Analytical approach to the Axelrod model based on similarity vectors

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AXELROD MODEL

Pairwise inteactions

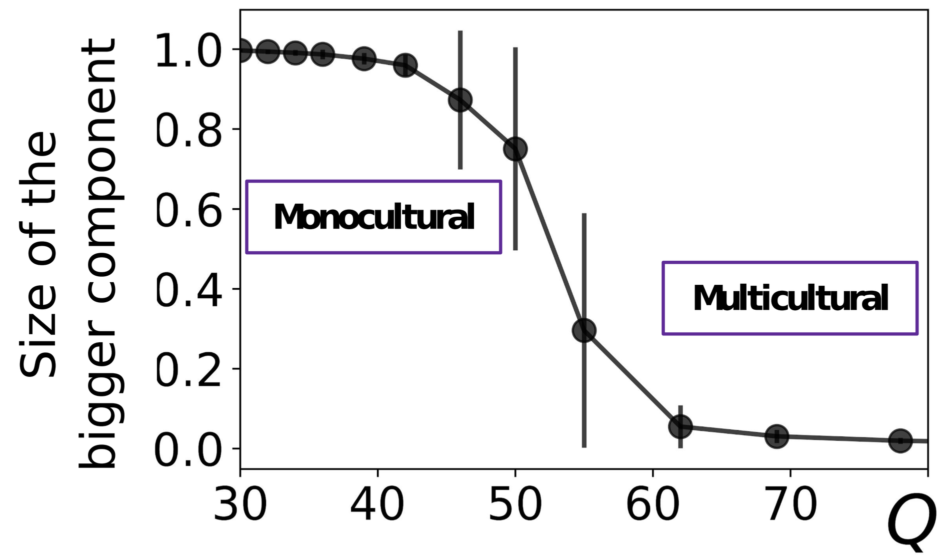
Copy a feature with probability h=S/F

Pameters

- N: #Agents
- F: #Features
- Q:#Number of features

How does this parameters play in the transition?

Non-equilibrium phase transition



R. Axelrod, The dissemination of culture: A model with local convergence and global polarization, Journal of conflict resolution 41, 203 (1997).

SIMILARITY VECTOR

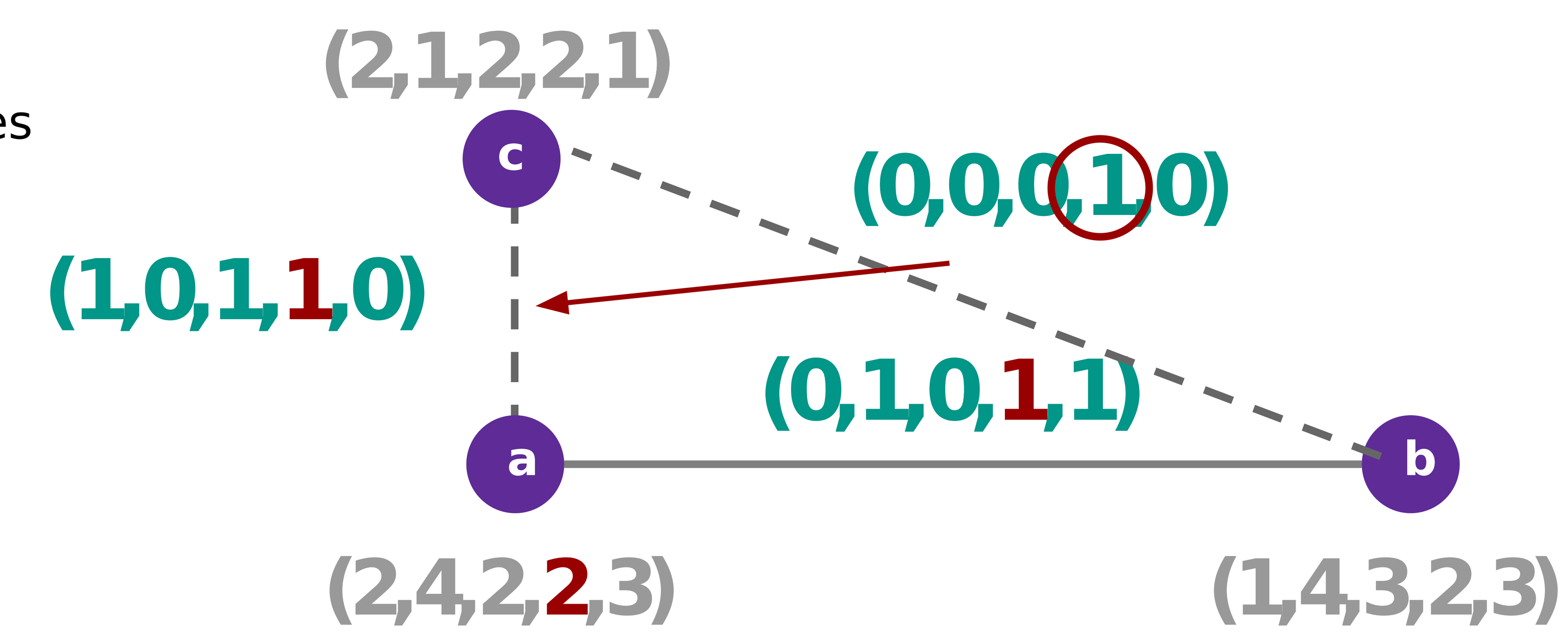
- F-dimensional binary vector.
- 1 if the agents share the cultural feature.
- 0 otherwise.

$$\frac{dP_i}{dt} = G(t) - L(t)$$

i-links — Gain: new posible i-links — Lost: Actual i-links posible chages

-Direct changes:
Chages related to a copy of a feature from one node to another

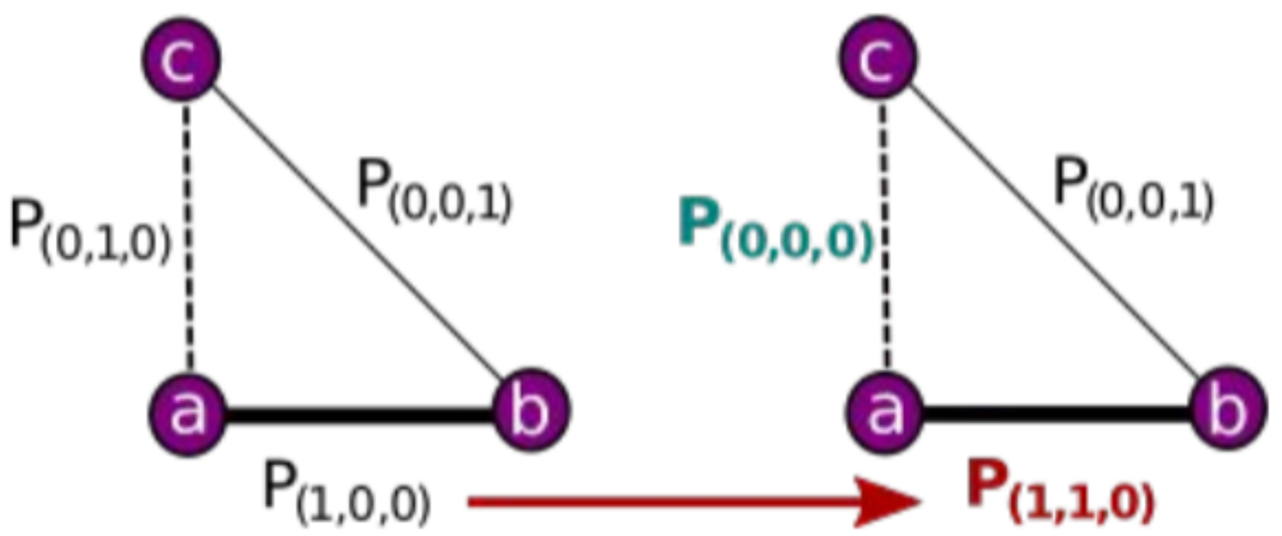
-Indirect changes
Coming from the updating of all the similarities involving the active agent. There are N-2 indirect changes in each interaction.



Q only sets the initial condition

MASTER EQUATIONS

Example F=3



Direct changes terms

$$\dot{P}_{(1,0,0)} \sim -\frac{1}{6}P_{(1,0,0)}$$

$$\dot{P}_{(0,0,0)} \sim \frac{1}{6}P_{(1,0,0)}$$

Indirect changes terms

$$\dot{P}_{(0,1,0)} \sim -\frac{1}{6}P_{(1,0,0)}P_{(0,1,0)}P_{(0,0,1)}$$

$$\dot{P}_{(0,0,0)} \sim \frac{1}{6}P_{(1,0,0)}P_{(0,1,0)}P_{(0,0,1)}$$

Equation

$$\frac{1}{\gamma}\frac{dP_0}{dt} = \frac{(N-2)}{27}\left(P_1^3 - 3P_0P_1P_2\right)$$

$$\frac{1}{\gamma}\frac{dP_1}{dt} = -\frac{P_1}{3} + \frac{(N-2)}{27}\left(-2P_1^3 - 3P_1^2P_3 + P_1P_2^2 + 6P_0P_1P_2\right)$$

$$\frac{1}{\gamma}\frac{dP_2}{dt} = \frac{P_1}{3} - \frac{2P_2}{3} + \frac{(N-2)}{27}\left(P_1^3 + 6P_1^2P_3 - 2P_1P_2^2 - 3P_0P_1P_2\right)$$

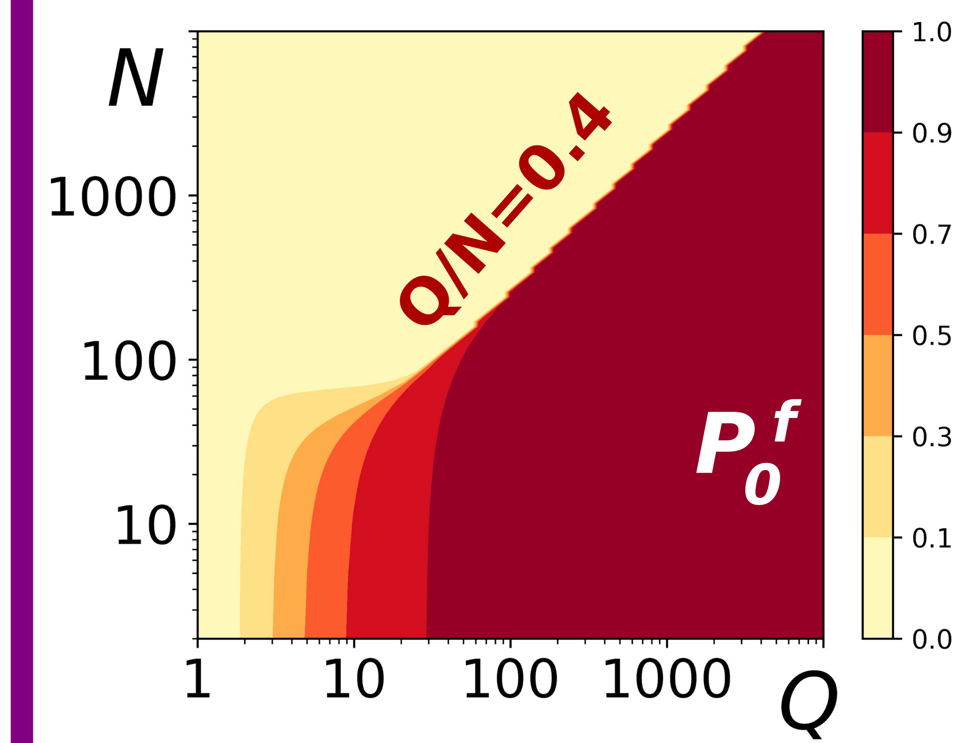
$$\frac{1}{\gamma}\frac{dP_3}{dt} = \frac{2P_2}{3} + \frac{(N-2)}{27}\left(-3P_1^2P_3 + P_1P_2^2\right)$$

$\gamma = \frac{\langle k \rangle}{N-1}$

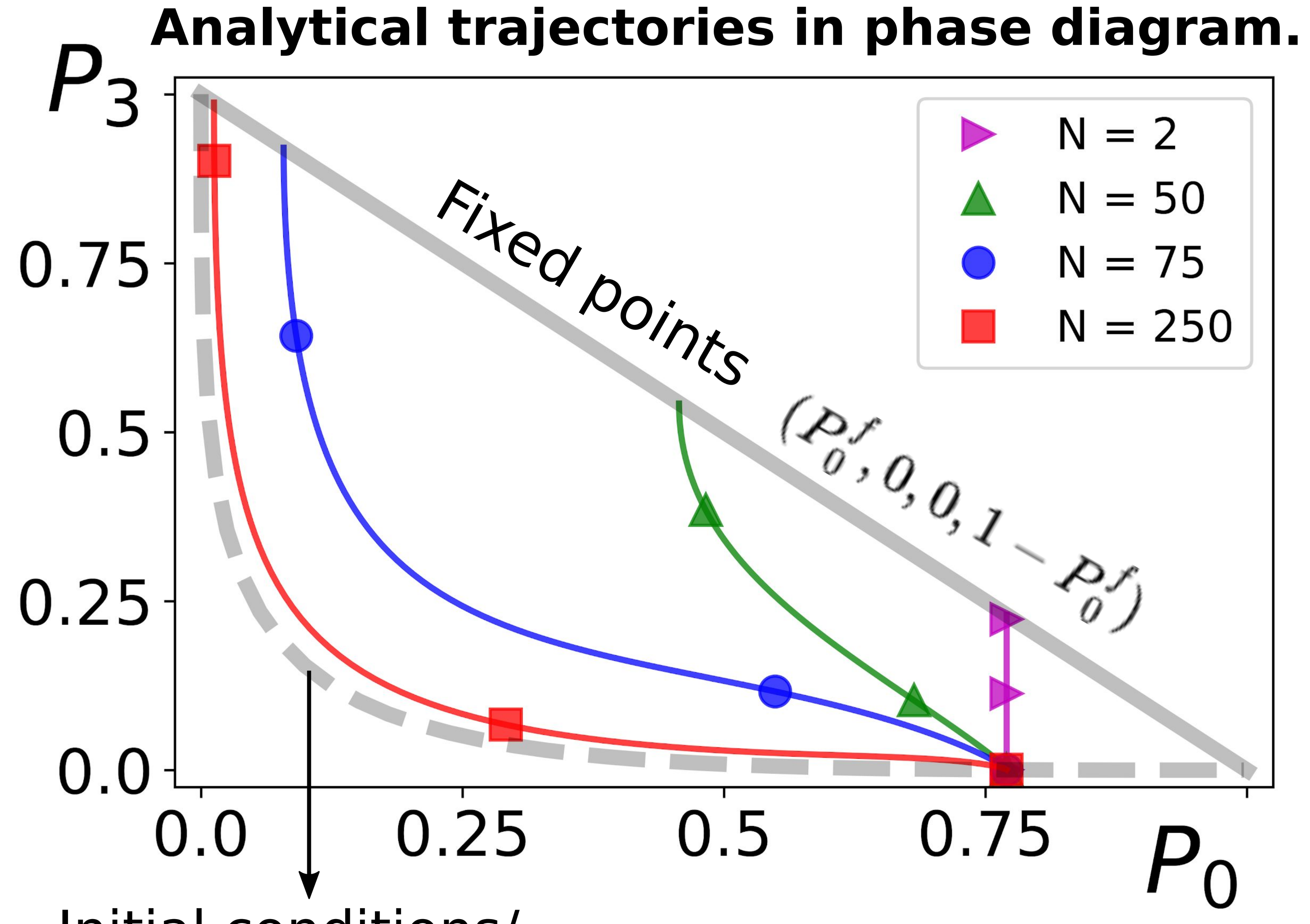
Linear terms Direct changes — **Non-lineal terms** Indirect changes

N couples the linear terms with cubic ones

TRANSITIONS



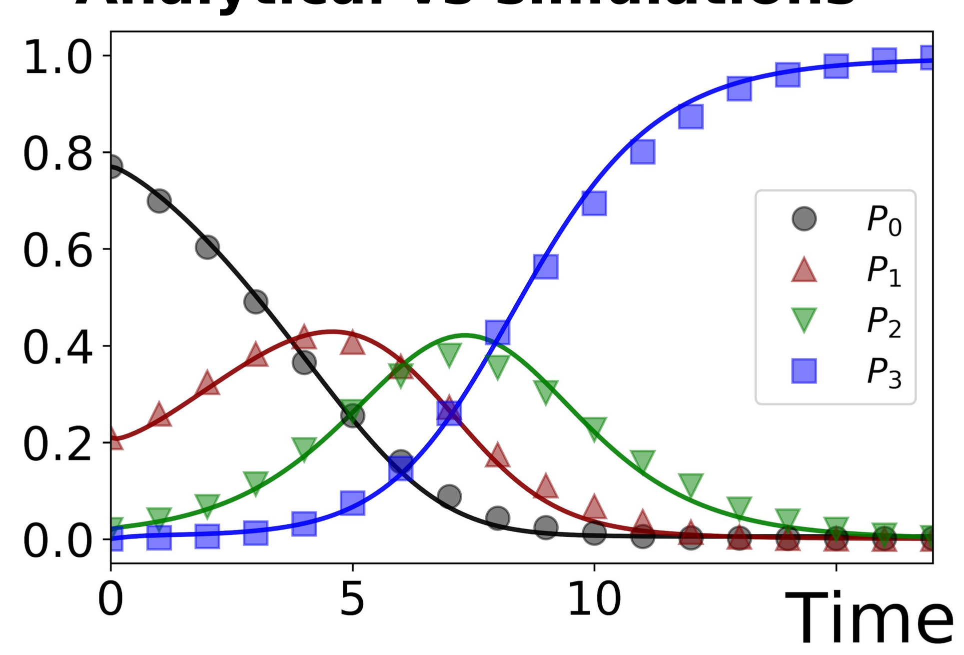
Analytical trajectories in phase diagram.



Initial conditions/
Non-lineal steady state

DINAMICS

Analytical vs simulations



F=3 agrees for little values of Q/N

The analysis accurately predicts for graphs with high mean degree

The transition depends of the initial value driven by Q and the decouple of the linal and cuadratic terms diven by N