Analytical approach to the Axelrod model based on similarity vectors
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**AXELROD MODEL**
Pairwise interactions

\[ [2,3,2,1,3] \quad [2,1,3,1,2] = \]  
Copy a feature with probability \( h = S/F \)

**Parameters**
- \( N \): #Agents
- \( F \): #Features
- \( Q \): #Number of features

**Non-equilibrium phase transition**

Gain: new possible i-links  
Lost: Actual i-links possible changes

- **Direct changes**:
  Chages related to a copy of a feature from one node to another
- **Indirect changes**:
  Coming from the updating of all the similarities involving the active agent. There are \( N-2 \) indirect changes in each interaction.

**SIMILARITY VECTOR**

\[ \frac{dP_i}{dt} = G(t) - L(t) \]

- **Forward transitions**
  \[ G(t) = \text{non-equilibrium phase transition} \]
- **Backward transitions**
  \[ L(t) = \text{size of the bigger component} \]

**F-dimensional binary vector.**
- 1 if the agents share the cultural feature.
- 0 otherwise.

**Example**

\[ (2,1,2,2,1) \quad (0,0,0,1,0) \]
\[ (1,0,1,1,0) \quad (0,1,0,1,1) \]
\[ (2,4,2,2,3) \quad (1,4,3,2,3) \]

**Q only sets the initial condition**

**MASTER EQUATIONS**

**Equation**

\[ \frac{dP_0}{\gamma} = \frac{(N - 2)}{27} \left( P_1^3 - 3P_1P_3P_2 \right) \]
\[ \frac{dP_1}{\gamma} = \frac{P_1}{3} + \frac{(N - 2)}{27} \left( -2P_1^2 - 3P_1^2P_3 + P_1P_2 + 3P_1P_2 - 3P_0P_1P_2 \right) \]
\[ \frac{dP_2}{\gamma} = \frac{2P_2}{3} + \frac{(N - 2)}{27} \left( P_3^3 + 6P_2P_3 - 2P_1P_2 - 3P_0P_1P_2 \right) \]
\[ \frac{dP_3}{\gamma} = \frac{2P_3}{3} + \frac{(N - 2)}{27} \left( -3P_1P_3 + P_1P_2 \right) \]

- **Linear terms**
  Direct changes
- **Non-linear terms**
  Indirect changes

**Fixed points**

- **Analytical trajectories in phase diagram.**
- **Initial conditions**
- **Non-linear steady state**

**TRANSITIONS**

- **Analytical vs simulations**
- **F=3 agrees for**
- **little values of Q/N**
- **The analysis accurately predicts for graphs with**
  - **high mean degree**

**DINAMICS**

- **Linear terms**
  Direct changes
- **Non-linear terms**
  Indirect changes

The transition depends of the initial value driven by \( Q \) and the decouple of the linal and quadratic terms driven by \( N \)