparent

Evolution of 1000 systems in the r - w plane, for f = 0.1



The critical risk increases with f: The social protection factor allows for risky agents to stay with $w > 0$.

This result does not depend on the size of the system: is an intrinsic property of the model.

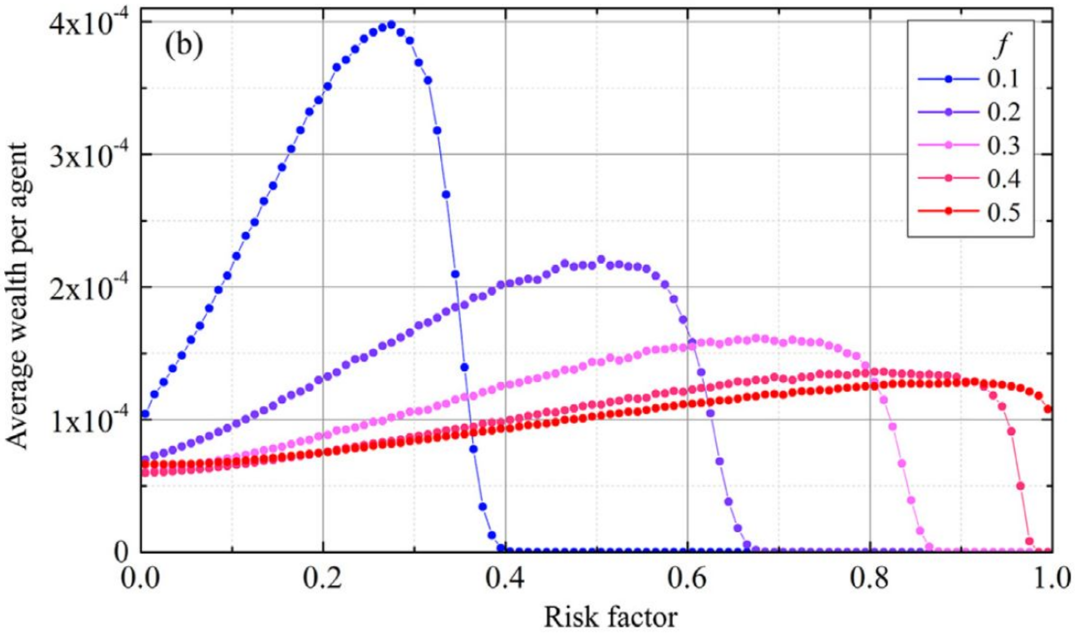
Average critical risk factor $\langle r_{crit} \rangle$ as a function of the social protection factor f

We have good and bad strategies, but …

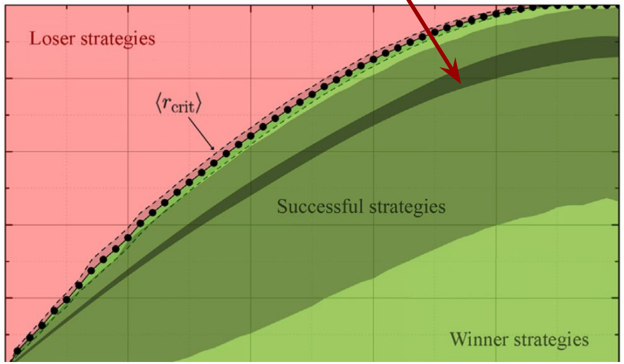
What are the best ones?

Optimal risk for $f > 0$

Average wealth per agent vs. risk factor in the equilibrium, 10^3 systems.

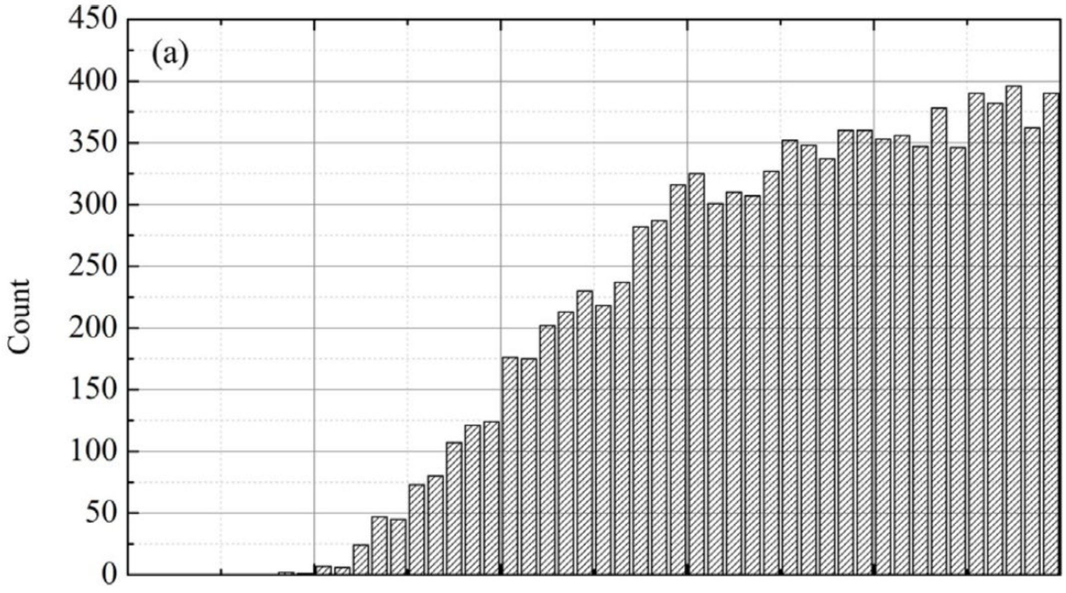


The presence of maxima show the existence of an optimal risk value



Optimal risk for $f = 0$

Risk histogram for the richest agent after 5×10^4 MCS, 10^4 systems



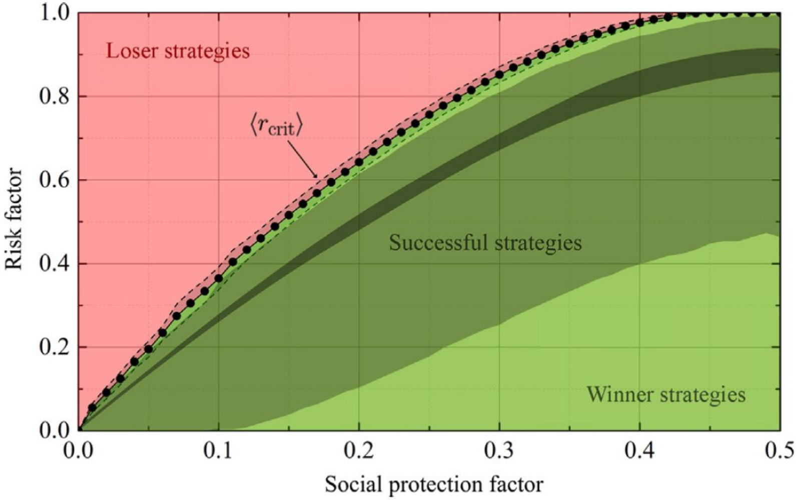
Winning strategy for this system:

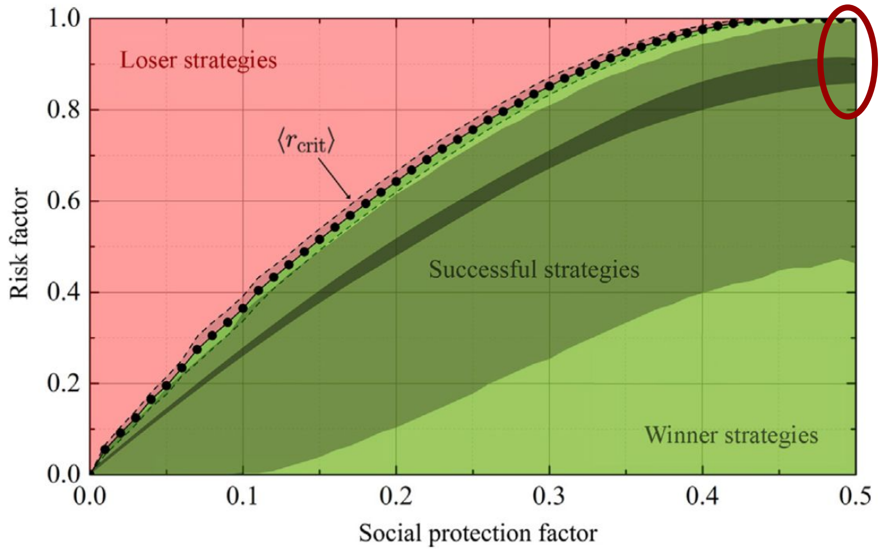
- ➢ Have a high risk, such that a high enough wealth can be achieved in a relatively small amount of transactions.
- ➢ Once high enough wealth is reached, interactions with the remaining agents will not put in risk the wealth of the richest agent.

- The microscopic dynamics of the agents unveils the existence of a critical risk, such that those agents with risks higher than this value will always end up losing everything when the system reaches equilibrium.

- This allowed for a classification of winner and loser strategies in this model, which depends on the values of the parameters f and r, and also turned out to be scale invariant.

- Another interpretation: It is possible to estimate how much wealth should be risked in different types of societies, characterized by their social protection factor f, to always obtain the maximum possible profit.

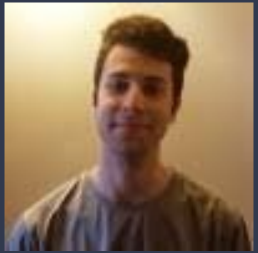




➢ Even in a society where the market regulation policies favor the poorest agents in every transaction as much as possible (f = 0.5), there will always be an individual optimal strategy that grants maximum average wealth.

In other words, in such a society there are no agents "outside the system" but there are still individuals who end up with more wealth than others, and they are those who have risks closer to the optimum.

Having found the existence of optimal strategies in this model, the next step was to provide the agents with rationality through neural networks, allowing them to change their risk factor to improve their performance.



Master thesis: Neural networks for the search for optimal strategies in econophysics problems.

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Wealth exchange models and machine learning: Finding optimal risk strategies in multiagent economic systems

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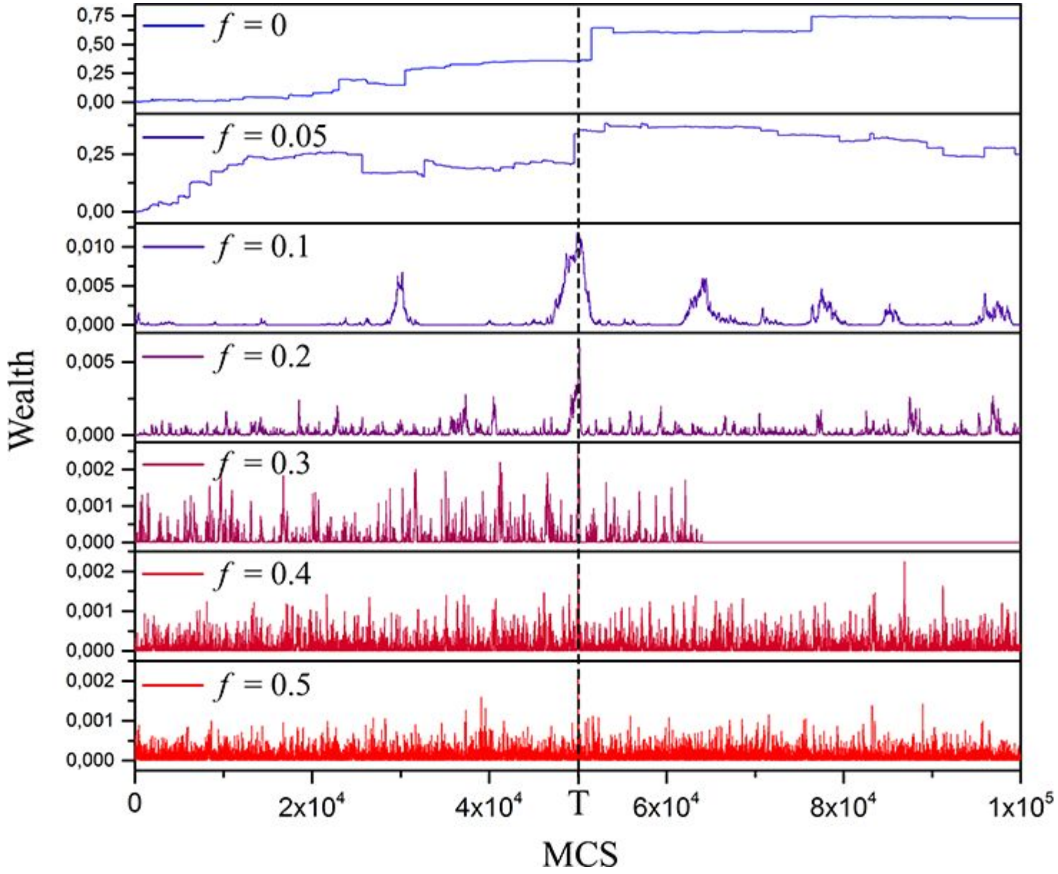
The *Yard-Sale* Model, a well known wealth exchange model whose observed macroscopic behavior hides many underlying aspects of particular complexity, was studied at the microscopic level. The performance of the agents during the successive transactions allows for the definition of successful or disadvantageous strategies according to the profit they achieve at the end of the process. Optimal strategies were found that maximize the individual wealth of each agent by performing their training through a genetic algorithm. The addition of different levels of rationality given by the amount of available information from their environment showed promising results, at both the microscopic and macroscopic levels. Remarkably, after the training process, the rational agents were able to determine when it would be convenient to interact with their opponents. Additionally, a region of parameters was found for which the distribution of wealth is a power law throughout the whole wealth range. As a general result, the incorporation of rational agents in this type of systems leads to greater inequality at the collective level.

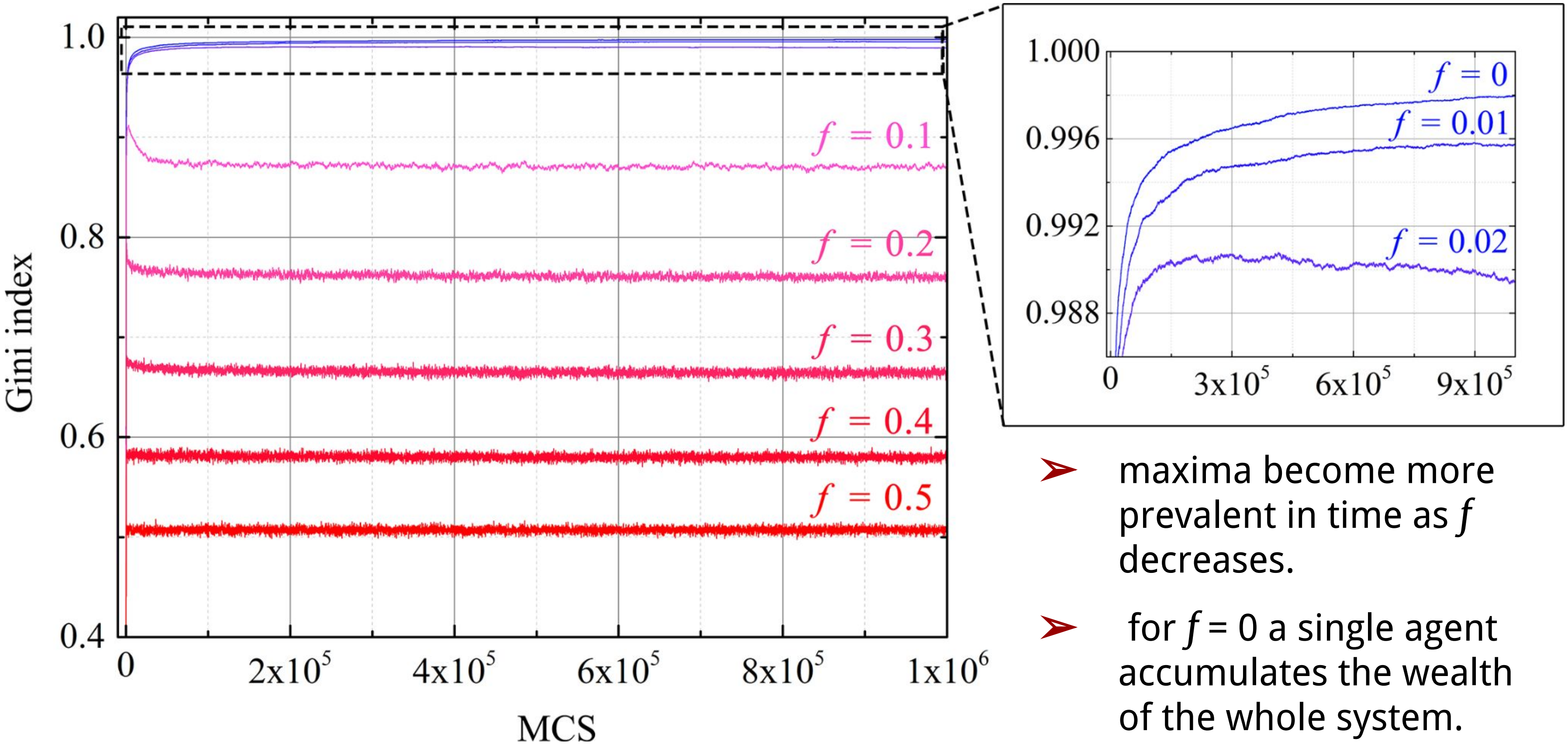
Thank you

- For $f \leq 0.1$, periods of richness and poverty can be distinguished.
- As f approaches higher values, w become erratic. Distinction between poor and rich agents becomes unclear.

We must be careful when interpreting macroscopic wealth distributions.



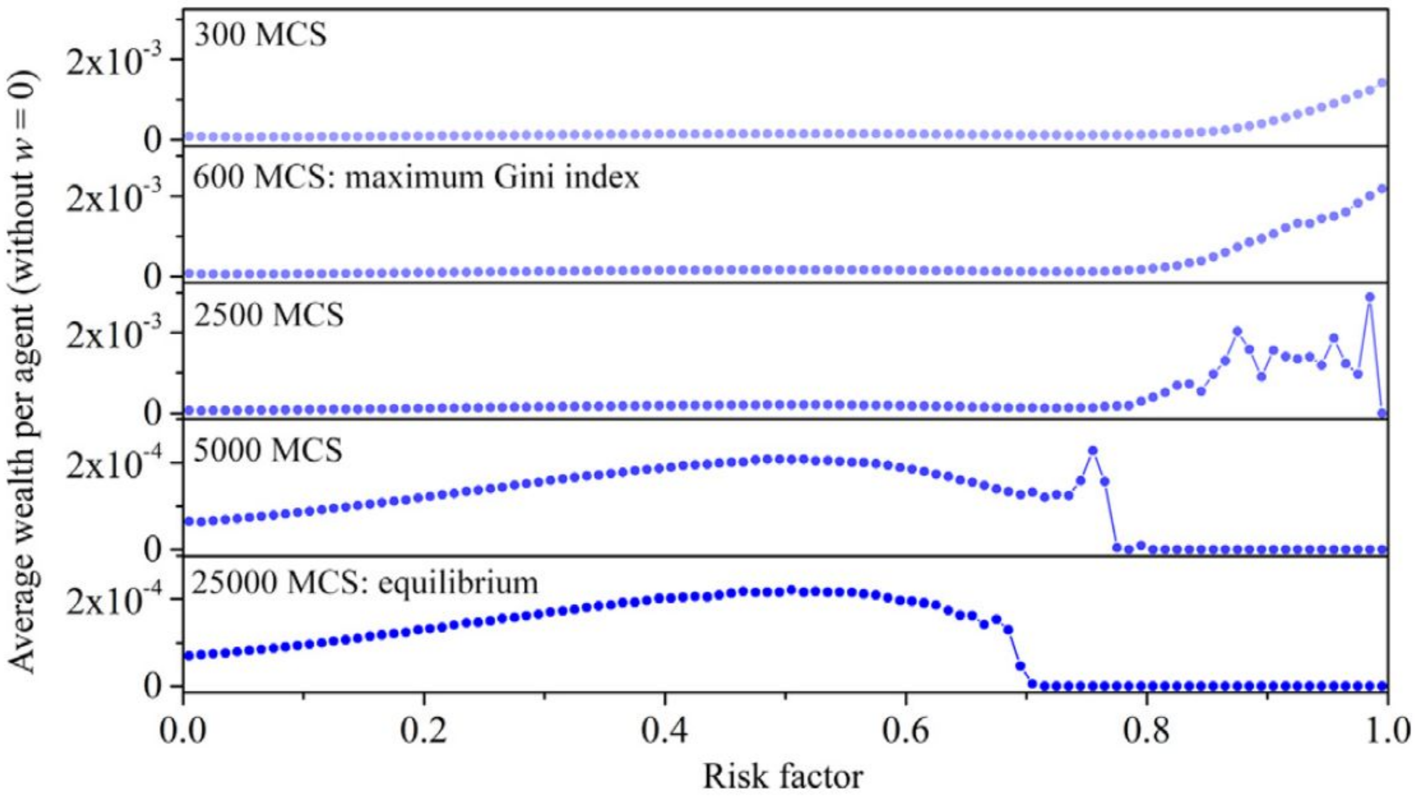
Transaction history of the richest agent at $T = 5 \times 10^4$ MCS



- maxima become more prevalent in time as *f* decreases.
- for *f* = 0 a single agent accumulates the wealth of the whole system.

Evolution of the Gini index

Cardoso, Gonçalves, Iglesias
Physica A 124201 (2020)



- A local maximum at $r \approx 0.5$ is present at all times (even if it is difficult to observe for shorter times).
- Higher risk agents accumulate most of the system wealth at the start of the simulation.

Average wealth per agent (10^3 systems) for $f = 0.2$

- The time evolution of the average wealth per agent, showed that agents with higher risks accumulate a high fraction of the total wealth in a relatively short time, but then end up losing everything, transferring their wealth to lower risk agents.

- The transaction histories of the wealthiest agents for systems with different social protection factors f showed how the social classes of the individuals cannot be defined when f is sufficiently high, and thus the position of the richest agent becomes irrelevant.

- However, a winner strategy could be determined depending on f : when $f = 0$, the probability of an agent ending up accumulating the wealth of the whole system increases with its risk. For $f > 0$, an optimal risk dependent of f could be found, such that the average wealth of the agents is maximized for all time.