Doubly Heavy Tetraquarks and other multiquarks in the quark model

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Critical Stability, Brazil, 2021

About critical stability

- ECT* (Trento) 1997
- Les Houches 2001
- ECT* Trento 2003
- Dresden 2005
- Erice 2008
- Erice 2011
- Santos 2014
- Dresden 2017
- Brazil 2021-22

Proceedings starting from 2001

Many thanks to Aksel Jensen and other co-organizers

About critical stability

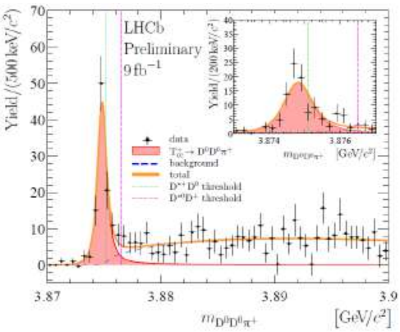
- Aims: Complementarity with big few-body conferences
- Interdisciplinary. Talks on nuclear physics, particle physics, atomic physics, quantum chemistry, solid state, mathematical physics, . . .
- Ample time for discussions in a pleasant surrounding
- Opportunity to discuss at length the onset of Efimov physics in our community

Table of contents

Based on recent or old work with J.-P. Ader, P. Taxil, J. Vijande, A. Valcarce, Cafer Ay, Hyam Rubinstein, S. Zouzou, C. Gignoux, B. Silvestre-Brac, M. Genovese, Fl. Stancu, J.-L. Ballot, . . .

Introduction

T_{cc}



Do narrow heavy multiquark states exist?

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(Received 11 August 1981)

We discuss the existence of states made of four heavy quarks in the context of poten-

Introduction

- Review the status of exotic hadrons
- Try to understand why multiquarks exist in specific configurations.
- Analogies and differences with respect to atomic physics
- In particular
  - $q_1 q_2 \bar{q}_3 \bar{q}_4$ vs. $m_1^+ m_2^+ m_3^- m_4^-$
  - Why Ps_2 $(e^+ e^+ e^- e^-)$ is stable if annihilation is disregarded, while $qq\bar{q}\bar{q}$ is unbound in a chromo-electric potential?

Experimental situation

- Rich history, mainly in the resonance sector
- i.e., concerning hadrons that can decay, but not too fast.
- Early pentaquarks (Z) in dubious KN analyses (early 60s).
- More recent θ^+ pentaquark (2003) ($B = 1$ and $S = +1$) and its partners. Not confirmed.
- Very recent hidden-charm pentaquarks $\bar{c}cqqq$ by LHCb, 2015...
- Baryonium mesons around 2 GeV preferentially coupled to baryon-antibaryons channels. Not confirmed in antiproton-induced experiments. Perhaps indication in heavy meson decays.
- Scalar mesons An abondant spectrum, leading to speculations: $q\bar{q}$ or $s\bar{s}$ orbitally excited, hybrid states $q\bar{q}g$, meson-meson molecules, tetraquarks $qq\bar{q}\bar{q}$ and all types of mixings.
- Dibaryons
- etc.

Experimental situation

- Most discussed:
- $X(3872)$ and partners: $c\bar{c}q\bar{q}$ states that do not decay immediately into $c\bar{c} + q\bar{q}$, and appear as metastable $c\bar{q} + \bar{q}c$ compounds. $X(3872)$ discovered in 2003 at Belle and confirmed in several experiments.
- $\Lambda(1405)$ baryon, too low with respect to naive quark-model predictions.
  Literature starts with Dalitz and Tuan in 1959.
  qqs vs $\bar{K}N$?
  Dual picture (Chew, ...) or double pole structure (Oset et al.)?

Phenomenological models

- Several interesting and innovative models to explain or even anticipate the exp. candidates
- Bags, strings, diquarks, etc.
- But should be examined critically
- Lack of rigor, e.g., clustering assumed, instead from being deduced!
- Tend to predict too many states, to explain a single one! (Pandora box syndrome)
- For instance, Frederickson et al. have set a warning against “demon-deuteron” states made of three diquarks
- Bag model, e.g., volume energy $4/3\,\pi R^3\,B$ increases less than N, the number of constituents, as the radius is slowly increasing.

Potential models

- Generic form

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m_i} - \frac{3}{16} \sum_{i<j} \left[\tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij}) + \frac{\tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j}{m_i\, m_j} v_{ss}(r_{ij}) \right] + \cdots \quad (1)$$

- where
  - $\cdots$ = spin-orbit, tensor,
  - $v(r)$ = quarkonium potential
  - $\tilde{\lambda}$ color operator (suitably changed for antiquarks)
  - v_{ss} short-range spin-spin
  - m_i = constituent mass

Potential models

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\, m_i} - \frac{3}{16} \sum_{i<j} \left[\tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij}) + \frac{\tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j}{m_i\, m_j} v_{ss}(r_{ij}) \right] + \cdots \qquad (1)$$

Questions about (1):

- Does (1) produce stable multiquarks?
- Does (1) produce resonances?
- Various corrections
  - String confinement
  - Relativistic kinematics
  - ...
- Do we need a careful handling of the 4-, 5-, 6-body problem?

Chromomagnetic binding

- $\sum \tilde\lambda_i.\tilde\lambda_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$ studied by Jaffe in the 70s for scalar mesons
- Again Jaffe demonstrated coherences of $\sum \tilde\lambda_i.\tilde\lambda_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$ for $H = uuddss$
- The H was searched for in many experiments, and not seen
- For instance ${}^{6}_{\Lambda\Lambda}\mathrm{He} \not\to \mathrm{He} + H$
- More precisely
  - For $N, \Lambda, \ldots, \delta = (-3/16) \sum \tilde\lambda_i.\tilde\lambda_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j = -3/2$
  - For $\Lambda\Lambda$ $\delta = -3$
  - For $\Delta - N$ $\delta = 3$ corresponding to 300 MeV
  - For H, minimal eigenvalue $\delta = -9/2$
  - Corresponding to 150 MeV below threshold
- IF
  - $\mathrm{SU(3)_F}$ flavor symmetry
  - Same $\langle v(r_{ij})\rangle$ in H and baryons

Chromomagnetic binding

- The H was discussed in dozens of paper
- $\mathrm{SU}(3)_\mathrm{F}$ breaking does not help
- $\langle v(r_{ij})\rangle$ smaller in H than in baryons
- Thus likely h unbound (Oka, Yazaki, Rosner, Karl, ...)
- But some recent lattice QCD finds it at the edge!
- Same mechanism and same conclusion for the 1987 pentaquark $\bar{Q}uuds$ (and usd permutations) by Gignoux et al. and Lipkin.
- Not conclusive search of this pentaquark at Fermilab

Chromoelectric binding

- What about

$$H=\sum_i \frac{\boldsymbol{p}_i^2}{2\,m_i}-\frac{3}{16}\sum_{i<j}\tilde{\lambda}_i.\tilde{\lambda}_j\,v(r_{ij})$$

- Which is relevant in the limit of large m_i?
- No obvious excess of attraction, as

$$-\frac{3}{16}\sum_{\text{mesons}}\tilde{\lambda}_1.\tilde{\lambda}_2=2=-\frac{3}{16}\sum_{i<j\in\text{ tetra}}\tilde{\lambda}_i.\tilde{\lambda}_j$$

- For equal masses, no binding, either for color $\bar{3}3$ or $6\bar{6}$ or mixing
- For unequal masses $QQ\bar{q}\bar{q}=MMmm$, binding if M/m large enough (or small enough). (Ader et al. 1981)

Chromoelectric binding

- Critical M/m depends on the potential (linear, Coulomb, etc.)
- In practice, critical M/m large!
- For $M \gg m$, color structure $([QQ]_{\bar{3}}[\bar{q}\bar{q}]_3)_1$ dominates
- Hence looks like $\bar{Q}'\bar{q}\bar{q}$ antibaryon (Lipkin, 1986)
- Sometimes referred to as "Heavy-quark-diquark symmetry"

Chromo-electric and -magnetic binding

- To get $T_{QQ} = QQ\bar{q}\bar{q}$ bound with realistic masses, one needs to combine CE and CM effects
- $QQ\bar{u}\bar{d}$ vs. $Q\bar{u} + Q\bar{d}$
  - Heavy-heavy chromoelectric attraction absent in the threshold
  - Light-light chromomagnetic attraction in $I = S = 0$ absent in the threshold
- $cc\bar{u}\bar{d}$ at the edge! (Janc & Rosina, Barnea et al., etc.)
- Till the almost bound T_{cc} discovered last summer!
- T_{bb} bound. Decays weakly. Large lifetime as compared to bbu.

Comparison with atomic physics

- What is the mechanism that binds $QQ\bar{q}\bar{q}$ when $M/m\ \nearrow$?
- Answer: same mechanism that makes ppe^-e^- more bound (in units of the threshold energy) than the positronium molecule $e^+e^+e^-e^-$ (internal annihilation disregarded)
- See Adamowski et al., Froehlich et al., review by Armour et al.

Solid State Communications Vol. 9, pp. 2037–2038, 1971. Pergamon Press. Printed in Great Britain

BINDING ENERGY OF THE BIEXCITONS

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BINDING ENERGY OF FOUR-PARTICLE COMPLEXES IN SEMICONDUCTORS

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Proof of Stability of the Hydrogen Molecule

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Stability of few-charge systems in quantum mechanics

E.A.G. Armour[a], J.-M. Richard[b,*], K. Varga[c,d]

Symmetry breaking

- Symmetry breaking lowers the ground state

$$H = H_0 + H_1 = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + x^2 + \lambda\, x \qquad \Longrightarrow \qquad E(\lambda) = 1 - \frac{\lambda^2}{4} < E(0)\,,$$

- More generally

$$H = H_0(\text{even}) + H_1(\text{odd}) \qquad \Longrightarrow \qquad E(H) < E(H_0)$$

for parity, charge conjugation, permutation, etc.

- But competition 4-body vs. threshold,
and often, the threshold benefits more!

Breaking particle identity

- Assume $\mu\mu\bar{\mu}\bar{\mu}$ is stable for some flavor independent interaction V
- Break particle identity in both sectors
- $Mm\bar{M}\bar{m}$ becomes unstable if $M/m \nearrow$
- QED
  - $\mu^+\mu^+\mu^-\mu^-$ stable
  - $M^+m^+M^-m^-$ unstable for $M/m \gtrsim 2.2$ (Bressanini, Varga, ...)
- Quark model with central forces
  - $q'q'\bar{q}'\bar{q}'$ unstable
  - $Qq\bar{Q}\bar{q}$ more and more unstable, vs. $Q\bar{Q} + q\bar{q}$
- However, some metastability can be envisaged, see
  - hydrogen-antihydrogen
  - $XYZ = (Q\bar{q}) + (\bar{Q}q)$

Breaking charge conjugation

- $(M^+M^+m^-m^-)$ vs. $(\mu^+\mu^+\mu^-\mu^-)$ with $2\,\mu^{-1} = M^{-1} + m^{-1}$
- Same threshold
- The decomposition $H = H_{\text{even}} + H_{\text{odd}}$

$$\frac{\boldsymbol{p}_1^2}{2M} + \frac{\boldsymbol{p}_2^2}{2M} + \frac{\boldsymbol{p}_3^2}{2m} + \frac{\boldsymbol{p}_4^2}{2m} + V = \left[\sum \frac{\boldsymbol{p}_i^2}{2\mu} + V\right] + \left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]$$

- Implies $E(H) < E(H_{\text{even}})$
- This explains why H_2 is more stable than Ps_2.
- Same reasoning holds for $QQ\bar{q}\bar{q}$ in a central interaction:
  - It starts unstable for $M = m$
  - It becomes stable if M/m large enough

The equal-mass case

- Interesting history
- 1945: Wheeler predicted the existence of $Ps_2 = e^+e^+e^-e^-$ as stable except for internal annihilation
- 1946 Øre (at Yale) concluded that it is unlikely
- 1947 Hylleraas and Øre demonstrated the stability
- Refined in more advanced variational calculations
- Erroneous criticism (Sharma, Phys. Rev. 171 (1986) 36)

> than the value obtained by Ore. There are four main reasons why our value differs so much from the value calculated by Ore. First, the Hamiltonian of the system was not transformed properly in Ref. 6 in order to eliminate the kinetic energy of the center of mass.

- The wf of Øre et al. is translation invariant!
- 2007 indirect experimental evidence for Ps_2

The equal-mass case

- Conflicting results also for $QQ\bar{Q}\bar{Q}$
- Simple QED $\rightarrow$ quarks extrapolation

$QQ\bar{Q}\bar{Q}$ STATES:
MASSES, PRODUCTION, AND DECAYS

Marek Karliner, Shmuel Nussinov, and Jonathan L. Rosner

existence of "dipositronium" thus implies that an analog di-quarkonium state exists

- Even for the simple NR, color-additive model

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m} - \frac{3}{16}\sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij})\ ,$$

  where v is the quarkonium potential, results differ!!!
- Vary et al., e.g., got binding
- Most 4-body calculations do not get binding!
- Why?

Ps_2 vs. tetraquark

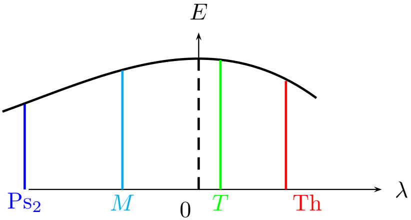
Meson-meson, atom-atom, Ps_2, tetraquark of frozen color given by

$$H = \sum \boldsymbol{p}_i^2/(2m) + \sum g_{ij} v(r_{ij}) \ . \quad \sum g_{ij} = 2 \quad v \text{ attrac.}$$

After suitable renumbering:

$$H = \sum \frac{\boldsymbol{p}_i^2}{2m} + \left(\frac{1}{3} - \lambda\right)[v_{12} + v_{23}] + \left(\frac{1}{3} + \frac{\lambda}{2}\right)[v_{13} + v_{14} + v_{23} + v_{24}] \ .$$

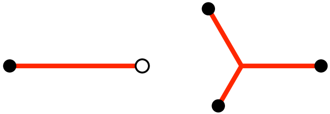
- Atomic physics Ps_2 vs. Threshold
- Quark model with frozen color $T = (\bar{3}, 3)$ or $M = (6, \bar{6})$



Tetraquarks penalized by the non-Abelian algebra!!!

Improved chromoelectric model

- Based on the string model
- Linear confinement interpreted as

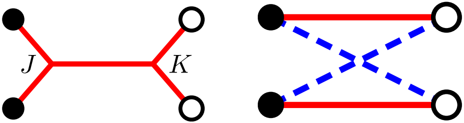


- Not very visible in baryon spectroscopy as compared to

$$V_{\text{conf}} = \frac{1}{2}(r_{12} + r_{23} + r_{31})$$

of the naive additive model.

- For tetraquarks, the minimum of



provides some extra attraction (Vijande et aL;, 2007, Bai et al., 2017). The connected diagram alone binds for $M \gg m$.

Relativistic kinematics

- Many relativistc effects
- If one concentrates on relativistic kinematics

$$\frac{\boldsymbol{p}^2}{2\,m} \to \sqrt{\boldsymbol{p}^2 + m^2} - m$$

- For given V, energy $\searrow$
- But more for twice 2-body than for 4-body
- Thus less binding

Outlook

- Binding? Strong competition between
  - Collective 4-body
  - $2 + 2$ threshold
- Interesting analogies between
  - 4 unit charges in atomic physics
  - tetraquarks in the quark model
- Better understanding of the role of symmetry breaking
- This is a new effect (chromo-electric) atop the more advertized chromomagnetic effect of Jaffe, . . .
- Thanks to the LHCb collaboration for discovering at CERN this state that was predicted 40 years ago at CERN

More shortly

Workshop

Double charm tetraquark and other exotics

in memoriam Gabriel Karl

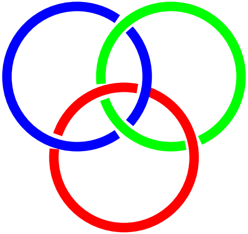
Lyon, November 22 & 23, 2021

https://indico.in2p3.fr/event/24937/page/2786-overview

The End

Backup slides

Borromean binding



- Obvious for tetraquarks: no $QQ\bar{q}$
- Cannot be built by adding the constituents one by one
- H_2 not Borromean, as $pe-$ and pe^-e^- are stable
- But $M^+m^+M^-m^-$ is Borromean for $M/m \sim 2$
- M^+m^- is stable, but none of the 3-body systems, such as $M^+m^+m^-$, is stable

Diquark approximation

- The Hamiltonian

$$H = \frac{\boldsymbol{p}_1^2}{2\,M} + \frac{\boldsymbol{p}_2^2}{2\,M} + \frac{\boldsymbol{p}_3^2}{2\,m} - \text{c.o.m} + v(r_{12}) + [v(r_{13}) + v(r_{23})] \;,$$

- is not very well approximated by

$$H' = \left[\frac{\boldsymbol{p}_x^2}{M} + v(x)\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + 2\,v(\sqrt{3}\,y/2)\right] \;,$$

with $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \;, \quad \boldsymbol{y} = (2\,\boldsymbol{r}_3 - \boldsymbol{r}_1 - \boldsymbol{r}_2)/\sqrt{3}$, which factorizes.

- The diquark internal energy is modified by the third quark.

Doubly-heavy baryons

- For instance, in the case of the harmonic oscillator this gives

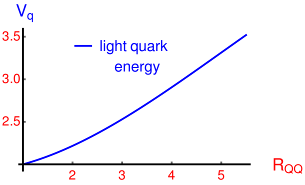
$$H' = \left[\frac{\boldsymbol{p}_x^2}{M} + x^2\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + \frac{3}{2}\,y^2\right] ,$$

instead of the exact

$$H = \left[\frac{\boldsymbol{p}_x^2}{M} + \frac{3}{2}\,x^2\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + \frac{3}{2}\,y^2\right] ,$$

- But the *Born-Oppenheimer* treatment is very good
- Especially if done in $\boldsymbol{y}$ at fixed $\boldsymbol{x}$, instead of $\boldsymbol{r}_3$ at fixed $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$
- For instance, with a linear potential, masses $M/m = 5$,
- $E_{\text{var}} = 4.940$ $E_{\text{BO}} = 4.938$ $E_{Dq} = 4.749$ (arbitrary units)

Doubly-heavy baryons



Born-Oppenheimer potential for (QQq), $M/m = 5$, $V \propto \sum_{i<j} r_{ij}$

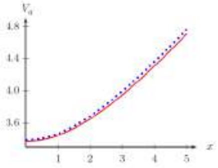
Fleck, R., PTP 82 (1989) 760

$(QQ\bar{q}\bar{q})$ Eichen-Quigg prescription

- Based on heavy-diquark–heavy-antiquark symmetry
- See Lipkin, Nussinov, …: analogies (QQq), (QQq) and $(QQ\bar{q}\bar{q})$
- But more quantitative (spin refinements omitted here)

$$(QQ\bar{q}\bar{q}) \stackrel{?}{=} (QQq) + (Qqq) - (Q\bar{q})$$

- Exact at $M/m \to \infty$ for our toy model H_T
- Overestimates $(QQ\bar{q}\bar{q})$ for finite M/m
- Linear case $m = 1$ and $M = 5$, lhs $= 4.362$ rhs $= 4.335$
- BO approach: exact at $R = 0$, but V_{BO} grows much faster for $(QQ\bar{q}\bar{q})$ than for (QQq) [shifted here by $(Qqq) - (Q\bar{q})$]

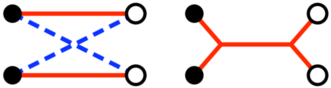


- Diff. vs. atomic physics for H_2^+ BO potential vs. H_2

String potential for $QQ\bar{Q}\bar{Q}$?

- Instead of $\propto \sum \tilde{\lambda}_i.\tilde{\lambda}_j r_{ij}$, use

$$V = \min\left\{ r_{13} + r_{24}, r_{14} + r_{23}, \min_{J,K}(r_{1J} + r_{2J} + r_{JK} + r_{K3} + r_{K4}) \right\} ,$$



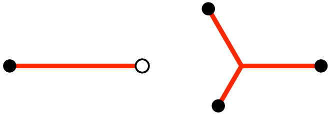
- Not so difficult (one does not need to compute the location of the junctions (Ay, R.,Rubinstein (2009), Bicudo et al.)
- gives more attraction (R., Vijande and Valcarce, 2007), and even binding for *equal masses* not submitted to the Pauli principle, say $(QQ'\bar{Q}\bar{Q}')$ with $M(Q) = M(Q')$ but $Q \neq Q'$.
- This restriction was forgotten in some recent papers

Summary for all-heavy

- $(cc\bar{c}\bar{c})$ and $(bb\bar{b}\bar{b})$ not bound in additive model nor in string-inspired variant
- Pity, would be suitable for J/ψ or Υ triggers.
- $(bb\bar{c}\bar{c})$ a little more favorable, mass ratio Q/q perhaps not large enough
- $(bc\bar{b}\bar{c})$ metastable, i.e., below its highest threshold, so a type of $(B_c\text{-}\bar{B}_c)$ molecule that can annihilate or rearrange itself into $(b\bar{b}) + (c\bar{c})$

Improved chromoelectric model

- Based on the string model
- Linear confinement interpreted as



- Not very visible in baryon spectroscopy as compared to

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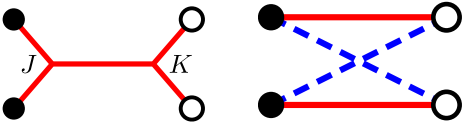
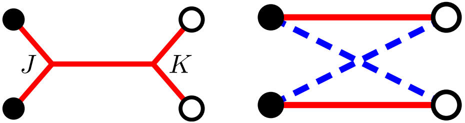
of the naive additive model.

- For tetraquarks, the minimum of



provides some extra attraction (Vijande et aL;, 2007, Bai et al., 2017). The connected diagram alone binds for $M \gg m$.

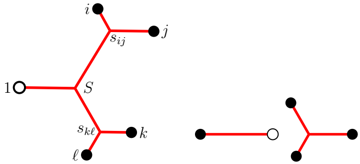
Adiabaticity and color-mixing



- The string potential corresponds to a Born-Oppenheimer treatment of the gluon field.
- With free rotations of the color wave function
- This is possible for $(bc\bar{b}\bar{c})$, not for $(bb\bar{b}\bar{b})$
- So the result by Bai et al corresponds to a fictitious $(bb'\bar{b}\bar{b}')$ state with $b' \neq b$, though same mass.

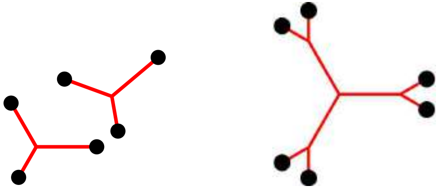
Higher configurations: pentaquark

- Same finding for pentaquark. In absence of constraints from antisymmetrization, pentaquark binding below the meson + baryon thresholds



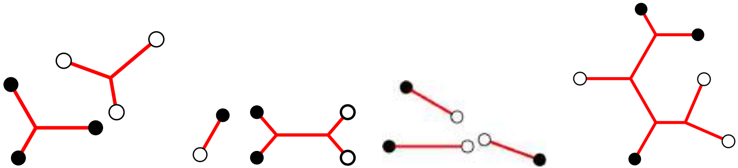
Higher configurations: dibaryon

- Same finding for pentaquark. In absence of constraints from antisymmetrization, dibaryon binding below the baryon+ baryon thresholds



Higher configurations: baryonium

- Same finding for $(3q, 3\bar{q})$. In absence of constraints from antisymmetrization, at least for some mass configurations, binding below the various thresholds (baryon-antibaryon, 3 mesons, meson + tetraquark)



Chromomagnetic binding

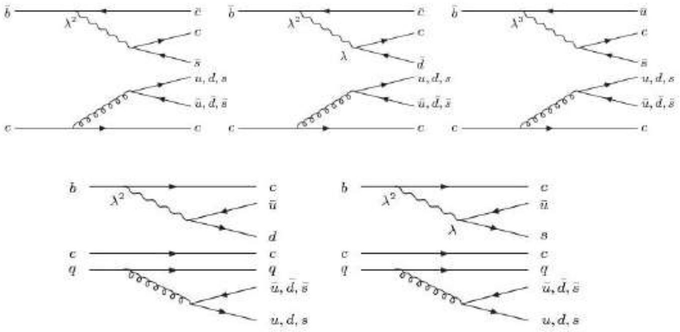
- In the 70s, the hyperfine splitting between hadrons ($J/\psi - \eta_c$, $\Delta - N$, etc.) explained à la Breit–Fermi, by a potential

$$V_{SS} = -A \sum_{i<j} \frac{\delta^{(3)}(\boldsymbol{r}_{ij})}{m_i\, m_j}\, \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \ ,$$

  a prototype being the magnetic part of one-gluon-exchange.
- Attractive coherences in the spin-color part: $\langle \sum \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \rangle$ sometimes larger for multiquarks than for the threshold.
- In particular $\langle \ldots \rangle$ twice larger (and attractive) in the best ($uuddss$) as compared to $\Lambda + \Lambda$.
- But $\langle \delta^{(3)}(\boldsymbol{r}_{ij}) \rangle$ much weaker for multiquarks than for ordinary hadrons, and needs to be computed. Hence uncertainties.
- Astonishing success with > 20 experiments on H and still lattice computations of H 40 years later!

Production of T_{cc} from B_c or Ξ_{bc}

Figs from the Roma group



The Born-Oppenheimer limit

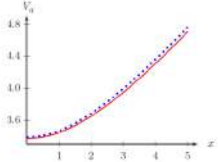
- Eichen-Quigg rule, following Lipkin, Nussinov, ...
- Based on heavy-diquark–heavy-antiquark symmetry
- Analogies of $Q\bar{q}$, QQq, Qqq and $QQ\bar{q}\bar{q}$

$$(QQ\bar{q}\bar{q}) \stackrel{?}{=} (QQq) + (Qqq) - (Q\bar{q})$$

- Exact at $M/m \to \infty$
- Works rather well for finite M/m
- Can be understood in the Born-Oppenheimer approach

The Born-Oppenheimer limit

- Already for QQq baryons, the Born-Oppenheimer method is very efficient. See, e.g. Fleck & R. (1989)
- $QQ\bar{q}\bar{q}$ for large M dominated by the $\bar{3}3$ color configuration, and one can compare the BO potentials
- BO approach: exact at $R = 0$, but V_{BO} similar for $(QQ\bar{q}\bar{q})$ and (QQq) [shifted here by $(Qqq) - (Q\bar{q})$]



- In atomic physics differences between $H_2{}^+$ BO potential vs. H_2

Hidden-charm pentaquarks

- Two recent contributions:
- Bound states below the threshold
  - Valcarce, Vijande, R., Phys. Lett. B774 (2017) 710-714 [arXiv:1710.08239]
  - $(\bar{c}cqqq)$ with $I = 1/2$ and $J = 5/2$ below the lowest S-wave threshold $\bar{D}^*\Sigma_c^*$ (but above $N\eta_c$ in D-wave)
  - For $I = 3/2$ and $J = 1/2, 3/2$ binding below S- and D-wave thresholds
  - Both chromo-electric and -magnetic parts necessary for binding
- Resonances in the quark model
  - Hiyama et al. (work in progress): real scaling, borrowed from electron-atom and electron-molecule scattering to separate, among the energies above the threshold, actual resonances from fictitious states produced by the variational method. Looks promising.
  - Similar to Luscher criteria for lattice, stability plateau in QCDSR
  - See Hiyama contribution at “Critical Stability”, Dresden, Oct. 2017