Deviations from the Majority: A Local Flip Model

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Summary
• Binary opinion/voting dynamics
• Contrarians do not adopt majority opinion
• Contrarian tendency depends on size of majority
• Phase transitions: number of fixed points and stability change

We study the effect of local distortions to the majority rule on the dynamics of opinions using the model of the Galam model. Each iteration of the local updates of opinion, the new model accounts for different probabilities of a local flip against the local majority as a function of the ratio of majority / minority within the discussing group. Depending on those probabilities, the model exhibits a wide variety of patterns for both monotonic and alternating dynamics. The cases of group sizes 3 and 5 are investigated in detail, and we find regimes that feature competition between three or five individuals for one of the opinions by the majority rule. This decision is carried to the next level of the hierarchy, where representatives form a group at the next level and for two opinions, their decision is passed to the next level, etc.

2. Model of opinion dynamics. The entire population random meets in groups of fixed size $r$. A discussion takes place in each group and the majority opinion is adopted by all members of the group. Next, the groups are broken up and the individuals randomly form new groups of size $r$ to resume discussion. This is repeated $n$ times.

Definition of the Model

For group size $r = 3$, the model is summarised by

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Group vote</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A A A</td>
<td>A</td>
<td>$(1 - b)p^3$</td>
</tr>
<tr>
<td>A A B</td>
<td>A</td>
<td>$a(1-p)^3$</td>
</tr>
<tr>
<td>A B B</td>
<td>A</td>
<td>$(1 - b)p (1 - p)^2$</td>
</tr>
<tr>
<td>B B B</td>
<td>B</td>
<td>$b(1-p)^3$</td>
</tr>
<tr>
<td>B B A</td>
<td>B</td>
<td>$(1 - b)p (1-p)^2$</td>
</tr>
</tbody>
</table>

For general $r$, there are flip parameters $a_{r/2}, \ldots, a_{r}$ with the subindex being the number of votes for $A$. Then the update equation is

$$R_{A}(\rho) = \sum_{i=r}^{\infty} \left( \frac{1}{i!} \cdot a_{i} \cdot (1-p)^{i-r} \cdot \sum_{k=0}^{r-i} \binom{r-i}{k} \cdot \rho^{k} \right).$$

Results

1. The unstable region $L$ with monotonic dynamics, given by the inequalities $1/3 < a < 1 - a$. This is a fixed point of the model.

2. The stable region $M_1$ with alternating dynamics, given by the inequalities $1/3 < a < 1 - a$. This is a fixed point of the model.

3. The stable region $M_2$ with alternating dynamics, given by the inequalities $1/3 < a < 1 - a$. This is a fixed point of the model.

4. The unstable region $H$ with alternating dynamics which lies above the line $b + 5/3 - a$.

We partition the parameter space into regions which exhibit either a single fixed point or fixed points, as well as a single point where every value $p \in [0, 1]$ is a fixed point.

1. The region in which every value $p \in [0, 1]$ is a fixed point is $L \cap M_1 \cap M_2$.

2. The region $F_1$ with only a single fixed point which is $1/2$ is given by the inequalities $1/3 < a < 1 - a$ and $b > 0$.

3. The region with three different fixed points is the complement of $F_1 \cup F_2 \cup F_3$. Here, the corner region around the origin and the $a$-axis excluding $(1/3, 0)$. The regimes of the model can now be determined by forming intersections of the regions given above.

1. The region $F_3$ is a regime of its own. Here the dynamics of the model are stationary.

2. The region $L \cap F_1$ is a regime characterised by having three fixed points. $1/2$ is unstable and there are two additional fixed points located at

$$p_{3} = \frac{1}{2} - \frac{1}{2} \sqrt{1 - 2a - 3b},$$

Note that in the fraction both the numerator and the denominators are positive. The points $p_{3}$ are attractors and their values are $1/3$ if and only if $b > 0$. The location not being at $1/3$, is a new feature when compared to $b = 0$ and similar to the contrarian model where $a > b > 0$.

3. The region $M_1 \cap F_1$ exhibits a single fixed point $1/2$ that is a global attractor. The dynamics of the model starting at any $p_{3}$ is monotonic convergence towards $1/2$.

4. The region $M_1 \cap F_3$ is the region where the $a$-axis with $a = 1/3$, has three fixed points, $0, 1/2, 1$, with $1/2$ being an attractor and $0$, $1$ repellers.

5. The region $M_1 \cap F_2$ also has a single fixed point $1/2$ that is a global attractor. However, the dynamics of the model are damped oscillations. Starting at any $p_{3}$, the orbits alternate around the limit $1/2$.

6. The region $H \cap F_1$ is characterised by having a single fixed point $1/2$ which acts as a repellor. The dynamics are oscillatory, so there are wild swings from one next to the next from large majorities for $A$ to large majorities of $B$ and back. The points $p_{3}$ and $p_{2}$ converge and their limits can be calculated explicitly.

7. The region $H \cap F_3$ is a regime of its own. Here the dynamics of the model are stationary.

8. The region $L \cap F_1$ is a regime characterised by having the three fixed points $0, 1/2, 1$. There is a period-doubling bifurcation when $b = 0$, then flips are only possible if the majority is not unanimous. We can analyse this model as the one for $r = 3$.

The regimes of this 2-parameter model are classified according to the stability of the universal fixed point $1/2$ and the number of fixed points of the update equation. As far as the stability of $1/2$ is concerned, we distinguish four regions:

1. The unstable region $L$ with monotonic dynamics, given by the inequalities $1/3 < a < 1 - a$. This is a fixed point of the model.

2. The stable region $M_1$ with alternating dynamics, given by the inequalities $1/3 < a < 1 - a$. This is a fixed point of the model.

3. The stable region $M_2$ with alternating dynamics, given by the inequalities $1/3 < a < 1 - a$. This is a fixed point of the model.

4. The unstable region $H$ with alternating dynamics which lies above the line $b + 5/3 - a$.

We can also compare 1-parameter models for $r = 5$ to the contrarian model introduced in [2]. We plot the update function $Q$ for the contrarian model and compare it to 1-parameter versions of the local flip model.

Scenarios Explained by the Model

The local flip model may provide a possible explanation of a series of social dynamics triggered by one or a few individuals acting against large local majorities, such as:

• At a party, initially people are standing around, having conversations. Then at some point in time, the first person starts dancing and others follow. After a short time, some fixed proportion of all people in attendance are dancing.

• The first person to applaud after a concert or a play will trigger others to join.

• A patient arrives at the waiting room of a hospital. If all or nearly all people already there face the same direction, chances are high the new arrival will do so as well. However, if a large proportion (but not necessarily close to half) faces a different direction, then new arrivals will select their orientation at random – some conforming to the majority, while others go against it. As more patients arrive, in the long run, the people will be evenly split, simply because there was no strong enough initial pressure to conform to the group’s behaviour.

• During a demonstration, one person starts throwing objects at the police and others close by follow suit. Rioters move among demonstrators and drag more and more people into violence.

• Sharing some confidential or sensational piece of information to convince others. This includes the spreading of violence.

References